ESTIMATION OF TIME-DOMAIN FREQUENCY STABILITY BASED ON PHASE NOISE MEASUREMENT

P. C. Chang, H. M. Peng, and S. Y. Lin
National Standard Time & Frequency Laboratory, TL, Taiwan
12, Lane 551, Min-Tsu Road, Sec. 5, Yang-Mei, Taoyuan, Taiwan 326
Tel: 886 3 424 5179; Fax: 886 3 424 5474; E-mail: *betrand@cht.com.tw*

Abstract

The time domain characterization of the frequency fluctuations is usually expressed in terms of the Allan variance, $\sigma_y^2(\tau)$, or the modified Allan variance, Mod $\sigma_y^2(\tau)$. Both variances can be accurately determined by the integral relations to $S_y(f)$, the power spectral density of fractional frequency fluctuations, which include five types of noise: White PM, Flicker PM, White FM, Flicker FM and Random Walk FM. These noise types are distinguished by the integer powers (α) in their functional dependence on Fourier frequency f. Because the noise is inherent to all kinds of oscillators and measurement systems, specifying their contributions to the time domain frequency stability is important and meaningful. In this paper, both the numerical integral and the curve-fitting methods are presented to estimate the frequency stability from the results of phase noise measurement of oscillators, amplifiers, etc. The numerical integral is a direct way to use and we calculate the integral approximation after smoothing some spike points. In addition, owing to the properties of power-law noise processes, the weighting coefficient h_α of each type of noise component could be estimated when curve-fitting skills are adopted. Cutler's formula is used to calculate the integral approximation using these coefficients. The approximations of frequency stability from these two ways are compared and analyzed. Lastly, the limitations and possible errors from the estimating methods are also discussed.

INTRODUCTION

In this paper, we are trying to use the phase noise measurement results to calculate the time domain frequency stability due the conversion between time and frequency domain. In general, if the spectral density of the normalized frequency fluctuations $S_y(f)$ is known, its mathematical relation to the Allan variance can be expressed as:

$$\sigma_y^2(\tau) = 2\int_0^{f_h} S_y(f)\frac{\sin^4 \pi\tau f}{(\pi\tau f)^2}df \tag{1}$$

where f_h is the high frequency cutoff of a low pass filter. From equation (1), the Allan variance can be straightforwardly calculated while numerical integration is adopted. Besides, the power-law model is frequently used for describing phase noise from oscillators, amplifiers, etc. It assumes that the spectral density of fractional frequency fluctuations is equal to the sum of terms, each of which varies as an integer power of frequency. Thus, there are two quantities that completely specify $S_y(f)$ for a particular power-law process: the slope on a log-log plot for a given range of f and the amplitude. The slope is denoted by α and, therefore, f^{α} is the straight line on a log-log plot that relates $S_y(f)$ to f. The amplitude is denoted by h_α and hence:

$$S_y(f) = \begin{cases} \sum_{\alpha=-2}^{+2} h_\alpha f^{\alpha} & for \quad 0 < f < f_h \\ 0 & for \quad f > f_h \end{cases} \tag{2}$$

The Allan variance derived by Cutler from equation (1) and (2) is as follows **[1-2]**:

$$\sigma_y^{\,2}(\tau) = h_{-2}\frac{(2\pi)^2}{6}\tau + h_{-1}2\,ln\,2 + \frac{h_0}{2\tau} + h_1\frac{1.038 + 3\,ln(2\pi f_h\tau)}{(2\pi)^2\tau^2} + h_2\frac{3f_h}{(2\pi)^2\tau^2} \tag{3}$$

If the value for each weighting coefficient h_α is appropriately determined, then the Allan variance could be calculated. This is carried out by the curve-fitting skills discussed below.

THE EXPERIMENTS

The measurement system consists of a FSS1000E phase noise detector, a FSSM100 noise standard, a FSS1011A delay line unit, a SDI LNFR-400 low-noise frequency standard, and a SRS-760 fast Fourier transformer (FFT) which is used to analyze the output signal from the phase noise detector. The measurement processes and data recording could be automated under software TestStation version 3.0.

All the measurement system including the DUTs should be warmed up for at least 24 hours before any test. The first experiment was the system noise floor test. We used a power splitter to divide the LNFR-400 10-MHz output into two signals and then followed the procedure for passive component measurement. In the second experiment, the phase noise of a 5-MHz frequency output from the hydrogen maser 76052 was measured using the phase-lockable LNFR-400 serving as a reference. Both experimental results are shown in Figure 1 and Figure 2.

CALCULATION OF EXPERIMENTAL RESULTS

In the frequency domain, $L(f)$ is the prevailing measure of phase noise among manufacturers and users of frequency standards, and it is defined as **[3]**:

$$L(f) = \tfrac{1}{2} S_\phi(f) \tag{4}$$

$$S_\phi(f) = \frac{\nu_0^{\,2}}{f^2} S_y(f) = \nu_0^{\,2} h_\alpha f^{\beta} \qquad (\beta \equiv \alpha - 2) \tag{5}$$

$$\frac{dB_C}{H_Z} = 10 \times log(L(f)) \tag{6}$$

where $S_\phi(f)$ is the spectral density of phase fluctuations. In Figure1, it is an $L(f)$ vs. f plot with its x-y axis in log scale. For f = 1Hz~1000 Hz, we see that when f increases by one decade, $L(f)$ also goes down by one decade. This noise process can be identified as flicker PM. For f = 10~99.75 kHz, we have white PM. In the region f = 1 kHz~10 kHz, it seems that flicker PM and white PM coexist and none of them could surpass each other. After smoothing some spike points in the raw data, a transformation from $L(f)$ to $S_y(f)$ was made. We used the function $S_y(f) = h_1 f^1$ and $S_y(f) = h_2 f^2$ to fit the data in the flicker PM and white PM region separately, and then got $h_1 = 9.9067 \times 10^{-29}$ and $h_2 = 6.1496 \times 10^{-32}$. To make sure the values of h_1 and h_2 were appropriately determined, we calculated their contributions of $S_y(f)$ in the flicker PM and white PM region, and verified that the interactions between these two coefficients were insignificant.

In Figure3, the blue line shows the results from $L(f)$ to $S_y(f)$ transformation and the red line is residuals of the former after the contributions of power-law model $h_1 f^1 + h_2 f^2$ and some outlying points have been removed. Figure4 shows the Allan deviations $\sigma_y(\tau)$ calculated from the numerical integration and the power-law method with the cutoff frequency f_h = 99.75 kHz and averaging period τ = 0.1~10 s. The Allan deviations from these two methods are in good agreement with each other (the relative errors are less than 6 %) except when τ is equal to 0.1 s and 10 s.

Following a similar procedure to deal with the experimental results in Figure 2, three kinds of noise processes including white FM ($h_0 = 5.6315 \times 10^{-25}$), flicker PM ($h_1 = 2.1948 \times 10^{-26}$), and white PM ($h_2 = 5.0359 \times 10^{-29}$) could be identified. The relative diagrams are shown in Figure 5 and Figure 6. We observed that the growing rate of the relative errors became faster when τ increases and that there are two abrupt plunges in the numerical integration when τ is equal to 0.1 s and 10 s.

CONCLUSION

In this paper, we calculated and compared the time domain frequency stability using the numerical integral and the curve-fitting methods. The curve-fitting method is useful to obtain the value of a weighting coefficient after identifying the Fourier frequency range for a certain power-law process. As for the numerical integration, it is straightforward to use, but its generated results change abruptly for some values of τ. In order to solve this problem, more research will be done in the future.

REFERENCES

[1] S. R. Stein, 1985, *"Frequency and Time–Their Measurement and Characterization,"* **Precision Frequency Control** (Academic Press, New York), volume **2**, chapter 12, pp. 203-204.

[2] *"Characterization of Frequency and Phase Noise,"* Report 580 of the CCIR, pp. 142-150, 1986.

[3] D. Allan, H. Hellwig, P. Kartaschoff, J. Vanier, J. Vig, G. M. R. Winkler, and N. F. Yannoni, 1988, *"Standard Terminology for Fundamental Frequency and Time Metrology,"* in Proceedings of the 42nd Annual Symposium on Frequency Control, 1-3 June 1988, Baltimore, Maryland, USA (IEEE Publication 88CH2588-2), pp. 419-425.

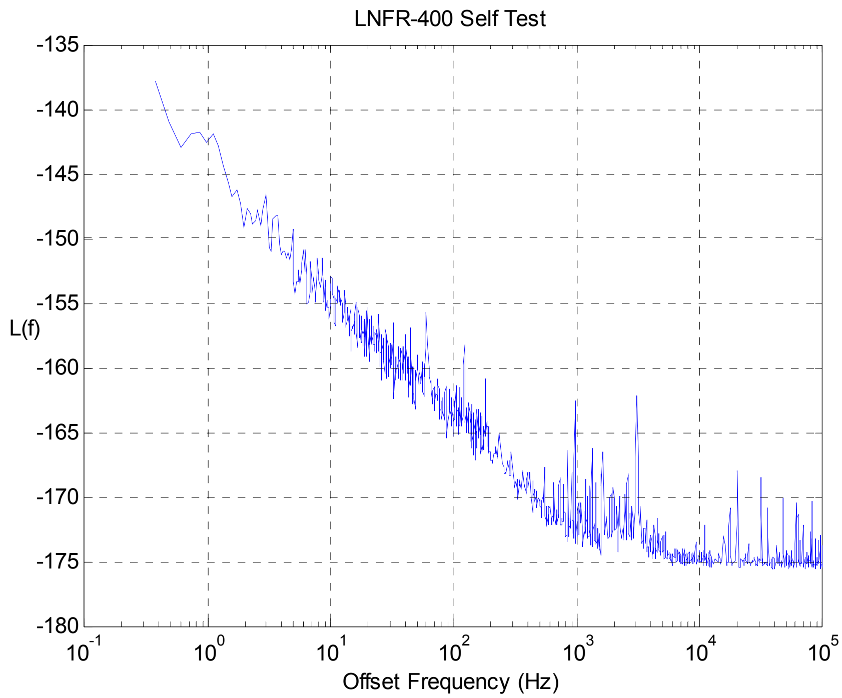

Figure 1. Phase noise measurement of LNFR-400 self test (10 MHz).

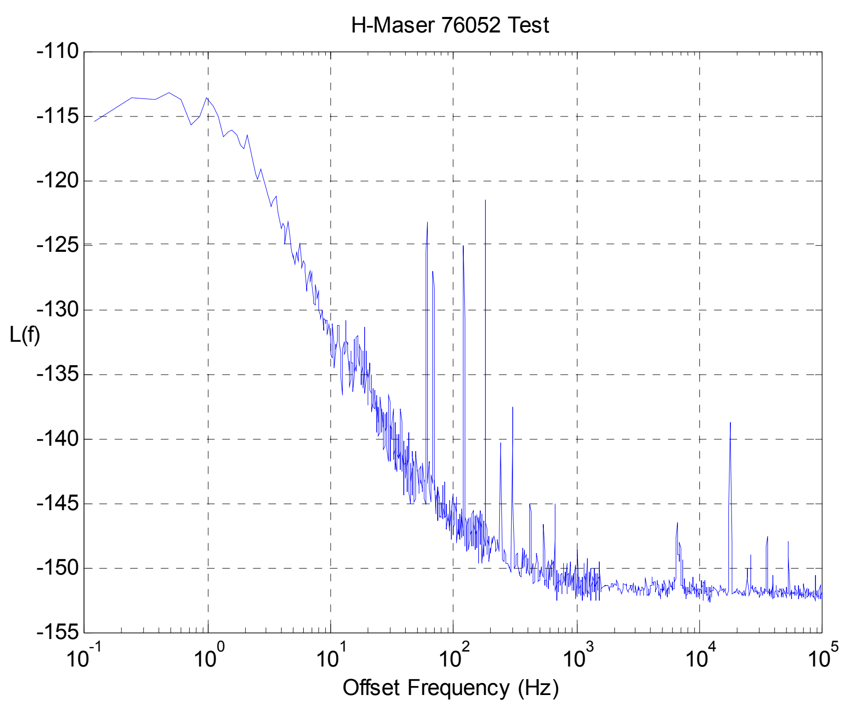

Figure 2. Phase noise measurement of H-maser 76052 (5 MHz).

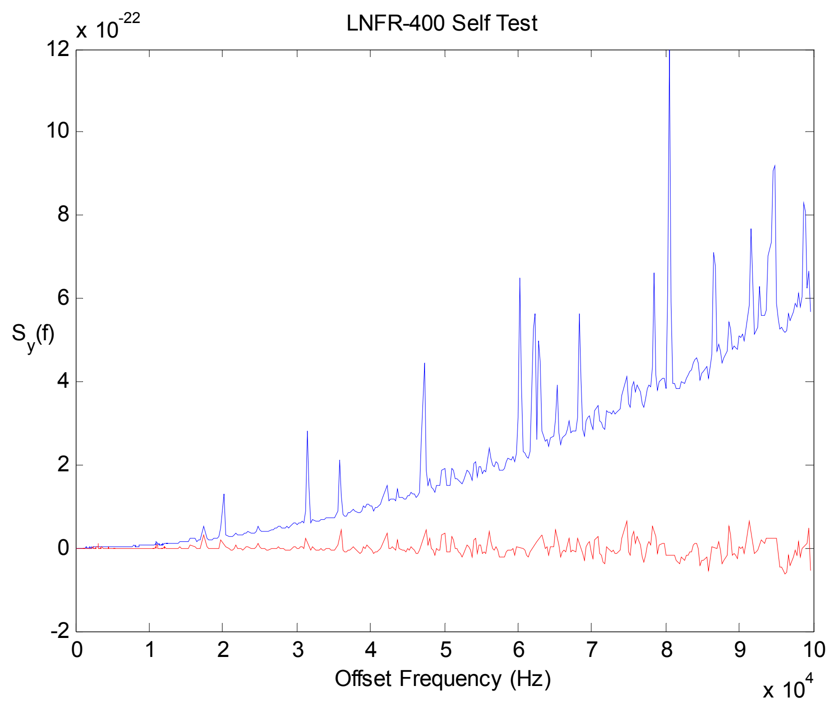

Figure 3. $S_y(f)$ and its residual after the contributions of power-spectral model and outlying points have been removed.

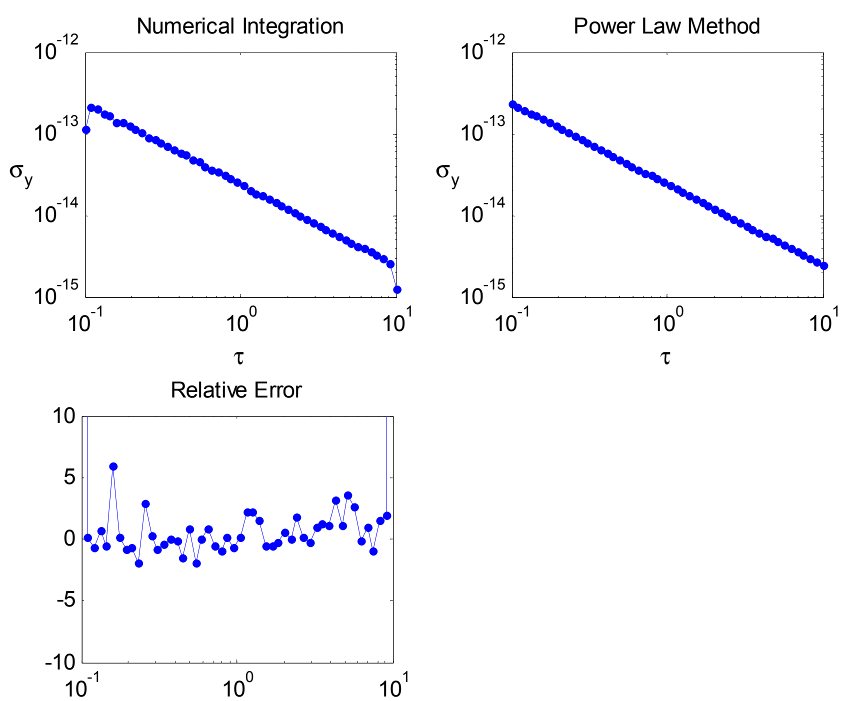

Figure 4. Allan deviations from two different methods (LNFR-400 self test).

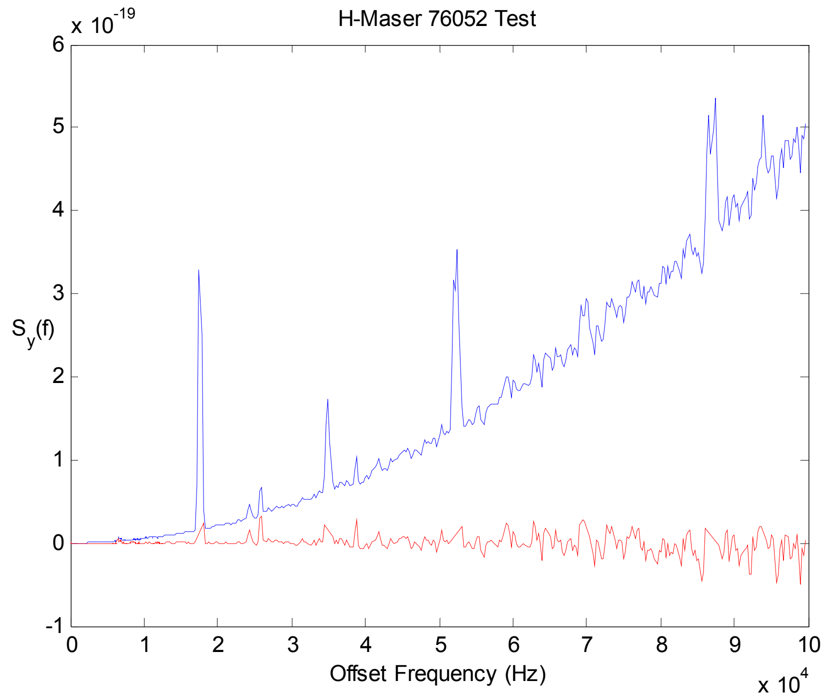

Figure 5. $S_y(f)$ and its residual after the contributions of power-spectral model and outlying points have been removed.

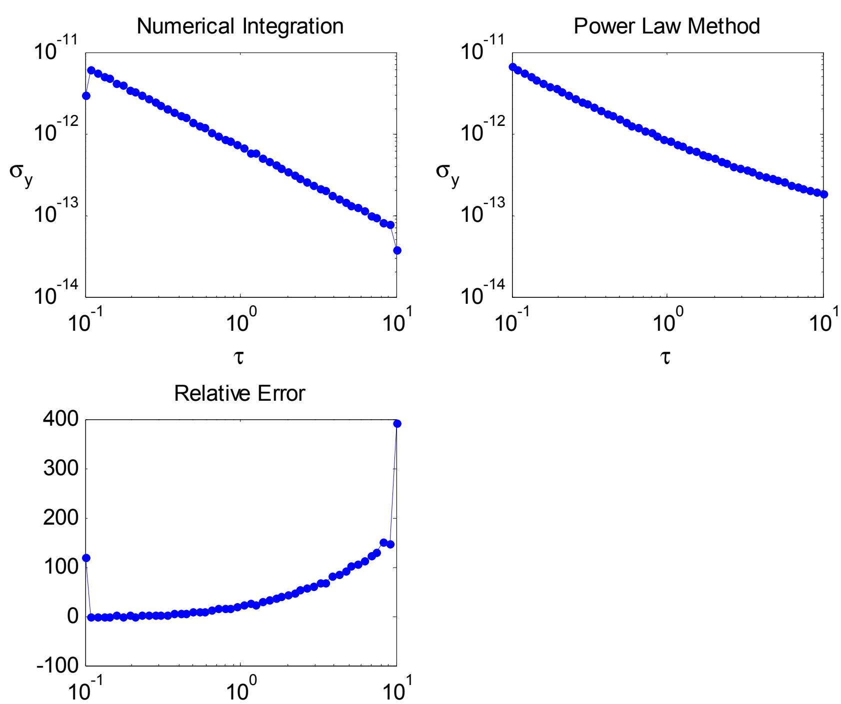

Figure 6. Allan deviations from two different methods (H-maser 76052).

QUESTIONS AND ANSWERS

DAVE HOWE (National Institute of Standards and Technology): One of the problems with doing a curve fit to something like L (f) – L (f) itself is smoothed, that is, the residuals are not white. What measures did you take to show that the residuals are white in the curve-fitting process? And what sort of FFT window function did you use?

PO-CHENG CHANG: We didn't consider many vectors. So it was a very easy way to calculate it.