Solving For Time: A Unified, Covariance-Based Comparison Of Celestial And Radionavigation Algorithms From 1770 To The Present

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Abstract—The United States has a long, but somewhat neglected history of using time-based algorithms for navigation, surveying, and mapping. In this paper, we show how Mason & Dixon, Maskelyne, Harrison, Lewis & Clark, and others developed algorithms that are today used in modern systems, including GPS.

Key words:* *celestial navigation, radionavigation, GPS, least squares, Mason & Dixon, Lewis & Clark, Andrew Ellicott, Nevil Maskelyne

I. INTRODUCTION

The United States has a long, history of using time-based algorithms for navigation, surveying, and mapping. We now take GPS for granted, but it is worth noting that GPS depends on the accurate dissemination of signals whose timing characteristics are based on a network of atomic clocks. This is not, however, a strictly modern approach to solving the world's radio navigation problems. Instead, the roots of this and other navigation systems can be traced back in time through centuries of activities whose results surround us, but which are often "hidden in plain sight."

To appreciate this, is it useful to list, by century, key events in the history of the United States that link Newton's early observation of the night skies to topics as mundane as laying out roads and defining state boundaries. Starting with the 1600's, we have:

1600's:

- Arthur Storer's measurements from Southern Maryland of Halley's Comet are cited in Newton's Principia, and help to exploit long baseline measurements for orbit determination [1]. Arthur Storer and Isaac Newton were childhood friends before Mr. Storer's parents moved to what is now called Huntingtown, a small town on modern Route 4 in Southern Maryland.

1700's:

- Mason and Dixon pioneered the Delta-time equal-altitudes celestial techniques used for surveying lines of constant latitude and longitude for state boundaries. This effort took four years, and now defines the boundaries of Maryland, Delaware, and Pennsylvania [2, 3].
- The famous Cavendish experiment, which yielded a value for the mass of the earth, was inspired by the question of whether gravitational perturbations caused by the Appalachian Mountains would affect the leveling of the optical transits used by Mason and Dixon [4].

1800's:

- Lewis and Clark measured lunar distances (the angles subtended by the moon and stars or planets) at the direction of Thomas Jefferson using techniques taught by Andrew Ellicott (for whom Ellicott City in Maryland is named) and Robert Patterson [5].

- Pierre L'Enfant, Andrew Ellicott, and Benjamin Banneker surveyed Washington, D.C., using the stars, such that lettered streets are nominally lines of constant latitude, while numbered streets are lines of constant longitude.

1900's:

- Latitude observatories were operated worldwide at a latitude of approximately 39 degrees North to measure the nutation and wobble of the earth's axis with respect to the North Star, Polaris. Although no longer used, several of these small buildings still exist and are protected as historical structures.

In the discussion that follows, we describe these events in more detail, and show how, one way or another, the measurement of time and time intervals was critical to their utility and success.

II. THE IMPORTANCE OF TIME

The algorithms upon which all of these major historical events are based share a key feature: they all depend on time. In particular, they depend on measurements of *absolute time* (e.g., GMT or UTC) or on measurement of *time intervals*. For example:

Newton's parallax technique is captured in the equations by which the Right Ascension and Declination of orbiting satellites and celestial objects are related to the azimuth and elevation angles used for pointing telescopes and antennas [6]. These calculations are an essential component of the geo-location algorithms used by the Global Positioning System (GPS).

Mason & Dixon used precise measurements of time intervals to determine true north in order to survey lines of constant latitude and longitude. The used what is known as the method of "equal altitudes."

Lewis and Clark measured "lunar distances" (the angle subtended by vectors to the moon and sun or the moon and a star or planet) which permits *a posteriori* determination of Greenwich time after correction of parallax and refraction using the Patterson/Ellicott algorithms [7, 5]. The method of lunars, espoused by Nevil Maskelyne, was introduced to modern readers in qualitative form in the book Longitude, by Dava Sobel [8]. Lewis and Clark's measurements have been criticized because they included only the lunar distances, but not the elevations above the horizon, of the relevant celestial bodies.

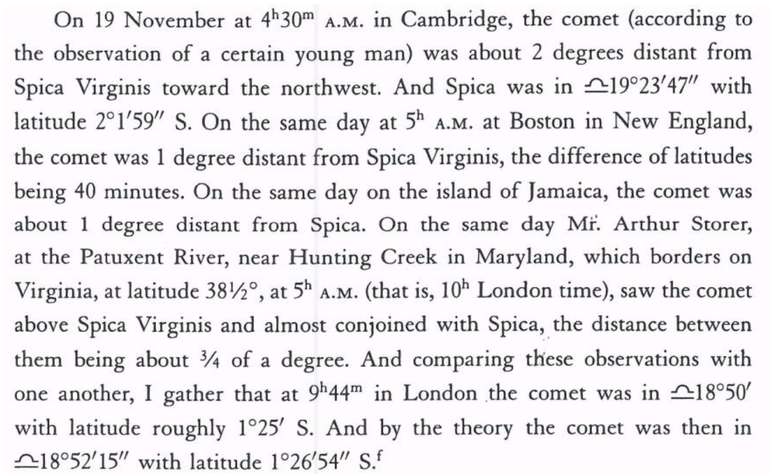
Patterson and Ellicott, however, showed that this was not important, provided that knowledge of the time interval between the lunar measurement and time of meridian passage (i.e., local apparent noon) was measured [6]. Despite often forgetting to wind their chronometer, and thus lacking a direct knowledge of Greenwich time, Lewis and Clark did set their chronometer to local noon prior to making lunar measurements later in the day, thus recognizing the ability to leverage time interval measurement in order to "solve" for Greenwich, or absolute time, and thence longitude [7]. However, Lewis and Clark did not reduce their measurements to an estimate of longitude during the expedition, despite measuring and recording latitude on a regular basis.

A century later, and for much of the 20^{th} century, observations of Polaris were performed for decades on a nightly basis from 5 locations world-wide. The result is the "POLARIS (Pole Star) Tables" in the Nautical Almanac [9]. These tables provide corrections to sextant measurements of the "altitude" of Polaris (i.e., the elevation angle) in order that sightings of Polaris would more accurately determine latitude.

As a more specific example of events in the past that connect to modern places and events, consider the excerpt from Newton's Principia, shown in Figure 1. Hunting Creek is located on Maryland Route 4 at present-day Huntingtown.

In the District of Columbia, MD Rt. 4 becomes Pennsylvania Avenue, the street upon which the White House resides at the intersection of 16^{th} Street (hence 1600 Pennsylvania Avenue). This north-south street was once used as a reference meridian (line of constant longitude) for surveying purposes, and hosts modern day Meridian Hill Park.

Storer's comet, described above, appeared in 1682 and most recently in 1986. It is now called Halley's Comet, as Halley predicted that the comet was not an isolated event, but that it would return on a regular basis.



On 19 November at $4^{h}30^{m}$ A.M. in Cambridge, the comet (according to the observation of a certain young man) was about 2 degrees distant from Spica Virginis toward the northwest. And Spica was in ♎19°23′47″ with latitude 2°1′59″ S. On the same day at 5^{h} A.M. at Boston in New England, the comet was 1 degree distant from Spica Virginis, the difference of latitudes being 40 minutes. On the same day on the island of Jamaica, the comet was about 1 degree distant from Spica. On the same day Mr. Arthur Storer, at the Patuxent River, near Hunting Creek in Maryland, which borders on Virginia, at latitude 38½°, at 5^{h} A.M. (that is, 10^{h} London time), saw the comet above Spica Virginis and almost conjoined with Spica, the distance between them being about ¾ of a degree. And comparing these observations with one another, I gather that at $9^{h}44^{m}$ in London the comet was in ♎18°50′ with latitude roughly 1°25′ S. And by the theory the comet was then in ♎18°52′15″ with latitude 1°26′54″ S.[f]

Figure 1. Isaac Newton's words in the Principia, 1687, describing celestial distances and the effects of parallax.

A second example of how past and present are linked is the Mason-Dixon line, surveyed over a four year period circa 1760. The line was made necessary because of a border dispute between Maryland and Pennsylvania. Both states were the result of land granted by royal charter, but by two different Kings of England. The confusion started, by some accounts, because of William Penn's desire for a deep-water port on the Delaware River, causing him to place the southern border of Pennsylvania south of the location specified in his royal charter [3]. This deep-water port is modern-day Philadelphia, home of the storied, but no longer active, Philadelphia Naval Shipyard. The shipyard is located near the point at which the Delaware River is no longer navigable by ocean-going vessels.

Mason and Dixon were instructed to survey a newly-agreed border under the supervision of the English Royal Society, and specifically Nevil Maskelyne. Mason and Dixon marked, at several mile intervals using stones quarried in the Scotland, a collection of lines and a small arc of a circle. The most important of these is a line of constant latitude referenced to a point 15 miles south of Philadelphia. This forms the modern border between Maryland and Pennsylvania.

Mason and Dixon located these lines by timing the motion of the stars. Figure 2 shows the result of their hard work.

Mason & Dixon were required to survey:

- A line of constant latitude from Fenwick Island to the Chesapeake Bay and to determine its "midpoint."
- A circle of radius 12 miles centered at a church tower in New Castle.
- A line from the "midpoint" that forms a perfect tangent to the circle.
- A line pointing due north from the circle, starting a few miles clockwise along the circle from the tangent point, with this line intersecting:
- The line of constant latitude we call the Mason-Dixon Line, which serves as the border between Maryland and Pennsylvania.

Figure 2. Lines of the Mason and Dixon survey.

An unanticipated result of this strange set of requirements is the "wedge," shown in Figure 3. This remained a no-man's land unclaimed by any state until the early 1900's, when it became part of Delaware.

Figure 3. An unintended result – "The Wedge."

The "midpoint" markers, as they are today, are shown in Figure 4. Figures 5 and 6 show Mason and Dixon's own description of the wedge, from their original journal and a modern depiction of the wedge. [2]. As of 2011, a restaurant located on the road that traverses the wedge sells "Wedge Burgers" in honor of this trigonometric nightmare.

Figure 4. The midpoint of the line from Fenwick Island to the Chesapeake. This is the southwest corner of Delaware.

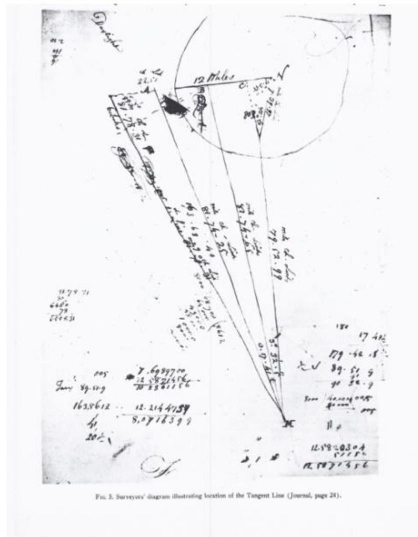


Figure 5. Mason & Dixon's actual journal, located in Archives II in College Park, MD, after having been found in a basement in Nova Scotia in 1860 and purchased from the Canadians in 1877 for $500 in gold [2].

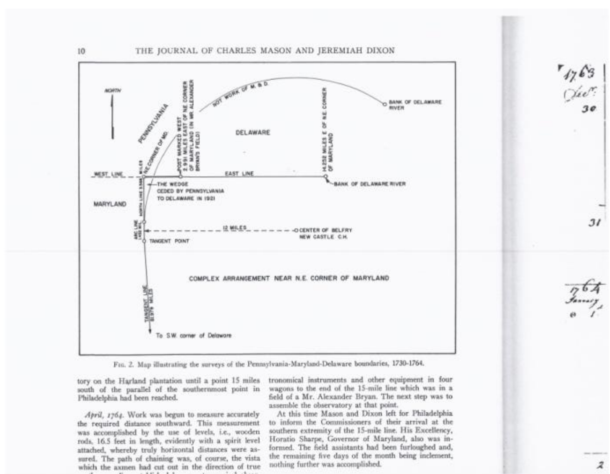

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Fig. 2. Map illustrating the surveys of the Pennsylvania-Maryland-Delaware boundaries, 1730-1764.

tory on the Harland plantation until a point 15 miles south of the parallel of the southernmost point in Philadelphia had been reached.

April, 1764. Work was begun to measure accurately the required distance southward. This measurement was accomplished by the use of levels, i.e., wooden rods, 16.5 feet in length, evidently with a spirit level attached, whereby truly horizontal distances were assured. The path of chaining was, of course, the vista which the axmen had cut out in the direction of true

tronomical instruments and other equipment in four wagons to the end of the 15-mile line which was in a field of a Mr. Alexander Bryan. The next step was to assemble the observatory at that point.

At this time Mason and Dixon left for Philadelphia to inform the Commissioners of their arrival at the southern extremity of the 15-mile line. His Excellency, Horatio Sharpe, Governor of Maryland, also was informed. The field assistants had been furloughed and, the remaining five days of the month being inclement, nothing further was accomplished.

Figure 6. A modern depiction of the Wedge [2]

III. HOW MASON AND DIXON USED THE STARS

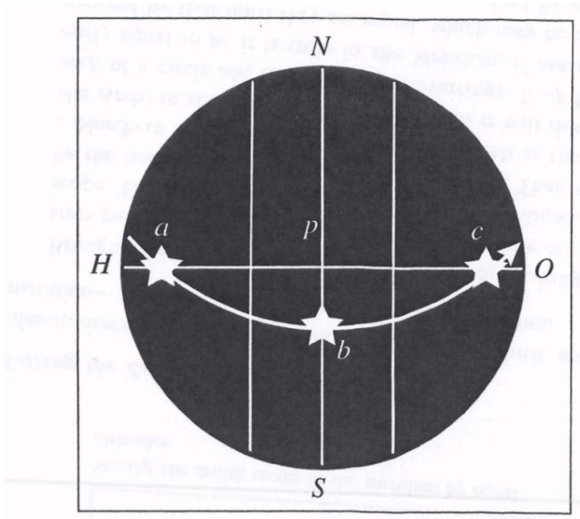
To complete their survey, it was essential for Mason and Dixon to determine, at dozens of surveying locations, the direction of true north. Simply defining north as the azimuth direction to the North Star was not adequate, as Polaris is not precisely aligned with the axis of rotation of the earth.

Instead, Mason and Dixon used the method of equal altitudes, which requires accurate stop-watch measurements of the relative times of ascent and descent of individual stars during their "meridian passage" across the northern sky as viewed through a transit-mounted telescope.

The transit, also called a zenith sector, was mounted on a tripod that was carefully leveled using a plumb bob. The pointing azimuth and elevation angles of the telescope could be measured with great precision.

After observing the movement of each of many stars using a telescope mounted on the transit, the transit would be adjusted slightly with the eventual goal that the next star, when observed, would trace an arc for which the time durations of ascent and descent, relative to crosshairs mounted in the telescope, were equal. Thus, an accurate measurement of time interval was essential to the determination of true north.

The concept is illustrated in Figure 7 excerpted from the book by Danson [3], which In turn is based on an illustration provided by Mason and Dixon in their journal of the survey. When the time required for a star (any star in the northern sky) to move from point *a* to point *b* was equal to the time required for the star to move from point *b* to point *c*, the transit was known to be pointing due north.



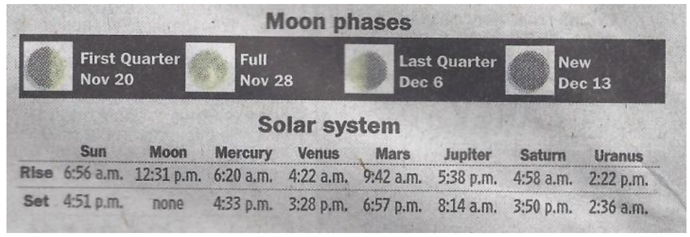
Setting the zenith sector in the meridian by equal altitudes.

Figure 7. The method of equal altitudes, from Danson [3]. The image is inverted because of the telescope lens used in the "zenith sector", or transit.

A more mundane example of the method of equal altitudes is shown using the data in Figure 8, which is a list of the rise and set times of the sun, moon, and several of the planets as provided by the Washington Post on a daily basis.

By averaging the times of sunrise and sunset, the local time of meridian passage of the sun (which is when ante-meridian, or A.M, becomes post-meridian, or P.M.) is easily determined. This occurs at "local apparent noon". By comparing this value for noon to Greenwich noon, as determined from one's wrist watch, telephone, or computer, one's longitude can be estimated. This illustrates, succinctly, the importance of John Harrison's chronometer as a tool for determining longitude [8]. Wrist-watch, or smart-phone time, is simply Greenwich Mean Time offset by an integer or half-integer number of hours, depending on one's geographic time zone. For the purist, one can adjust to GPS time by adding the appropriate number of leap seconds.

However, when using the sun as a reference, things aren't quite this simple. Because the earth's orbit is elliptical, Kepler's third law causes the rate of the earth's orbital progression to vary with the distance of the earth from the sun, hence the use of Greenwich Mean Time to average this effect over the span of a year. And, because the earth's axis is inclined with respect to its plane of motion, the change in position of the sun with respect to the equator as the seasons change, which is captured in the "equation of time", must be considered. Preston argues that these effects were known, at least by the time of the Lewis and Clark expedition, and could be applied to their data [5].



Moon phases

First Quarter	Full	Last Quarter	New
Nov 20	Nov 28	Dec 6	Dec 13

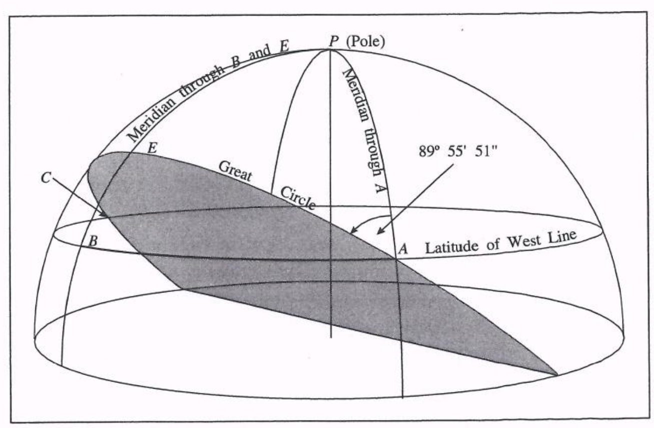
Solar system

	Sun	Moon	Mercury	Venus	Mars	Jupiter	Saturn	Uranus
Rise	6:56 a.m.	12:31 p.m.	6:20 a.m.	4:22 a.m.	9:42 a.m.	5:38 p.m.	4:58 a.m.	2:22 p.m.
Set	4:51 p.m.	none	4:33 p.m.	3:28 p.m.	6:57 p.m.	8:14 a.m.	3:50 p.m.	2:36 a.m.

Figure 8. Information for the sun, moon, and planets, published daily in The Washington Post, including Uranus, but not Neptune or Pluto.

Returning to the problem of the Mason and Dixon survey, once true north was ascertained, the transit was leveled, and then rotated in azimuth by a precise angle determined using Napier's rules of spherical trigonometry. The angle was computed so as to define a segment of a great circle that would cross the *small circle*, or line of constant latitude upon which the transit was stationed, both at the location of the transit, and again at a known distance, such as 12 miles.

The geometry of this is shown in Figure 9. Mason and Dixon surveyed the great circle segment from A to C. They did this by using optical sighting through the transit's telescope of candles held at a distance of several miles by other members of the surveying team. Light, by Fermat's principle, follows great circles, but never lines of constant latitude. Thus, optical surveying techniques, including the use of present-day laser range finders, are based on great-circle techniques.



A 10-minute arc of the great circle. The great circle is the arc passing through *A, E,* and *C*. The distance *B* to *E* equals 1,714 feet.

Figure 9. The spherical trigonometry used by Mason and Dixon to relate great circles to small circles (i.e.,circles that define lines of constant latitude or, in the case of GPS, a circle of constant pseudorange that, when viewed close-up and linearized, form "lines of position"). EPA forms a right spherical triangle [2].

For the same reason, straight lines drawn on aviation charts represent the great circles that are nominally followed by radio-navigation signals, including GPS. Straight lines on nautical charts, however, typically represent lines of constant heading. This follows the century's old nautical tradition of navigating by compass, rather than by following vectors defined by ground based radio beacons (e.g., VOR and DME).

Once the segment of great circle from points A to C, typically a distance of 10 – 12 miles, was surveyed, a Gunter chain (the 1700's version of a tape measure) was used to transfer the surveyed great circle segment to the desired line of constant latitude. This was typically a distance of several tens of feet or less.

This extremely tedious activity took four years, as opposed to the 2 years it took Lewis and Clark to go from St. Louis to the Pacific Ocean and back.

The spherical trigonometry required to calculate the angle from north to sight the great circle, as well as that required for computing the distance between the great circle segment and the desired line of constant latitude, is given by Napier's rules, illustrated in Figure 10 and by equation 1.

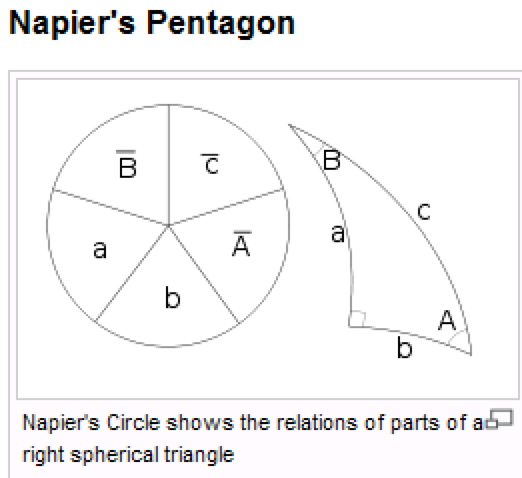


Figure 10. Napier's Pentagon, showing a mnemonic for the angular trigonometry of right spherical triangles (from Wikipedia).

Equation 1 gives the mathematical relationship by which all of the interior angles of a right spherical triangle can be computed by, in this case, measurement of latitude, a sighting of true north, and knowledge of the radius of the earth. Note that the sum of the interior angles of a spherical triangle is greater than 180 degrees, and that arc lengths are specified as angles. This is in contrast to the lunar measurements of Maskelyne and Lewis and Clark, which although angles, are referred to as distances.

$$\cos(B) = \tan(a)\cot(c) = \cos(b)\sin(A) \tag{1}$$

Some additional points are worth mentioning. First, in order to create the clear sight-lines needed for optical surveying, the Mason and Dixon expedition required the services of a large number of "sawyers" to chop down trees to create "vistos." A great deal of effort was also required to transport and place the numerous survey stones, such as those shown in Figure 4, which were placed at several mile intervals.

When Mason and Dixon encountered the first serious mountain, Sideling Hill near Hancock, Maryland, they apparently left the remainder of their significant stockpile of stones in the woods near George Washington's famous Fort Frederick, where they were discovered in the early 1900's.

Mason and Dixon also left one of their transits, very carefully protected in oilskins, beneath the floor boards of a building in Philadelphia before returning to England. This instrument, discovered in the 20th century, is on display in at the Independence Hall National Historic Site [3].

Finally, Henry Cavendish was concerned that the gravitational pull of the Appalachian Mountains (the same mountains that discouraged Mason and Dixon from placing all of the marker stones), would affect the accuracy with which the transit could be leveled, thus causing a bias error in the survey measurements. This inspired Cavendish to perform his famous experiment, which indirectly resulted in a measurement of the mass of the earth [4].

IV. The Legacy of Mason & Dixon

Mason's and Dixon's techniques, which yielded a boundary line with errors on the order of 400 feet, left a much larger legacy than just a line on a map. Their use of lines of constant latitude and longitude are reflected to this day in the boundaries of many of the western states. Lines of constant latitude figure prominently in the Missouri Compromise of 1820, a failed attempt to deal with the issue of slavery that used the Mason and Dixon line and the small circle at 36 degrees and 30 minutes North (the Southern border of Missouri) as two of the boundary lines that separated the "slave" states from "free" states. It could be argued that this postponed the civil war for another 40 years, permitting the development of the rifled musket barrel and the ironclad ship, two engineering innovations whose introduction in the Civil War have forever changed the world [10].

Other noteworthy contributions of the Mason and Dixon survey include:

- The nation's border with Canada is, for much of its length, a line of constant latitude;
- Many of the more recent states have borders that are lines of constant latitude and lines of constant longitude;
- The District of Columbia has a grid based on the same concept, with the US Capitol at its origin and a grid of east-west and north-south streets placed using measurements of the motion of the stars;
- To bring this concept up to date, the position of the statue named Freedom, on the dome of the US Capitol, has been surveyed with GPS to an accuracy of centimeters;
- The point of intersection of the borders of Nebraska, Colorado and Wyoming is 27 degrees west of the old Naval Observatory in Washington, DC, within approximately 75 feet.

Figures 11 and 12 show the relation between the boundary point, at which Colorado, Wyoming, and Nebraska meet, with respect to a reference meridian drawn through the "old" Naval Observatory. This is "the American Meridian," which is memorialized by the cobblestones on 23rd Street, Northwest. This street is a meridian that is based on the concept, in Washington, DC, that numbered streets are nominally lines of constant longitude based on a Cartesian grid whose origin is the United States Capitol building.

Figure 11. The author's children at the boundary of Wyoming, Nebraska, and Colorado at 104 deg 03' 09" W in 1998.

Figure 12. The 1850 Meridian at 77° 03' 6.119" W, the site of the old Naval Observatory on 23rd St. in Washington, DC between what are now the Kennedy Center and the State Department (Wikipedia photo credited to D B King).

Figure 13 is a photograph of a latitude observatory. It is the epitome of a well-ventilated building, with fully-louvered walls to prevent temperature gradients (and hence convection currents that would cause optical diffraction). The roof slides open to permit observation of the northern sky, thus allowing equal altitude measurements from which to compare the motion of Polaris with respect to the direction of true north.

Figure 13. The Gaithersburg, Maryland latitude observatory. Its sister observatories are in Ohio, California, Japan, the former Soviet Union, and Italy.

V. GPS, LEAST SQUARES, AND KALMAN FILTERS

The small circle defined by Mason and Dixon's line of constant latitude, when extended around the earth, is identical, after a transformation of coordinates, to the circle formed at a distance of constant "pseudo-range" when a GPS signal lands on the earth. Viewed at close range, a segment of a small circle looks linear, and becomes a line of position.

Note that it doesn't matter whether one measures the range to a satellite (as in GPS), or the angle to a star (as in Mason and Dixon). The small circles and the resulting lines of position are the same. However, these lines of position are "noisy", due to measurement error, and are typically not orthogonal (i.e., their dot products are non-zero). A statistical treatment of these linearized lines of position immediately results in the concept of the covariance matrix.

To determine one's position from multiple lines of position, each from a separate satellite signal, or in the case of stars, observation of multiple stars using a sextant, *least squares* techniques are used. To illustrate this, a real-world representation of three intersecting lines of position is shown in Figure 14, where contrails from three aircraft intersect in the sky over West Virginia.

This is an example of a system of three equations in two unknowns, and the best estimate of the solution to this over-determined system of equations will be a point within the triangle described by the contrails.

Figure 14. Aircraft contrails that demonstrate the concept of "intercepting lines of position."

This use of least squares is described in great detail in the Nautical Almanac [9]. Extending this technique to a situation where the contrails are measured at different points in time while the position of the observer on the ground is moving leads immediately to the need to add a motion model to the mathematical description of the system.

If additional lines of position are needed, one can use a "running fix," in which observations of one star (or the sun, or GPS satellite) are made at different instants in time. This yields the additional lines of position that are required in order compute a navigation fix.

Combining the concepts of least squares, a covariance matrix, and a motion model leads one quickly into the world of Kalman filters [11]. These filter algorithms are the bread and butter of the techniques used by GPS receivers, radars, and other over-determined systems to produce optimal estimates of location and velocity.

The use of measurements of actual time, when using equal altitudes to determine longitude, or the measurements of relative time, when using equal altitudes to determine direction, also introduces errors. These time-keeping errors are typically of two types: drift and offset. When including these uncertainties in the analysis, the estimation, and hence attempt at elimination, of these timing errors is called "solving for time."

One can also make the case, however, that the method of lunars, in which Greenwich Mean Time and longitude are determined by lunar measurements, is perhaps more appropriately what is meant by "solving for time." That is, we distinguish here between solving for time errors (i.e., residuals of the Kalman filter computations) and solving for a value of actual time, which would otherwise be unknown even in the absence of model or measurement errors.

In any case, there is no question that time measurement and time interval measurement have been of considerable importance to the development of surveying, mapping, and navigation for hundreds of years.

VI. ELIMINATION OF COMMON-MODE ERRORS

The previous discussion would be incomplete with addressing a critical technique for improving the accuracy of measurements. When using time interval measurements instead of measurement of absolute time, offset errors due to, for example, a lack of knowledge of one's longitude, disappear. This is because time interval, or Delta-time, techniques eliminate these "common-mode" errors.

The elimination of common-mode errors by using *differential* measurements is a fundamental principle of many techniques and systems. For example, operational amplifiers, or op-amps, are a well-known example of *differential amplifiers*.

The mechanical counterpart of the same differential technique is found in the design of a sextant. Unlike the tripod mounted transit of Mason and Dixon, sextants are hand held, and are used on aircraft and ships. These moving platforms exhibit considerable vibration, pitch, roll, and yaw. The sextant cleverly removes these motion-induced, but nevertheless common-mode, errors by enabling differential measurement of the angle between a star and the horizon.

In GPS, common mode errors are eliminated by the use of differential GPS, and by the use of multiple GPS antennas for attitude determination when relative, but not absolute, position is the desired measurement.

Radio astronomers use a related differential measurement, of *closure phases*, to eliminate common-mode errors caused by radio-propagation delays induced by the ionosphere [12].

And, older versions of the FAA's Very-High-Frequency Omni-directional Range (VOR) radio transmitter stations use a mechanical common-mode reduction technique for making signal phase differences, which are the basis of operation of VOR, independent of the rate at which various mechanical parts rotate [13].

The point here is straightforward. Measurement of time-intervals, rather than of absolute time, reduces common-mode errors in the same manner as do very large numbers of mechanical, electrical, and mathematical systems. The net effect is that when instruments are used to make measurements, *measurement noise* and *model noise* overlap. The combined effects are captured in the canonical model of the covariance matrix that is at the core of the Kalman filter algorithm.

Many individuals who are unconcerned with navigation or time-keeping also understand these principles. Kalman's covariance-based estimation and optimization techniques, as used in time-keeping and navigation, have found their way onto Wall Street and elsewhere outside of the scientific and engineering communities. The jargon is different, but the principles are the same [14].

VII. LEAP SECONDS

The discussion presented herein would not be complete without consideration of the *leap second*. Leap seconds are introduced into the definition of Universal Coordinated Time in order to keep the world's network of atomic clocks aligned with the annual motion of the earth around the sun. But, it is a matter of international debate, to be resolved at the 2015 World Radio Conference in Geneva, whether GPS time, rather than Universal Time Coordinated (UTC), should be used as the world's calendar.

GPS time is not concerned with leap seconds. It differs from UTC by an integer number of leap seconds, as GPS time does not change when additional leap seconds are added, as necessary, to align time-keeping with the motion of the earth around the sun. Use of GPS-time would be a significant departure from the assumption that the earth's clocks should be synchronized to the earth's motion around the sun.

Would Mason and Dixon have cared? Yes and No. Their measurements of the Maryland-Pennsylvania border depended on time intervals, for which leap seconds could be regarded as a common-mode error to be eliminated. However, Mason and Dixon also conducted optical measurements of the transit of Venus across the sun, for which leap second corrections to celestial computations are indeed helpful.

However, for Mason and Dixon, a more significant time error occurred in the form of unanticipated transportation delays. The French Navy attacked their ship as they were leaving Portsmouth, England, to travel to St. Helena, an island in the South Atlantic. The delay in procuring a new, undamaged ship caused a delay that made it impossible to reach St. Helena in time, and observations were conducted from Africa instead. This apparently increased the "dilution of precision" of the desired parallax measurements due to the use of a shorter baseline [3]. Isaac Newton would have been sympathetic with respect to this interesting situation.

Would Lewis and Clark have cared about leap seconds? Probably not, since they often forgot to wind their chronometer, and thus appeared to make absolute measurement of time irrelevant. However, they also measured the time of meridian passage of the sun when later in the same day they measured lunar distances [7]. By so doing, they cleverly trumped the future presumption that their lunar measurements were somehow defective [5]. In the process, they turned time interval measurements into measurements of absolute time, using the solar system and background stars as the GPS-independent celestial clock upon which all of the events of history described in this paper are aligned. Furthermore, their data permits one to "solve for time," as opposed to "solve for time error."

But, if leap seconds are removed by setting the world's systems to GPS instead of UTC, the offset correction will simply be tabulated in one set of tables rather than another. Just as a business might, legitimately, keep two sets of books, so will the world's time keepers. It is simply a question of whether one converts from GPS-time to UTC, or from UTC-time to GPS.

VIII. CONCLUSION

In the previous discussion, we have taken a walk through several hundred years of history, and have given several examples of how the measurement of time and of time intervals has influenced our current technology and way of life.

However, there is a different time scale, which might be referred to as the acceleration of time. This new scale is evident in the time-line of the historical material presented above.

Consider that it is often said that it took Newton 20 years to invent calculus, and that Kepler took 16 years to derive his third law. Or, that:

- Mason and Dixon took four years to survey boundary lines that, using GPS, could now be surveyed in a day or two by a teenager with a car, and with a precision much better than the 400 feet that Mason and Dixon achieved.
- Lewis and Clark travelled to the Pacific Ocean and back in two years.
- The first trans-Atlantic telephone cables were laid on the ocean floor in approximately a week. (The time required for ocean travel hasn't changed a great deal over the past century, presumably because the development of aircraft removes much of the financial incentive to build faster ships.)
- The Apollo missions to the moon took 3-4 days in each direction.
- The New Horizons spacecraft, currently en-route to a rendezvous with Pluto and its moons, took just 6 hours to travel from the earth to the moon hours from its time of launch, as it did not need to slow down when it got there, but was simply passing by. (On the other hand, New Horizons will arrive at Pluto more than ten years after its launch. Depending on one's perspective, this is either a very fast or very slow mission, or both.)
- The International Space Station orbits the earth in approximately 90 minutes.

In conclusion, perhaps we have come full circle, and are closer to the style and philosophical bent of Newton and his successors than we realize. In any case, it seems worthwhile to have revisited the many accomplishments of people who thought long and hard about difficult problems, and did so without the use of computers or the Internet.

And, like Mason and Dixon, not all of us would have been willing to haul the famous marker stones over Sideling Hill in Western Maryland. Indeed, this illustrious mountain is considered, at least by the engineers of the Interstate Highway System's Interstate I-68, to be "in the way." As a result, much of the mountain just south of the Mason and Dixon line has simply been removed. The result is one of the largest road-cuts in the nation, as shown in Figure 15.

Compared to driving over the same mountain on the storied US Route 40, which is still maintained as a federal highway and is the approach route to the famed National Road of the 1800's, one saves approximately 15 minutes, or one sixth of the *time interval* that it takes the ISS to orbit the earth.

Figure 15. The road cut on sideling Hill, south of the Mason-Dixon line. (Used under the terms and conditions of Rocks and Minerals (*http://www.rocksandminerals.org/Terms-conditions/terms.html*)

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