AN ALGORITHM FOR THE DETECTION OF FREQUENCY JUMPS IN SPACE CLOCKS

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Abstract

In this paper, we present a frequency jump detector for space clocks based on two steps: a time-varying configurable deterministic trend, and a filter on the noises to improve the sensitivity.

We prove and discuss the validity of the method by analyzing numerical simulations representing typical GNSS clocks.

1. INTRODUCTION

In a Global Navigation Satellite System (GNSS), the user position is estimated from the time of flight of the signals generated by the system satellites. Any anomaly of the atomic clock behavior on board the satellites causes errors in the localization, and clock anomalies may arise due to different reasons. It is, therefore, fundamental to detect the clock anomalies as soon as possible and to inform the users. At the same time, it is important to limit the number of false alarms, which are mainly caused by the natural random fluctuations of the atomic clock noise or by noise of the measurement system.

In this paper, we present a frequency jump detector for space clocks. Frequency jumps represent in fact one of the most common anomalies in atomic clocks on board satellites. In addition, clocks on board are measured by a complex measurement system that may suffer from missing data, outliers, additive noises, and fluctuations. An efficient detector needs to deal with all these issues aiming to distinguish between genuine frequency jumps and false alarms.

The proposed detector is based on two steps. First, a time-varying configurable deterministic trend is estimated and removed from the clock frequency data. Second, a filter on the noises is applied, aiming to improve the sensitivity of the detector at the level of the frequency jumps under test. Then the filtered frequency data are compared to a given threshold and, if the difference is larger, a warning is raised.

We prove and discuss the validity of the method by analyzing numerical simulations and experimental data from GNSS clocks.

2. MODELLING CLOCK ANOMALIES

We model clock noise in discrete time as

$$\bar{y}[n] = \mu[n] + \xi[n]$$

where $\bar{y}[n]$ is the average frequency deviation, $\mu[n]$ is a nonstationary deterministic drift, and $\xi[n]$ is a white frequency noise (WFN). We now describe each of these terms in detail. The average frequency deviation is defined as

$$\bar{y}[n] = \frac{1}{\tau_0}\int_{(n-1)\tau_0}^{n\tau_0} y(t')dt'$$

where $y(t)$ is the normalized frequency deviation. The white frequency noise $\xi[n]$ is a Gaussian random process with zero mean and autocorrelation function given by

$$R_\xi[n_1, n_2] = E[\xi[n_1]\xi[n_2]] = \sigma^2\delta[n_1 - n2]$$

where E is the expected value and $\delta[n]$ is the discrete-time delta function, defined as $\delta[n]=1$ when $n=0$ and $\delta[n]=0$ when $n \neq 0$. Finally, when the clock is behaving according to the specifications, that is, in nominal mode, we model the deterministic drift $\mu[n]$ as

$$\mu[n] = a_0[n] + a_1[n]n + A\sin(2\pi f_0 n + \varphi) \tag{1}$$

where f_0 is a known value. The functions $a_0[n], a_1[n]$ are slowly varying, and we assume that they are approximately constant on a window of N_w samples, that is

$$\begin{aligned} a_0[n'] &\approx a_0[n] \\ a_1[n'] &\approx a_1[n] \end{aligned} \tag{2}$$

when $n' = n - N_w + 1, \ldots, n$.

When the clock experiences a sudden frequency jump with magnitude Δa_0 at time n_a, we replace the parameter $a_0[n]$ by

$$a_0'[n] = a_0[n] + \Delta a_0 u[n - n_a]$$

where $u[n]$ is the discrete-time step function, defined as $u[n]=0$ for $n<0$, and $u[n]=1$ for $n \geq 0$.

When a slow frequency jump with amplitude Δa_0 and duration Δn occurs at time n_a, we model the parameter $a_0[n]$ by

$$a_0'[n] = a_0[n] + \frac{\Delta a_0}{\Delta n}(n - n_a)u[n - n_a] - \frac{\Delta a_0}{\Delta n}(n - n_a - \Delta n)u[n - n_a - \Delta n].$$

3. THE ANOMALY DETECTOR

We give the formal description of the developed anomaly detector for a real-time implementation. If n is the current time instant, then the detector operates on the $N_w = N_{w1} + N_{w2}$ samples of the average frequency deviation $\bar{y}[n']$ in the time interval

$$n' = n - N_w + 1, \ldots, n.$$

First, the estimates $\hat{a}_0, \hat{a}_1, \hat{A}, \hat{\varphi}$ of the parameters defined in Eq. (1) are obtained from the N_{w1} samples of $\bar{y}[n']$ in the time interval

$$n' = n - N_w + 1, \ldots, n - N_{w2}.$$

These estimates are obtained by solving the least-squares minimization problem given by

$$\hat{a}_0, \hat{a}_1, \hat{B}, \hat{C} = \underset{a_0, a_1, B, C}{\arg\min} \sum_{n'=n-N_w+1}^{n-N_{w2}} \left[a_0 + a_1 n' + B\sin(2\pi f_0 n') + C\cos(2\pi f_0 n') - \bar{y}[n']\right]^2$$

where the identity

$$A\sin(2\pi f_0 n + \varphi) = B\sin(2\pi f_0 n) + C\cos(2\pi f_0 n)$$

is used so that the minimization problem is linear in the parameters a_0, a_1, B, C. We point out that the parameters A, φ can be recovered by noting that

$$A\cos(\varphi) = B$$
$$A\sin(\varphi) = C$$

It is now straightforward to write the estimated drift as

$$\hat{\mu}[n'] = \hat{a}_0 + \hat{a}_1 n' + \hat{B}\sin(2\pi f_0 n') + \hat{C}\cos(2\pi f_0 n')$$

for $n' = n - N_w + 1, \ldots, n - N_{w2}$.

Then, we extrapolate the estimated drift in the time interval $n' = n - N_{w2} + 1, \ldots, n$, and we subtract it from the average frequency deviation

$$\bar{y}_0[n'] = y[n'] - \hat{\mu}[n'].$$

Finally, we smooth the obtained detrended average frequency by a moving average evaluated over N_{w3} samples

$$s[n'] = \frac{1}{N_{w3}} \sum_{k=n'}^{n'+N_{w3}-1} \bar{y}_0[n']$$

for $n' = n - N_{w2} + 1, \ldots, n - N_{w3} + 1$. We detect an anomaly at time $\hat{n}_a$ if

$$|s[\hat{n}_a]| \geq s_{\text{th}}$$

where s_{th} is a configurable threshold. We note that if the anomaly occurs at time n_a, then the detection lag is given by $l = \hat{n}_a - n_a$. In a forthcoming publication, we will discuss the choice of the threshold s_{th} and of the window lengths N_{w1}, N_{w2}, N_{w3}.

4. RESULTS

In Fig. 1, we show $N = 1000$ samples of the average frequency deviation of a clock experiencing three anomalies. The parameters of the deterministic trend, Eq. (1), are $a_0 = 2$, $a_1 = 8 \times 10^{-3}$, $A = 0.5$, $\varphi = 0$, and $f_0 = 1/250$. The (sudden) frequency jumps are located at $n_a = 279, 569, 849$, and the corresponding magnitudes are $\Delta a_0 = 2.1, 2.5, 3$. Finally, the variance of the white frequency noise $\xi[n]$ is $\sigma^2 = 1$.

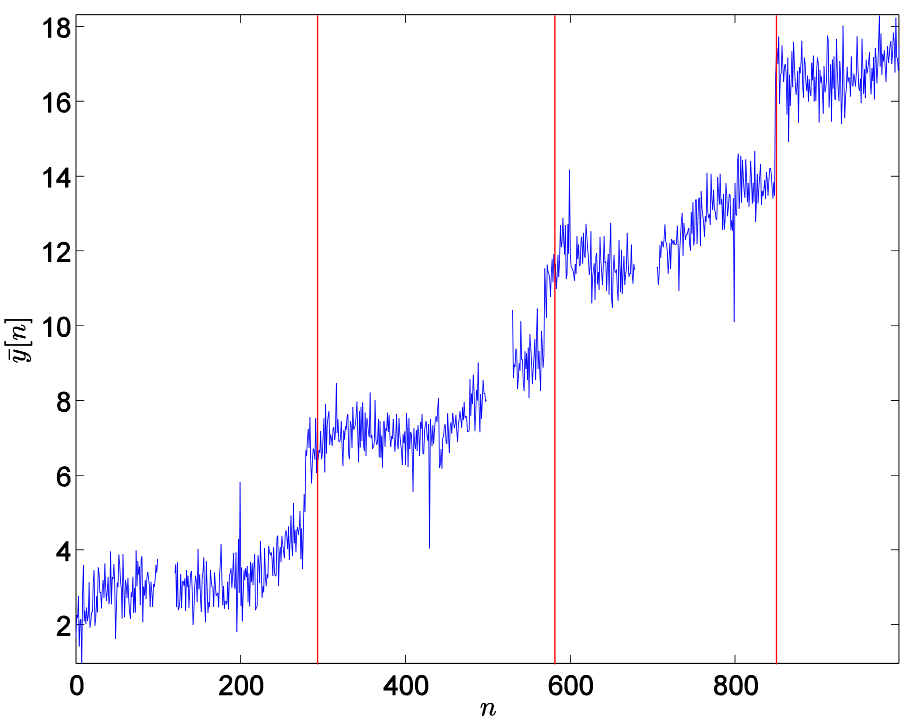

Fig. 1. Simulated clock noise (blue line) and detected anomalies (red lines).

The detector successfully identifies the three anomalies, and the detection times are $n_a = 293$, 581, 850. The corresponding detection lags are $l = 14$, 12, 1. Generally, the detection lags decrease as the magnitude of the frequency jumps increase.

The detector assumes that large outliers have already been removed by a preprocessing algorithm, such as a sigma filter, but it is nonetheless robust to those minor outliers which are not easily removed by these procedures. The simulated data contain four such outliers, which are clearly visible in Fig. 1. The obtained results prove that our detector operates correctly in presence of these outliers.

In addition, the simulation results in Fig. 1 show three sets of missing data. The developed detector can be straightforwardly extended to the case of missing data, which is common with space clocks. We will give this extension of our method in a future publication.

5. CONCLUSION

We have presented a method for the detection of anomalies in atomic clocks. The method is designed for space clocks, where frequency jumps are common anomalies, and where the clock noise can be effectively approximated by a white frequency noise plus a time-varying deterministic trend made by linear and sinusoidal components. The method is robust and can operate with missing data and outliers. We prove the validity of the method with simulated data.

6. ACKNOWLEDGMENT

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