Estimation of the duration of the scale unit of TAI with primary frequency standards

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Abstract— **Frequency measurements of primary frequency standards (PFS) are reported more or less regularly to the BIPM to improve the accuracy of TAI. Data from the several PFS are combined to compute an estimation of the duration of the scale unit of TAI. The algorithm used to this purpose "weights" the different evaluations via filter coefficients that depend on the uncertainty of the evaluation, on the time elapsed between the interval of evaluation of the PFS and that of the calculation, and on the model that represents the instability of the free atomic scale (échelle atomique libre, EAL). We discuss in this paper problems encountered with the filtering algorithm, and we propose a tentative solution. We discuss the model that could be chosen to represent the instability of EAL.**

I. INTRODUCTION

International Atomic Time TAI is a realization of Terrestrial Time TT, a coordinate time of a geocentric reference system. TAI gets its stability from some 250 atomic clocks kept in some 50 laboratories worldwide and its accuracy from a small number of primary frequency standards (PFS) developed by a few metrology laboratories. The free time scale, EAL, is first established from a weighted average of some 250 atomic clocks, then the frequency of EAL is compared with that of the primary frequency standards using all available data and a frequency shift (frequency steering correction) is applied to EAL to ensure that the frequency of TAI conforms to its definition. Changes to the steering correction are expected to ensure accuracy without degrading the long-term (several months) stability of TAI, and these changes are announced in advance in the BIPM Circular T.

The accuracy of TAI therefore depends on PFS measurements, which are reported more or less regularly to the BIPM. Data from the several PFS are combined to compute an estimation of the duration of the scale unit of TAI. The algorithm used for this purpose in recalled in section II and the 'memory' of the algorithm, by which the weight of a given PFS evaluation varies over time, is emphasized in section III. In section IV, we discuss a specific problem encountered with the filtering algorithm, and we propose a tentative solution. Finally in section V, we discuss a model that could be chosen to better represent the instability of EAL.

II. PROCEDURE TO ESTIMATE THE DURATION OF THE SCALE UNIT OF TAI

Time laboratories maintaining PFS report to the BIPM measurements of the frequency of the primary standard with respect to that of a clock participating into TAI (unless the PFS be a participating clock). A report of the evaluation of a primary standard j over an interval T_{ji} contains, at present, the following information:

the interval of comparison T_{ji},

the time average of the frequency difference between the reference and the PFS during the interval T_{ji}, indicated by W_{ji},

u_{Bj}, the combined uncertainty from systematic effects,

u_{Aij}, the combined uncertainty from that originated in the instability of the PFS ,

$u_{link/lab}$, the uncertainty of the link between the PFS and the clock in the laboratory participating into TAI used to refer the frequency of the primary standard (if applicable),

$u_{link/TAI}$ is the uncertainty of the link of the laboratory to TAI, as estimated by the BIPM.

The algorithm used at the BIPM to estimate the duration of the scale unit of TAI [1] combines the individual calibrations of PFS and calculates the frequency of the time scale during a given interval (usually the month of calculation of Circular T).

The calibrations are usually referred to a local independent time scale. When using them to improve the accuracy of TAI, we should account for the transfer resulting from de local time scale to the reference time scale (in this case, EAL), and for the transfer of the frequency measurements from the various calibration dates to the period of interest T.

A standard j carries out n_j calibrations. If N is the number of standards considered, the number of available calibrations will be $\sum_{j=1}^{N} n_j$

We calculate the rate of EAL over an interval T as:

$$y = \sum_{j=1}^{N} \sum_{i=1}^{nj} a_{ji} W_{ji}$$

where W_{ji} is the rate difference between EAL and a PFS j on a given interval T_{ji}, and a_{ji} are the filter coefficients. The filter coefficients aji, are normalized and depend on

1. The uncertainty of the evaluation i of the standard j.

2. The distance between T_{ji} and T

3. The instability of EAL, which transfers the evaluation from T_{ji} to T

III. MEMORYOF THE ESTIMATION ALGORITHM

The filtering process applied to the measurements of PFS makes consideration of the correlation terms of successive calibrations conducted with the same standard. The filter coefficients keep a “frequency memory”, assigning to each calibration a sort of “maximum weight” when it overlaps with the interval of calculation. The values of the coefficients decrease when the interval of evaluation of a standard moves away from the interval of calculation T. Fig.1 shows, for a sample of primary standards contributing to TAI, the evolution of the filter coefficients. Calibrations 360 days before and after the calculation interval have been considered.

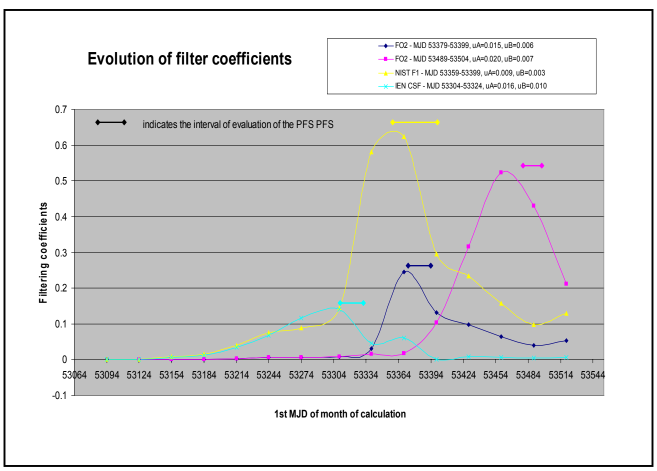

Figure 1. Evolution of filter coefficients.

IV. PROBLEM ENCOUNTERED WITH THE WEIGHTING ALGORITHM:

Frequency calibrations with PFS reported to the BIPM cover typically intervals of operation of the standard of 20 – 40 days.

In 2004 the NIST contributed with an evaluation of the caesium fountain NIST F1 over a period of 60 days, from MJD 53109 to 53169 [2]. The uncertainty components, as reported and published in Circular T are $u_A = 0.8 \times 10^{-15}$ and $u_B = 0.3 \times 10^{-15}$ (u_A includes $u_{link/lab}$ and $u_{link/TAI}$). In presence of such a calibration over a very long interval (60 days), and with a small u_A, some of the coefficients a_{ji} were negative. They have been detected for calibrations overlapping with the long one and external to the interval of calculation. This could be the consequence of having PFS with better stability than that adopted for the scale of reference, indicating that a refined model to represent the stability of EAL should be adopted.

When using a model better representing the stability of EAL (e.g. model 2 or 3 below) the negative filter coefficients disappear.

V. STABILITY MODEL FOR EAL

1. The instability of EAL is expressed as the quadratic sum of three components :

- white frequency noise of $6{\times}10^{-15}/\sqrt{\tau}$,

- flicker frequency noise of $0.6{\times}10^{-15}$,

- random walk frequency noise of $1.6{\times}10^{-16}{\times}\sqrt{\tau}$

with τ given in days.

This is a very conservative model notably for the short term (τ < 1-2 months) if one considers that a single commercial Cs clock has a typical white frequency noise of $3{\times}10^{-14}/\sqrt{\tau}$. It has been used in the monthly estimation of y published in Circular T since 1998.

2. An updated model has been used since 2003 for the computation of TT(BIPM) [3]. It has the same form as the previous one but is composed of

- white frequency noise of $3{\times}10^{-15}/\sqrt{\tau}$

- flicker frequency noise of $0.5{\times}10^{-15}$,

- random walk frequency noise of $1.0{\times}10^{-16}{\times}\sqrt{\tau}$

with τ given in days.

3. The most recent estimation of EAL stability is [4]. It is estimated that the 1-month instability is of order 0.4×10^{-15}

and that long-term systematic trends drive the 6-month instability to close to $2x10^{-15}$. Unfortunately, this cannot be represented by a 3-component noise model as above. It is proposed that a model similar to the model (2) above be used but that possible systematic trends be accounted for by estimating a frequency drift of EAL. This will be discussed in the WG on primary frequency standards established by the CCTF in 2004.

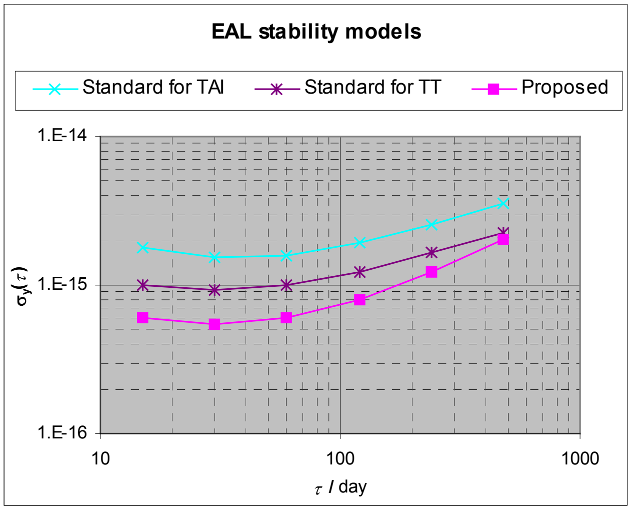

Figure 2. EAL instability models

VI. CONCLUSION

The accuracy of TAI is improved by making use of the measurements of frequency of PFS reported by a few time laboratories to the BIPM. In the process of estimating the duration of the scale unit of TAI, filter coefficients provide a way to weight the calibrations included in the estimation and keep a memory for successive calibrations of the same standard.

The inclusion of a calibration over a 60-day period provoked the occurrence of negative filter coefficients, indicating that the model that represents the frequency instability of EAL might be inadequate if we consider the present stability of primary standards.

To solve this inconsistency, a more refined model will be developed, similar to that used for the computation of TT(BIPM) since 2003, and possibly including a frequency drift of EAL.

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