USING SIGN PATTERNS TO DISTINGUISH FEARED CLOCK EVENTS

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Abstract

The first and second difference of clock measurements is investigated to detect and identify clock events. A hypothesis test of the difference functions is described which is calibrated by given Allan deviations of the observed clock differences. The impact of four defined clock events, namely time step, frequency step, drift step, and outlier on the first and second differences is calculated. The analysis outlines that the sign of these calculations generate a pattern of "1", "0", and "-1", which are called event patterns. In order to identify the underlying event type, the identification method interprets the detection process again as a sequence of "1", "0", and "-1" and maps the event patterns to the detection patterns. The detection and identification method are assessed by analyzing the time offset measurements between a commercial rubidium frequency standard and an active hydrogen maser. Both clocks are operated at the DLR timing laboratory.

INTRODUCTION

There are different applications in which time offset measurements are processed. A task is to predict the time offset based on former time offset measurements. It is obvious that a clock event affects the following time offsets and the prediction error increases using clock predictions based on measurements without the event. Hence, it is fundamental to detect clock events in order to adapt parameters of statistical applications.

Another important task is the empirical calculation of Allan deviations. It is disturbed by clock anomalies and any interference alters its statistical meaning.

In the following, both a detection method and an identification method, which differs between difference clock events, are described that are based on first or second difference of clock measurements.

DETECTION METHOD BASED ON FIRST AND SECOND DIFFERENCES

The time offset measurement between two clocks is called $Z(t)$. Besides stochastic components, several clock types, e.g., rubidium frequency standards, are affected by a deterministic drift. Since the deterministic drift affects the first and second difference of clocks in a different manner, it is reasonable to distinguish the detection methods between these two clock types.

In **[1]**, further detection methods are presented.

CLOCKS WITHOUT DETERMINISTIC DRIFT

Assuming clocks which have got no deterministic drift component, either the first difference

$$\Delta_\tau(Z(t)) = \frac{Z(t) - Z(t-\tau)}{\tau}$$

(First difference)

or the second difference

$$\Delta^2{}_\tau(Z(t)) = \Delta_\tau(Z(t)) - \Delta_\tau(Z(t-\tau)) = \left(\frac{Z(t) - Z(t-\tau)}{\tau} - \frac{Z(t-\tau) - Z(t-2\tau)}{\tau}\right)$$

(Second difference)

can be used as detection statistic. It is assumed that both differences are Gaussian with zero mean for any t. The standard deviation is approximated by the given specification of the Allan deviation **[2]**. Table 1 summarizes the used approximations of the standard deviation. Notice, the performance of the standard deviation approximation depends on the present noise type. In case of white frequency noise, the approximations used are optimal **[2]**.

Table 1. Statistic parameters of clocks without drift.

	First difference	**Second difference**
Approximated mean	0	0
Approximated standard deviation	$ADEV(\tau)$	$\sqrt{2}ADEV(\tau)$

CLOCKS WITH DETERMINISTIC DRIFT

The first difference of clocks with deterministic drift grows linearly, whereas the impact on the second difference is constant and, additionally, its level depends on the averaging interval τ. So it is of advantage to focus on the second difference, since the drift is only present for long averaging time τ − for short averaging times, it can be neglected.

The statistic parameters of the second difference depend on the τ of interest. Limiting τ to values in which drift can be neglected, the standard deviation can be approximated by $\sqrt{2}ADEV(\tau)$. Its mean is assumed to be zero.

For the remaining τ values, the deterministic drift value has to be estimated. On the one hand, this can be done by an empirical method, e.g., median or average on the other hand, clock specifications can be used. The standard deviation is again approximated by a given Allan deviation with drift removed. Table 2 summarizes the results of clocks with drift.

Table 2. Statistic parameters of clocks with drift.

	Second difference with drift negligible	**Second difference with drift present**
Approximated mean	0	Median or average
Approximated standard deviation	$\sqrt{2}ADEV(\tau)$	$\sqrt{2}ADEV(\tau)$

Hypothesis Test

The hypothesis H_0 is that both the first and the second differences are Gaussian random variables. Using the specifications of Tables 1 and 2, the random functions are normalized and the test statistic or decision rule is

$$X := \frac{|\bullet - Mean(\bullet)|}{Std(\bullet)} \leq level$$

The hypothesis is accepted if the test statistic is true, and rejected if the test statistic is false. The type I error describes the situation when the hypothesis is rejected although it is true. The probability of committing a type I error in dependency to the level is shown in Table 3 (Figure 1).

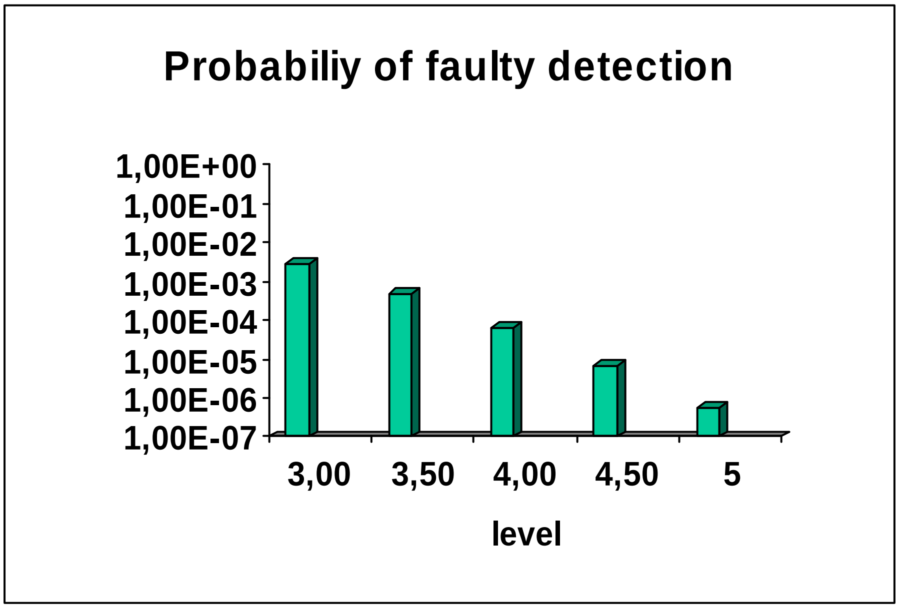

Figure 1. Probability of faulty detection.

Table 3. Probability of faulty detection.

Level	3	3.5	4	4.5	5
Probability of faulty detection	0.00269	0.000465	0.0000633	0.00000679	0.000000573

By definition, a measurement $Z(t)$ is called detected if its hypothesis H_0 is rejected.

DETECTION PATTERN

Each measurement is assigned one of the values "1", "0", and "-1". The assignment depends on the hypothesis test. "0" is applied to an accepted measurement, whereas "1" or "-1" is used in case of rejection. Since the hypothesis test defines a positive and a negative threshold, "1" is assigned to measurements which violate the positive threshold and, "-1" is assigned vice versa. Applying this method on a measurement process, a sequence of "1", "0", and "-1" is generated which is called the detection pattern.

EXAMPLE: COMMERCIAL RUBIDIUM CLOCK WITH DRIFT

The detection method is applied on the time offset measurements between a low-cost rubidium frequency standard (LPFRS) and an active hydrogen maser (AHM). The LPFRS was manufactured by Temex Time and the reference clock AHM (CH1-75) was manufactured by

Kvarz. Both clocks are operated at the temperature-controlled DLR clock laboratory. The measurements are realized every 300 s using a phase comparator of Kvarz (CH7-48).

The datasheet specifies the short- and long-term performances of LPFRS by: ADEV (100 s) < $1*10^{-12}$ (white frequency noise) and ADEV (30 d) < $5*10^{-11}$ (deterministic drift). Assuming white frequency noise (WFN) characterizes the Allan deviation, ADEV (1 s) is related to the clock specifications by:

$$\sqrt{AVAR(1s)100^{-1}} = 10^{-12} \Leftrightarrow AVAR(1s) = 10^{-22}$$

The measurements are analyzed using the fixed time interval $\tau = 1800s$. At this $\tau = 1800s$, the impact of the drift on the second difference is negligible. The standard deviation is

$$\sigma(\Delta_{1800}{}^{2}(Z(t))) = \sqrt{2}ADEV(1800s) = \sqrt{2}ADEV(1s)\sqrt{1800^{-1}s} = 3.33*10^{-13}$$

It is applied to calibrate the hypothesis test.

Figure 2 shows the first differences of the observed measurements. Clearly, a linear component is present, which results from the deterministic drift.

Figure 3 shows the second difference along with the threshold $\sigma(\Delta_{1800}{}^{2}(Z(t)))*5$ (green line).

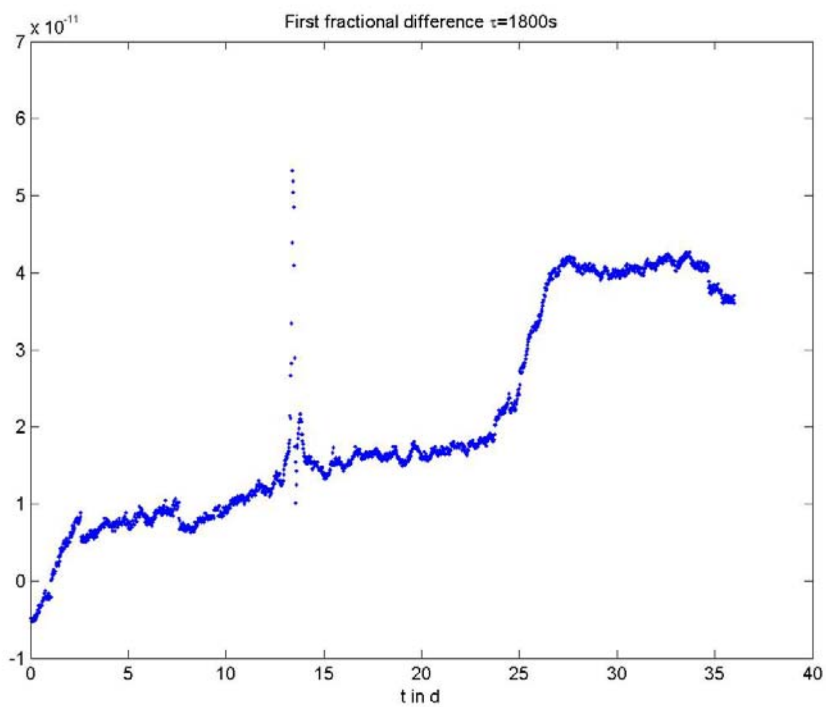

Figure 2. First difference of LPFRS.

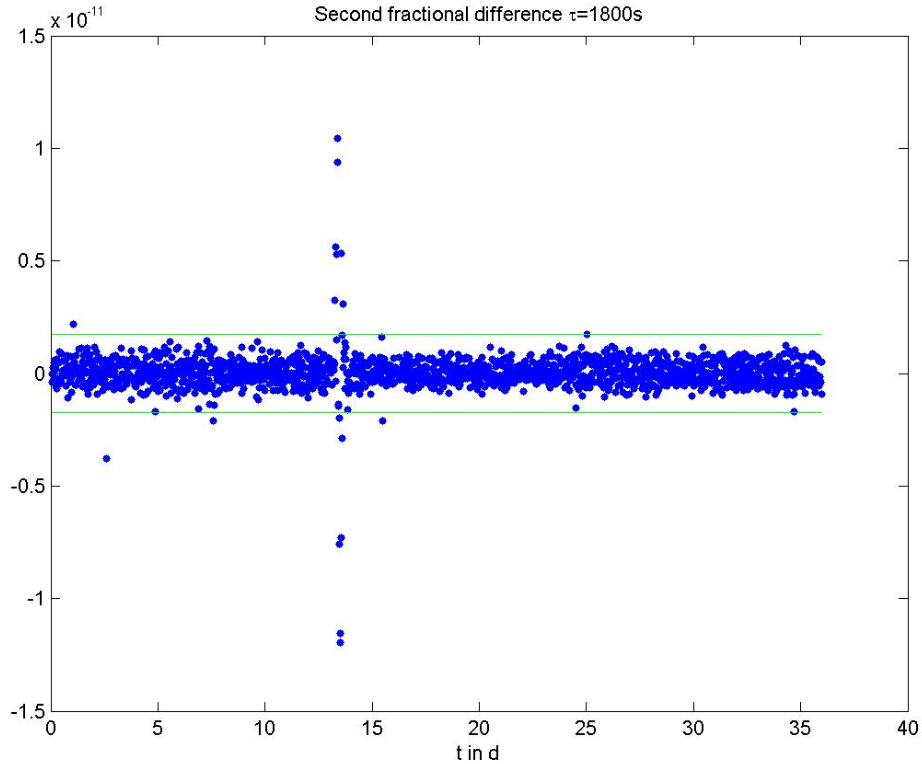

Figure 3. Second difference with detection level.

Table 4 outlines the dependency of the level to the number of detections. Using the level 3, 67 measurements of 1726 are detected. Thirty measurements are detected using level 4 and 18 are detected using level 5.

Table 4. Dependency of level to number of detections.

level	3	4	5
Detects of 1726	67	30	18

The level 3 and 4 result in a lot of detections, which potentially are not clock events. The level 5 states a reasonable compromise without changing the standard deviation approximation.

AUTOMATIC IDENTIFICATION OF CLOCK EVENTS

Besides the detection of measurements $Z(t)$ whose hypothesis H_0 is rejected, it is of interest to automatically determine the underlying event type.

DEFINITION OF DETERMINISTIC EVENT TYPES

Depending on which clock state is disturbed, different event types result. A time step disturbs the first clock state and represents a shift of the clock offset. A frequency step is understandable as a linear function which is added to the clock offset. Consequently, a drift step is understandable as a quadratic function which is added to the clock offset.

Table 5. Time offset of event types.

Event types	$t < t_{ev}$	$t = t_{ev}$	$t > t_{ev}$
time step: $E^{time}(t)$	0	e_{phase}	e_{phase}
frequency step: $E^{frequ}(t)$	0	0	$e_{frequ}(t-t_{ev})$
drift step: $E^{drift}(t)$	0	0	$0.5e_{drift}(t-t_{ev})^2$
Outlier: $E^{outlier}(t)$	0	$e_{outlier}$	0

An outlier states an exception, since its impact is only punctual. Table 5 summarizes the impact of the four event types on the time offset.

EVENT PATTERN OF THE EVENT TYPES

Applying the sign-function on the calculations of the first and second difference of the four event types (see APPENDIX), a “1”, “0”, and “-1” sequence is generated. The results of the sign-function are summarized in Tables 6 and 7. The assigned name depends on the number of non-zero values. The names are listed at the table.

Table 6. $\mathrm{sgn}(\blacksquare)$-pattern of first difference.

event	t_{ev}	$t_{ev}+\tau$	$t_{ev}+2\tau$	$t_{ev}+3\tau$	$t_{ev}+4\tau$	**Name**
time step	$\mathrm{sgn}(e_{phase})$	0	0	0	0	1-pattern
frequency step	0	$\mathrm{sgn}(e_{frequ})$	$\mathrm{sgn}(e_{frequ})$	$\mathrm{sgn}(e_{frequ})$	$\mathrm{sgn}(e_{frequ})$	4-pattern
drift step	0	$\mathrm{sgn}(e_{drift})$	$\mathrm{sgn}(e_{drift})$	$\mathrm{sgn}(e_{drift})$	$\mathrm{sgn}(e_{drift})$	4-pattern
outlier	$\mathrm{sgn}(e_{outlier})$	$-\mathrm{sgn}(e_{outlier})$	0	0	0	2-pattern

In case of the first difference, three different patterns result. Also, the event pattern of a frequency and a drift step are equal to each other. The 1-pattern implies a time step and the 2-pattern implies an outlier.

The second difference generates four different patterns and each pattern uniquely determines its event type.

Table 7. $\mathrm{sgn}(\blacksquare)$-pattern of second difference.

event	t_{ev}	$t_{ev}+\tau$	$t_{ev}+2\tau$	$t_{ev}+3\tau$	$t_{ev}+4\tau$	**name**
phase step	$\mathrm{sgn}(e_{phase})$	$-\mathrm{sgn}(e_{phase})$	0	0	0	2-pattern
frequency step	0	$\mathrm{sgn}(e_{frequ})$	0	0	0	1-pattern
drift step	0	$\mathrm{sgn}(e_{drift})$	$\mathrm{sgn}(e_{drift})$	$\mathrm{sgn}(e_{drift})$	$\mathrm{sgn}(e_{drift})$	4-pattern
outlier	$\mathrm{sgn}(e_{outlier})$	$-\mathrm{sgn}(e_{outlier})$	$\mathrm{sgn}(e_{outlier})$	0	0	3-pattern

MAP DETECTION PATTERN TO EVENT PATTERN IN ORDER TO IDENTIFY EVENT TYPE

In order to identify the event type, the detection pattern of the measurement process is analyzed and mapped to one of the event patterns. Using this procedure, event types are distinguished from each other. Notice, the first difference only differs between time step and outlier, whereas the pattern of a frequency and drift step are equal. The second difference distinguishes between all event types. Figures 4 and 5 schematically outline the four different event patterns.

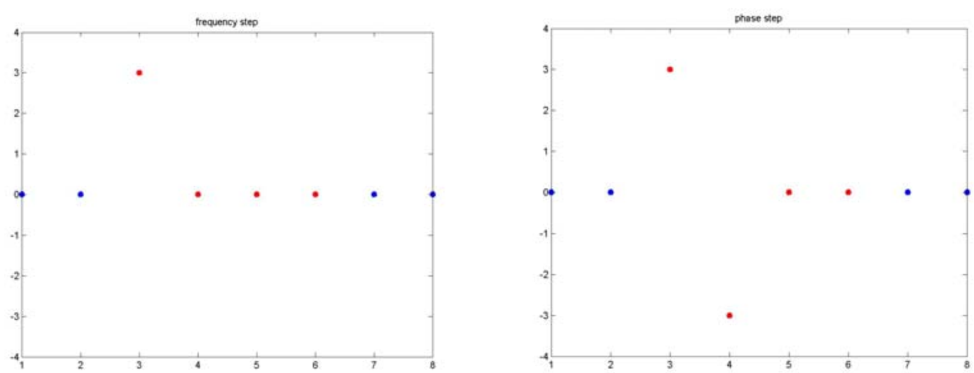

Figure 4. 1-pattern and 2-pattern.

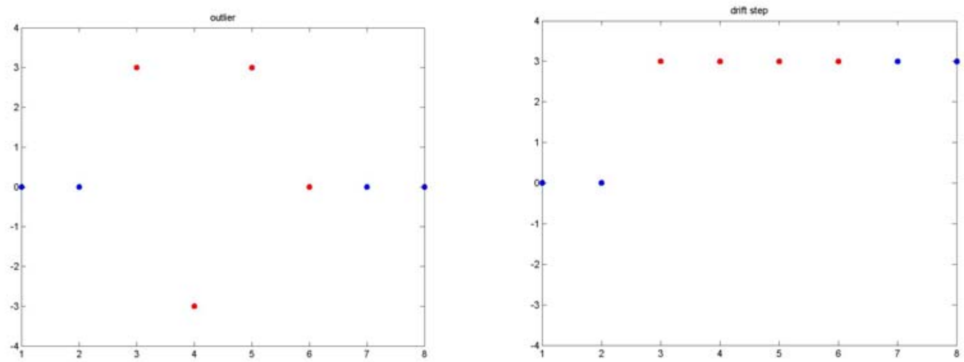

Figure 5. 3-pattern and 4-pattern.

The identification starts at the moment of the first detection and uses the following sign values to generate the detection pattern. The detection pattern is compared with the 1, 2, 3, 4-patterns.

The method determines the event type as long event types are not overlapped. Since the first and second differences are both linear functions, ambiguities are possible. E.g., the pattern of a frequency step with positive level at t_1 followed by a frequency step with negative level at $t_1 + \tau$ generates a 2-pattern using the second difference. As a result, the identification method might output a time offset step.

EXAMPLE: EVENT IDENTIFICATION OF A COMMERCIAL RUBIDIUM FREQUENCY STANDARD

The measurements of the LPFRS which are presented in the previous chapter are used to test the identification method. Figure 6 shows the second difference of the measurements along with the detection level (green line). Additionally, the occurrence of patterns is marked with black arrows.

The 1-pattern emerges several times. Since the second difference is analyzed, the 1-pattern belongs to a frequency step; thus, the method outputs a frequency step. In order to verify the result, Figure 7 shows the first difference. The arrows signalling the patterns are copied from Figure 6 to connect both difference figures. A visual check shows that the 1-patterns agree with frequency steps.

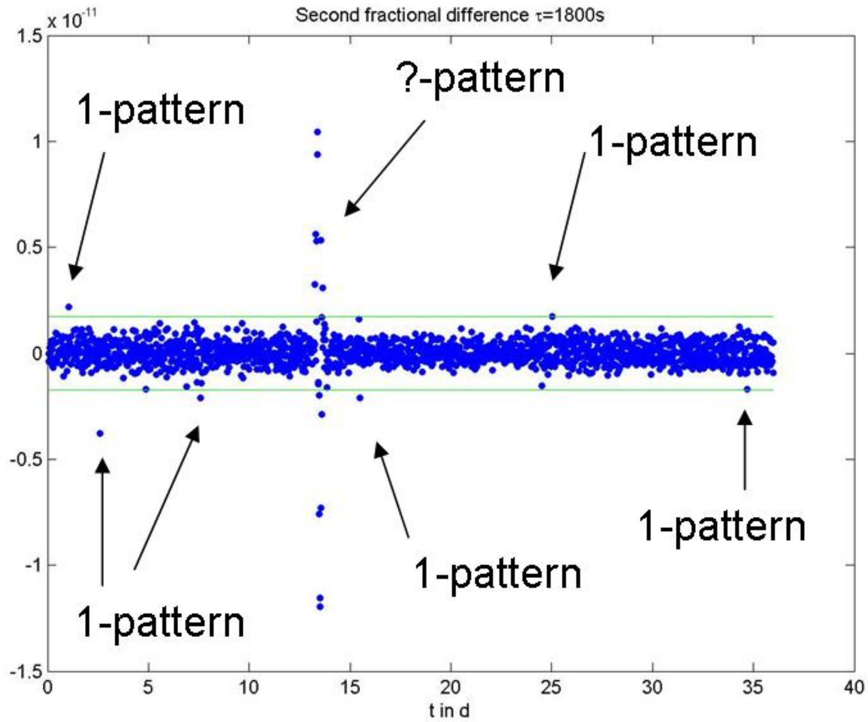

Figure 6. Occurrence of patterns at the second difference.

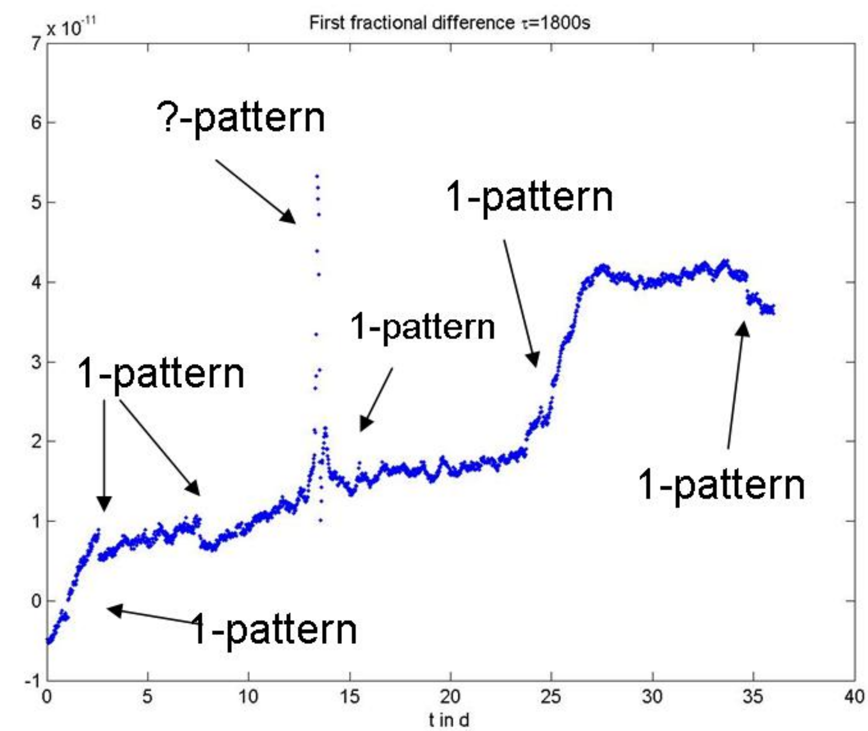

Figure 7. Agreement of patterns with event types.

The ?-pattern marks a couple of detections which do not generate one of the previously defined patterns (Figures 6 and 7). Figures 8 and 9 snapshot the corresponding time offset and detection pattern. A visual check of the time offset plot points out that the time offset starts to grow in a quadratic manner from time START till time MARK 1. From MARK 1 to time END, the growth decreases and results in a regular growth. Comparing the time offset of START and END, the event looks like a time step. However, the detection pattern does not fit to the event pattern of a time step (Figure 9). It is reasonable, because the method only identifies instantaneous time offset steps.

It is remarkable that the level of the growth from START till MARK 1 and from MARK 1 till END is about the same, and, thus, the impact on the time offset corresponds to a time offset step. The identification method does not account for event levels and, thus, is not able to identify the event.

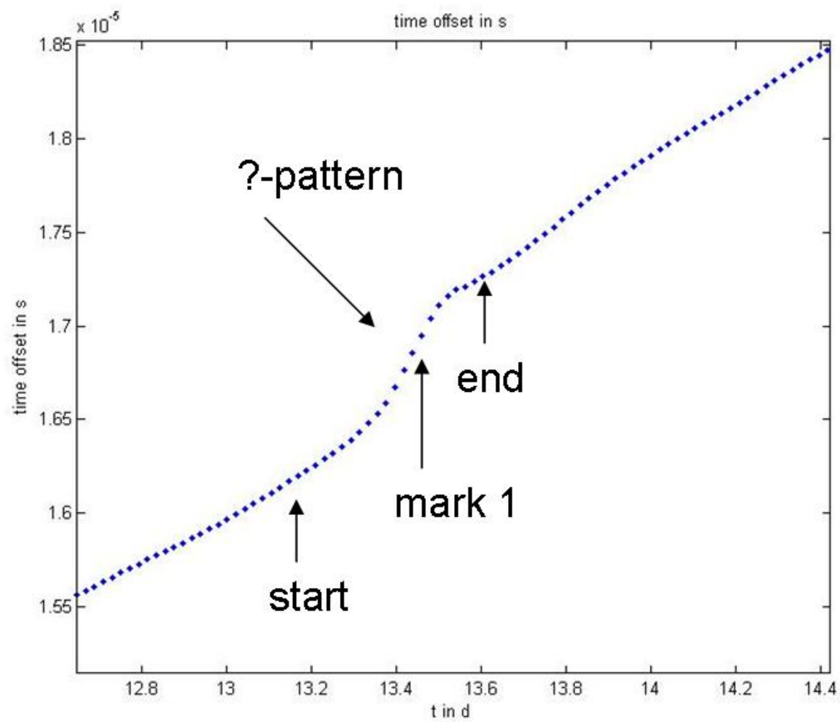

Figure 8. Time offset of the ?-pattern.

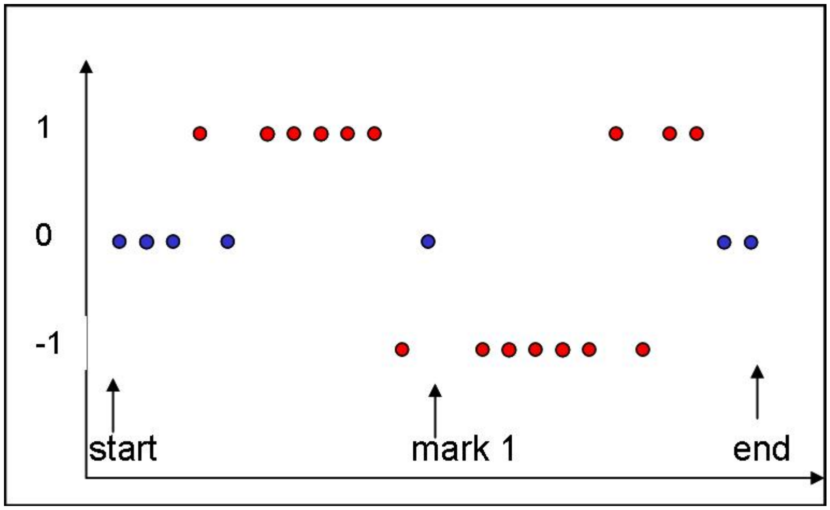

Figure 9. Detection pattern of ?-pattern.

CONCLUSION

A detection method which uses the statistics of the first and second differences was analyzed. The method assumes that the both differences are Gaussian for the investigated τ interval. Its standard deviation is approximated by using the given Allan deviation. Since the impact of a deterministic drift on the first differences is linear in t, it is recommended that one to use the second differences for clocks with drift. However, either the first or second difference is reasonable for clocks without a deterministic drift component.

Besides the detection of faulty measurements, it is of interest to algorithmically identify the event type. The impact of four basic clock events namely time step, frequency step, drift step, and measurement outlier on the first and second difference is calculated. The sign of the calculations is used to generate “1”, ”0”, ”-1”-patterns. It was shown that four different patterns occur, which are called event patterns. In case of the first difference, the patterns of a time step and an outlier are different and can be used to distinguish the event types. The patterns of the second difference distinguish between all four event types.

The detection and identification methods were applied to the time offset measurements between a commercial rubidium clock and an active hydrogen maser. The identification of feared clock events was not always straightforward. The identification algorithm assumes the simplified situation that, after an event, the next four measurements are only disturbed by the past event and not by another one. An overlap of events is not accounted for. As a result, ambiguities of the event types are possible. E.g., two successive frequency steps with different sign will have the same pattern as a phase step.

This ambiguity effect also emerged in the measurements. The rubidium clock exhibited an event which was ongoing for about 7 hours. Although the detection pattern of this event was similar to that of a time step, the design of the identification method is not able to identify it. Nevertheless, steps in frequency were detected and identified and the algorithms performed reasonably.

APPENDIX

A: First and Second Difference of Event Types

The definition of events (Table 5) is used to calculate its first difference. The results are outlined at Table 7. The linearity of the second difference is utilized to derive the second differences (Table 8).

Table 8. First difference of event type.

	$\Delta_\tau(E^{ev}(t_{ev}))$	$\Delta_\tau(E^{ev}(t_{ev}+\tau))$	$\Delta_\tau(E^{ev}(t_{ev}+k\tau))$ with $k \geq 2$
Phase event	$\frac{e_{phase}}{\tau}$	0	0
Frequency event	0	e_{frequ}	e_{frequ}
Drift event	0	$0.5 e_{drift}\tau$	$(k-0.5)e_{drift}\tau$
Outlier with $t_{event} = t_{k_{event}}$	$\frac{e_{outlier}}{\tau}$	$-\frac{e_{outlier}}{\tau}$	0

Table 9. Second difference of event type.

	$\Delta^2_\tau(E^{ev}(t_{ev}))$	$\Delta^2_\tau(E^{ev}(t_{ev}+\tau))$	$\Delta^2_\tau(E^{ev}(t_{ev}+2\tau))$	$\Delta^2_\tau(E^{ev}(t_{ev}+k\tau))$ with $k \geq 3$
Phase step	$\frac{e_{phase}}{\tau}$	$-\frac{e_{phase}}{\tau}$	0	0
Frequency step	0	e_{freq}	0	0
Drift step	0	$0.5 e_{drift}\tau$	$e_{drift}\tau$	$e_{drift}\tau$
Outlier with $t_{event} = t_{k_{event}}$	$\frac{e_{outlier}}{\tau}$	$-2\frac{e_{outlier}}{\tau}$	$\frac{e_{outlier}}{\tau}$	0

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