EFFECTS OF PARAMETER ESTIMATION AND CONTROL LIMITS ON STEERED FREQUENCY STANDARDS

Jim Skinner and Paul Koppang
U.S. Naval Observatory

Abstract

Data estimation and frequency steer limits are important aspects that influence the overall response and design of a robust control system. We study the effects that frequency steer limits have on the response of a control system. Additionally, we attempt to find a balance between the performance of the control system and the robustness of maintaining parameter limits. Performance will be evaluated utilizing both simulated and archived measurement data.

INTRODUCTION

The U.S. Naval Observatory (USNO) maintains several steered clock systems. Each system is comprised of a Datum-Sigma Tau Hydrogen Maser that provides a frequency signal to a Datum-Sigma Tau Auxiliary Output Generator (AOG) frequency synthesizer. This AOG is steered toward its target based on measurement of target minus AOG in phase (time error). The principal differences between any two of these systems are the measurement systems that provide the phase value, the algorithms used, and the target of the control. We look at the step response of the control system along with the combination of the filter and control responses.

The motive for this study is to determine a methodology by which the limiting parameters for a steering algorithm may be determined. The objective is to determine how finely to restrict the control in order to keep the system from steering overly aggressively without limiting the ability of the control to react to nominal changes, to the underlying clock.

DESCRIPTION OF CONTROL SYSTEM

The control system that we are working with receives a current phase measurement. The two-state Kalman filter uses this data point and the one-step-ahead filter estimates from the previous data point to determine the current state estimates [1]. These state estimates are the filter's approximation of how far from the target the steered clock is in phase and frequency. The proportional control uses these state estimates to calculate frequency steers that are sent to the AOG. We chose to study only critically damped control systems because the choice of a time constant uniquely determines the gain vector and, therefore, gives a baseline for control response characteristics as the time constant τ is varied [2].

The time constant of the control corresponds to the amount of time it takes the control to remove a frequency offset. Determination of the time constant of the control is also dependent on the level of noise in the measurements of phase. The method by which the time constant for a USNO system is determined is by taking the intersection of the noise level of the system with the stability plot of the steered device, a hydrogen maser. Figure 1 below shows that for a measurement of noise level 1, the time constant should be between 1 and 2 days. Similarly, for noise level 2, the time constant should fall between 3 and 5 days.

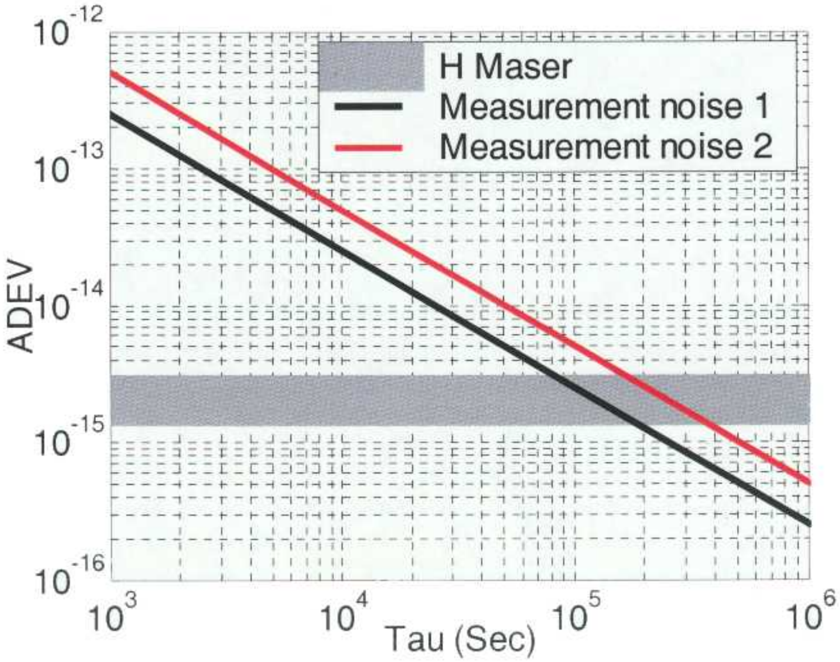

Figure 1: Determination of control time constant.

DETERMINATION OF MAXIMUM STEER LIMITATION

An unlimited control system should remove any frequency offset, assuming the system is controllable. While this is advantageous for any changes that are known to be true, the desire is to gain robustness and minimize perturbations to the steered clock caused by possible outlying events. This is the main reasoning for the use of a maximum steer limitation. The question arises as to what is an appropriate level of limitation. Using a steered system driven by a hydrogen maser as an example, the stability of a given maser is known. Therefore, the maximum steer limitation is determined in such a manner that the control can, at the least, react to a nominal change in the system's underlying maser. If an additional limitation must be added to limit the visibility of steering to users of the frequency of the system, an accumulated limitation may be added. This can be determined in a similar fashion.

The determination of the hourly maximum allowable steer was done by applying a critically damped control to sets of simulated maser data [2]. The hypothesis was that one half of the peak-to-peak values of the steers would provide an appropriate value for the maximum steer. The standard deviation of the steers was also determined, and the peak-to-peak value was in the neighborhood of three sigma for every time constant tested. As this seemed reasonable, the peak-to-peak value was accepted. The values determined for different time constants are shown in Figure 2.

The concern became that the use of only this hourly maximum steer limitation might allow the system to accumulate a large change. By a method analogous to the one used to determine an hourly maximum, a maximum allowable accumulated steer was determined. The hypothesis was that one half of the peak-to-peak value of the accumulated steers over a 1-day period would be an appropriate limit. Again it was determined to be roughly a three-sigma event, hence acceptable. These values are shown in Figure 3 for various different time constants.

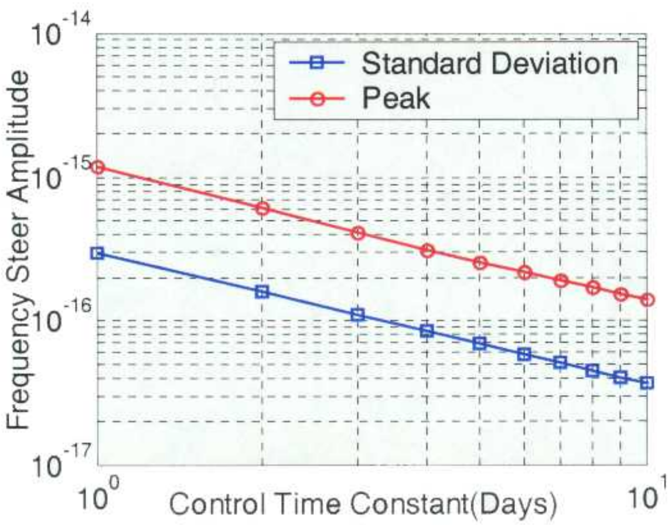

Figure 2

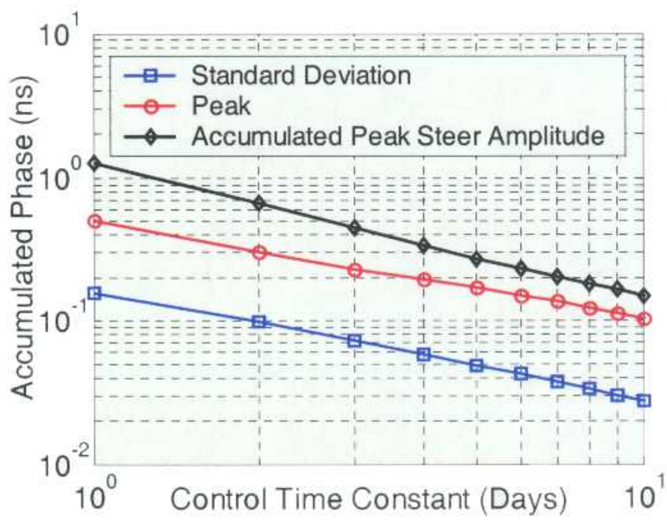

Figure 3

TWO MODELS

Two different models of a limited control system were used in the testing phases. "Model 1" is the control system settings currently in use at the USNO Alternate Master Clock in Colorado Springs, Colorado. This system steers the Alternate Master Clock to the USNO Master Clock in Washington, DC via Two-Way Satellite Time Transfer (TWSTT). The time constant is approximately 4 days with an hourly maximum steer of 8.33×10^{-17} and no accumulated maximum steer. While a critically damped control has real and equal poles, the poles for the actual control system are complex conjugates. The poles are $0.9894 \pm 0.0009\ i$, which implies that the control is slightly under damped. Since the imaginary parts are so small, the analysis done using a critically damped control remains valid. It is worthy to note that a full day of maximum steers accumulates to approximately 0.090 nanoseconds and 7 days worth of maximum steers accumulates 4.24 nanoseconds (see Figure 4).

"Model 2" is designed based on the principles set forth in this paper up to this point. The time constant of 4 days is within the acceptable range of time constant values based on the stability of the maser and the measurement noise of TWSTT, so that did not change. The maximum steer limitation based on Figure 2 is 3.0×10^{-16} with an accumulated maximum steer of 0.200 nanoseconds over any 24-hour period (Figure 3). While the daily maximum steer allowed under Model 2 is greater than that under Model 1, the cumulative effect of continued maximum steers under Model 1 is quadratic in phase (as the accumulated limit is linear in frequency) while the cumulative effect of continued maximum steers under Model 2 is linear in phase. This is illustrated in Figure 4.

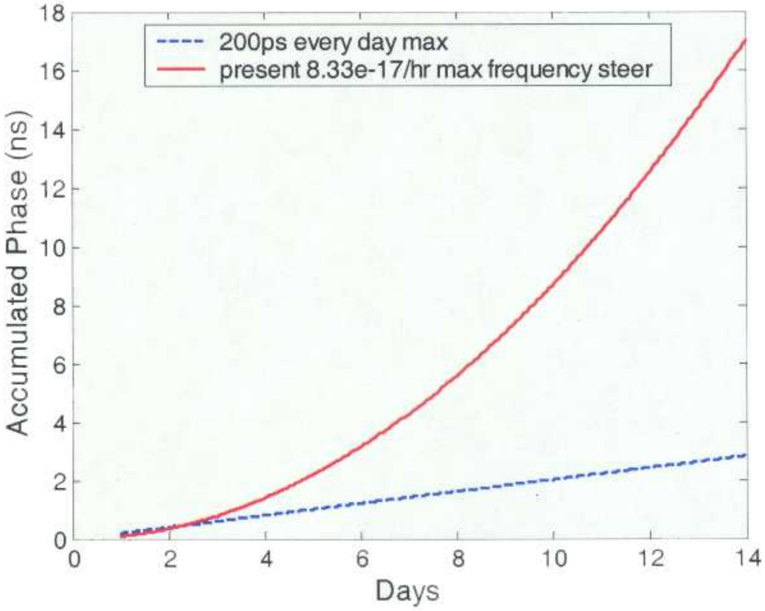

Figure 4

Note: The parameters of the Kalman filter used to provide the state estimates for the control were not addressed, and were identical for the two models.

CONTROL SIMULATIONS WITH SIMULATED CLOCKS

Both Model 1 and Model 2 were applied to the same sets of 10,000 hourly points for each of 10 simulated masers. One measure of how well the controls are performing is the percentage of the time the control is required to apply a maximum steer. These values are given in Table 1. This information supports the hypothesis that the maximum steer limitation of Model 1 is somewhat too strict, since nominal changes directly attributable to the characteristics of a maser have forced the control to limit nearly one out of every four steers.

Table 1: Percentage of total number of steers that were limited by the maximum.

	Model 1	Model 2
Percentage over Hourly Max	24.21%	0.00%
Percentage over Daily Max	N/A	0.38%

Next we compare the variance of their state estimates. To test for a significant improvement in the variances, i.e. if $\sigma_1^2 \geq \sigma_2^2$, one must examine the equivalent relationship given by the hypotheses:

$$H_0:\ \sigma_1^2/\sigma_2^2 \geq 1$$
$$H_1:\ \sigma_1^2/\sigma_2^2 < 1.$$

The ratio of the Model 1 variance of phase estimates to that of Model 2 is 0.714. The ratio of frequency variances is 0.794. Using the test statistic F with $\alpha = 0.01$ and degrees of freedom for both models equal to 9,999, the critical value is 0.955. Since both ratios are less than the critical value, the null hypotheses are rejected. Therefore, the improvement to the control is significant [3].

Up to this point, all signs point to Model 2 as an improvement to the control system. It is necessary to test the scatter of the hourly steers to insure that the steers are not so aggressive as to overly perturb the underlying steered maser. The frequency stability of the maser is approximately 2×10^{-15} at 1 hour. The standard deviations of the steers for both models are given in Table 2 below:

Table 2: Standard deviation of hourly steers of both models.

	Model 1	Model 2
Standard Deviation of Hourly Steers	3.708×10^{-17}	4.348×10^{-17}

As should be expected, the standard deviation of the steer values for Model 2 is somewhat larger than that of Model 1. This is intuitive based on the increased scatter of the steers in Model 2. Regardless, the steers from both models are over an order of magnitude less than the underlying maser stability. So, working with simulated masers, Model 2 adds a level of robustness to the control system, decreases the steered variance with respect to its target, and does not overly perturb the stability of the maser.

CONTROL SIMULATIONS WITH HISTORICAL DATA

Satisfied that Model 2 performed significantly better than Model 1 on simulated data, the next step was to test the same two models on historical measured data. TWSTT data measuring the difference between the USNO Master Clock and the Alternate Master Clock, spanning from MJD 50209 until 51970, were desteered for this portion of the testing. Both models were then applied to these desteered data.

The analysis of the percentage of maximum steers used by the two models over the entire period of the data showed some improvement in the performance of Model 1 as compared with the simulated clocks (see Table 1 above). For Model 2, the use of the accumulated maximum decreased while the use of the hourly maximum increased slightly, although neither change in the percentage of maximum steers in Model 2 is significant. The values are given in Table 3.

Table 3: Percentage of total number of steers that were limited by the maximum.

	Model 1	Model 2
Percentage over Hourly Max	19.64%	0.89%
Percentage over Daily Max	N/A	0.02%

It is also useful to determine the variances of the state estimates of the control system. The idea behind this is that a lower variance implies that the control is more closely following its target. As is shown in Table 4, Model 2 shows an improvement in these statistics.

Table 4: Variance of the components of the state estimates.

	Model 1	Model 2
Phase Variance	8.678×10^{-19}	5.430×10^{-19}
Frequency Variance	1.258×10^{-29}	1.045×10^{-29}

To test whether or not the improvement in variance is significant, the assumption is made that the improvement is not significant. In order to reject this hypothesis, the test statistic must be compared to the appropriate critical F distribution value:

$$H_0: \sigma_1^2/\sigma_2^2 \geq 1$$
$$H_1: \sigma_1^2/\sigma_2^2 < 1.$$

The ratio of the Model 1 variance of phase estimates to that of Model 2 is 0.626. The ratio of frequency variances is 0.962. Using the test statistic F with $\alpha = 0.01$ and degrees of freedom for both models equal to 29,999, the critical value is 0.831. Since both ratios are less than the critical value, the null hypotheses are rejected. Therefore, the improvement is significant using historical data as well.

An examination of the standard deviation of the steers reveals that the steers for both models are still well within the stability of a maser at 1 hour. The sample standard deviations for both models are in Table 5:

Table 5: Standard deviation of hourly steers of both models.

	Model 1	Model 2
Standard Deviation of Hourly Steers	5.109×10^{-17}	7.256×10^{-17}

Again, we see that Model 2 adds a level of robustness, decreases the variance of the steered system with respect to its target, and does not overly perturb the stability of the underlying maser.

CONCLUSIONS

While the proposed modifications may appear at first glance to provide more opportunity for the introduction of a large error in the system, it has been shown that the modifications actually decrease this possibility. At the same time, the proposed modifications significantly decrease the variance of the steered system state estimates. The proposed changes improve the system robustness, allow for tightened control, and do not overly perturb the underlying maser. These results are supported by simulations with both simulated clocks and historical data. Therefore, the modifications proposed to the control that steers the Alternate Master Clock to the USNO Master Clock would improve the system performance and robustness.

ACKNOWLEDGMENTS

Thanks to Dr. Ken Senior of the USNO for providing the authors with several sets of simulated maser data.

REFERENCES

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