GPS-BASED TIME ERROR ESTIMATES PROVIDED BY SMOOTHING, WIENER, AND KALMAN FILTERS: A COMPARATIVE STUDY

Y. S. Shmaliy*, A. V. Marienko,**
M. Torres-Cisneros*, and O. Ibarra-Manzano*
***Electronics Dept., Guanajuato University, Salamanca, 36730, Mexico**
****"Sichron" Center, 4 Skrypnika St., Kharkiv, 310057, Ukraine**
E-mail: shmaliy@salamanca.ugto.mx

Abstract

GPS timing plays a critical role in modern practice of time errors estimate and synchronization. Big noise of the GPS-based measured data and inherent non-stationarity of a time error cause major difficulties here. In spite of theoretical separation of the application fields for the filters (stationary and non-stationary signals), GPS-based time error processes require more explicit practical answer. Indeed, what process may be practically treated as a stationary one and, to opposite, how to recognize a non-stationary case? In this report we answer these questions by numerically and show that for the same transient time the following filter should be used to get the best accuracy for the known initial fractional frequency offset y_0 (time error rate) of oscillator, namely an average smoother for $|y_0| < r_1$, the Wiener filter for $r_1 \le |y_0| \le r_2$, and the Kalman filter for $r_2 < |y_0|$, where r_1 and r_2 are coordinates dependent on the required accuracy. We prove this conclusion by the example of a time error estimate of the rubidium standard based on the reference timing signals of the Motorola GPS UT+ Oncore Timing receiver.

INTRODUCTION

GPS timing plays a critical role in modern practice of time errors "on-line" estimate and synchronization. Major difficulties here are caused mainly by big noise of the measured data provided by GPS receiver and inherent non-stationarity of a time error. Statistically, once a random signal exhibits stationary nature, then the optimal Kolmogorov-Wiener approach (Wiener filter [1]) is efficiently used and, conversely, the Kalman-Bucy technique (Kalman filter [2]) yields the best estimator for the non-stationary random signals. In spite of both approaches evidentally covering all cases, test and measurement still use an average smoother [3] owing to its transparency and small variance, and despite of an estimate bias caused inevitably by non-stationarity.

Modern timekeeping systems employ all three filters. Observing them even in the past Proceeding of PTTI'99, we realize that, for instance, to detect the failure of a single satellite clock three different space-segment timekeeping subsystems are designed [4]. Two subsystems use a direct average and the three-state Kalman filter is implemented in the third case,yielding the most accurate estimate as compared to the average filters. In the synchronization algorithm for the WAAS network they are based on the two-state Kalman filter [5], providing a control error of 50ns. The same two-state filter was used in the new steering strategy for the USNO Master Clock [6]. Contrarily, while steering the cesium-based primary clock of the Geo Uplink Subsystem they employ an average smoother [7]. Finally, to estimate a time error of the master clock for the WAAS test transmissions the algorithm was designed based on the IIR digital filter of the 2[nd] order [8], which may be considered as a non-optimal Wiener filter.

The result is following, for rather the same quality master clocks and time errors the different estimators are used, and it seems obvious that aiming to obtain the smallest estimate and synchronization error, one must follow the rules to select the filter in the optimal way. To work the rules out for the GPS-based time error processes, first, we must realize what process may be practically considered as a stationary one and, conversely, how to recognize a non-stationary case? Finally, what type of the digital filters should be used to be the most accurate in practice for the certain transient time and known rate of change of a time error caused by the crystal oscillator, rubidium standard, cesium standard, or even hydrogen maser?

In this report we answer the questions in the following way. We numerically study all three filtering algorithms based on an average smoother, the Wiener and Kalman filters for the same common transient time t_{tt} taken as an average time of a smoother. We then simulate the GPS-based time error random process with a constant initial fractional frequency offset y_0 between reference and local oscillators and study the filtering errors for the proper y_0 . In this way we determine the ranges for y_0, in which each filter exhibits a minimal either total (RMS variance plus mean bias) or a maximum error. We show that for the same t_{tt} the following filter should be used to get the best accuracy, depending on y_0:

- ***Average smoother is for the range of*** $|y_0| < r_1$,
- ***Wiener filter is for*** $r_1 \le |y_0| \le r_2$, and
- ***Three-state Kalman filter is for*** $r_2 < |y_0|$,

where r_1 and r_2 are determined for the total error as r_{1t} and r_{2t}, and for the maximal error as r_{1m} and r_{2m}, respectively. Because processing time influences the error strongly then we study the errors for the different t_{tt}, finding out correspondent dependencies $r_{1,2}(t_{tt})$ and presenting the simple approximation function $r_i = a_i t_{tt}^{-1.5}$, where $i \equiv 1t, 2t, 1m, 2m$, and a_i is a constant.

We, finally, consider the example of the filter selection to get the most accurate estimate of the time error of the rubidium standard with known y_0 employing the Motorola UT+ Oncore receiver. First, through the equality $y_0 = a_i t_{tti}^{-1.5}$ we establish the critical transients for the total and maximal errors, t_{tr1t}, t_{tt2t} and t_{tt1m}, t_{tt1m}, respectively, and expect that an average smoother will give the smallest error for $t_{tt} < t_{tt1t}$ or $t_{tr} < t_{tt1m}$, the Wiener filter should be the most accurate for $t_{tr1t} < t_{tr} < t_{tt2t}$ or $t_{tt1m} < t_{tt} < t_{tt2m}$, and, finally, the Kalman filter must be the best for $t_{tt2t} < t_{tr}$ or $t_{tt2m} < t_{tt}$. As a matter of the fact we conclude that the methodology holds true at least for the considered case.

MATHEMATICAL MODELS OF THE SIGNALS

Consider mathematical presentation of a timing signal of local oscillator and a noisy time error.

An Oscillator Timing Signal

A model of total instantaneous phase $\Phi(t)$ of oscillator is truly legalized in [9] as

$$\Phi(t) = \Phi_0 + 2\pi v_{nom}(1 + y_0)t + \pi D v_{nom} t^2 + \varphi(t), \tag{1}$$

where: Φ_0 is initial phase offset,

y_0 is the fractional frequency offset from the nominal frequency v_{nom} (mainly due to finite frequency settability of the clock), y_0[in ns/hour] = 3.6× y_0[in parts of 10^{-12}],

D is the linear fractional frequency drift rate (basically representing oscillator aging effects),

$\varphi(t)$ is the random phase deviation component.

While subtracting from (1) the same type phase model of a reference source and then dividing the result by $2\pi v_{nom}$, one **comes to the time error error model**

$$x(t) = x_0 + (y_0 - y_{0ref})t + \frac{D - D_{ref}}{2} t^2 + \frac{\varphi(t) - \varphi_{ref}(t)}{2\pi \nu_{nom}}. \tag{2}$$

In (2) one can also take that all the degradation sources y_{0ref}, D_{ref}, and $\varphi_{ref}(t)$ of the reference source are negligible as compared to those of the clock under test. As a result, the $x(t)$ model reduces to the practical form of a time error of local oscillator

$$x(t) = x_0 + y_0 t + \frac{D}{2} t^2 + \frac{\varphi(t)}{2\pi \nu_{\text{nom}}}. \tag{3}$$

A Noisy Time Error

Basically, we measure a time error $x(t)$ in discrete time, providing values x_ν for discrete time points t_ν for the constant time interval $\Delta = t_\nu - t_{\nu-1}$, where $\nu = 0, 1, 2, \ldots$. GPS-based measurements add a noise to a time error, which has a normal histogram, thus, may be modeled as a Gaussian noise. Both a time error and a noise are summed (3) allowing presentation of a measured noisy time error (observation) and a clock state with respect to (3) in the matrix form as follows

$$\xi_\nu = \mathbf{H}_\nu \lambda_\nu + n_{0\nu}, \tag{4}$$

$$\lambda_\nu = \mathbf{A}_{\nu-1} \lambda_{\nu-1} + \mathbf{n}_{\lambda\nu}, \tag{5}$$

where ξ_ν is a measured noisy time error (observation), λ_ν is 3-dimensional oscillator (clock) state vector (time error, frequency, and aging), $\mathbf{H}_\nu$ is 1×3 dimensional measurement matrix, $\mathbf{A}_\nu$ is 3×3 dimensional clock state transition matrix, $n_{0\nu}$ and $\mathbf{n}_{\lambda\nu}$ are jointly independent white noises with zero expectations and covariances V_ν and Ψ_ν of 3×3 dimension, respectively,

$$V_\nu = E\{n_{0\nu} n_{0\nu}^T\}, \tag{6}$$

$$\mathbf{\Psi}_\nu = E\{\mathbf{n}_{\lambda\nu} \mathbf{n}_{\lambda\nu}^T\}. \tag{7}$$

In discrete time the model (3) is transferred to the form of

$$x_\nu = x_{\nu-1} + y_{\nu-1}\Delta + \frac{D_{\nu-1}}{2}\Delta^2 + n_{x\nu}, \tag{8}$$

where $y_\nu = y_{\nu-1} + D_{\nu-1}\Delta + n_{y\nu}$; $D_\nu = D_{\nu-1} + n_{D\nu}$; and $n_{x\nu}$, $n_{y\nu}$, and $n_{D\nu}$ are correspondent discrete noises.

Correspondingly, (8) allows writing all the matrixes for (4) and (5) as

$$\boldsymbol{\lambda}_v = \begin{bmatrix} x_v \\ y_v \\ D_v \end{bmatrix}, \mathbf{A}_v = \begin{bmatrix} 1 & \Delta & \Delta^2/2 \\ 0 & 1 & \Delta \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{n}_{\lambda v} = \begin{bmatrix} n_{xv} \\ n_{yv} \\ n_{Dv} \end{bmatrix}, \text{ and } \mathbf{H}_v = [1 \quad 0 \quad 0], \tag{9}$$

present (4) in a form of

$$\xi_v = x_v + n_{0v}, \tag{10}$$

and describe the noise matrix (7) as

$$\boldsymbol{\Psi}_v = E\{\mathbf{n}_{\lambda v}\mathbf{n}_{\lambda v}^T\} \cong S_{Dv}\Delta \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{11}$$

where S_{Dv} is two-side spectral density of a continuous white noise of aging depending on Δ [10] and expressed through the time error noise straightforward.

ESTIMATION OF A TIME ERROR

Consider the three statistical algorithms, namely an average smoothing, Wiener's, and Kalman's.

Average Smoother

Average smoother allows a non-optimal estimate $\hat{x}_v$ of a time error x_v (in a sense of a minimal RMS error) based on an observation ξ_v (10). The algorithm does not require any a priori knowledge about an oscillator state model, time error, and even an observation and is straightforward

$$\hat{x}_v = \frac{1}{N}\sum_{i=v-N+1}^{v} \xi_i, \tag{12}$$

where $\xi_v = 0$ if $v < 0$, N is number of average points. According to (12), the first estimate appears with delay on N points of the process, thus, a filter transient time equals $t_{tr} = \Delta(N-1)$.

Wiener Filter

In discrete time domain the realizable Wiener filter provides estimate through a convolution of its impulse response h_v and an observation (10)

$$\hat{x}_v = \sum_{i=v-M+1}^{v} \xi_i h_{v-i}, \tag{13}$$

where $\xi_v = 0$ if $v < 0$, M determines the length of h_v that is taken to be equal zero apart the time interval 0, ..., $(M\text{-}1)\Delta$. To get a minimal RMS error for estimate, first, in the tradition of Wiener define an optimal unrealizable response $H_{0k} = S_{xk}/(S_{xk} + S_{n0})$, where S_{xk} is discrete power spectral density of a time error (oscillator phase) that in spirit of Leeson [11] is taken here as $S_{xk} = af_k^{-2}$, where f_k is Fourier frequency; a is a constant; S_{n0} is constant power spectral density of a white noise; $k = 0, \ldots, K\text{-}1$, and K limits the length of the time error sequence taken at the early stage to estimate the spectral densities with enough accuracy. Then use a proper approximating filter with response $H_k \exp(j\phi_k)$ and come through the inverse discrete Fourier transform to the optimal impulse response h_{ov} of a realizable filter

$$h_{ov} = B\mathscr{D}^{-1}\{H_k e^{j\phi_k}\}, \quad (14)$$

where B is a constant. In our experiment we increase (reduce) the level of S_{n0}, increasing (reducing) in this way a filter transient time.

Kalman Filter

The three-state Kalman filter is matched with a clock model (3) [12], allowing the following algorithm for (4) and (5) to get estimates in discrete time

$$\hat{\lambda}_v = \mathbf{A}_{v-1}\hat{\lambda}_{v-1} + \mathbf{K}_v(\xi_v - \mathbf{H}_v\mathbf{A}_{v-1}\hat{\lambda}_{v-1}), \quad (15)$$

where a filter gain is defined as

$$\mathbf{K}_v = \widetilde{\mathbf{R}}_v\mathbf{H}_v^T(\mathbf{H}_v\widetilde{\mathbf{R}}_v\mathbf{H}_v^T + V_v)^{-1}, \quad (16)$$

where $\widetilde{\mathbf{R}}_v = \mathbf{A}_{v-1}\mathbf{R}_{v-1}\mathbf{A}_{v-1}^T + \Psi_v$ is a matrix of predicted errors, and the filtering errors are calculated as

$$\mathbf{R}_v = (\mathbf{I} - \mathbf{K}_v\mathbf{H}_v)\widetilde{\mathbf{R}}_v, \quad (17)$$

where $\mathbf{I}$ is a unit matrix. Transient of the on-line operating Kalman filter is due to time expended to get estimate (17). It may be varied around the optimal value by changing S_{Dv} in (11), so that since S_{Dv} rises then t_{tr} decreases, and **vice versa.** The total filtering error rises once transient is a non-optimal.

NUMERICAL STUDIES OF THE FILTER ERRORS

All three filters, namely an average smoother (12), Wiener's (13), and Kalman's (15), are examined here for the same time error process. To show the effect, the noisy process (3) is simulated with variance σ = 40ns and with both a stationary part of a deterministic function ($0 \le t < 25$ hours), in which case $x_0 = y_0 = D = 0$ in (3), and a non-stationary part ($25 \le t$ hours) with $y_0 = -2\cdot10^{-12} = -7.2$ ns/hour and $D = 0$. Because we consider a transient time as a **principal** performance of a filter, then, to know trade-off, we obtain the same t_{tr} = 10hours for all three filters. While providing, the transient of a smoother was evaluated by its average time $t_{tr} = \Delta(N-1)$ and that of the Wiener and Kalman filter finished at the level of 0 9. Figure 1 shows the simulated processes and estimates extracted by the filters. Figure 2 gives correspondent errors calculated as difference between estimated and simulated functions

$$\varepsilon_v = \hat{x}_v - x_v. \quad (18)$$

Just as it had been expected based on the filter strategies for the dynamic range (25hours < t), the Kalman filter showed the smallest error, the Wiener filter was less accurate, and, the smoother stayed hors-concours with its biggest error. **Conversely,** for the range of a stationary noisy error (t < 25hours), the smoother was the best, the Wiener filter exhibited more big error, and the Kalman filter looked like the worst. Nevertheless, it is obviously speaks in favor of the Kalman filter that its error remains say rather the same for the both stationary and non-stationary ranges (Figure 2).

Excited by the curiosity of the different filtering errors for the stationary and non-stationary processes with the constant transient, we come to another experiment, while simulating only a non-stationary process and evaluating (18) for t_{tr} = const and various y_0 by the total filtering error

$$\sigma_t = \left|\langle \varepsilon_v \rangle\right| + \sqrt{\left\langle \left[\varepsilon_v - \langle \varepsilon_v \rangle\right]^2 \right\rangle} \tag{19}$$

and the maximal filtering error

$$\varepsilon_{\max} = \max\left|\varepsilon_v\right| . \tag{20}$$

Before going on to analyze the results, let us study Figure 3, which shows the total (*a*) and maximal (*b*) errors as functions of y_0 for t_{tr} = 6hours for all three filters. There an average smoother yields the smallest total error for $y_0 < r_{1t} = 1.57\times10^{-13}$, the Wiener filter is the most accurate for $r_{1t} = 1.57\times10^{-13} < y_0 < r_{2t} = 4\ 035\times10^{-13}$, and the Kalman filter is for $r_{2t} = 4.035\times10^{-13} < y_0$. The same filters provide the smallest maximal error for the ranges of $y_0 < r_{1m} = 7.59\times10^{-13}$, $r_{1m} = 7.59\times10^{-13} < y_0 < r_{2m} = 9.22\times10^{-13}$, and $r_{2m} = 9\ 22\times10^{-13} < y_0$, respectively. We then estimate coordinates r_{1t} , r_{2t} , r_{1m} , and r_{1m} by changing t_{tt} , and come to the correspondent dependences (Table 1).

Table 1

DEPENDENCIES OF THE COORDINATES (FIGURE 3) ON THE FILTER TRANSIENT

Transient, hours	Measure	Total error coordinates		Maximal error coordinates	
		r_{1t}	r_{2t}	r_{1m}	r_{2m}
1 5	y_0, ns/hour	**4.032**	**11.376**	**20.844**	**32.782**
	y_0	1.12×10^{-12}	3.16×10^{-12}	5.79×10^{-12}	$9\ 106\times10^{-12}$
3 0		**1.804**	**4.054**	**8.28**	**11.376**
		5.01×10^{-13}	1.126×10^{-12}	2.3×10^{-12}	3.16×10^{-12}
4 5		**0.984**	**2.07**	**3.996**	**6.091**
		2.733×10^{-13}	5.748×10^{-13}	1.11×10^{-12}	1.692×10^{-12}
6 0		**0.565**	**1.453**	**2.733**	**3.318**
		1.57×10^{-13}	4.035×10^{-13}	7.593×10^{-13}	9.217×10^{-13}
7 5		**0.3636**	**1.1506**	**1.878**	**2.107**
		$1\ 01\times10^{-13}$	3.196×10^{-13}	5.217×10^{-13}	5.854×10^{-13}
9 0		**0.3**	**0.8028**	**1.1088**	**1.5329**
		8.36×10^{-14}	2.23×10^{-13}	3.08×10^{-13}	4.258×10^{-13}
10 5		**0.2578**	**0.703**	**1.036**	**1.272**
		7.16×10^{-14}	1.952×10^{-13}	2.878×10^{-13}	3.533×10^{-13}
12 0		**0.2542**	**0.4896**	**1.1614**	**1.6718**
		7.06×10^{-14}	1.36×10^{-13}	3.226×10^{-13}	4.644×10^{-13}
13 5		**0.2031**	**0.4913**	**0.7423**	**0.9086**
		5.643×10^{-14}	1.3647×10^{-13}	2.062×10^{-13}	$2\ 524\times10^{-13}$
15 0		**-**	**0.398**	**0.7636**	**0.7758**
		-	1.106×10^{-13}	2.121×10^{-13}	2.155×10^{-13}
16 5		**0.0295**	**0.3207**	**0.583**	**0.6548**
		3.19×10^{-14}	8.91×10^{-14}	1.619×10^{-13}	1.819×10^{-13}
18 0		**-**	**0.2497**	**0.5526**	**0.6073**
		-	6.937×10^{-14}	1.54×10^{-13}	1.69×10^{-13}

The curves provided in this way (Figure 4) were noisy because of different length-limited samples of the simulated random process. Nevertheless, it seems obvious that the following approximating function $G\times t_{tt}^{-1\ 5}$ is accurate enough to be used in practical calculations for each coordinate. We approximate those as follows

$$r_{1t} \cong 2.3\times t_{tr}^{-1\ 5},\ r_{2t} \cong 6.0\times t_{tt}^{-1\ 5},\ r_{1m} \cong 11\times t_{tt}^{-1\ 5}, \text{and } r_{1m} \cong 15\times t_{tt}^{-1\ 5}, \tag{21}$$

and use (21), while considering the below-given example of the filter selection for the GPS-based time error process generated by the rubidium standard.

EXAMPLE: FILTER SELECTION FOR THE GPS-BASED TIME ERROR OF A RUBIDIUM STANDARD

Measurement of the time error of the rubidium standard had been carried out based on the Motorola GPS Timing Receiver Oncore UT+ with average time Δ = 100s for about 30 hours with the initial error of $x_0 \cong 2.1$ ns and offset $y_0 \cong -4.7$ ns/hour$= -1.3 \times 10^{-12}$. To separate the ranges for each filter, substitute the known y_0 for each coordinate in (21) and come to the following prediction:

For the total error (19)

If $t_{tr} < 1.455 hours$ then **an average smoother** should be the most accurate

If $1.455 hours < t_{tr} < 2.757 hours$ then **the Wiener filter** should be the most accurate

If $2.757 hours < t_{tr}$ then **the three-state Kalman filter** should be the most accurate

For the maximal error (20)

If $t_{tr} < 4.129 hours$ then **an average smoother** should be the most accurate

If $4.129 hours < t_{tr} < 5.078 hours$ then **the Wiener filter** should be the most accurate

If $5.078 hours < t_{tr}$ then **the three-state Kalman filter** should be the most accurate

Then tune the filters step by step for several transients to satisfy the above-determined conditions and estimate total (19) and maximal (20) errors (Table 2).

Table 2

TOTAL AND MAXIMAL ERRORS OF THE FILTERS FOR THE DIFFERENT TRANSIENT TIMES

t_{tr},	Total error, ns			Maximal error, ns		
hours	Smoother	Wiener	Kalman	Smoother	Wiener	Kalman
1 0	8.367	8.210	10 83	17.204	16.626	36 363
2 2	9.429	8.225	9 446	14.447	14 842	18 088
3 0	10 447	8.677	8.279	15 536	14.763	16 850
4 5	13 363	10 25	6.622	19 021	14 486	13.805
6 0	16 942	12.376	5.306	26 099	16 204	12.759

An analysis of Table 2 shows that just as it had been predicted the three-state Kalman filter allows the smallest both total error for $3.0 hours \le t_{tr}$ and maximal error for $4.5 hours \le t_{tr}$. The Wiener filter exhibits the smallest those errors for $2.2 hours = t_{tr}$ and $3.0 hours = t_{tr}$, respectively. An average smoother gives the smallest both those errors for $t_{tr} \le 1.0 hours$ and $t_{tr} \le 2.2 hours$, respectively. In the range of $t_{tr} \le 1.0 hour$ we watched also for the small error of the Wiener filter. This is because of the limited processing sequence available with small average time. Thus, we have proved in this way the above-given methodology generalized by Figure 4, except the case of t_{tr} = 1.0hour, and, finally, to illustrate the real filtering process, we bringFigures 5—8, those show four cases of a time error estimate provided by all three filters.

CONCLUDING REMARKS

We have numerically examined in this report the errors of the three filtering approaches, namely an average smoothing, Wiener's, and Kalman's once they are employed to get "on-line" time error estimate in the modern timekeeping systems. As a major result we present Figure 4, which, at first, practically answer the question "What are stationary and non-stationary time error processes?" separating space for them left and right, respectively, by correspondent curves. And, it is the most important, Figure 4 allows selection of the filter type for the initial frequency offset y_0 (rate of change of a time error) and the filter transient t_{tr}. Based on this, once interesting of the maximal filtering error, we conclude that the following filter seems to be the most accurate depending on y_0 and t_{tr} , namely

- For $y_0 \cong \mathbf{10^{-11}}$ (crystal) the three-state Kalman filter is the most accurate once t_{tr} **> 1hour**
- Once $y_0 \cong \mathbf{10^{-12}}$ (crystal or rubidium) then an average smoother is accurate for t_{tr} **< 5hours**, the Wiener filter is for **5hours <** t_{tr} **< 6hours**, and the three-state Kalman filter is for **6hours <** t_{tr}
- For $y_0 \le \mathbf{10^{-13}}$ (cesium and hydrogen) an average smoother is accurate for t_{tr} **< 24hours**

The results are readily extended to the general case, including aging. Just account the maximally possible frequency offset of your oscillator for the measurement (observation) and follow the above-given methodology, for which more satisfactory justification we plan to revise the results further analytically.

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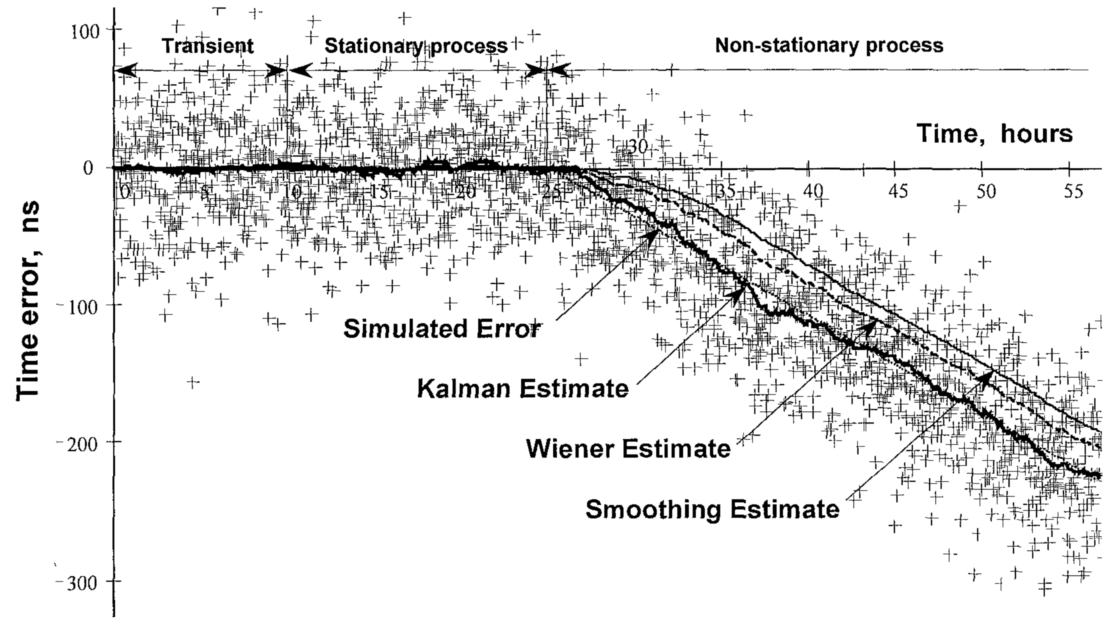


Figure 1. Simulated error, noisy observation, and estimates provided by the average smoothing, Wiener, and three-state Kalman filters for the stationary and non-stationary processes

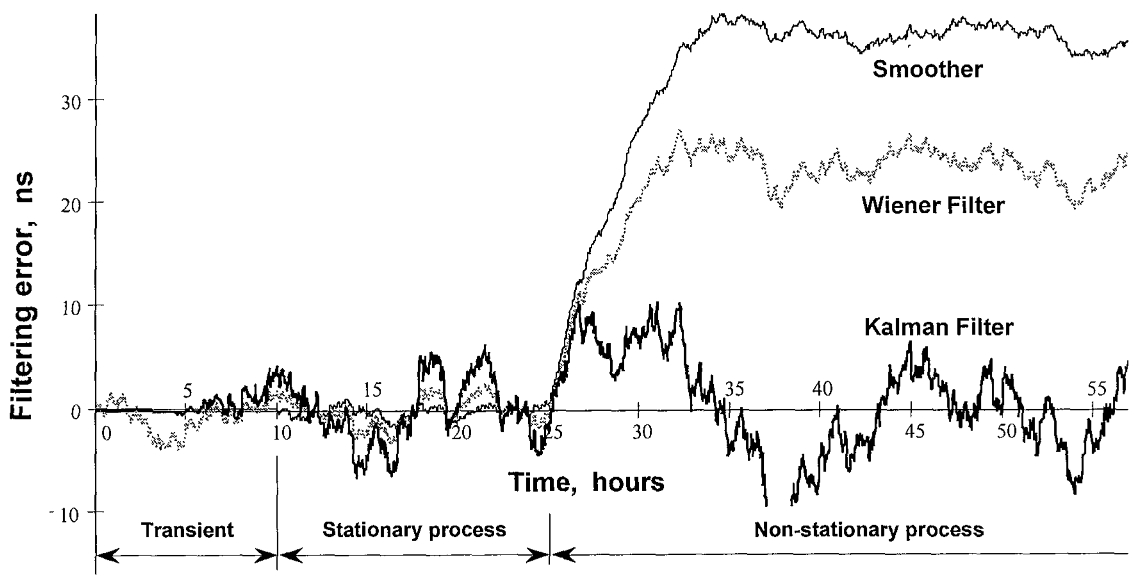


Figure 2. Errors of the average smoothing, Wiener, and three-state Kalman filters in the stationary and non-stationary ranges (see Fig.1)

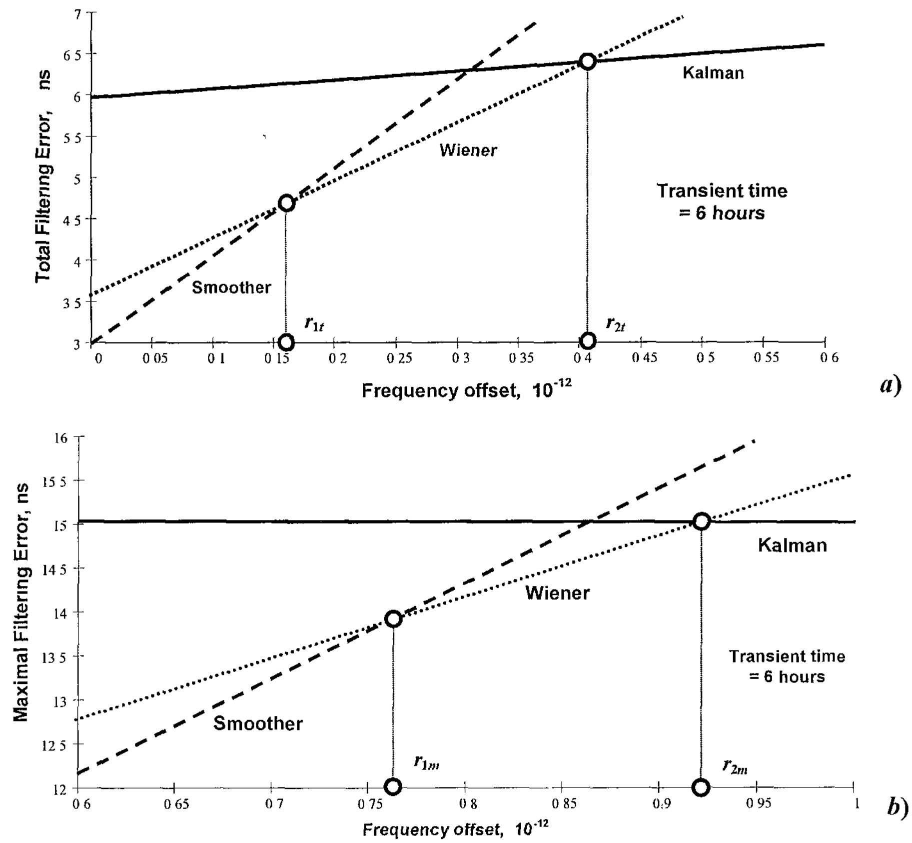


Figure 3. The total (a) and maximal (b) errors of the average smoothing, Wiener, and three-state Kalman filters as functions of the initial frequency offset y_0

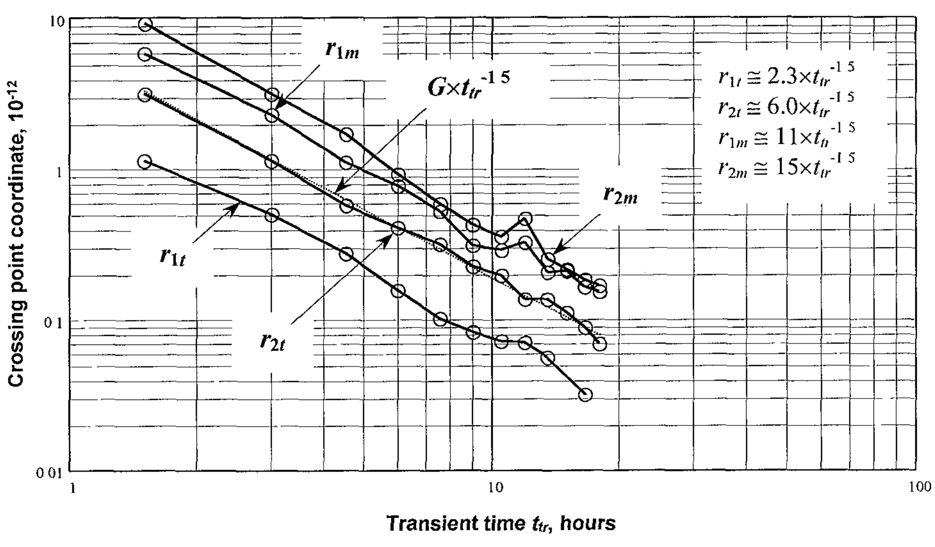
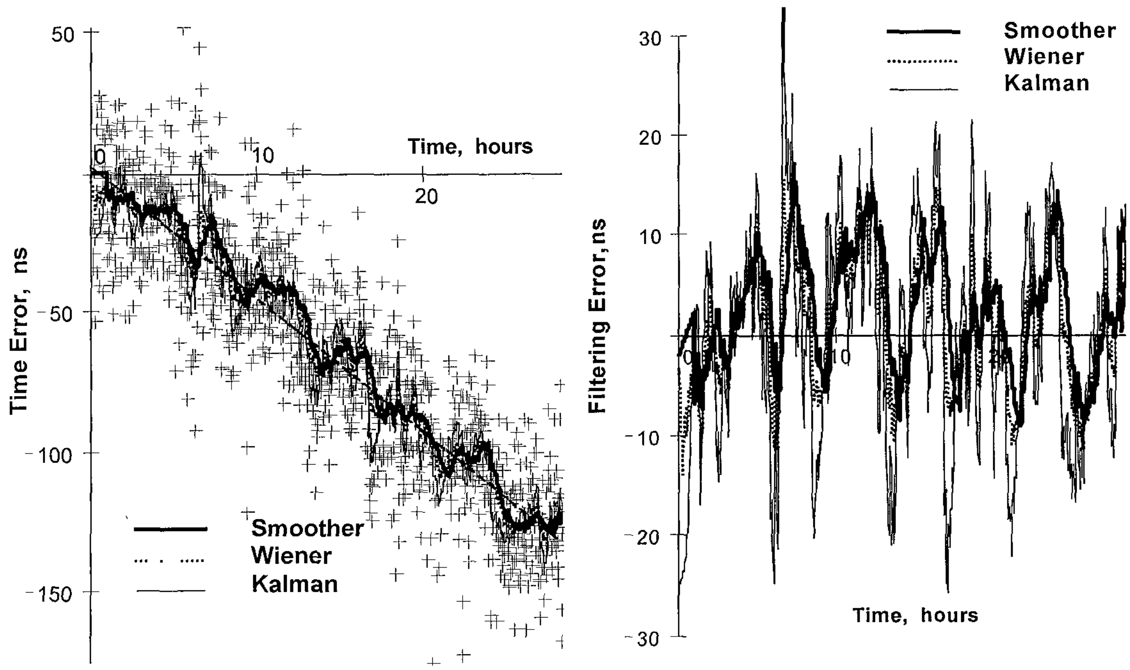
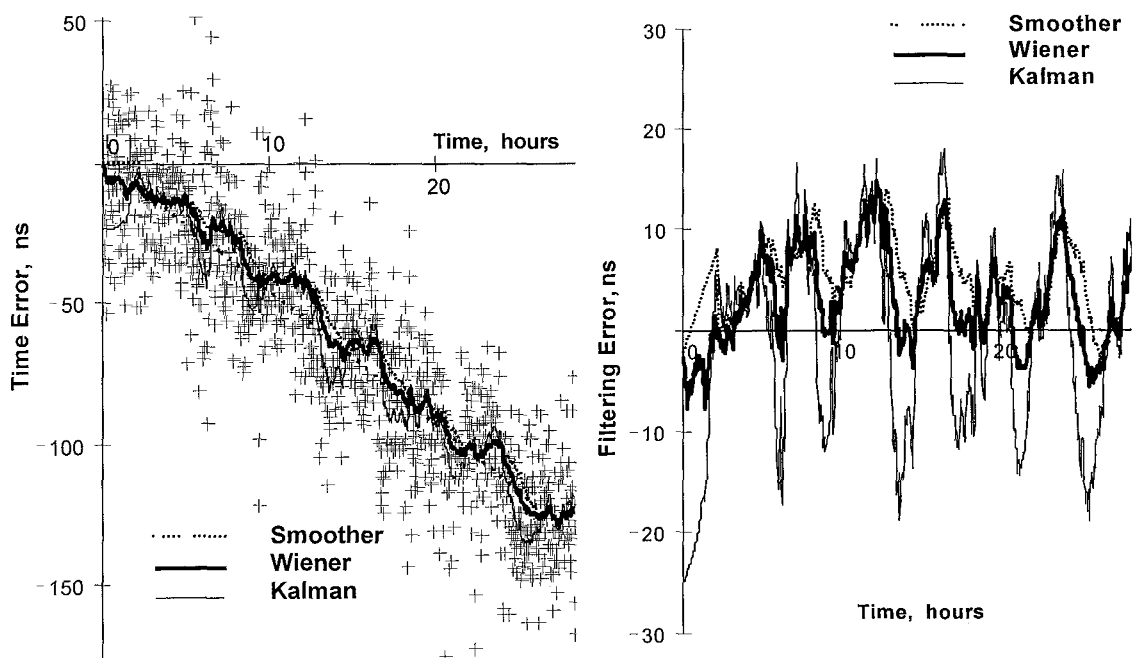


Figure 4. Coordinates r_{1t}, r_{2t}, r_{1m}, and r_{2m} as functions of the filters transient time t_{tr}, and correspondent approximating functions



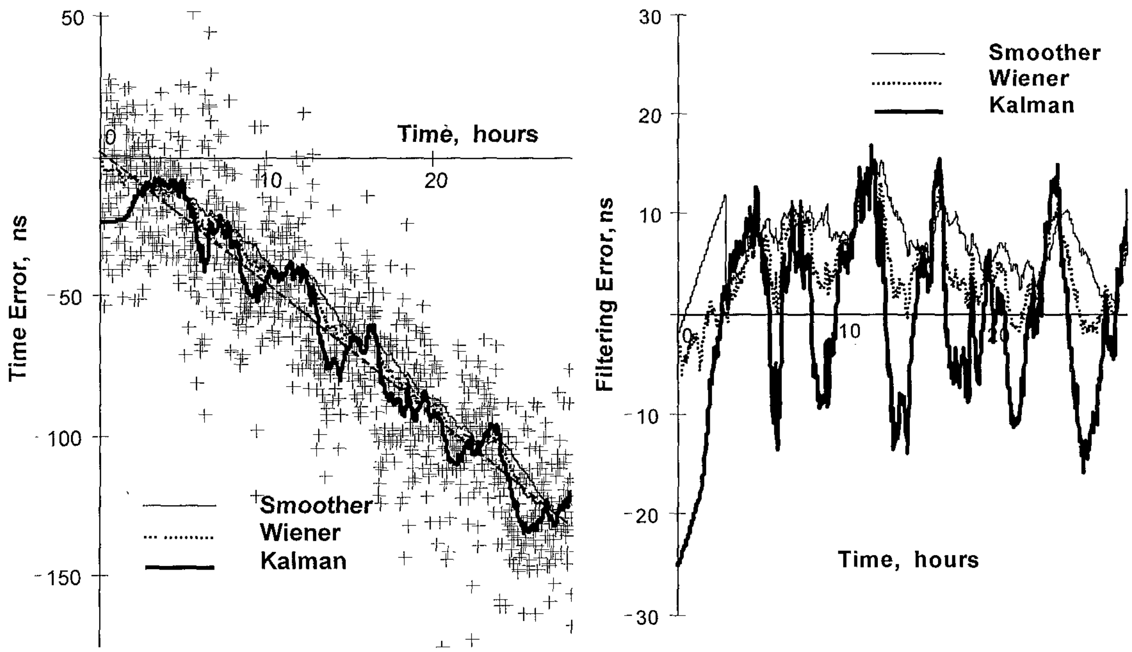
a) *b)*

Figure 5. Estimates for t_{tr} = 1.0hour (*a*) and filtering errors (*b*): the average smoother is the most accurate



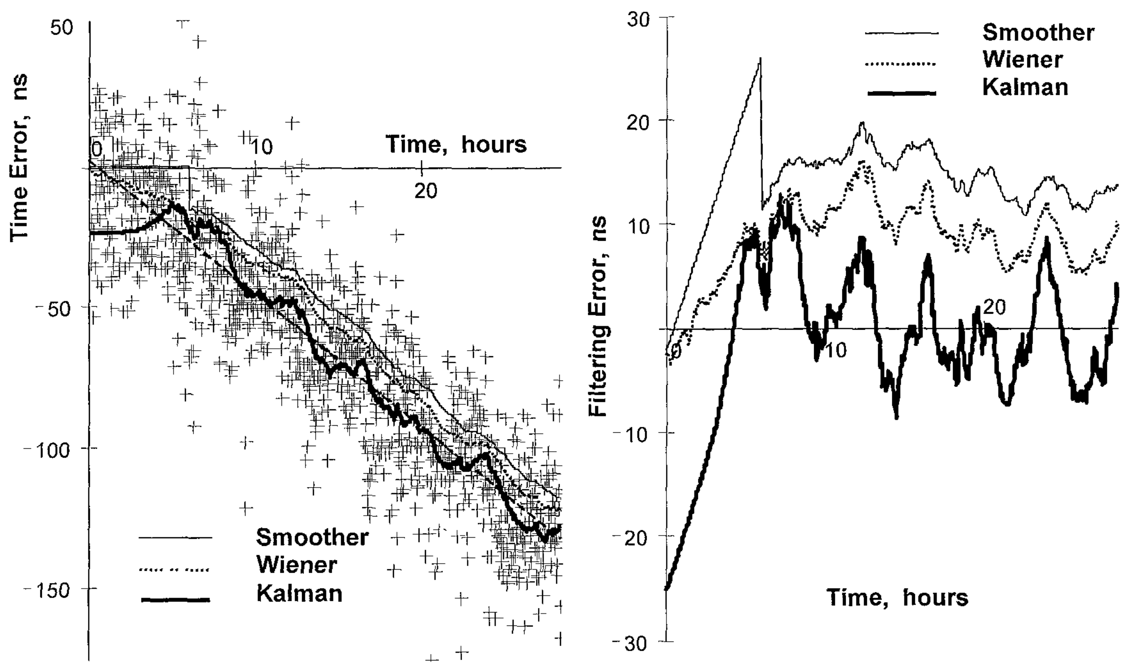
a) *b)*

Figure 6. Estimates for t_{tr} = 2.2hours (*a*) and filtering errors (*b*): the Wiener filter exhibits the smallest error



a) *b)*

Figure 7. Estimates for t_{tr} = 3.0hours (*a*) and filtering errors (*b*): the Kalman filter is the most accurate



a) *b)*

Figure 8. Estimates for t_{tr} = 6.0hours (*a*) and filtering errors (*b*): the Kalman filter seems obviously like the best estimator