Time Scales Comparisons Using Simultaneous Measurements in Three Frequency Channels

Petr Pánek and Alexander Kuna

Institute of Photonics and Electronics AS CR, Chaberská 57, Prague, Czech Republic

panek@ufe.cz

Abstract—Simultaneous time transfer in three frequency channels can be performed after implementing new GPS signals. It leads to improving the accuracy on the one hand by increasing the number of measurements entering the statistical processing, and on the other hand by improving the characteristics of the time transfer based on ionospheric free solution.

This paper describes the design of optimal methods for processing the results of measurements in several frequency channels based on the Best Linear Unbiased Estimation (BLUE). The optimal estimate was derived for a number of elementary models (no ionospheric delay, first-order ionospheric delay, first- and second-order ionospheric delay), and the contribution of using measurements in three frequency channels was evaluated compared to procedures used up to now. We show that the optimal estimates are in some cases equal to already used linear combination of measurements (e.g. L3P) and thus these classical procedures can be considered optimal. We confirm the fact that the ionospheric free solution is always redeemed with significant increase of impact of random errors. The impact is increased by factor of three concerning the measurements in two frequency channels compared to measurement in a single frequency channel. We show that improvement of 15 % can be achieved if measurements in three frequency channels are performed.

Results of experimental measurement are also presented. The measurement aimed to look into the real behavior of random errors when using five GPS signals broadcast in three frequency channels, namely the amount of their correlation, which plays an important role in processing the results of several simultaneous measurements. We show that the correlation between measurement errors caused by multipath propagation is low despite the small span between the L2 and L5 frequency channels, and that one can benefit by including the measurements in the L5 frequency channel in the processing.

Key words: Time Transfer, GNSS

I. INTRODUCTION

Two new signals – L2C and L5C – available since modernization of the GPS, allows simultaneous time transfer in three frequency channels L1, L2, and L5 without any limitations [1], [2]. The benefit in case of time scales comparisons via common-view is: a) the number of measurements entering statistical processing can be increased up to five (L1C, L1P, L2C, L2P, L5C), b) the time transfer when the ionospheric delay is suppressed can be improved.

This paper gives a hint on possibilities in finding optimal processing of the results from the simultaneous code measurements in the three frequency channels. Results from two experimental measurements are also presented. The first measurement is focused on evaluation of the amount of correlation among measurement errors in individual GPS signals. The second measurement aimed at practical evaluation of triple-frequency comparison using CGGTTS V02 data.

II. PROCESSING OF SIMULTANEOUS MEASUREMENTS IN THREE FREQUENCY CHANNELS

We assume code measurement of time difference between local time scale and satellite time scale in the three frequency channels L1, L2, L5 with frequencies $f_1 = 1575.42$ MHz, $f_2 = 1227.60$ MHz, and $f_3 = 1176.45$ MHz. We denote the ratios of the frequency f_1 to these frequencies as

$$\begin{aligned} r_1 &= 1 \\ r_2 &= \frac{f_1}{f_2} = \frac{77}{60} \\ r_3 &= \frac{f_1}{f_3} = \frac{154}{115} \end{aligned} \tag{1}$$

We consider the error due to ionospheric delay as non-random deviation with slow variations and describe its frequency dependence with a model of first- and optionally second-order ionospheric delay [3]. Since the tropospheric delay cannot be separated from the measured time difference, we include it formally in the measured time difference between local time scale and satellite time scale. We describe the errors due to receiver noise and multipath propagation as unbiased random errors which are independent among the particular frequency channels.

When neglecting the ionospheric delay, the result of a single measurement in N frequency channels can be described with a simple model

$$y_i = x_1 + w_i, \quad i = 1 \ldots N \tag{2}$$

where x_1 is the measured time difference between the local time scale and the satellite time scale, and $w_i, \quad i = 1, \ldots, N$ are random errors. We assume that

$$E[w_i] = 0, \quad i = 1 \ldots N, \tag{3}$$

$$\mathrm{Var}[w_i] = \sigma^2, \quad i = 1 \ldots N, \tag{4}$$

$$E[w_i w_j] = 0, \quad i \neq j, \quad i = 1 \ldots N, \quad j = 1 \ldots N. \tag{5}$$

In case that we consider the random errors as well as first-order ionospheric delay, the result of a single measurement in the three frequency channels is

$$y_i = x_1 + r_i^2 x_2 + w_i, \quad i = 1 \ldots 3 \tag{6}$$

where x_2 is the group ionospheric delay of the first order in the L1 frequency channel.

We can also consider first- and second-order ionospheric delay

$$y_i = x_1 + r_i^2 x_2 + r_i^3 x_3 + w_i, \quad i = 1 \ldots 3 \tag{7}$$

where x_3 is the group ionospheric delay of the second order in the L1 frequency channel.

The models (2), (6), and (7) have the form of a linear combination and the measurement errors in these models are unbiased. Thus the best linear unbiased estimation (BLUE) [4] can be used to estimate the unknown parameters. BLUE is optimal in terms of minimal estimation variance and if the measurement errors have Gaussian distribution then it is also the best nonlinear estimation, i.e. the best estimation at all. We determine the BLUE for four instances:

A. Code measurements in several frequency channels and the ionospheric delay included in the estimated time difference

This case corresponds to common-view at a short baseline. Then the residual ionospheric delay is nearly zero and suppressing the ionospheric delay by using its frequency dependence makes no sense.

The measurement model can be described with (2) and BLUE leads to

$$\hat{x}_1 = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{8}$$

with variance

$$\sigma_{\hat{x}_1}^2 = \frac{\sigma^2}{N}. \tag{9}$$

A simple average of the results from individual measurements and the standard deviation of this estimate decreases with the square root of the number of frequency channels. The standard deviation of the estimate is 0.707σ and 0.577σ when measuring in two and three frequency channels respectively. It is possible to measure simultaneously on two orthogonal signals in L1 and L2 frequency channels. If the measurement errors of these orthogonal signals are uncorrelated then two other measurements can be added into processing and the estimation is based on five measurements. Standard deviation of the estimate is then 0.447σ. When the standard deviations of the measurement errors are not equal in all measurements, BLUE leads to a weighted average of the measurement results.

B. *Code measurements in two frequency channels and the ionospheric delay included in estimated parameters*

Only first-order ionospheric delay is estimated. Measurement model can be described with (6) where $N = 2$. Then BLUE of the time difference is

$$\hat{x}_1 = (a+1)y_1 - ay_2 \tag{10}$$

and BLUE of the first-order ionospheric delay

$$\hat{x}_2 = a(y_2 - y_1). \tag{11}$$

Values of coefficient a for frequency channel pairs L1/L2, L1/L5, and L2/L5 are summarized in Table 1. Standard deviations of the estimates normalized by the standard deviation of the random measurement error are also listed in Table 1.

Table 1. Values of coefficients for estimating the time difference and the first-order ionospheric delay based on measurements in two frequency channel and normalized standard deviation of the estimates.

f_1	f_2	a	$\sigma_{\hat{x}_1}/\sigma$	$\sigma_{\hat{x}_2}/\sigma$
L1	L2	1.5457277	2.98	2.19
L1	L5	1.2606043	2.59	1.78
L2	L5	11.2553191	16.6	15.9

BLUE is the linear combination of results from a pair of individual measurements. The linear combination of L1 and L2 frequency channels is already commonly used and denoted as L3. It is obvious from Table 1 that suppressing the first-order ionospheric delay is paid by significant increase of the random errors impact. Concerning the L3 combination, the impact of random errors is increased three times compared to measurement in a single frequency channel.

C. *Code measurement in three frequency channels and the ionospheric delay included in estimated parameters*

Only the first-order ionospheric delay is estimated. The measurement model can be described with (6) where $N = 3$. Then BLUE of the time difference is

$$\hat{x}_1 = (a_1 + b_1 + 1)y_1 - a_1 y_2 - b_1 y_3 \tag{12}$$

and BLUE of the first-order ionospheric delay in channel L1

$$\hat{x}_2 = a_2 y_2 + b_2 y_3 - (a_2 + b_2) y_1. \quad (13)$$

Values of coefficients a_1, b_1, a_2, b_2 and standard deviations of the estimates normalized by the standard deviation of the random measurement error are listed in Table 2.

Table 2. Values of coefficients for estimating the time difference and the first-order ionospheric delay based on measurements in three frequency channel and normalized standard deviation of the estimates.

i	a_i	b_i	$\sigma_{\hat{x}_1}/\sigma$
1	0.3596456	0.9672985	2.55
2	0.4682064	0.8787628	1.68

Results in the Table 2 show that the standard deviation of the triple-frequency measurement with ionospheric delay suppression is reduced approximately by 15 % compared to usual dual-frequency measurement L3.

D. Code measurement in three frequency channels and both the first- and the second-order ionospheric delays in estimated parameters

The measurement model can be described with (7) where $N = 3$. Then BLUE of the time difference is

$$\hat{x}_1 = a_1 y_1 + (1 - a_1 - b_1) y_2 + b_1 y_3, \quad (14)$$

BLUE of the first-order ionospheric delay in channel L1

$$\hat{x}_2 = (a_2 + b_2) y_2 - a_2 y_1 + b_2 y_3, \quad (15)$$

and BLUE of the second-order ionospheric delay in channel L1

$$\hat{x}_3 = a_3 y_1 - (a_3 + b_3) y_2 + b_3 y_3. \quad (16)$$

Values of coefficients a_1, b_1, a_2, b_2, a_3, b_3 and standard deviations of the estimates normalized by the standard deviation of the random measurement error are shown in Table 3.

Table 3. Values of coefficients for estimating the time difference and the ionospheric delay of the first and second order based on measurements in three frequency channels and normalized standard deviation of the estimates.

i	a_i	b_i	$\sigma_{\hat{x}_1}/\sigma$
1	7.0805833	20.0497660	33.7
2	12.3677333	47.8468779	77.9
3	6.2871500	27.7971119	44.4

Apparently, suppressing the second-order ionospheric delay causes enormous uncertainty increase of the resulting time difference estimation by a factor of ten compared to L3. Since the second-order ionospheric delay is in order of magnitude lower than the standard deviation of the random error of the code measurement, one can conclude that such processing of the code measurement results makes no sense in usual applications.

III. FIRST EXPERIMENT

The main goal of this experiment was to evaluate the amount of correlation among measurement errors in individual frequency channels. The measurement was done in the Institute of Photonics and Electronics

(IPE) in Prague. The GTR51 time transfer receiver with a Novatel GPS-703-GGG antenna were used. The antenna was installed in the height of ~1.2 m (~4 ft) above the edge of the flat roof. GPS satellite PRN 25 with flyover of ~5 h was tracked.

The multipath combination was used to determine measurement error caused by receiver noise and multipath signal propagation, i.e. the time difference and the ionospheric delay were estimated based on results from the carrier phase measurements. These estimates were then subtracted from the time difference from the code measurements. The estimate based on carrier phase measurements is ambiguous and includes an unknown systematic error. This error can be excluded by subtracting the average over the whole measurement period assuming that measurement error caused by receiver noise and multipath propagation is unbiased.

Figure 1 displays plots of measurement error caused by receiver noise and multipath propagation. Standard deviation for all five signals derived from the whole measurement period is listed in the first row of the Table 4. Analysis showed that the white noise prevailed in the plots.

Table 4. RMS of measurement error caused by receiver noise and multipath propagation evaluated from the whole measurement period.

	L1C	*L1P*	*L2C*	*L2P*	*L5C*
Noise and Multipath	0.81 ns	0.78 ns	1.24 ns	0.88 ns	0.46 ns
Multipath	0.29 ns	0.32 ns	0.41 ns	0.37 ns	0.34 ns

Amount of correlation among measurement errors in individual signals evaluated over the whole measurement period is shown Table 5. One can see that the correlation is very low. Slight increase can be observed in case of using two signals within a single frequency channel. This is caused by the receiver noise that prevails in the measurement error and it is independent in the individual signals, even if the signals are from the same frequency channel.

Table 5. Amount of correlation among measurement errors caused by the receiver noise and multipath propagation evaluated from the whole measurement period.

	L1P	*L2C*	*L2P*	*L5C*
L1C	0.10	−0.03	−0.01	0.03
L1P	–	−0.02	−0.01	0.03
L2C	–	–	0.11	−0.08
L2P	–	–	–	−0.13

Measurement error consists mainly of the receiver noise. It has character of white noise which can be effectively suppressed by filtering in most cases even in relatively short period of measurement. However this is not applicable on measurement error due to multipath propagation. It shows quite long correlation time and its suppression with filtering can be problematic. Although its size can be neglected compared to the receiver noise, it becomes decisive after the receiver noise is filtered out. Thus we tried to separate the measurement error due to multipath propagation and inspect it individually. We applied 60-s moving average to separate the multipath from the noise. Such window length allowed reducing the noise approximately 8 times while the multipath didn’t change significantly.

The resulting graph of measurement error due to just multipath propagation is plot in Figure 2. RMS of the measurement error evaluated from the whole measurement for both receivers and all five signals is listed in row 2 of Table 4.

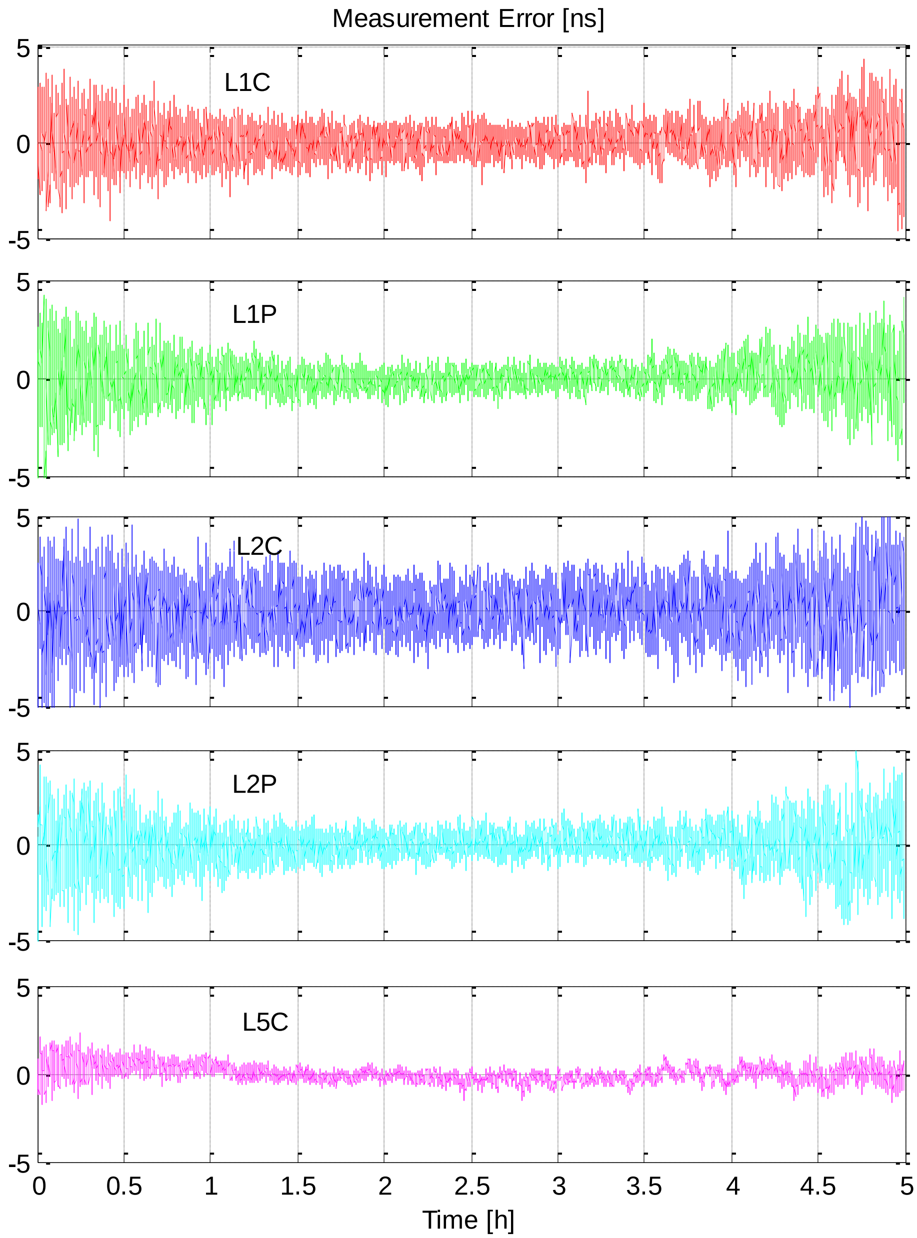

Figure 1. Plot of the measurement errors due to receiver noise and multipath propagation.

Note that there is apparent slow variation with peak-to-peak value of 0.3 ns in the L5 signal error plot. We did not observe similar variation in any of the other signals. We found an explanation in [5]. The carrier phase signal delay at satellite varies with temperature changes as the satellite is exposed to the Sun. This phenomenon should not degrade the common-view comparison because the fluctuations are the same at both sites.

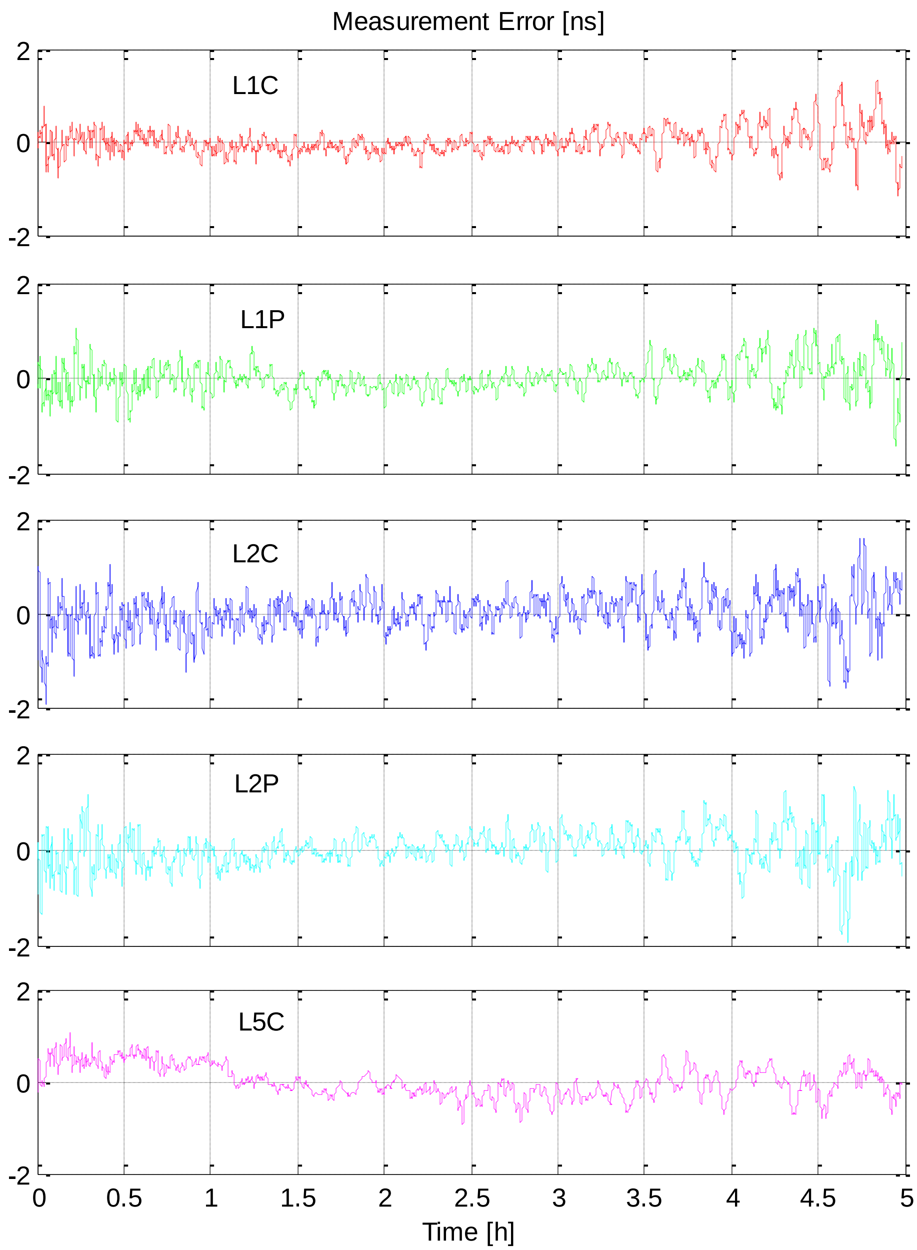

Figure 2. Plot of the measurement errors due to receiver noise and multipath propagation.

The correlation coefficients among the measurement errors caused by multipath propagation are summarized in Table 6. High correlation between two signals within a single frequency channel can be observed. The correlation coefficient is approximately 0.5 in this case and it reached 0.8 for a limited amount of time when the influence of the multipath propagation was really high.

The correlation between measurement errors in signals in L2 and L5 frequency channels is also slightly increased because the frequencies of these channels are relatively close.

Table 6. Amount of correlation among measurement errors caused by the multipath propagation evaluated from the whole measurement period.

	L1P	***L2C***	***L2P***	***L5C***
L1C	0.55	−0.15	−0.14	0.09
L1P	–	−0.06	−0.15	0.02
L2C	–	–	0.42	−0.25
L2P	–	–	–	−0.34

IV. SECOND EXPERIMENT

This experiment aimed at practical evaluation of common-view comparison based on simultaneous measurement in the three frequency channels. The measurement was performed in IPE with a pair of Dicom GTR51 receivers denoted as TPX and TPY. Both receivers were provided with the UTC(TP) reference. Novatel GPS-704-X with antenna amplifier was plugged to the receiver TPX and Novatel GPS-703-GGG to TPY respectively. Both antennas were installed on the roof of the institute. The distance between the antennas was ~5 m (~16 ft).

GPS satellite PRN 1 was tracked during the comparison. The satellite flyover took about 5 h and 19 tracks were processed. The satellite elevation was always at least 20 deg.

Both receivers generated measured data L1C, L2C, and L5C in CGGTTS V02 format [6, 7]. We compared the data by using the standard common-view procedure. We used the individual signals and after that we calculated BLUE with and without suppressing the ionospheric delay according to (12) and (8) respectively.

Figure 3 shows the comparison results. The displayed time differences are in fact equal to comparison error because both receivers were fed with the same reference. RMS of the comparison error in L1C, L2C, and L5C was around 280 ps. The resulting accuracy of the triple frequency comparison without suppression of the ionospheric delay was 170 ps RMS and 650 ps RMS with the suppression. The expected theoretical values are 162 ps RMS and 714 ps RMS respectively.

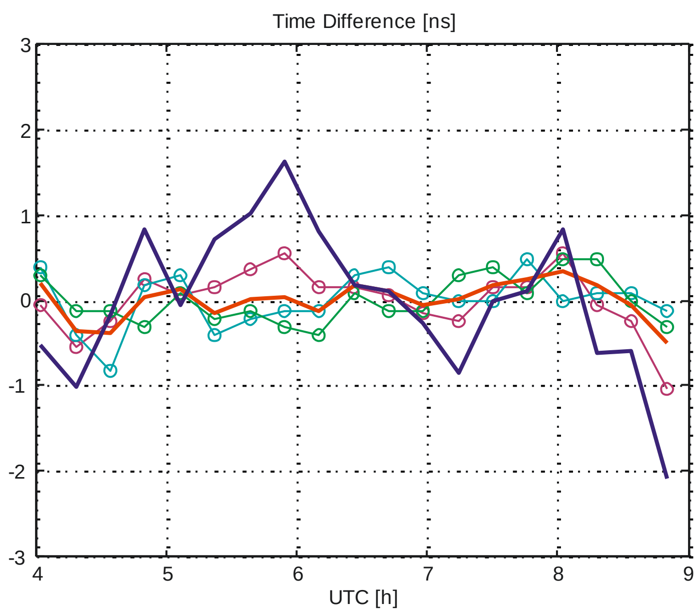

Figure 3. Measured time differences L1C (magenta), L2C (cyan), L5 (green), and the result of the triple frequency comparison with (blue – 650 ps RMS) and without (red – 170 ps RMS) ionospheric delay suppression.

V. CONCLUSION

We derived the best linear unbiased estimation (BLUE) for four cases with code measurement in several frequency channels:

- The ionospheric delay is included in estimated time difference. This case corresponds to common-view at a short baseline when the ionospheric delay is eliminated. BLUE leads to average of the individual measurements. Standard deviation of the estimation decreases with square root of the number of measurements.
- Measurements in two frequency channels are considered. The ionospheric delay is included in estimated parameters. Only first-order ionospheric delay is estimated. BLUE of the time difference leads to linear combination of results from two individual measurements. Suppression of the first-order ionospheric delay is paid by a significant increase of random errors.
- Measurements in three frequency channels are considered. The ionospheric delay is included in estimated parameters. Only first-order ionospheric delay is estimated. BLUE of the time difference leads to linear combination of results from three individual measurements. The standard deviation of the triple-frequency measurement with first-order ionospheric delay suppression is reduced approximately by 15 % compared to the previous case.
- Measurements in three frequency channels are considered. The ionospheric delay is included in estimated parameters, and both the first- and the second-order ionospheric delays are estimated. BLUE of the time difference leads again to linear combination of results from three individual measurements. Suppressing the second-order ionospheric delay causes enormous impact of random errors and this case has no practical usage.

Two experimental measurements were performed. The first aimed at evaluation of the amount of correlation among errors of measurements on signals L1C, L1P, L2C, L2P, and L5C. It appears that the correlation is very low even when using two orthogonal signals from the same frequency channel. This is caused by the receiver noise that prevails in the measurement error. The impact of this noise can be reduced typically by factor of 2.2 by statistical processing of the results from measurements in the five signals at given time. The white noise can be usually suppressed by filtering, but not the measurement error due to multipath propagation. Although the multipath error is smaller compared to receiver noise, it becomes decisive after the receiver noise is filtered out. Thus we separated the measurement error due to multipath propagation and inspected it individually. We observed higher correlation between errors of measurement on two signals within the same frequency channel. The correlation coefficient was about 0.5. It shows that the impact of the multipath propagation cannot be reduced with statistical processing of the results from measurements on orthogonal signals within the same frequency channel. On the other hand the correlation between signals in different frequency channels stayed at low level thus the triple frequency measurements can be used to reduce the impact of the multipath propagation in this case.

The goal of the second experiment was to practically evaluate the common-view comparison with simultaneous measurement in the three frequency channels. Data measured on signals L1C, L1P, L2C, L2P, and L5C in CGGTTS V02 format were used in this experiment. The standard deviation of the comparison error in individual frequency channels was around 280 ps RMS. The resulting comparison accuracy without ionospheric delay suppression was 170 ps RMS and 650 ps with the suppression. The expected theoretical values are 162 ps and 714 ps respectively.

ACKNOWLEDGMENT

This work was supported by Project No. FR-TI3/453 of the Ministry of Industry and Trade of the Czech Republic.

REFERENCES

[1] Navstar GPS Space Segment / Navigation User Interfaces, IS-GPS-200E, Global Positioning System Wing (GPSW) Systems Engineering & Integration Interface Specificication, 8 June 2010.

[2] Navstar GPS Space Segment / User Segment L5 Interfaces, IS-GPS-705A, Global Positioning System Wing (GPSW) Systems Engineering & Integration Interface Specificication, 8 June 2010.

[3] D. Odijk, "Fast precise GPS positioning in the presence of ionospheric delays," Netherlands Geodetic Commission, Delf 2002 (ISBN 90 6132 278 2).

[4] S. M. Kay, Fundamentals of Statistical Signal Processing – Estimation Theory. Prentice Hall, 1993.

[5] O. Montenbruck, et al.,"Threse's challenge - A Close Look at GPS SVN62 Triple-Frequency Signal Combinations Finds Carrier-Phase Variations on the New L5," GPS World, vol.21 (2010), no.8, pp.8-19.

[6] D. W. Allan and C. Thomas, "Technical Directives for Standardization of GPS Time Receiver Software," Metrologia, vol.31 (1994), p.69-79.

[7] J. Azoubib and W. Lewandowski, "CGGTTS GPS/GLONASS data format Version 02," Document of the 7th CGGTTS Meeting, 1998.