# An Improvement of the Controlling Algorithm for Taiwan's Time Scaling System

Shinn-Yan Lin National Standard Time and Frequency Laboratory Telecommunication Laboratories Chung-Li, Taiwan sylin@cht.com.tw

Abstract—Telecommunication Laboratories (TL, Taiwan) developed a new controlling algorithm to improve the shortterm and mid-term stability of TL's time scaling system. The controlling algorithm is used for controlling a micro-phase stepper to constrict its output followed a reference paper clock, Taiwan's local atomic time scale - TA(TL). The original system used frequently p-control adjustment to keep the system almost synchronized with the reference paper clock. The new algorithm used a derivate control mode to set the long term frequency offset of the micro-phase-stepper, and used a proportional control mode to adjust the daily phase deflection. The new algorithm improve the short-term stability of the system from 1e-13 into 4.6e-14 when the average time = 600 seconds; and the long-term Allan deviation still constrained with paper clock, up to 2e-15 when the average time is more than 7 days.

# I. INTRODUCTION

Telecommunication Laboratories (TL, Taiwan) began to study and publish the time scale algorithm in 2003, we developed an inversely exponential weighting process to replace the traditional inversely square root weighting process[5]. In the next year, we developed a controlling algorithm to convert the virtual paper clock time scale into the physical output: the controlling algorithm simulated the phase locked loop to let a hydrogen maser follow the phase variation of the virtual paper clock[6]. We also announced Taiwan's local atomic time scale, TA(TL), in November, 2004[7]. The purpose of this paper is to modify the controlling algorithm for the micro-phase stepper in this system; we hope the new algorithm can improve the shortterm and mid-term stability of the system physical output, and maintain the long-term accuracy with TA(TL).

Our system including one cesium clock ensemble, two hydrogen masers, two micro-phase steppers, one measurement sub-system, and one controlling sub-system (Figure 1). The cesium clock ensemble including 7~9 Agilent 5071a with high performance tube (the 7~9 clocks means we always have 1 or 2 clocks under fixing or Po-Cheng Chang National Standard Time and Frequency Laboratory Telecommunication Laboratories Chung-Li, Taiwan betrand@cht.com.tw

changing tube); the TA(TL) is generated by this ensemble through the weighting process we designed in 2003. Two hydrogen masers (Kvarz CH1-75) are used as the reference input of two micro-phase steppers (AOG-110 and SDI HROG-5). This paper only used one hydrogen maser (Serial number 76502, HM6052) and one micro-phase stepper (SDI HROG-5) to test our new controlling algorithm.

In this paper, we introduced our original system at first, then point out the imperfection of the original controlling algorithm. That's the stability of the original system would be limited by the performance of the reference paper clock, TA(TL). In our time scaling model, its stability is roughly proportional to the inverse square of the total number of clocks,  $1/\sqrt{N}$ . Since the typical Allan deviation of clocks of our ensemble was about 2.5e-13 (average time = 600 seconds) and 1e-13 (average = 10,000 seconds) and our clock ensemble has 8 cesium clocks in the testing period, the Allan deviation of TA(TL) was about 1.0e-13 (average time = 600 seconds) and 3e-14 (average time = 10,000 seconds) when it was compared with a hydrogen maser.

In order to have a more stable system, we modified the controlling algorithm. In section III, we described the modification of the controlling algorithm. We used both proportional and derivative control mode to control the micro-phase stepper. We found the most stable relative stability between TA(TL) and HM6052 is about 3e-15 when the average time is 3.5 days, or it means there is about 1 ns variation in 3.5 days. So that we set the frequency offset of the micro-phase stepper to be the 6 day's average frequency offset between the HM6052 and TA(TL) and made the adjustment every 3 days. We still keep the proportional control term, but ran once per day to modify the long-term phase shift between system output and TA(TL).

The stability of the result system output was improved very much. The short-term and mid-term Allan deviation is basically followed the HM6052 when average time below 3 days, it is about 4e-14 (average time = 600 seconds) and 8e-15 (average time = 10,000 seconds). The long-term stability is about 2~3e-15 ( $\tau = 15$  days), it is tightly followed the long-term behavior of TA(TL).

### II. THE IMPROVEMENT OF THE NEW CONTROLLING ALGORITHM

### A. The original system

To convert our paper clock time scale (TA(TL)) into a physical output, we designed a phase locked mechanism simulating the phase locked loop to synchronize a hydrogen maser with TA(TL). In the mechanism, a micro-phase stepper was used for replacing the part of voltage control of the phase locked loop, and the frequency reference input of the micro-phase stepper comes from a hydrogen maser. Since the paper clock has no physical output, we calculate







the time scale of the paper clock every 600 seconds and compare the phase difference between the paper clock and micro phase steppers at the same time (Figure 1).

While we treat the micro-phase stepper as a feedback voltage control, we need a control law to adjust the frequency offset of the micro-phase steppers. Here we use a quasi-proportional control law to set the step of frequencyoffset adjustment. Proportional control means the more phase error the system has, the more frequency offset adjustment we add. We set the control effort (fractional frequency step) to be:

$$3 \cdot 10^{-14} e^{\frac{-3}{\alpha^2}}$$
 (1)

Where  $\alpha = (\text{phase difference between paper clock and micro-phase-stepper output})/100 pico-seconds.$ 

An advantage of equation (1) is the formula of control effort has an upper limit itself because of its inverse exponential form; it can avoid a huge frequency offset change caused by an incident phase error.

The quasi-proportional control law can tightly sync the system output with TA(TL), the phase difference between physical outputs and TA(TL) can be kept in  $\pm 200$  picoseconds [6]. But it also means the short-term and mid-term stability of this system output would be the same as the reference paper clock. Unfortunately, the short-term and mid-term stability of paper clock (TA(TL)) is worse than the hydrogen maser (HM6052) about one order of magnitude, the typical Allan deviation of TA(TL) is about 1e-13 when average time is 600 seconds but our hydrogen masers can achieve 3e-14 in the same average time.

# B. The stability limition of TA(TL)

The TA(TL) is a time scaling result of 7~9 cesium clocks ensemble, all the cesium clocks are the same type (Agilent 5071a with high performance tube). The stability of TA(TL) will correlated with the number of clock in the ensemble and the scaling algorithm. For the scaling algorithm, we tested 3 kinds of time scaling algorithm: there were equal weighting, inversely exponential weighting, and inversely square root weighting. The equal weighting process is slightly unstable than the other two, but we did not find any significant difference [5]. In our system, the main factor influencing the stability of TA(TL) is the number of cesium clocks in the ensemble. Since all the clocks in our ensemble are the same type and operated independently, we expect the combination of their Allan deviation will decrease by the increasing of N, where N is the number of clocks in the ensemble.

Figure 2 showed a test result for the relation between the number of clocks in our ensemble and the relative Allan deviation, we found the stability of TA(TL) seem to follow the inverse-square-root rule, it may be simply described as:

$$\sigma_{\tau}(paper \ clock) \cong \frac{Average[\sigma_{\tau}(Cs)]}{\sqrt{N}}$$
(2)

Where  $\sigma$  = Allan deviation.

In this time, we have only 8 clocks in our cesium clock ensemble. The Average Allan deviation of our cesium clock (Agilent 5071a with high performance tube) is about 2.5e-13 and 6e-14 when average time is 600 and 10000 seconds. The stability of our TA(TL) is about 1e-13 and 2.1e-14, it is the same as the equation (2) expects. The equation (2) also infers if we hope TA(TL) to have the same short-term performance of hydrogen maser, we have to have more than 50 Agilent 5071a cesium clocks in our ensemble. It means we can not use the phase locked mechanism to do any improvement in near future.

## C. The modification of the controlling algorithm

Use the hydrogen maser directly can improve the short term and mid term stability immediately, but we all know the hydrogen maser is not accurate in long term. Figure 3 showed the Allan deviation between HM6052 and TA(TL), the minima value of Allan deviation is about 4.5e-15 when the average time is about 3~6 days. Beyond 3~6 days, the HM6052 will turn aside the right accurate path because of its agency.

In order to keep the long term accuracy, we modified the controlling algorithm for the micro-phase-stepper. Our new policy is keeping the frequency offset of the micro-phase stepper unchanged until the HM6052 become unstable than TA(TL). We set the frequency offset of the micro-phase stepper to be the 6 day's average frequency offset between HM6052 and TA(TL), and set it to the micro-phase stepper every 3 days. In the control law's point of view, we can treat it as the derivative control:

frequency offset = 
$$f_{t-6 days}(Hm6052) - f_{t-6 days}(TA(TL))$$
 (3)

The period to set the frequency offset is 3 days.

To avoid the phase error accumulating, we still keep the proportional control mode, but decrease its adjusting period from 10 minutes into 24 hours; the proportional control effort is set to be:

Control effort = 
$$5 \times 10^{-15} \times e^{\left[ phasediff (TATL-MPS02)/100 ps \right]^{1/2}}$$
 (4)



The period of each adjustment is 24 hours.

The new controlling algorithm is the combination of equation (3) and (4): We calculate the 6 day's average frequency offset between HM6052 and TA(TL) and set it to the micro-phase stepper every 3 days; and compared the phase difference between the output of micro-phase stepper and TA(TL) every 24 hours, adjust the frequency offset according to equation (4).

#### III. RESULT

To appraise this stability and accuracy of this system, we compare the 5 MHz of the system output and our maser clock (the name of the station is TWTF) with USNO via GPSCP. Here we choose USNO as the reference station because of we think the master clock of USNO is both stable and accurate enough to evaluate our system. We record and analyze the data from MJD 53544 to 53603. The GPS receiver is Ashtech Z12T and the analyze software is Bernese 5.0.

The result of frequency domain is showed in figure 4. In the short-term (average time less than 1000 seconds), the system using new controlling algorithm (blue line of figure 4, MPS02) is almost the same as its reference hydrogen maser (pink line of Figure 3, HM6052); much better than the paper clock (yellow line of figure 3, TATL). The Allan deviation of HM6052 and MPS02 are about 4.6e-14 when the average time equal to 600 seconds and the TATL is about 1e-13. In the range of mid-term (average time from 1000 seconds into 1 day), the system output is slightly worse than hydrogen maser, but still more stable than paper clock until the average time approaches 1 day. The paper clock showed its very stable long-term stability (better than 2e-15) when the average time beyond 1 day. The minima Allan deviation of HM6052 occur when the average time is 3.5 days and become worse and worse away from 3.5 days, it's the same as our measurement system. The MPS02 is much more accurate than HM6052 in long-term; it basically follows the reference paper clock TATL. The Allan deviation is about 2e-15 when the average time over 7 days.

In the phase domain, the pink and blue lines of figure 5 showed the phase difference between HM6052, MPS02, and TATL. We removed their drifts so that we could distinguish the variance more easily. the HM6052 had an obvious  $2^{nd}$  order term, the phase variance is about  $\pm 20$  ns in 60 days. MPS02 showed more accurate performance, the phase variance is about  $\pm 5$  ns in 60 days. The yellow line showed the phase comparison result between MPS02 and USNO, it is under  $\pm 5$  ns in 60 days, the same as the result of TATL.

## IV. CONCLUSION

The original controlling algorithm needs 144 times adjustments per day to lock the hydrogen maser with the reference paper clock, and its mid-term stability is limited by the reference paper clock. Base on equation (2), our clock ensemble may need 40 extra Agilent 5071a with highperformance tube to improve it stability into hydrogen maser



level. The new controlling algorithm just needs 1.3 adjustments per day and both keep the short-term and midterm property of hydrogen maser and long-term accuracy of our paper clock. By the way, our test system used 8 5071a and 1 hydrogen maser only.

We estimate the minima requirement of that kind of system is 1 hydrogen maser with a stable micro-phase stepper plus a cesium clock ensemble with at least 5 5071a cesium clocks. We estimate Allan deviation of this system can achieve 3e-15, or means 8 ns phase variance in one month.

#### References

- Allan D. W. [1987] Time and Frequency (time domain) characterization, estimation, and prediction of precision clocks and oscillators. IEEE trans. Ultrasonics. Ferroelectrics and Frequency Control, UFFC-34, 647-654.
- [2] ITU [1997] Handbook selection and use of precise frequency and time system, Chapter 6.
- [3] Azoubib J., Granveaud M. and Guinot B. [1977] Estimation of the scale unit duration of time scales. Metrologia, Vol. 13, 87-93.
- [4] Tavella P. and Thomas C. [1991a] Comparative study of time scale algorithms. Metrologia, Vol. 28, 57-63.
- [5] Lin S. and Peng H. [2003] A paper clock model for cesium clock ensemble of TL. 35<sup>th</sup> Annual Precise Time and Time Interval Meeting, 2003.
- [6] Lin S [2004] A Phase Locked Mechanism for Time Scaling Frequency Control System. 2004 IEEE International Ultrasonics, Ferroelectrics, and Frequency Control Joint 50th Anniversary Conference, 2004.
- [7] Lin S., Peng H, Tseng W., Lin H., Liao C. [2004] The Future Model of TA(TL), 36<sup>th</sup> Annual Precise Time and Time Interval Meeting, 2004.