Dynamic Frequency Response Of The Auto-Tuned Hydrogen Maser To Systematic Perturbations

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Abstract--- **This paper discusses the frequency stability of atomic hydrogen masers which use the cavity frequency-switching automatic cavity frequency stabilization system. The theoretical response of the maser output frequency when the cavity frequency is perturbed by systematic perturbations is presented and compared with experimental measurements. A new and improved implementation of the auto-tuner design is also introduced which has promise to significantly improve the long term stability of the hydrogen maser.**

I. Introduction

This paper gives the calculated transient and steady state frequency response of hydrogen masers to systematic perturbations when using the cavity frequency-switching tuner. A plot of the theoretical noise introduced by the tuner when using different servo gain settings is given as well as equations giving the effect on the maser output frequency when different types of cavity frequency disturbances occur. The frequency stability of operational hydrogen masers is shown in relation to the theoretical performance. In addition, as a result of a new study of the auto-tuner system, an improved design configuration of the tuner has been conceived and is introduced in this paper.

II. Potential Auto-Tuner Noise

Reference [1] gives a complete description of the frequency-switching tuner and the derivation of the equations presented in this paper. Equation (1) gives the "potential" noise, the standard deviation of the fractional frequency, that the cavity frequency-switching automatic cavity stabilization system could contribute to the hydrogen maser output signal.

$$\sigma(\tau)_{na}=(1/Q_m)[3KT^oN/4\pi P_cT_A]^{1/2}[1+(f_w/f_b)^2](f_b/4f_w) \qquad (1)$$

Here Q_m is the maser line Q, $(f/\Delta f_l)$, K is Boltzmann's constant, T^o the absolute temperature, N the noise figure of the receiver isolator and amplifier, P_c the power coupled from the maser cavity, T_A the averaging time and f_w and f_b the cavity modulation width and cavity band width respectively. $\sigma(\tau)_{na}$ is characterized as "potential" since it does not contribute to the maser instability until after averaging times of the order of 1,000 seconds or longer; but for times longer than 10,000 seconds the effect is negligible compared to environmental effects the tuner corrects for.

III. Actual Noise Due To The Tuner

Figure 1 shows the noise sources due to random processes which limit the short term stability of the auto-tuned hydrogen maser. The total standard deviation of the fractional frequency is given by (2), which consists of three parts.

$$\sigma(\tau) = \{(\sigma(\tau)_A)^2 + (\sigma(\tau)_P)^2 + (\sigma(\tau)_C)^2\}^{1/2} \qquad (2)$$

The first two, $\sigma(\tau)_A$, and $\sigma(\tau)_P$ are respectively the receiver additive noise and the atomic thermal perturbing noise which are present in all types of hydrogen maser [2]. $\sigma(\tau)_C$ is the noise contributed by the auto-tuner.

The cavity tuner response to frequency offsets is characterized by a time constant T_1 which is established by settings of a cavity register "Gain", defined as "C". C is a clock frequency which establishes the maximum rate at which the cavity frequency can step up or down. Using typical maser design parameters in the current configuration of these masers, there are three gain setting available. Here it is assumed that C can be set to 10, 30 or 90 Hz corresponding to T_1 = 2,500, 833 or 278 seconds respectively.

The result of using different register clock frequencies is clearly apparent in Fig. 1; the benefits of using higher clock frequencies will be seen in the following sections. It should be noted here that in the presently produced hydrogen masers the tuner controls the cavity frequency by changing the voltage bias on a varactor diode located in a coupling loop within the maser cavity. The temperature of the cavity is independently controlled by a thermistor input servo system. Thus cavity frequency changes due to temperature (via the thermal expansion coefficient of the cavity structure) are corrected by the tuner.

Equation 3 gives the rate at which the auto-tuner responds to an offset of the average cavity frequency from the tuned position or else the rate at which the tuner corrects the cavity frequency when a noise source, f_n, is present.

$$df/dt = -(f - f_n)/T_1 \qquad (3)$$

Integration of (3) with f_n identified by various types of cavity frequency disturbances or noise processes gives the calculated frequency response of the cavity. Dividing (3) by the nominal oscillation frequency, f_o, and multiplying by the ratio of the cavity quality factor divided by the maser "line Q", Q_c/Q_l, gives the maser fractional frequency response.

IV. Maser Response To Perturbations

A. Response to a step offset in cavity frequency.

Equation (4) gives the tuned maser response to a step offset, F_X, in cavity frequency (referred to the maser output frequency).

$$(df/f)_K = \sigma(\tau) + (F_X - \sigma(\tau)_C)e^{-t/T1} \qquad (4)$$

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Equation (4) applies for maser offsets less than one part in 10^{14}. For offsets larger than a few parts in 10^{14} the tuner responds at nearly the maximum rate. F_X could be due to an initial offset of the register setting, or from a change in cavity frequency due to mechanical slippage or a voltage regulator change, for example. Calculations of the factor $e^{-t/T1}$ for T_1 = 2,500, 833 and 278 seconds and t = 3,600 seconds (1 Hour) gives 0.24, 0.013 and 2 x 10^{-6} respectively. So step offsets, which occur infrequently, are quickly corrected.

B. Response to cavity drift

Cavity frequency drift may be due to any of several things, such as structural changes, varying voltages or temperature sensor drift. When the cavity frequency has stabilized at the external drift rate (D_R in terms of the maser output frequency) a maser frequency offset, F_{cD}, results as given by (5).

$$F_{cD} = D_R \times T_1 \qquad (5)$$

Assuming $D_R = 10^{-13}$ per day, the calculated offset for three different register gain settings is 3 x 10^{-15} (T_1 = 2,500 seconds), 1 x10^{-15} (T_1 = 833 seconds) and 3.2 x 10^{-16} (T_1 = 278 seconds). It is apparent that if the drift rate decreases or increases over time there would also be a related change in the maser output frequency. Temperature, the major potential problem in this case, will be considered next.

C. Response To Temperature Changes With The Tuner Off

The cavity sensitivity to temperature is due primarily to the thermal expansion coefficient of the copper cavity and secondarily to temperature variations of the dielectric constant of the quartz maser storage bulb. The expansion coefficient of copper is $\alpha_c = 1.7 \times 10^{-5}/^oC$, the long term inverse temperature gain of the cavity is $\Delta T^o_c/\Delta T^o_o = 2.5\times10^{-5}$ and $Q_c/Q_m = 2 \times 10^{-5}$. The steady state temperature response in terms of the maser output frequency, without use of a cavity tuner, is then given by (6).

$$\Delta f/f_{mo} = \alpha_c Q_c/Q_m \, \Delta T^o_c/\Delta T^o_o = 8.5 \times 10^{-15}/^oC \equiv \alpha_m \qquad (6)$$

The temperature servo for the vacuum enclosure responds relatively rapidly to external temperature changes with a time constant of 400 seconds, approximately. The cavity, with three level thermal shielding between the cavity and the vacuum enclosure and a very high thermal mass of 1.4 x 10^4 Watt-Seconds/^{o}C has a long servo response time, T_c, of approximately 9,000 seconds, so calculations of the maser frequency response relate mainly to the long thermal delay time of the cavity. Using the above factors, analysis of the maser response to room temperature variations is given by $\Delta f/f_{mo} = \alpha_m(1 - e^{-t/Tc})$ with the tuner off.

D. Response To Temperature Changes With The Tuner On

If the room temperature is constant for a long period of time and then there is an instantaneous change in temperature which lasts for another long period the tuned maser response is given by (7).

$$\sigma(\tau)_{st} = \alpha_m[e^{-t/Tc} - e^{-t/T1}]T_1/(T_c - T_1) \qquad (7)$$

Chart 1 gives the calculated maser frequency excursion for several times after a 1^{o}C step change and the cavity gain set at three different values. This chart shows that the theoretical frequency instability induced by temperature changes is quite small in comparison to the demands of most applications, however, the possible advantage of using higher servo gains is apparent.

CHART I

FREQUENCY EXCURSION FOR A 1^{o}C STEP CHANGE IN AMBIENT TEMPERATURE

C (Hz)	T_1 (Sec.)	t (Sec.)	$\sigma(\tau)_{st}$
10	2,500	2,500	1.3 x 10^{-15}
"	"	5,000	1.8 x 10^{-15}
"	"	10,000	1.0 x 10^{-15}
"	"	20,000	3.5 x 10^{-16}
30	833	2,500	6.2 x 10^{-16}
"	"	5,000	5.0 x 10^{-16}
"	"	10,000	2.9 x 10^{-16}
90	278	2,500	2.0 x 10^{-16}

Calculations have also been made for the case where the ambient temperature varies periodically, however the results show that theoretically the tuner corrects the cavity frequency in those cases well below the noise levels due to phenomena unrelated to the cavity tuner. Observation of cavity register changes in seven masers over a period of a year or longer shows an average rate of 5.6 x 10^{-15}/day, most probably related to drifting cavity temperature [3]. When the maser is new the maser frequency may be offset, significantly for some purposes, because of that.

The effect of the cavity temperature drift may be calculated from (5). If D_R in Equation 5 is 5 x 10^{-13}/Day and the tuner gain is set to the lowest value, the calculated offset would be 1.4 x 10^{-14}. If then the drift decreases with time, as is usual, the offset frequency will change accordingly. However, the possibility of very badly drifting thermistors can be easily determined by observing the rate of change of the cavity register voltage and the possible problem can then be fixed when the maser is new.

V. EXPERIMENTAL VERSUS THEORETICAL FREQUENCY STABILITY

Fig. 2 shows the relative stability plot for two hydrogen masers. This plot agrees quite well with the theoretical curves in Fig. 1 for averaging times up to about 1,000 seconds, but for longer averaging times the stability clearly departs from the theoretical $t^{-1/2}$ behavior. There are of course several well known fundamental perturbations to the hydrogen maser oscillation frequency. There is, for example, the magnetic sensitivity, the 2nd order Doppler shift, the "wall" shift and the spin-exchange shift. But analysis shows that in a good laboratory environment these factors theoretically should not result in the flattening out of the stability curve at the level shown in Fig. 2.

In view of the above, a new study has been made of ways to improve the cavity frequency-switching tuner. This has resulted in a new and improved design which is described in [4]. The new tuner configuration is illustrated in Fig .3. The system is the same as the present one described in [1] up to the point at which the modulation signal exits from the demodulator and band pass filter circuit. The new design controls the cavity frequency through the temperature expansion coefficient of the cavity assembly by controlling the cavity heater directly from the tuner electronics; this eliminates both the previous cavity temperature control system and the cavity coupling loop with the varactor diode. The new system has a second order servo loop characteristic producing proportional plus integral control. Thus cavity frequency drift, frequency offset, and long term cavity temperature variation or drift are theoretically minimized or eliminated. The dynamic analysis of the new system is given in [4] which also includes a discussion of the fundamental perturbations to the hydrogen maser oscillation frequency and shows that very significant improvement in the hydrogen maser long term stability is possible.

VI. Conclusion

In conclusion, I hope to see the new tuner design incorporated in future atomic hydrogen masers. I would also like to see more experimental work on other aspects of the hydrogen maser. These masers are used in many important government programs. They have extremely long lives. Masers delivered over 17 years ago are still operating and contributing to the stability of international time. It would be very interesting to see how much more stable they can be.

VII. Aknowlegment

I wish to acknowledge some very useful technical discussions with Mr. Bryan Owings and Mr. Tim Oakley of the Sigma Tau Group of Symmetricom Corporation where we reviewed operating details of production masers to help confirm the validity of the theoretical model of the present design auto-tuner. I also wish to thank all the people as well as the institutions that have supported our hydrogen maser work in the past.

References

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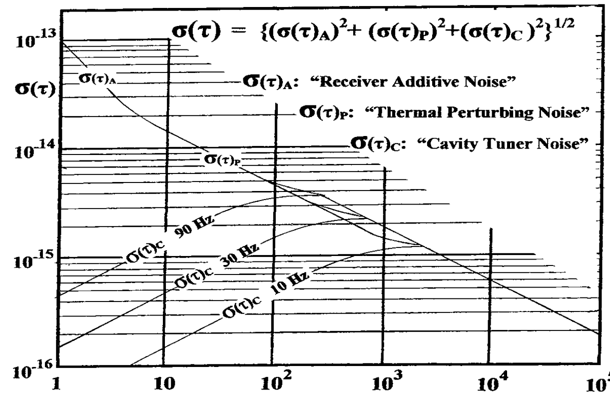
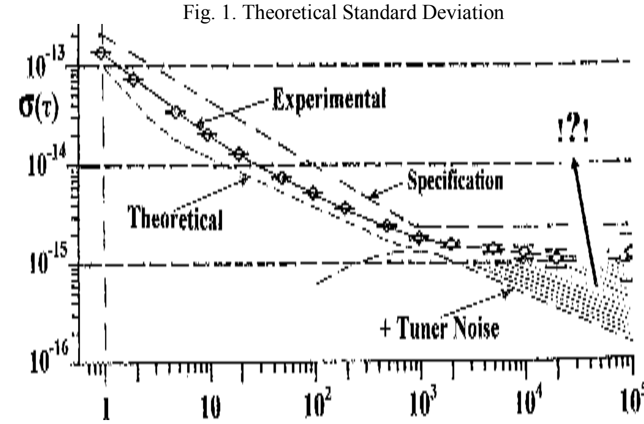


Averaging Time (Seconds)
Fig. 1. Theoretical Standard Deviation



Averaging Time (Seconds)
Fig. 2. Relative Stability Plot For Two Hydrogen Masers

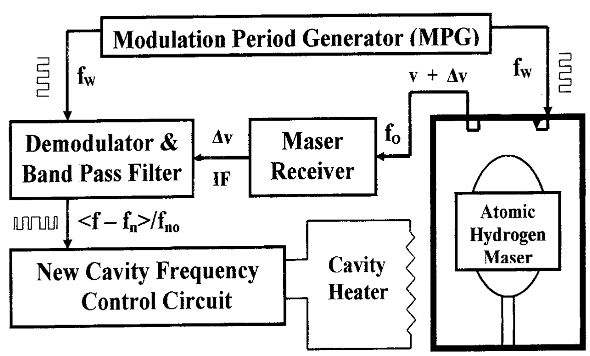


Fig. 3. New Tuning System Block Diagram