# Determining Geo-stationary Satellite Position Using TWSTFT Links

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***Abstract*—Based on several two-way satellite time and frequency transfer (TWSTFT) links and multi-channel time transfer modems, a positioning method to geo-stationary communication satellite is presented, moreover, the accuracy of the positioning is concisely estimated. The characteristics of the method are simple and practical. Especially, the positioning system is completely established on TWSTFT links, so it does not need an extra investment.**

## Ⅰ. INTRODUCTION

Two-way satellite time and frequency transfer is currently one of the most precise and accurate time comparison techniques over the large distance [1]. The precision of the time transfer is on sub-nanosecond which is better than that of GPS Common View method one order, the accuracy is expected to reach one nanosecond, and the Bureau International des Poids et des Mesures (BIPM) has already adopted TWSTFT data for the computation of the International Atomic Time (TAI). Since 1998, some TWSTFT links in the Asia-Pacific region have being built [2]. The time laboratories involved in the region include the National Institute of Information and Communications Technology of Japan (NICT), the National Metrology Institute of Japan (NMIJ), the National Time Service Center of China (NTSC), the Korea Research Institute of Standards and Science (KRISS), the Telecommunication Laboratories of Taiwan (TL), and the Standards Productivity and Innovation Board of Singapore (SPRING). Geo-stationary communication satellite JCSAT-1B at 150 deg. E is used for the TWSTFT links. In addition, a link of NICT to the National Measurement Laboratory of Australia (NML) has been established using the geo-stationary communication satellite PAS-8 at 166 deg. E.

Recent years, multi-channel time transfer modems have been developed at NICT for the TWSTFT links [3]. The modems can perform time transfer experiments among more than three stations simultaneously. Each station transmits a time signal that is modulated using the spread-spectrum method. All of the signals received at the satellite are combined and retransmitted back to the ground. The multi-channel receiving section at each station demodulates the signal from the satellite and makes arrive-time measurements using the satellite reference clock. The time differences for all the pairs of participating stations can be calculated. Thus the time transfer of all pairs of stations can be performed simultaneously. Table Ⅰ shows the main specifications of the modem.

With the development of the TWSTFT and the use of the multi-channel modem, we have explored a method of determining the geo-stationary communication satellite position. That is, the satellite position can be determined when TWSTFT among the stations is performed.

TABLE I

SPECIFICATIONS OF MODEM

| | |
|---|---|
| Modulation | Direct-sequence spread spectrum method using PRN code |
| Modulation Channels | 3 (one for time transfer, two for Earth station delay calibration) |
| Demodulation or Receiving Channels | 8 (six for time transfer, two for Earth station delay calibration) |
| Clock rate of PRN code | 2.04775MHz |
| Bit length of PRN code | 8191bits (13 stage FSR) |
| Communication function | Data transmission among the participating stations |
| Remote control function | Modems on slave stations can be controlled from the master station |

## II. POSITIONING METHOD

We choose at least three TWSTFT links in which each link includes NICT, and these links use a common geo-stationary communication satellite. When time transfer among the links is performed by the multi-channel modems, the signal delays from each station to NICT via the satellite can be obtained simultaneously. If the stations are denoted by Gi, and the coordinates are denoted by $(x_i, y_i, z_i)$, where $i$ is from 1 to N (N>3), and NICT is denoted by G1, the delays from each station to NICT can be expressed by

$$\begin{cases} \sqrt{(x_2-x_s)^2+(y_2-y_s)^2+(z_2-z_s)^2}+\sqrt{(x_1-x_s)^2+(y_1-y_s)^2+(z_1-z_s)^2}=\tau_1\cdot C \\ \sqrt{(x_3-x_s)^2+(y_3-y_s)^2+(z_3-z_s)^2}+\sqrt{(x_1-x_s)^2+(y_1-y_s)^2+(z_1-z_s)^2}=\tau_2\cdot C \\ \ldots\ldots \\ \sqrt{(x_N-x_s)^2+(y_N-y_s)^2+(z_N-z_s)^2}+\sqrt{(x_1-x_s)^2+(y_1-y_s)^2+(z_1-z_s)^2}=\tau_{N-1}\cdot C \end{cases} \quad (1)$$

where $(x_s, y_s, z_s)$ is the satellite coordinates, $C$ is the speed of light, and $\tau_i$ is the delay from Gi to G1.

Since locations of the stations are known and time among the stations can be synchronized by the TWSTFT itself, the satellite position including 3 unknowns can be calculated by (1).

For satellite JCSAT-1B, we can use the Asia TWSTFT links to determine its position. Fig. 1 simply shows the configuration.

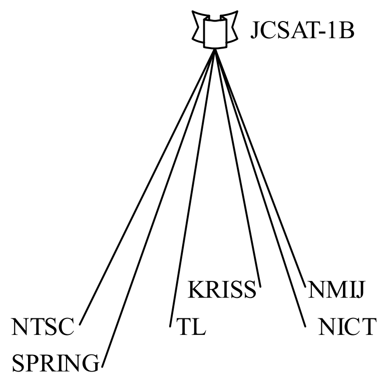


Fig. 1 Distribution of stations in the Asia region

In addition, we may also choose two TWSTFT links in which each link includes NICT, moreover, another TWSTFT equipment at NICT is installed, which enables us to obtain the delay from NICT to NICT via the satellite. In this case, the positioning can be achieved using only 3 stations. When the time transfer among three stations is performed, 3 delays (two are from two stations to NICT, and one is from NICT to NICT) are obtained. The positioning equations are

$$\begin{cases} \sqrt{(x_1-x_s)^2+(y_1-y_s)^2+(z_1-z_s)^2}=\frac{\tau_1}{2}\cdot C \\ \sqrt{(x_2-x_s)^2+(y_2-y_s)^2+(z_2-z_s)^2}=(\tau_2-\frac{\tau_1}{2})\cdot C \\ \sqrt{(x_3-x_s)^2+(y_3-y_s)^2+(z_3-z_s)^2}=(\tau_3-\frac{\tau_1}{2})\cdot C \end{cases} \quad (2)$$

If the NICT-USNO (U.S. Naval Observatory) TWSTFT link can be constructed using the satellite PAS-8 at 166 deg. E, ones may use the link and NICT-NML link established to determine the satellite position. Fig. 2 shows the configuration. Since the three stations (NICT, NML, USNO) are spread out in the Pacific-rim region, an ideal geometric distribution is formed among the stations and satellite.

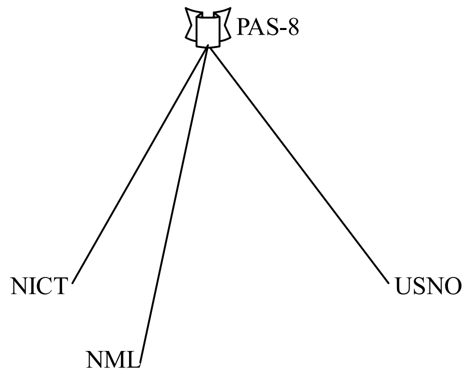


Fig. 2 Distribution of stations in the Pacific-rim region

## III. POSITIONING ACCURACY

The positioning accuracy for satellite depends on two aspects. One is the geometric distribution among stations and the satellite. The other is the ranging accuracy.

### *A. Geometric distribution*

As an example, there are 3 equations in (1), that is, there are 4 stations attending the positioning. By linear developing and to delete the high order terms, we obtain

$$\begin{bmatrix} A11 & A12 & A13 \\ A21 & A22 & A23 \\ A31 & A32 & A33 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} b1 \\ b2 \\ b3 \end{bmatrix} \qquad (3)$$

where

$$Ai1 = (xi-x0)/di + (x4-x0)/d4 \qquad i = 1, 2, 3$$
$$Ai2 = (yi-y0)/di + (y4-y0)/d4 \qquad i = 1, 2, 3$$
$$Ai3 = (zi-z0)/di + (z4-z0)/d4 \qquad i = 1, 2, 3$$

$$di = \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2 + (z_i - z_s)^2} \qquad i = 1, 2, 3, 4$$

$$bi = Ri \times C - (di + d4) \qquad i = 1, 2, 3$$

$\triangle x = xs - x0$; $\triangle y = ys - y0$; $\triangle z = zs - z0$

where (x0, y0, z0) is the normal position of the satellite. Equation (3) can be indicated as

$$AX = B \qquad (4)$$

where A = (Aij), the i and j are from 1 to 3, and the A is called a coefficient matrix.

$X = (\triangle x, \triangle y, \triangle z)$,

$B = (b1, b2, b3)$.

The solution of (4) is

$$X = (A)^{-1} B \qquad (5)$$

The coordinate variance matrix of X is defined as

$$Cov(X) = (A^T A)^{-1} \qquad (6)$$

where $A^T = (Aji)$. Assuming

$$(A^T A)^{-1} = \begin{bmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 & \sigma_{xz}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 & \sigma_{yz}^2 \\ \sigma_{zx}^2 & \sigma_{zy}^2 & \sigma_{zz}^2 \end{bmatrix} \qquad (7)$$

The square root of the sum of elements in diagonal line of (7) is usually defined as a judgment standard of the geometric distribution among ground stations and satellite, and the value is called position dilution of precision (PDOP).

$$PDOP = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2} \qquad (8)$$

Moreover, the positioning accuracy equal the ranging accuracy multiplied by PDOP. Generally speaking, the larger the area surrounded by these ground stations is, the less the PDOP is. Thus the stations should be distributed in wider region.

For satellite JCSAT-1B, the Table II gives some values of PDOP among the stations in the Asia TWSTFT links, where the number of attending the positioning stations is from 4 to 6, and "Y" stands for the station that attends the positioning.

TABLE Ⅱ

SOME PDOP VALUES

| NICT | NMIJ | SPRING | TL | NTSC | KRISS | PDOP |
|---|---|---|---|---|---|---|
| Y | Y | Y | Y | Y | Y | 30.9 |
| Y | Y | Y | | Y | Y | 31.0 |
| Y | Y | Y | Y | | Y | 54.4 |
| Y | Y | Y | Y | Y | | 31.3 |
| Y | Y | Y | | Y | | 31.4 |

*B. Ranging accuracy*

There are 4 main factors that affect the ranging accuracy. We primarily give an estimate as follows.

a. Propagation path: when the signal through the ionosphere and troposphere, the delay will increase. The correction of the delay can be calculated by mathematics formulas. In fact, the delay is related to the elevation at station. Table Ⅲ gives the elevations at some stations. When the elevation from station to satellite is bigger than 30 deg., the error of the correction for the troposphere should be less than 1ns. Moreover, due to the Ku-band frequency used in the TWSTFT, the error of the correction for the ionosphere is relative small, and it should be less than 0.5 ns.

TABLE Ⅲ

ELEVATIONS FROM STATIONS TO SATELLITE

| Station | Elevation | Station | Elevation |
|---|---|---|---|
| | unit: deg. | | unit: deg. |
| NICT | 47.2 | KRISS | 41 |
| NMIJ | 47 | SPRING | 37.1 |
| TL | 46.7 | NTSC | 31.3 |

b. Equipments at stations: the delay of the equipments may be calibrated with the uncertainty less than 1 ns.

c. Time synchronization among the stations: the time among the stations can be synchronized by the TWSTFT itself, and the accuracy is about 1 ns.

d. Coordinates of station: by the GPS and other means, it is easy to determine the station' location with error of 0.3m.

Table Ⅳ gives total systematic errors. Thus, we obtain the ranging accuracy is about 0.62m.

TABLE Ⅳ

SYSTEMATIC ERROR BUDGET

| Items | Error (unit: ns) |
|---|---|
| 1. Ionosphere | 1 |
| 2. Troposphere | 0.5 |
| 3. Equipments at station | 1 |
| 4. Time synchronization among the stations | 1 |
| 5. Coordinates in stations | 1 |
| Total | 2.06 |

When NICT, NMIJ, SPRING, NTSC attend the positioning of JCSAT-1B, the positioning accuracy (PA) is

$$PA \approx 19.4\ (m) \quad (9)$$

If NICT, NML, USNO attend the positioning of PAS-8, the PDOP is 10.5, thus its positioning accuracy (PA) is

$$PA \approx 6.5(m) \quad (10)$$

## Ⅳ. ADVANTIGES OF THE METHOD

The TWSTFT positioning method to geo-stationary satellite has following advantages.

(1) Since the stations in the TWSTFT links are all major time institutes with keeping precise time scales, they are able to maintain a consecutive long-term operation of TWSTFT. Thus the positioning has relative high reliability.

(2) For accurate positioning, the time among the stations must be synchronized accurately. The TWSTFT itself just possesses such ability.

(3) Due to the signal frequency being at Ku band in TWSTFT, ionosphere effect to the ranging error is relative small.

(4) There are relative ideal geometric distributions among the stations and satellite, which is greatly contributed to the accurate positioning.

(5) The positioning system is completely established on the TWSTFT links, it does not need a special extra investment.

## Ⅴ. CONCLUSIONS

Method of determining geo-stationary communication satellite position using Asia-Pacific TWSTFT links possesses some advantages. Especially, it is completely established existent TWSTFT links, so a special large investment is not needed. Moreover, the method has the ability of potential high accurate positioning. The position of satellite JCSAT-1B can be determined using the Asia TWSTFT links, and the position of satellite PAS-8 can be determined using the Pacific-rim TWSTFT links. As next step, some researches can be done, such as, the precise orbit determining and improvement using consecutive TWSTFT data, and to explore the applications of precise orbit in satellite time service, navigation and its augmentation systems.

## REFERENCES


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