Theoretical Studying About the Measurement of the C-Field Intensity In the Optical Pumped Cesium Frequency Standard

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***Abstract*—In the paper the measurement of the C-field intensity is analyzed theoretically in optical pumped Cesium frequency standard. The transition probability from the F=3 m_F=0 level state to the F=4 m_F=0 level state has been discussed as the function of the frequency and power of the signal injected to the low frequency coils. The result shows out the fluorescence intensity in the probing region increases to 1.37 time of the normal condition without the low frequency signal at the Ramsey cavity.**

I. Introduction

Optically pumped cesium beam frequency standard has been realized as the primary frequency standard [1]. We have known the accuracy of the optically pumped cesium beam frequency standard relates with the value of the C-field intensity. Usually series low frequency coils are placed on the drift region of the Ramsey cavity at the Cesium beam tube of the traditional frequency standard, in order to acquire the value of the C-field intensity [2], but to the best of our knowledge in optically pumped cesium beam frequency standard nobody has discussed how to determine the C-field intensity with the low frequency signal. In our optically pumped cesium atomic beam frequency standard with a sharp angle of incidence detecting laser beam, as shown in Fig.1, the pumping laser frequency is locked to the F=4—F'=4 transition line of the cesium D_2 saturated absorption spectrum, and the detecting laser light is locked to the F=4, F'=5 and F=4, F'=4 cross over line of the cesium D_2 saturated absorption spectrum. Because there is a 66.5 degree angle between the probing laser beam and the atomic beam, for the atoms in the F=4 state with velocity close to the most probable velocity, the probing laser frequency is Doppler shifted up 125.7 MHz to the F=4, F'=5 cycling transition, thus effectively providing cycling transition detection for these atoms. Atoms with velocity far from the most probable velocity do not interact with the detecting laser light and have no contribution to the signal [3].

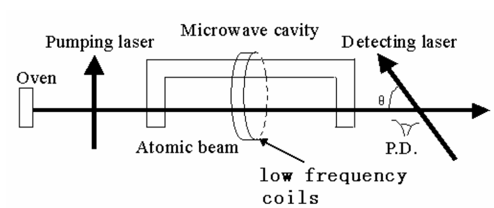


Fig.1 Diagram of experimental set-up of the optically pumped cesium beam frequency standard with sharp angle detecting laser beam

II. Analyzing

We consider the transition of the Zeeman sublevels of F=3, neglecting the millman effect. According to the selective rule, the low frequency transition only takes place between the sublevels with $\Delta F=0$, $\Delta m_F=\pm1$, then the transition frequency as follows[4]:

$$v(3,m_2-3,m_1)=350.975\times10^3 H_0-13.358(2m_1-1)H_0^2 \qquad (1)$$

Where H_0 is the intensity of the C-field (unit is gauss), m_1 and m_2 represent the magnetic quantum numbers respectively. Assuming the intensity of the C-field is 60mGs, we then have:

$$v(3,m_2-3,m_1)\approx 21058\text{Hz}$$

Assuming that sublevels of the F=3 are equal distance, so the transition from F=3 $m_F=0$ to F=3 $m_F=\pm1$ tends to balance the population of all the F=3 sublevels via cascading $\Delta F=0$, $\Delta m_F=\pm1$ transition. The Resulting population may be described by the Majorana formula [2,4].

When passing the pumping area, the atoms interact with the pumping laser, and almost all atoms would be optically pumped to the F=3 ground state; then the atoms pass through the U-shaped microwave cavity, after they go through the first interaction with the microwave, part of atoms transit to the F=4 $m_F=0$ ground state; after later they interact with the low frequency signal at the drift region in the Ramsey cavity, meanwhile the Majorana transitions take place among all the sublevels F=3.We assume that: before all atoms entered into the Ramsey cavity, the probabilities were equal at all sublevels of the F=3 state. Interacting with the microwave at the first interaction area, it is possible to transit to the F=4,$m_F=0$ level state from the F=3 $m_F=0$ level state, the atomic quantities of others sublevels except the F=3 $m_F=0$ state are unchangeable. So that at the low frequency coils not only exists the transition from the F=3,$m_F=0$ state to other sublevels, but also there is the transition from other sublevels of the F=3 to the F=3,$m_F=0$, but the change of the atomic quantities is proportional to the difference among the F=3 $m_F=0$ state and other sublevels of the F=3 state. So we can consider the transitions from the F=3 $m_F=0$ state to other sublevels of the F=3 state, the probabilities of the transition are as follows:

$$P_3(0,-1)=P_3(0,+1)=3!\times3!\times4!\times2!\times\left(-\frac{\cos\frac{\alpha}{2}\cdot\sin^5\frac{\alpha}{2}}{2!\times3!}+\frac{\cos^3\frac{\alpha}{2}\sin^3\frac{\alpha}{2}}{2!\times2!}-\frac{\cos^5\frac{\alpha}{2}\sin\frac{\alpha}{2}}{2!\times3!}\right)^2$$

$$P_3(0,-2)=P_3(0,+2)=3!\times3!\times5!\times\left(\frac{\cos^2\frac{\alpha}{2}\cdot\sin^4\frac{\alpha}{2}}{3!\times2!}+\frac{-\cos^4\frac{\alpha}{2}\sin^2\frac{\alpha}{2}}{2!\times3!}\right)^2$$

$$P_3(0,-3)=P_3(0,+3)=3!\times3!\times6!\times\left(\frac{\cos^3\frac{\alpha}{2}\sin^3\frac{\alpha}{2}}{3!\times3!}\right)^2$$

Where

$$\sin^2\frac{\alpha}{2}=\frac{|2d|}{(\omega_0-\omega)^2+(2d)^2}\times\sin^2\frac{t}{2}\sqrt{(\omega_0-\omega)^2+(2d)^2}$$

$$2d=-\frac{H}{2\hbar}\left|\mu_{m,m'}\right|$$

Here ω_0 is the resonance angle frequency corresponding to all the possible $\Delta m=\pm1$ transition, ω stands for the angle frequency of the low frequency signal at the coils. d is the function of the power of inducing to the low frequency coils. The probability of the F=3,$m_F=0$ level state could be obtained, the result is as follows:

$$P_3(0,0)=3!\times3!\times3!\times3!\times\left(\frac{\sin^6\frac{\alpha}{2}}{3!\times3!}-\frac{\cos^2\frac{\alpha}{2}\cdot\sin^4\frac{\alpha}{2}}{2!\times2!}+\frac{\cos^4\frac{\alpha}{2}\sin^2\frac{\alpha}{2}}{2!\times2!}-\frac{\cos^6\frac{\alpha}{2}}{3!\times3!}\right)^2$$

The equation of interaction between the atoms and the microwave at the Ramsey cavity could be solved, the wavefunction of the F=4,$m_F=0$ is given by[2,4]:

$$C_n(2\tau+T) = -i\sin b\tau \cdot e^{[-i\omega(T+\tau)]} e^{[-\frac{i}{\hbar}E_m T]} \cdot$$

$$C_m(\tau) + \cos b\tau \cdot e^{[-\frac{i}{\hbar}E_n T]} C_n(\tau) \qquad (2)$$

Where

$\tau = l/u \qquad T = L/u$

Here l and L represent the lengths of the interacting and drift regions; u is the velocity of the atom; b represents the perturbation parameter of the microwave field in the cesium beam frequency standard; E is the atomic energy; m and n stand for the lower state F=3 m_F=0 and higher state F=4 m_F=0. $C_m(\tau)$ and $C_n(\tau)$ are the atom wavefunctions after the first interaction with the microwave at the Ramsey cavity. Assuming that the frequency of the microwave field is equal to atomic transition frequency and atoms enter the Ramsey cavity at t=0. Then $C_m(\tau) = \cos(b\tau)$ and $C_n(\tau) = \sin(b\tau)$. Considering the low frequency coils are laid at the drift of the Ramsey cavity and the Majorana transitions occur in the Zeeman sublevels, we correct the equation (2) and can obtain:

$$C_n(2\tau+T) = -i\sin b\tau \cdot e^{[-i\omega(T+\tau)]} e^{[-\frac{i}{\hbar}E_m T]}$$

$$C_m(\tau)P_m(t) + \cos b\tau \cdot e^{[-\frac{i}{\hbar}E_n T]} C_n(\tau)$$

$$= -i\sin b\tau \cdot \cos b\tau [e^{-i\omega T} \cdot e^{-\frac{i}{\hbar}(E_m - E_n)T} \cdot P_m(t)$$

$$+1] \qquad (3)$$

$P_m(t)$ is correct terms because of the Majorana transitions of the sublevels of F=3; t is the duration of passing through the low frequency coils. With $E_n - E_m = \omega\hbar$, the equation (3) thus becomes:

$C_n(2\tau+T) = -i\sin b\tau \cdot \cos b\tau [P_m(t)+1]$ So the atomic probability at the F=4 m_F=0 can be written as:

$$\left|C_n(2\tau+T)\right|^2 = \sin^2 b\tau \cdot \cos^2 b\tau [P_m(t)+1]^2 \qquad (4)$$

The atomic quantity of the F=3,m_F=0 also can be calculated, after interaction with the low frequency signal, the result is as follows:

$$\cos^2 b\tau + \sum_{k\neq 0}(1-\cos^2 b\tau) \cdot P_3(0,k)$$

$$= 1 - \sin 2b\tau \cdot P_3(0,0) \qquad (5)$$

The left first term of the equation (5) represents the atomic quantity before interaction with the low frequency signal; the left second term is the atomic quantity of the Majorana transitions to F=3,m_F=0 level state. So we can find out the correct term of the lower state F=3 m_F=0 is given by

$$P_m(t) = \sqrt{\frac{1-\sin^2 b\tau \cdot P_3(0,0)}{\cos^2 b\tau}} \qquad (6)$$

According to the equation (6), the equation (4) can be written as:

$$\left|C_n(2\tau+T)\right|^2 = \sin^2 b\tau \cdot \cos^2 b\tau \cdot$$

$$[\sqrt{\frac{1-\sin^2 b\tau P_3(0,0)}{\cos^2 b\tau}} + 1]^2 \qquad (7)$$

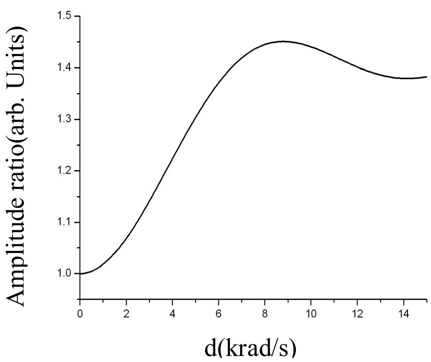
Considering the atomic velocity distribution in the atomic beam, the measured result should be an average of atomic velocity distribution [5]:

$$< P_n(2\tau+T) >= \int_0^\infty \rho(u)\left|C_n(2\tau+T)\right|^2 du \qquad (8)$$

The $\rho(u)$ is the velocity distribution of atoms in the cesium beam. Because probing laser only interacts with atoms with a narrow velocity distribution in our cesium beam frequency standard with sharp angle incident detecting laser beam, so $\rho(u)$ can be expressed as [5]:

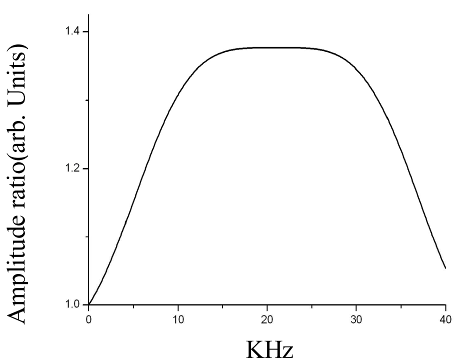
$$\rho(u) = \rho_0 e^{-4\ln 2\frac{(u-u_0)^2}{\Delta u_0^2}} \qquad (9)$$

In the above equation ρ_0 symbolizes the normalized constant. u_0 and Δu are the most probable velocity and the effective velocity width(FWHM) respectively. In our experimental setup L=0.126m, l=0.01m, the length of interaction between the atom and the low frequency signal is 0.01m. The equation (8) contains two variables, which correspond to the frequency and power of the low frequency signal, so the equation (8) has been discussed in detail here. First when the frequency of the low frequency signal is equal to the frequency of the Majorana transition, the atomic probability of the F=4 level state as function of the power of the low frequency signal is obtained as shown in Fig.2. Second when the power of the low frequency signal is the optimum power, then we can obtain the relation between the atomic probability of the F=4 level state and the frequency of the low frequency signal with scanning the frequency of the low frequency signal. The result is showed in Fig.3



Transition matrix element

Fig.2: the atomic probability of the F=4 as a function the power of the low



Frequency of the low frequency signal

Fig.3: the atomic probability of the F=4 as a function the frequency of the low frequency signal

III. Result

Fig.2 shows d=8.6Krad/s at the optimum power of the low frequency signal, Fig.3 shows if the frequency of the low frequency signal is 21KHz, then it is evident that the Majorana transitions take place in the our optically pumped cesium beam frequency standard; here the fluorescence intensity in the probing region increases to 1.37 time of the normal condition without the low frequency signal at the Ramsey cavity.

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