OBSERVING A GRAVITATIONAL WAVE BACKGROUND WITH LISA

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Abstract

LISA (Laser Interferometer Space Antenna) is a proposed space mission which will use coherent laser beams exchanged between three remote spacecraft, to detect and study low-frequency cosmic gravitational radiation.[1] The multiple Doppler readouts available with LISA, which incorporate frequency standards for measuring phase differences between the received and transmitted laser beams, permit simultaneous formation of several observables.[2,3,4] All are independent of lasers and frequency standard phase fluctuations, but have different couplings to gravitational waves and to the various LISA instrumental noises. Comparison of the conventional Michelson interferometer observable with the fully-symmetric Sagnac data-type allows unambiguous discrimination between a gravitational wave background and instrumental noise. The method presented here can be used to detect a confusion-limited gravitational wave background.

INTRODUCTION

The Laser Interferometer Space Antenna (LISA) is a space mission, jointly proposed by NASA and ESA, aimed to detect and study gravitational radiation in the millihertz frequency band. With its three spacecraft, each carrying lasers, beam splitters, photodetectors and drag-free proof masses on each of their two optical benches, LISA will have the capability of measuring six time series of Doppler shifts of the one-way laser beams between spacecraft pairs, and six shifts between adjacent optical benches on each spacecraft. By linearly combining, with suitable time delays, these twelve data sets, it will be possible to cancel the otherwise overwhelming phase noise of the lasers ($\Delta\nu/\nu \simeq 10^{-13}$) to a level $h \simeq \Delta v/c \simeq 10^{-23}$. This level is set by the buffeting of the drag-free proof masses inside each optical bench, and by the shot noise at the photodetectors[4].

LISA is expected to detect monochromatic radiation emitted by galactic binary systems. Particularly at low Fourier frequencies (say $0.1 - 8$ mHz), however, there will be many galactic binaries radiating within each Fourier resolution bin[1]. These latter signals will not be detectable individually, forming a continuum which could be confused with instrumental noise. The level of this stochastic background is uncertain, but could be in the range 10^{-20} – 10^{-23}. Since these galactic binary populations are virtually guaranteed, the detection of their signals could be the first direct detection of gravitational waves.

For this measurement it is very desirable that competing proof-mass and other instrumental noises be both characterized and calibrated before flight, and measured in the actual flight configuration while data are being taken. In contrast to Earth-based, equal-arm interferometer detectors of gravitational radiation, LISA will have multiple readouts, and the Doppler data they generate can be combined differently to give measurements not only insensitive to laser phase fluctuations and optical bench motions, but also with different sensitivities to gravitational waves and to the remaining system noise[3,4].

In this article we discuss two laser-and-optical-bench-noise-free combinations of the LISA readouts, previously denoted ζ (Sagnac) and X (Michelson), that have very different responses to the gravitational wave background but comparable responses to instrumental noise sources[4,5].

THE SAGNAC AND MICHELSON INTERFEROMETERS

The six Doppler beams exchanged between the LISA spacecraft imply the six Doppler readouts y_{ij} $(i,j=1,2,3)$ recorded when each transmitted beam is mixed with the laser light at the receiving optical bench. Delay times for light travel between the spacecraft must be carefully accounted for when combining these data. Six further data streams, denoted z_{ij} $(i,j=1,2,3)$, are generated internally to monitor both lack of rigidity and laser synchronization between the independent optical benches at each spacecraft. The combination ζ uses all the Doppler data symmetrically[4,5]

$$\begin{aligned}\zeta &= y_{32,2} - y_{23,3} + y_{13,3} - y_{31,1} + y_{21,1} - y_{12,2} \\ &\quad + \frac{1}{2}(-z_{13,21} + z_{23,12} - z_{21,23} + z_{31,23} - z_{32,13} + z_{12,13}) \\ &\quad + \frac{1}{2}(-z_{32,2} + z_{12,2} - z_{13,3} + z_{23,3} - z_{21,1} + z_{31,1}) \ . \end{aligned} \tag{1}$$

The comma notation indicates time-delays along the arms of the 3-spacecraft configuration

$$y_{32,2} \equiv y_{32}(t - L_2) \ , \tag{2}$$

and so forth (units in which $c = 1$).

The transfer functions of ζ to instrumental noises and to gravitational waves were calculated in references [3, 4]. The resulting instrumental noise power spectrum for ζ is shown in Figure 1. Also shown there is the computed power spectrum of ζ, averaged over the sky and elliptical polarization states, that would result from a stochastic background originated by an ensemble of galactic binary systems[1].

The laser-and optical-bench-noise-free combination, X, only requires four data streams. This combination is equivalent to an (unequal arm) Michelson interferometer. Its expression is equal to[4,5]

$$\begin{aligned} X &= y_{32,322} - y_{23,233} + y_{31,22} - y_{21,33} + y_{23,2} - y_{32,3} + y_{21} - y_{31} \\ &+\frac{1}{2}(-z_{21,2233} + z_{21,33} + z_{21,22} - z_{21}) \\ &+\frac{1}{2}(+z_{31,2233} - z_{31,33} - z_{31,22} + z_{31}) \ . \end{aligned} \tag{3}$$

The expected instrumental noise power spectrum in X is shown in Figure 1. Also shown is the anticipated galactic binary confusion spectrum[1], which would be observed in X. Comparison of X and ζ allows the background to be discriminated from instrumental noise.

DETECTING THE GALACTIC STOCHASTIC BACKGROUND

The flight configuration of the three spacecraft forming LISA will be essentially equilateral, with $L_1 = L_2 = L_3 = L = 16.67$ sec. In the frequency band of interest $(0.1 - 8$ mHz), the expressions for the Fourier transforms of the gravitational wave signals $\widetilde{X}^{gw}(f)$, $\widetilde{\zeta}^{gw}(f)$ and the power spectral densities of the system noises in X and ζ, $S_{X^{noise}}(f)$, $S_{\zeta^{noise}}(f)$, can be Taylor-expanded in the dimensionless quantity fL. The first non-zero terms are equal to

$$\widetilde{X}^{gw}(f) \simeq 2\ (2\pi i f L)^2\ \left[\hat{n}_3 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_3 - \hat{n}_2 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_2\right] \ , \tag{4}$$

$$\begin{aligned} \widetilde{\zeta}^{gw}(f) &\simeq \frac{1}{12}\ (2\pi i f L)^3\ \Big[(\hat{k}\cdot\hat{n}_1)(\hat{n}_1 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_1) + (\hat{k}\cdot\hat{n}_2)(\hat{n}_2 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_2) \\ &\qquad + (\hat{k}\cdot\hat{n}_3)(\hat{n}_3 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_3)\Big] \ , \end{aligned} \tag{5}$$

$$\begin{aligned} S_{X^{noise}}(f) &\equiv S_{X^{proofmass}}(f) + S_{X^{opticalpath}}(f) \\ &\simeq 16\,[S_1(f) + S_{1^*}(f) + S_3(f) + S_{2^*}(f)]\ \ (2\pi f L)^2 \\ &\quad +4\,[S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f)]\ \ (2\pi f L)^2 \end{aligned} \tag{6}$$

$$\begin{aligned} S_{\zeta^{noise}}(f) &\simeq [S_1(f) + S_2(f) + S_3(f) + S_{1^*}(f) + S_{2^*}(f) + S_{3^*}(f)]\ \ (2\pi f L)^2 \\ &\quad + [S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f) + S_{13}(f) + S_{12}(f)] \ , \end{aligned} \tag{7}$$

where we have denoted by $S_{X^{proofmass}}(f)$, and $S_{X^{opticalpath}}(f)$ the aggregate contributions to the power spectrum of the noise in the response X from the proof mass and optical path noises respectively. The expressions in square brackets in Equations (4, 5) incorporate LISA antenna responses[3,4], and are of the same order of magnitude. The proof mass Doppler noise spectra $S_i(f)$, $S_{i^*}(f)$ $(i = 1, 2, 3)$ will be designed to a nominal power spectral level[4] $S^0(f) = 2.5 \times 10^{-48}[f/1Hz]^{-2}$ Hz^{-1}, while the optical path noise spectra $S_{ij}(f)$,$(i, j = 1, 2, 3 \, , i \neq j)$, which include shot noises at the photo detectors and beam pointing noise[1], are expected to be equal to a nominal spectrum $S^1(f) = 1.8 \times 10^{-37}[f/1Hz]^2$. Both these noise sources will be estimated before launch, but could be larger when the in-orbit data will be taken.

First consider the responses to the gravitational wave signal, given in Equations (4, 5). At $f = 10^{-3}$ Hz, for instance, (where $2\pi f L \simeq 10^{-1}$) the absolute value of the coefficient in front of the squared-bracket in the ζ response (Eq. 5) is about three orders of magnitudes smaller than the corresponding coefficient given in the expression for X (Eq. 4). The power spectral densities of the noises due to the proof masses and the optical-path noise (Eqs. 6, 7) will only differ at most by an order of magnitude. We conclude that in this lower frequency range the LISA Sagnac response, ζ, can be used as a *gravitational wave shield.* In what follows we will ignore the gravitational wave background contribution to ζ.

To take quantitative advantage of this property of ζ, consider the observed power spectral densities of X and ζ

$$S_X^{obs}(f) = S_{X^{gw}}(f) + S_{X^{proofmass}}(f) + S_{X^{opticalpath}}(f) \tag{8}$$

$$S_\zeta^{obs}(f) = \frac{1}{16}\left[S_{X^{proofmass}}(f) + \frac{S_{X^{opticalpath}}(f)}{(\pi f L)^2}\right] + [S_{13}(f) + S_{12}(f)] + [S_2(f) + S_{3^*}(f)] \ (2\pi f L)^2 \ , \tag{9}$$

where in Equation (9) we have written the power spectra of the noises in ζ in terms of the power spectra of the noises in X and of some remaining terms that are not present in X, to emphasize commonality of some noise sources. We suppose that the noise contributed by any one of the proof masses and optical-path noise sources will be greater than or equal to the design values, $S^0(f)$ and $S^1(f)$ respectively. From Equation (9), if the magnitude of the <u>measured</u> power spectral density of the response ζ is at its anticipated level $S_\zeta^{obs}(f) = 6 \ S^0(f)(2\pi f L)^2 + 6 \ S^1(f)$, then the level of the power spectral density of the noise entering into X is known. The spectrum

$$S_{X^{gw}}(f) = S_X^{obs}(f) - 64 \ S^0(f)(2\pi f L)^2 - 16 \ S^1(f)(2\pi f L)^2 \ , \tag{10}$$

should then be attributed to a galactic binary background of gravitational radiation. In any event, the RHS of Equation (10) is an upper bound to $S_{X^{gw}}$.

On the other hand, if the measured spectrum of ζ is above its anticipated design level, consider the following combination of the measured spectra

$$
\begin{aligned}
S_X^{obs}(f) - 16\, S_\zeta^{obs}(f) &= S_{X^{gw}} - 16\,[S_2(f) + S_{3^*}(f)]\,(2\pi f L)^2 \\
&\quad -16\,[S_{13}(f) + S_{12}(f)] \\
&\quad -16\,[S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f)] \times \\
&\quad \times \left[1 - (\pi f L)^2\right]. \qquad (11)
\end{aligned}
$$

The coefficient of S_ζ^{obs} has been chosen so that the noise terms on the right hand side are all now negative-definite and can thus be bounded from above by their design, or nominal, values $S^0(f)$ and $S^1(f)$ respectively. The result is a lower bound for observational discrimination of the gravitational wave background spectrum

$$
\begin{aligned}
S_{X^{gw}}(f) &\geq S_X^{obs}(f) - 16\, S_\zeta^{obs}(f) + 32\, (2\pi f L)^2 S^0(f) \\
&\quad +16\,[6 - (2\pi f L)^2] S^1(f). \qquad (12)
\end{aligned}
$$

Equations similar to (11) and (12) can be written for the other two interferometer combinations, Y and Z[4]. In those equations, there will be different mixes of canceled and bounded noise sources, resulting, in general, in different gravitational wave spectrum lower bounds.

CONCLUSIONS

The response of the Sagnac interferometer to a gravitational wave signal is several orders of magnitudes smaller than that of the Michelson interferometer. In the frequency band of interests (0.1 − 8) mHz, however, the Sagnac response to the noise sources is of the same order of magnitude as that of the Michelson interferometer. As a consequence of these facts we have shown that it is possible to estimate the magnitude of the noise sources affecting the Michelson interferometer response in the low-frequency region of the accessible band by using the Sagnac interferometer. This in turn allows us to discriminate a gravitational wave background of galactic origin from instrumental noise affecting the Michelson interferometer response.

ACKNOWLEDGMENTS

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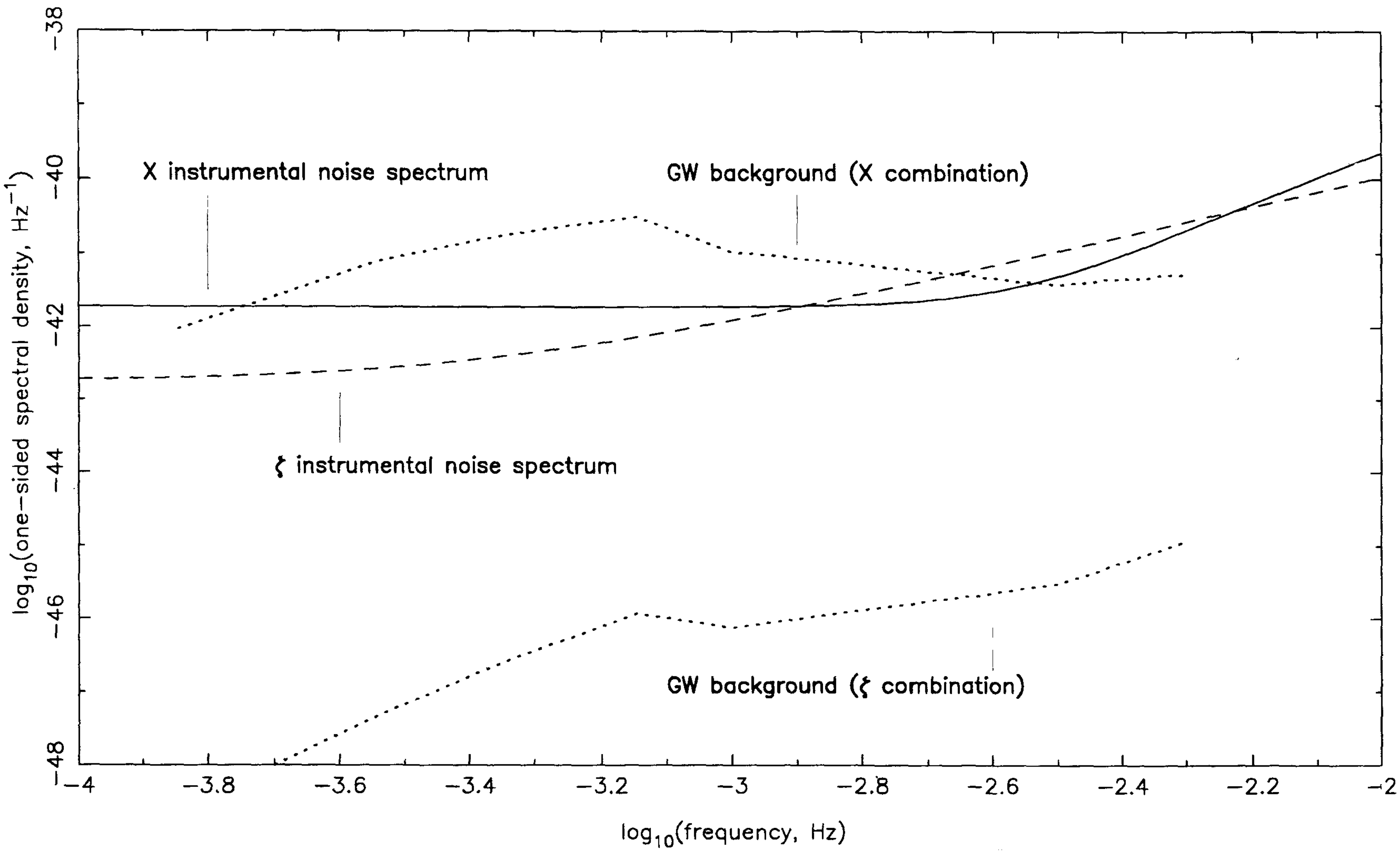

Figure 1. Fractional Doppler frequency instrumental noise power spectra for data combinations X (solid line) and ζ (dashed line). These are derived from the transfer functions for X and ζ to instrumental noises and the nominal LISA spectra for individual proof mass noise $[3x10^{-15}(m/sec^2)/\sqrt{Hz}]$ and one-way optical path noise $[20x10^{-12}m/\sqrt{Hz}]$, converted to fractional Doppler spectra. Also plotted are the spectral responses of X and ζ to the expected stochastic gravitational wave background, as discussed by Bender and Hils [3] and Hils [4]. Using ζ to measure on-orbit instrumental noise allows a gravitational wave background in X to be either uniquely determined or bounded.

Questions and Answers

ROBERT NELSON (Satellite Engineering Research Corporation): It might be appropriate to note that Dr. Joseph Webber of the University of Maryland passed away this past September. Dr. Webber founded the whole subject of experimental gravitational wave detection at the University of Maryland. In addition, in the late 1940s, he published one of the first papers on the notion of a population inversion concept that was the foundation for the invention of the maser.

Over the last decade or so, Dr. Webber did analyses that indicate some evidence of gravitational wave detection by VAR detectors at the University of Maryland and in Rome which were correlated to observations of neutrinos that were associated with the supernova, I believe it was 1985. One of the things that you might consider in the future is, in light of that type of analysis, is to correlate your own gravitational wave measurements with other sources such as neutrinos which, according to theory, are produced in gravitational collapse events.

MASSINO TINTO: Yes, I would like to add and I should have said it earlier, this particular instrument will work in the millihertz frequency band. So we are expecting to observe sources that will not be in the kilohertz frequency band where less detectors were operating. So supernova light is frozen and will not very likely be in this span of observations. Usually supernova explosions are accompanied by neutrino emissions.

The reason for this analysis, in a sense, is really trying to assess where the sensitivity is without relying on other instruments. In a sense, it would corroborate your observation. So once you know what the sensitivity curve is, anything that is above it, you know, sort of classifies as other sources.

DEMETRIOS MATSAKIS (USNO): Is there any timing requirements that you could talk about with this system?

TINTO: Yes, and I am getting to that. You see, when you actually phase for the phase differences from these Doppler measurements, you have to take into account that the spacecraft are moving relative to each other and is not stationary. So at 10^{-14} hertz, which is the frequency of the laser, the relative speed of 10 meters per second introduces big nodes of several megahertz. You have to track that bit node in your phase measurement. To do that, we rely on USOs, so we have a timing system aboard this spacecraft that allows us to measure the phase and then, on the other hand, introduce noise. It is so noisy, and we want to remove that noise. There are techniques to eventually remove this noise measurement introduced by the USOs which are incorporated into this interferometric technique that we have. So the system is certainly has on-board timing systems and timing requirements. In fact, we rely on state-of-art USOs in order to do these measurements. We need parts in 10^{15} or better USO performances.

But I didn't want to present all the details of the timing system on board the spacecraft.

MATSAKIS: It is covered in your published paper though.

TINTO: Yes, this is published and will be in the Proceedings anyway.

THOMAS CLARK (NASA Goddard Space Flight Center): I had looked at a similar type interferometer at one time, and maybe I don't understand the measurement. If you are talking about the laser measuring carrier phase of the laser—if I did the calculations you are talking about, travel times of about 16 seconds—that means that you have to have an intrinsic oscillator stability at the laser frequency to something better than one sixteenth of a hertz. Probably more like 10 millihertz at 10^{14} hertz. So that says you are expecting a laser to have an intrinsic frequency stability over the Allan variance at the travel time, 16 seconds, of something like a part in 10^{16}.

TINTO: No, no, we don't need that. Even parts in 10^{13} you have to know accuracy of the uplink. The separation of the spacecraft is only 50 meters or so. If you know the uplink, then you can combine to six measurements in such a way as to remove the fluctuations of the laser to the level that you want to.

CLARK: The other question I was going to ask has to do with the background noise, which is a different one. I will have to think about your comment on that one. When we had thought about this in terms of a slightly different program, rather than the gravitational wave background noise, the gravity field background noise looked to me to be a serious problem. For instance, the gravity field changes at this interferometer location due to Venus orbiting around Mars are many times the effect of the signal that you are seeing. The gravity noise due to essentially all of the undetected and unmodeled asteroids looked to me like they would give so much position noise across the interferometer that gravity field could not be detectable. Have you thought in terms of just the solar system gravity noise?

TINTO: I think that has been included in the Phase A report, which you will find at that Web site that I gave. I believe the conclusion was that that was not a problem. Also, you have to take into account the time scales that you are talking about. LISA will be 1,000 to 10,000 seconds or so up to 1 second integration time. Most of the facts we're concerned about is the time scale completely outside the band. It will not be affecting the performance of the instrument. Other things have been analyzed and found not to be significant. To get the details, I can point out the references.

But I think the key point here for LISA is that you can actually synthesize an interferometer, which means you can remove the fluctuations of the laser. It requires a knowledge of the uplink. So going to your first question, if you know the uplink with an aperture of 30 meters or something, then you can combine these six Doppler measurements in such a way as to remove those fluctuations. So you need the laser at part in the 10^{13} or so. Even worse, if your knowledge of the uplink is better.