A REVISED WAY OF FIXING AN UPPER LIMIT TO CLOCK WEIGHTS IN TAI COMPUTATION

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Abstract

The free atomic time scale Èchelle Atomique Libre (EAL), from which International Atomic Time (TAI) is derived by frequency steering, is obtained as a weighted average of a large number of free-running and independent atomic clocks spread worldwide, using the algorithm ALGOS which is optimized for long-term stability. Since January 1998, a new procedure for implementing an upper limit of clock weights has been used. The use of an absolute maximum weight p_{MAX}, was replaced by the choice of a relative maximum weight, ω_{MAX}. This new technique is more robust than the former one and it optimizes the stability of the time scale at the expense of a more complicated computation. The chosen value $\omega_{MAX} = 7.00x10^{-3}$ corresponded to the value of the maximum relative weight assigned to clocks in the EAL computation, with $p_{MAX} = 2500$, in the 60-day interval November/December 1997. In this paper, we show that $\omega_{MAX} = 7.00x10^{-3}$ is no longer appropriate. No efficient discrimination is made between the HP 5071A units: more than 80% of such clocks reach the maximum relative weight. The value of $omega_{MAX}$ really needs to be updated from time to time in order to obtain an efficient discrimination between the HP 5071A units and to improve the stability of EAL. To avoid frequent redefinition of $omega_{MAX}$, we suggest here making $omega_{MAX}$ a function of the number, N, of clocks that participate in TAI. A relation such as $\omega_{MAX} = A/N$, where A is an empirical constant, could be used. Such a relation has been tested using ALGOS with values 3.0, 2.5, and 2.0 for A. The resulting computed time scales (over 2.5 years) using real data show that all the HP 5071A units are not equivalent. We also obtain an improved stability for the computed time scales, which is the underlying aim of this study.

1 INTRODUCTION

The free atomic time scale EAL (échelle atomique libre), from which TAI is derived by frequency steering, is obtained as a weighted average of a large number of free-running and independent atomic clocks spread worldwide. The computation uses the algorithm ALGOS, which is optimized for long-term stability.

Since January 1998, a modified form of the algorithm has been used for the calculation of TAI. This algorithm is based on the same defining equations as ALGOS [1], but includes two changes:

(a) reduction of the time interval T of the computation from two months to one month [2];

(b) application of an upper limit to the relative weights, ω_{MAX}, instead of to the absolute weights, p_{MAX}, attributed to the contributing clocks [3, 4].

These changes are described in more detail in [5] and [6].

The value $\omega_{MAX} = 7.00 \times 10^{-3}$ was fixed at the time of the first computation (January 1998) to ensure continuity of the time scale. It corresponds to the value of the maximum relative weight assigned to clocks in the EAL computation (with $p_{MAX} = 2500$) in the 60-day interval November/December 1997. Our analysis of the distribution of the relative weights attributed to clocks shows, however, that this value is no longer appropriate.

The underlying aim of this study is to improve the stability of EAL and, hence, of TAI. To achieve this we need to take best advantage from the HP 5071A clocks and also from the hydrogen masers, by discriminating efficiently between them. Fixing an appropriate upper limit to the clocks weights in the EAL computation can ensure this.

As the choice of ω_{MAX} is empirical, we could simply update its value, but, as we demonstrate that fixing ω_{MAX} to a constant could lead to a situation where the weights ω_i attributed to clocks are not normalized.

To avoid such a situation, we suggest making ω_{MAX} a function of the number, N, of clocks participating in TAI. A relation such as $\omega_{MAX} = A/N$ is proposed, where A is an empirical constant.

Three test time scales (E2, E25 and E3) have been established over 2.5 years using real clock data. They are based on the algorithm ALGOS and use the values 2.0, 2.5 and 3.0, respectively, for the constant A. By comparing the distributions of the resulting relative weights attributed, we show that E2, E25, and E3 allow a much better discrimination between clocks than does EAL. As they rely more heavily on the very best clocks, the time scales E2, E25, and E3, are thus more stable than EAL. We conclude that the stability of EAL, and hence that of TAI, can be improved.

2 THE UPPER LIMIT OF RELATIVE WEIGHTS IN EAL COMPUTATION

The computational process used to apply an upper limit to relative weights in ALGOS is described in [4]. The most important feature of this process is that it does not independently assign a weight to each clock; instead, the set of clocks is treated globally.

The classical variance of the frequency of clock H_i relative to EAL [$\sigma_i^2(12,T)$, computed from 12 consecutive 30-day samples] plays an important part in the computation of its relative weight, ω_i, and therefore has an effect on the stability of EAL. This variance $\sigma_i^2(12,T)$ (calculated over twelve consecutive samples of duration T) is an estimate of the zero-dead-time sample variance $\sigma_i^2(12,T,T)$ [7, 8] of the frequencies of clock H_i relative to EAL, and is linked to the usual Allan variance $\sigma_{y_i}^2(T)$ by the relation:

$$\sigma_i^2(12,T,T) = B_1 \sigma_{y_i}^2(T) \tag{1}$$

where $B_1 = 1.924$ in the case of flicker-frequency noise modulation over averaging times $T = 30$ d.

When EAL is computed, $\sigma_i^2(12,T)$ values of the clocks reaching ω_{MAX}, differ significantly. We define σ^2_{MIN} to be the largest of these variances, corresponding to the least stable clock attributed ω_{MAX}.

Figure 1 shows the distribution of relative weights, $\{\omega_i, i = 1, N\}$, attributed to clocks in the EAL computation. The given values are the means calculated over 2.5 years (from January 1998 until June 2000). The number of clocks reaching $\omega_{MAX} = 7.00 \times 10^{-3}$ and falling within five other classes of relative weight ω_i are presented. The classes are defined below:

class	I	$80\ \%\ \omega_{MAX}$	$\leq \omega_i$	$< \omega_{MAX}$
class	II	$60\ \%\ \omega_{MAX}$	$\leq \omega_i$	$< 80\ \%\ \omega_{MAX}$
class	III	$40\ \%\ \omega_{MAX}$	$\leq \omega_i$	$< 60\ \%\ \omega_{MAX}$
class	IV	$20\ \%\ \omega_{MAX}$	$\leq \omega_i$	$< 40\ \%\ \omega_{MAX}$
class	V		ω_i	$< 20\ \%\ \omega_{MAX}$

We observe that 123 clocks reach ω_{MAX} and that there are less than 10 clocks within each class, apart from class V which includes 29 clocks. Fixing ω_{MAX} to 7.00×10^{-3} thus allows a large number of clocks (70 % of the total number) to reach this upper limit.
The clocks that contribute to the construction to EAL can be separated into three different clock types: hydrogen masers, HP 5071A clocks, and other caesium clocks. Figure 2 shows the distribution of relative weights for these three clock types. Again the same classes of relative weights are presented. For each clock type there are less than 5 units within each class, apart from class V which includes 3 HP 5071A clocks, 6 hydrogen masers and 20 other caesium clocks. Among the clocks reaching ω_{MAX}, we have: 93 HP 5071A clocks (83 % of the total number of such clocks), 17 hydrogen masers (60 % of the total number of hydrogen masers), and 31 % other cesium clocks (31 % of this group). These data are summarized in Table 1. It is clear that the discrimination is not efficient, especially for the HP 5071A clocks and the hydrogen masers.

3 NEED TO UPDATE ω_{MAX}

When attributing relative weights, efficient discrimination between clocks must be made in order that the resulting time scale relies most heavily upon a maximum number of the very best clocks. With $\omega_{MAX} = 7.00 \times 10^{-3}$, however, clocks with $\sigma_i\,(12,T) = 15.9 \times 10^{-15}$ are given the same relative weight as clocks with $\sigma_i\,(12,T)$ values many times less than this. Figure 3, a histogram of the $\sigma_i(12,T)$ data, clearly shows that $\omega_{MAX} = 7.00 \times 10^{-3}$ yields a value of σ_{MIN} that does not achieve the required discrimination. It is, therefore, necessary to update ω_{MAX} such that the corresponding σ_{MIN} value is small enough that such discrimination is possible.

Figures 4 and 5 show the relative frequency stabilities of some of the best HP 5071A clocks and hydrogen masers used in the establishment of TAI. On each figure, the indicated value of $\sigma_{yMIN}(30)$, computed from (1), differs significantly from the plotted $\sigma_{y_i}(30)$ values.

4 REVISED WAY OF FIXING AN UPPER LIMIT TO CLOCK WEIGHTS

We propose not to fix ω_{MAX} to a given value, but to make ω_{MAX} a function of the number, N, of clocks contributing to the construction of TAI. A relation such as $\omega_{MAX} = A/N$ is suggested, where A is an empirical constant. There is no fundamental difference between this function and a fixed value; in both cases the choice of ω_{MAX} is empirical. Nevertheless, a fixed value of ω_{MAX} could lead, if N were small, to a situation where N_1 clocks reach ω_{MAX}, while the weights of the remaining $(N - N_1)$ clocks is zero. This situation is very unlikely to occur, but if it did, then the relative weights would not be normalized and the relation of definition of EAL would no longer be valid. Such a situation could not occur with the proposed alternative: if N decreased suddenly, then ω_{MAX} would increase so that discrimination among clocks would still be made and the normalization requirement fulfilled.

5 TESTS ON REAL DATA

In this section three time scales are considered and compared to EAL. Each is calculated by running the ALGOS algorithm on real clock data, using different values of $\omega_{MAX} = A/N$. These time scales are E2, E25, and E3, with values of 2.0, 2.5, and 3.0, respectively, for the constant A.

The results for our set of clocks over the period from January 1998 to June 2000 are summarized in Table 1, where they are compared with EAL, and the properties of the test time scales are described in more detail below.

Table 1. Summary of the results of the four time scales EAL, E2, E25, and E3 calculated over the period January 1998 to June 2000. The values of ω_{MAX} and σ_{MIN} are indicated, along with the fraction of clocks attributed the maximum weighting.

Time scale	$10^3 \times \omega_{MAX}$	$10^{15} \times \sigma_{MIN}$	100 × Fraction of clocks reaching ω_{MAX}			
			Total	HP 5071A	Hydrogen masers	Other Cs clocks
EAL	7.00	15.9	70	83	60	31
E2	9.51	7.8	41	50	41	12
E25	11.89	5.8	27	33	36	7
E3	14.27	4.9	18	20	3	1

5.1 TIME SCALE E2

Figures 6 and 7 show the distribution of relative weights attributed to clocks in the computation of E2. The results seem more satisfying than those obtained with EAL (cf. Fig. 1) in the following respects:

- it reduces reasonably the amount of clocks reaching this upper limit;
- it allows a given clock H_i, providing it does not show abnormal behavior, to reach ω_{MAX} if its $\sigma_i(12,30)$ value is less than or equal to 7.8×10^{-15};
- it allows a reasonable discrimination between the clocks;
- it relies more heavily upon the best clocks.

5.2 TIME SCALE E25

Figure 8 shows the distribution of relative weights, attributed to clocks in the computation of E25. The value σ_{MIN}, corresponding to the least stable clock attributed ω_{MAX}, is nearly three times smaller than that obtained for EAL (Table 1). The conditions for a given clock to reach the upper limit of relative weights are more severe here than for EAL or E2.

The distribution of relative weights for the three clock types defined in Section 2 is presented in Figure 9. We observe that an efficient discrimination among the clocks is made, and that the number of clocks at ω_{MAX} is still great enough to ensure the reliability of the time scale. Figure 10 shows a histogram of the standard deviation $\sigma_i(12,30)$ and the position of σ_{MIN}.

For the group of clocks considered, the time scale E25 has the following advantages over EAL:

- it substantially reduces the number of clocks attributed ω_{MAX};
- it allows a given clock H_i, providing it does not show abnormal behavior, to reach ω_{MAX} when its $\sigma_i(12,30)$ value is less than or equal to 5.8×10^{-15};
- it allows efficient discrimination between the clocks;
- it relies more heavily upon the very best clocks.

5.3 TIME SCALE E3

Figures 11 shows the distribution of relative weights attributed to clocks in the computation of E3, and Figure 12 shows the distribution of relative weights for the three different clock types. Only 18 % of the clocks are attributed the maximum relative weight, and this fraction is not enough large to ensure the reliability of TAI. Here the conditions required for a clock to obtain the maximum relative weight are too severe. Figure 13 shows a histogram of the standard deviations $\sigma_i(12,30)$ and the position of σ_{MIN}.

With the considered clock ensemble, E3

- reduces excessively the number of clocks attributed ω_{MAX};
- allows a given clock H_i, providing it does not show abnormal behavior, to reach ω_{MAX} if its $\sigma_i(12,30)$ value is less than or equal to 4.9×10^{-15};
- provides severe discrimination between the clocks;
- relies more heavily upon a selection of the very best clocks than the other time scales considered here, but the small fraction selected is not large enough to ensure the reliability of the resulting time scale.

5.4 STABILITY OF TIME SCALES EAL, E2, E25, AND E3

The stabilities of the time scales EAL, E2, E25, and E3 have been compared to three of the best independent time scales in the world: those maintained at the NIST (Boulder, Colorado, USA), the BNM-LPTF (Paris, France), and the AMC (Colorado Spring, USA). Intrinsic values of $\sigma_y(\tau)$ computed using the N-cornered-hat technique are shown in Figure 14.

As expected, because the time scales E2, E25, and E3 allow more efficient discrimination between the clocks, they rely more heavily upon the very best units and are consequently more stable than EAL.

6 CONCLUSIONS AND PROPOSAL

The fixed value 7×10^{-3} for the upper limit of clock weights ω_{MAX} is no longer appropriate because it does not allow efficient discrimination between the clocks. It is suggested that this fixed value be replaced by a function $\omega_{MAX} = A / N$, where A is an empirical constant and N is the number of clocks participating in the construction of TAI. Such a function would avoid a possible problem described in Section 4 which could arise if an insufficient number of clocks were available.

The suggested function has been tested successfully, with $A = 2.0$, $A = 2.5$, and $A = 3.0$, using real clock data over 2.5 years, and the resulting time scales E2, E25, and E3 are more stable than EAL.

We, therefore, propose to adopt a function such as $\omega_{MAX} = A/N$ to define the upper limit for clock weights in the algorithm ALGOS used to calculate TAI. Regarding the choice of the constant A, among the three tested values, A=3.0 must be set aside because it causes an excessive reduction in the number of clocks reaching ω_{MAX}. A more appropriate value seems to be $A = 2.5$; the corresponding time scale E25 allows a more efficient discrimination between clocks and shows a better stability. Adoption of this value for TAI computation would, however, change ω_{MAX} from 7.00×10^{-3} to 11.89×10^{-3}. Such a discontinuity could influence the behavior of TAI and should be avoided. It would be preferable to move more gradually towards this value, for example by using $A = 2.0$ for one year and then moving to $A = 2.5$.

We, thus, propose to set $A = 2$ from January 2001 until December 2001, and to set A=2.5 from January 2002 onwards. In this way ω_{MAX} will change from 7.00×10^{-3} to 9.51×10^{-3} at the beginning of January 2001, and from 9.51×10^{-3} to 11.89×10^{-3} in January 2002. Based on the experience of the BIPM Time Section, such small changes will not perturb the behavior of TAI.

7 REFERENCES

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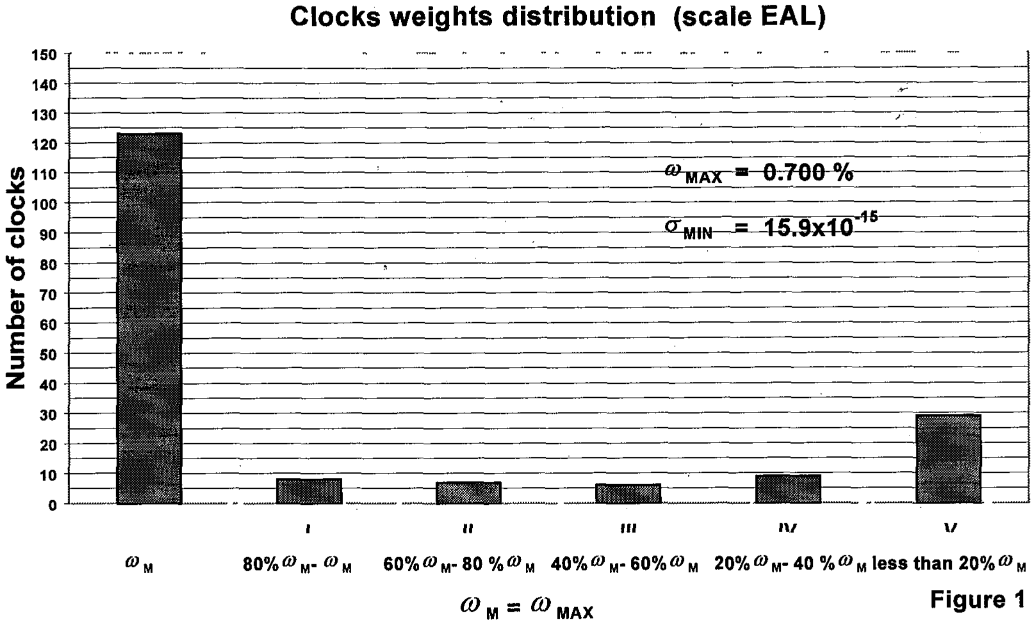


Figure 1

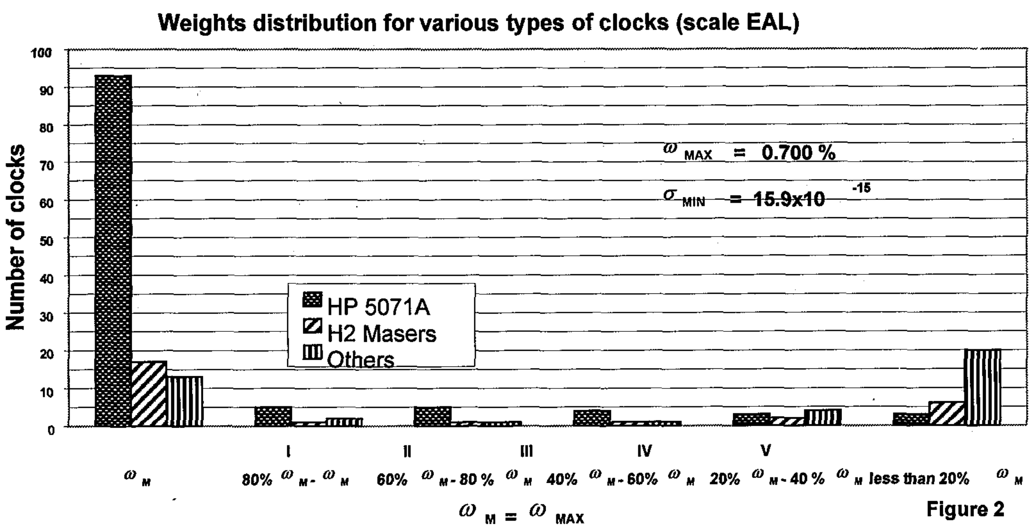


Figure 2

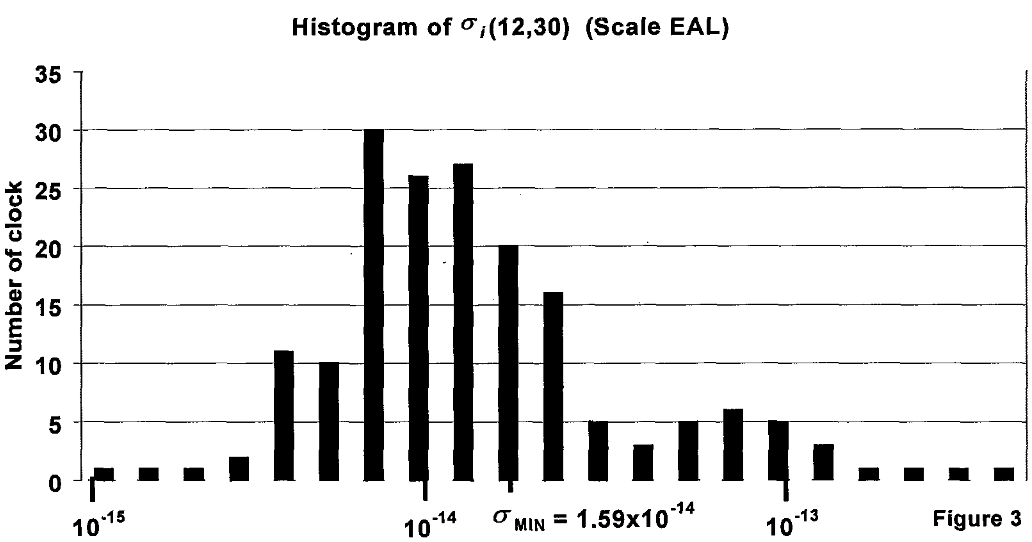
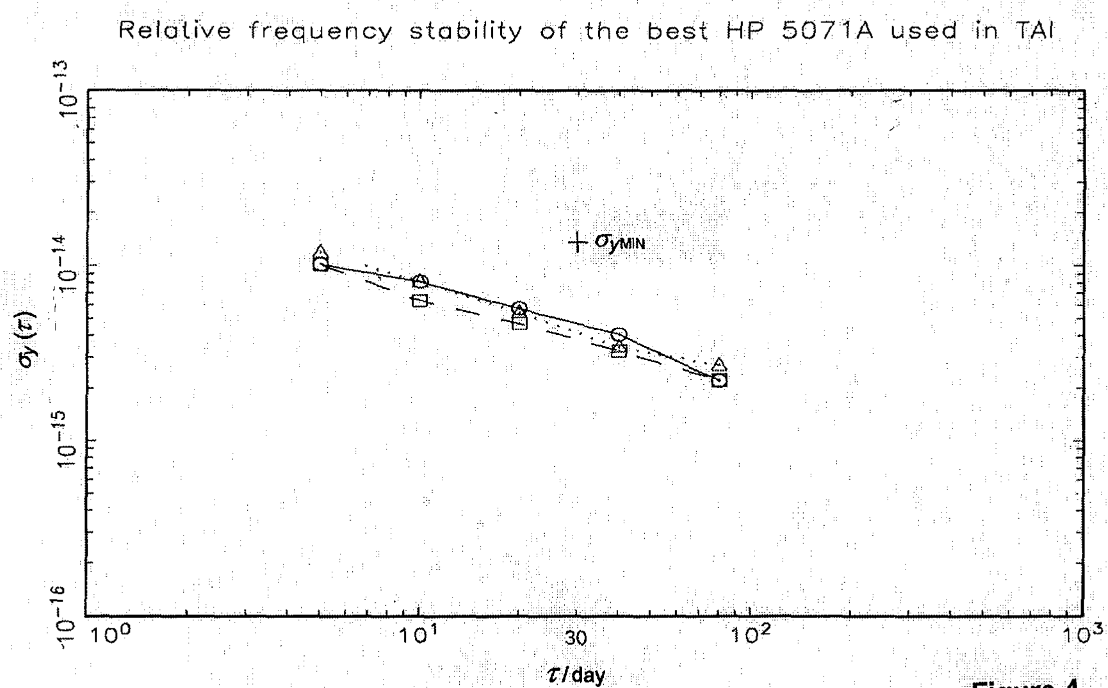


Figure 3



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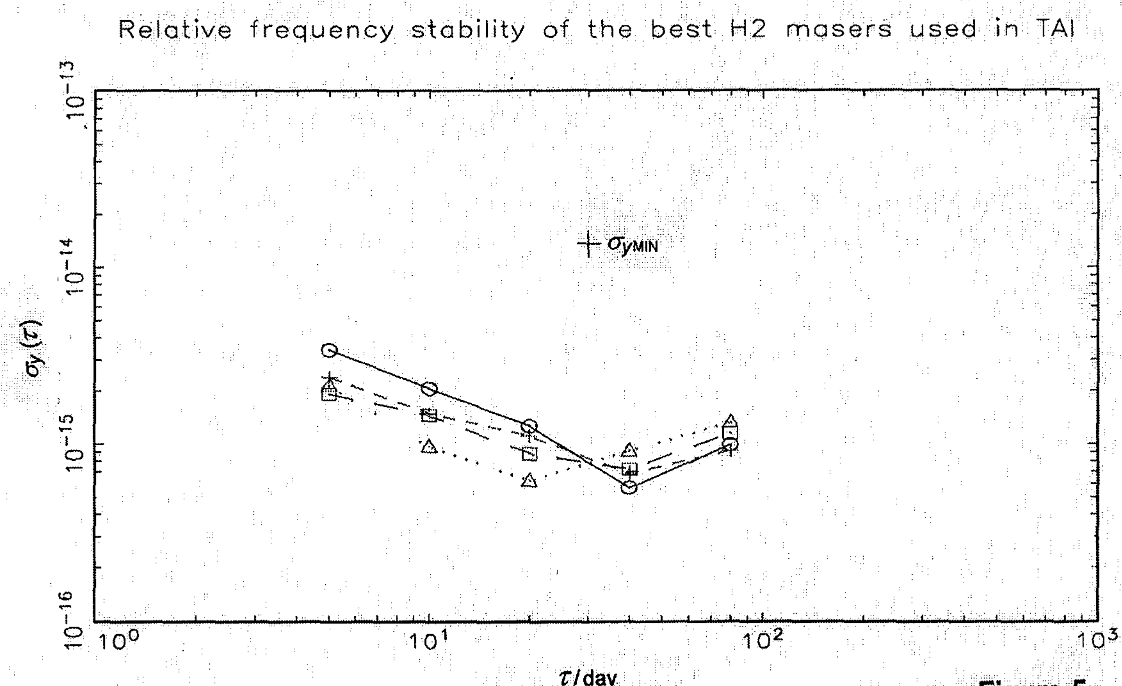


Figure 5

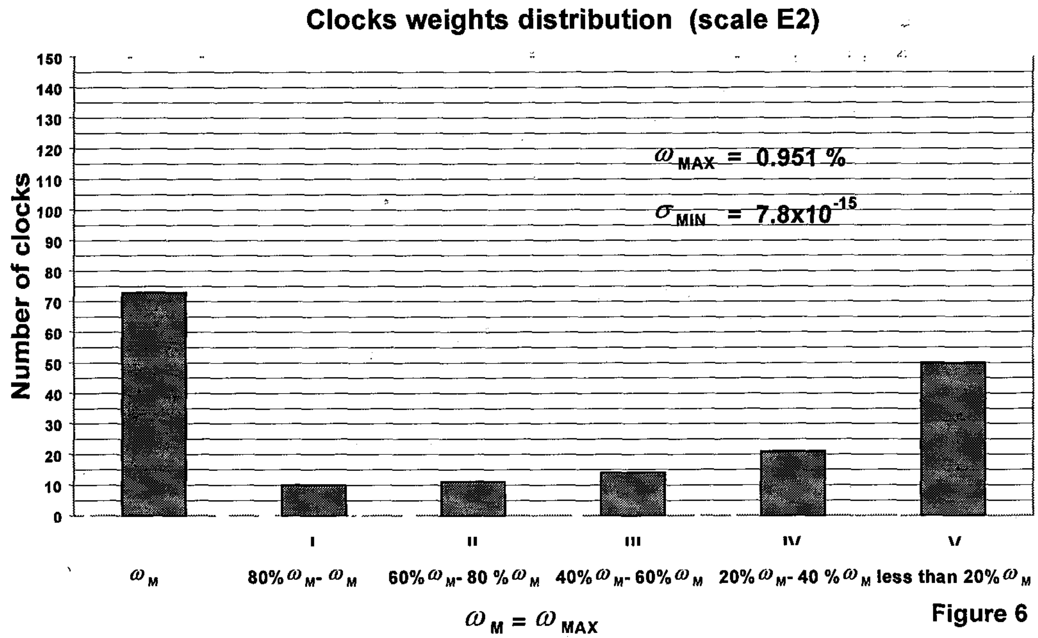
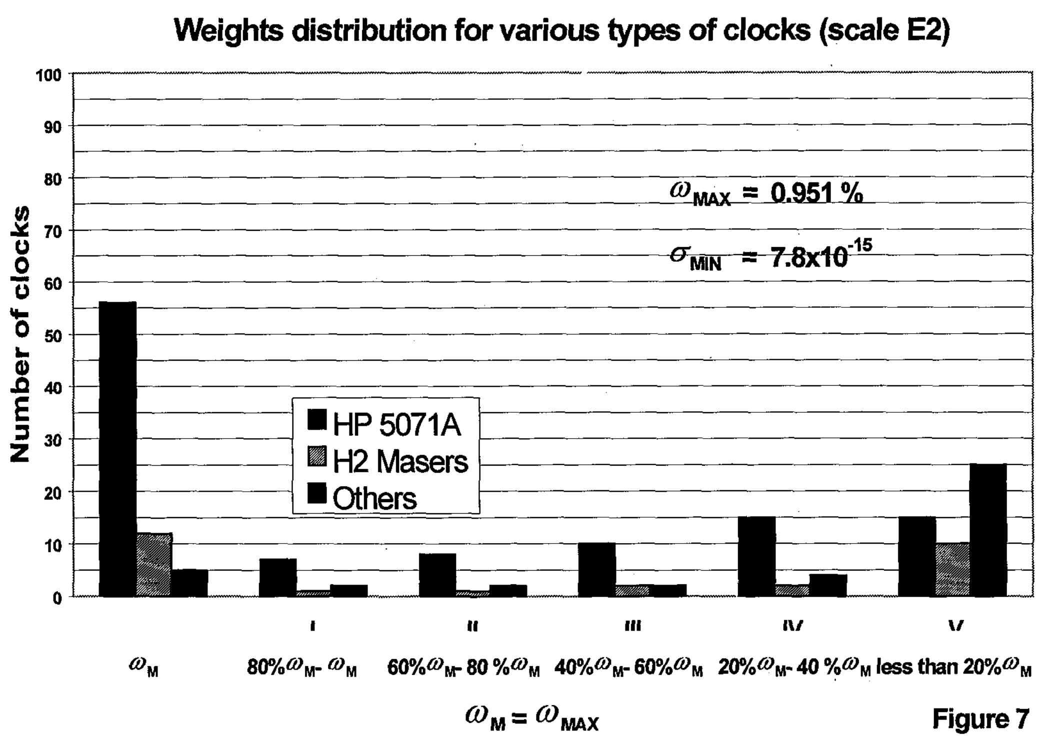


Figure 6



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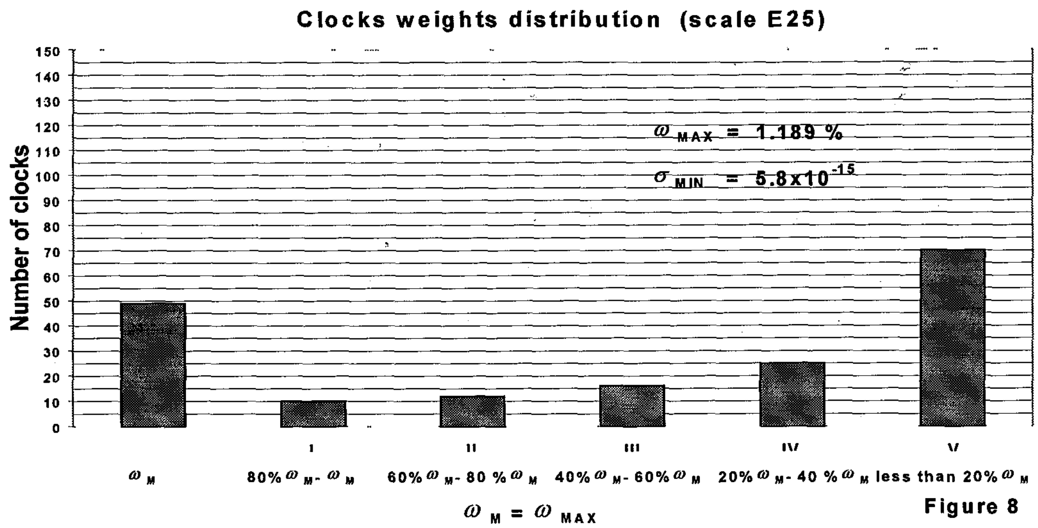


Figure 8

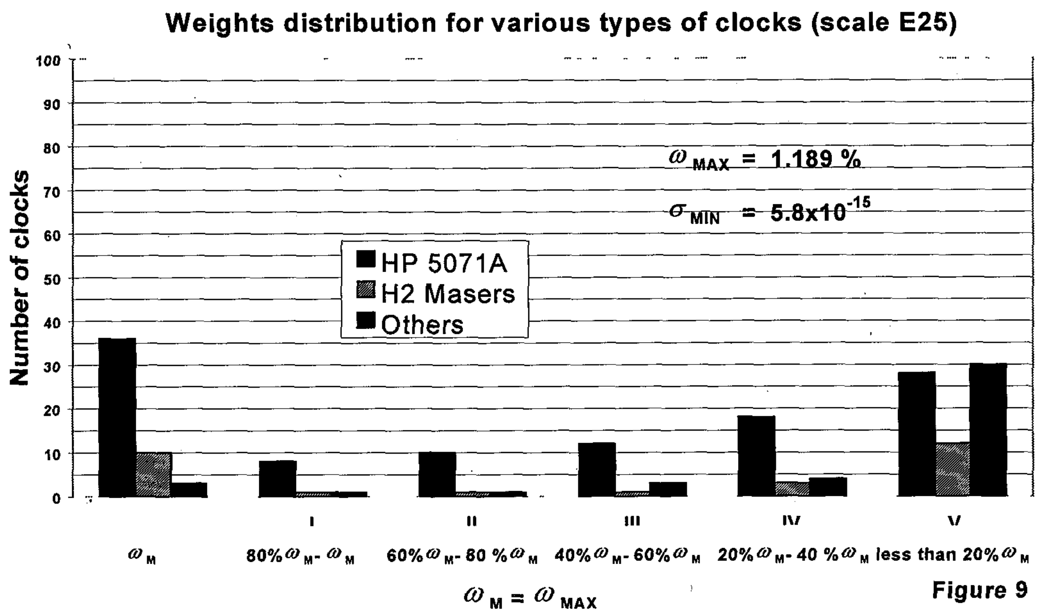


Figure 9

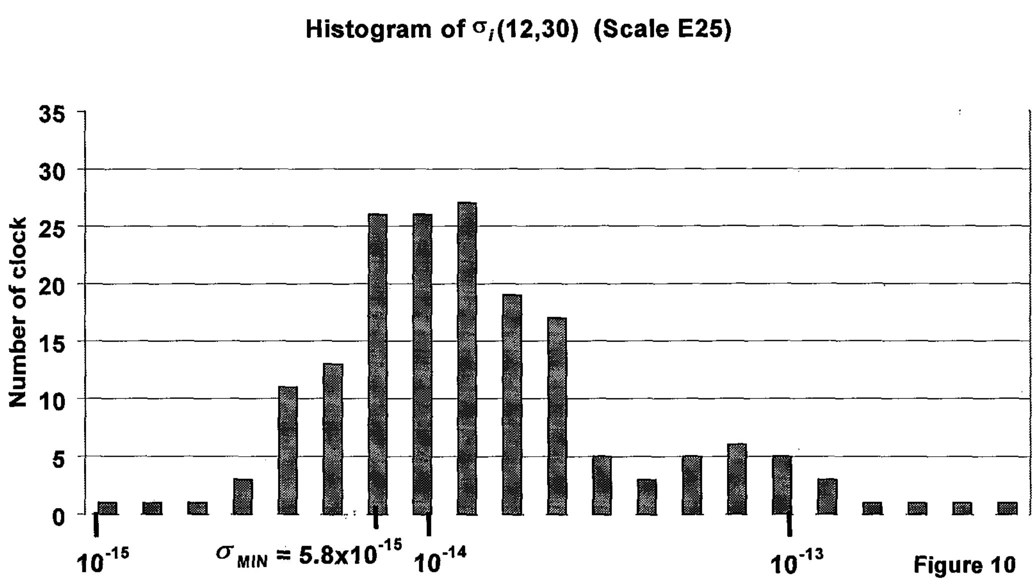


Figure 10

Clocks weights distribution (scale E3)

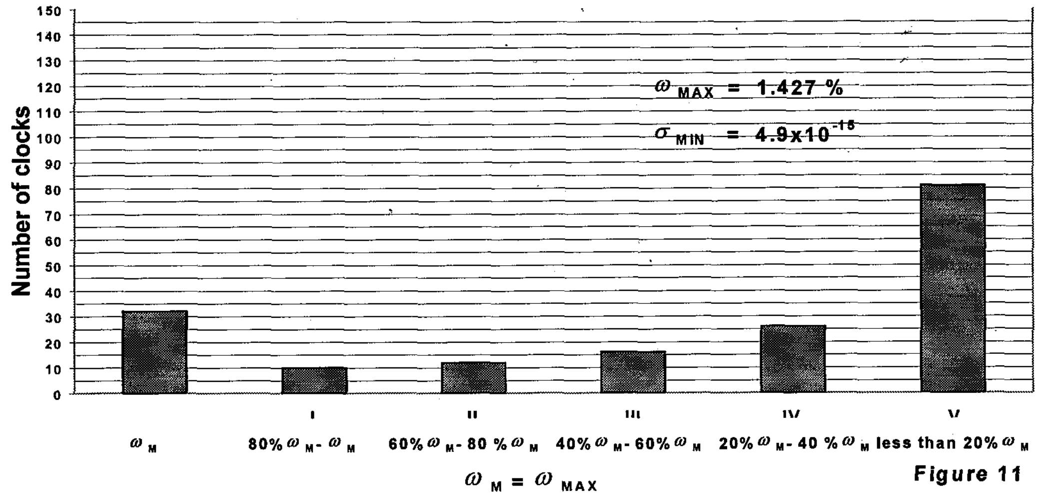


Figure 11

Weights distribution for various types of clocks (scale E3)

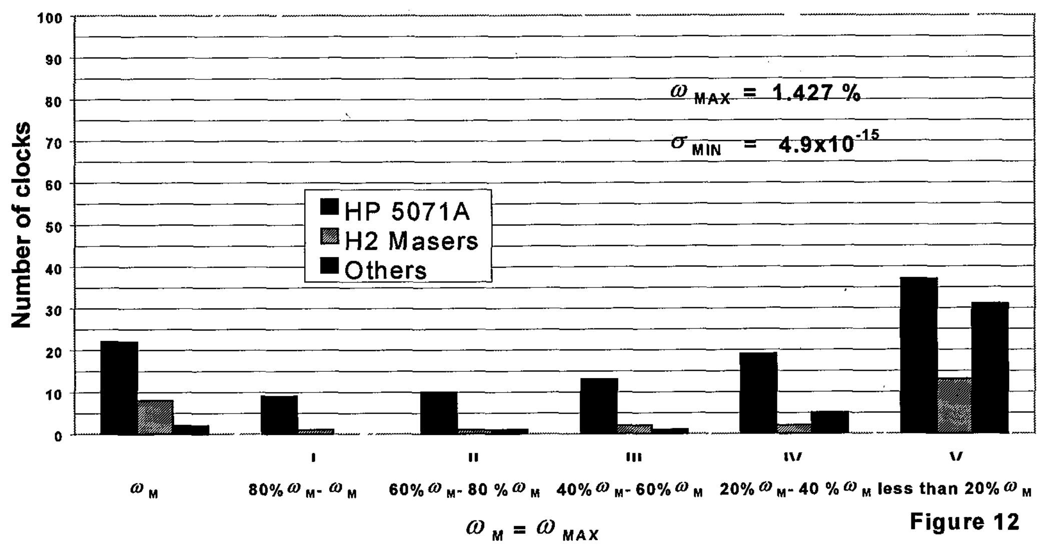


Figure 12

Histogram of $\sigma_i(12,30)$ (Scale E3)

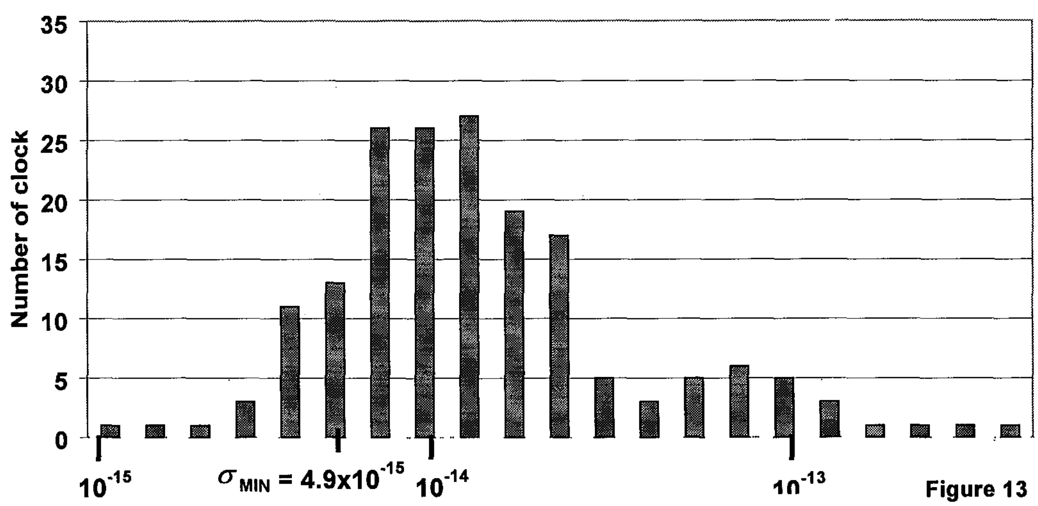
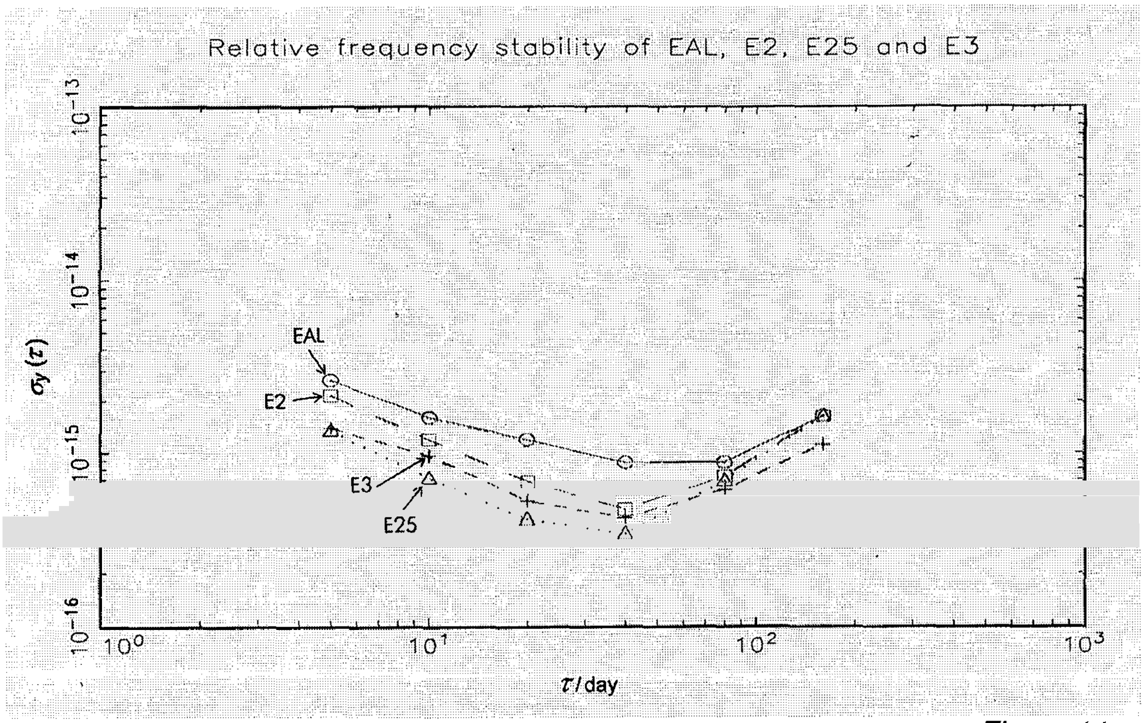


Figure 13


Relative frequency stability of EAL, E2, E25 and E3
$\sigma_y(\tau)$
τ / day
EAL
E2
E3
E25
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{0}
10^{1}
10^{2}
10^{3}

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Questions and Answers

JUDAH LEVINE (NIST): I think that's a good idea. One of the things that you will have to be concerned about is as you raise the maximum weight that a clock can have, the hydrogen masers will begin to take over, because in short term, they are much more stable than anything else. But that will introduce frequency drift into TAI, because almost all the hydrogen masers drift. That will mean that you may need more steering to TAI as the hydrogen masers begin to pull the scale. As you make the weight larger and larger, that will become an effect that you will begin to see. I mean, we have that effect on our hydrogen masers; they drift.

JACQUES AZOUBIB: The drift of hydrogen masers is not taken into account in the algorithm because we have a test that analyzes if a clock shows an abnormal behavior. And if a hydrogen maser shows a frequency drift, its weight is set to zero.

LEVINE: The drift would be too small for you to see. It will be like 10^{-16}/day. In short term, it will look much, much better than a cesium.

AZOUBIB: Yes, in short term, yes.

WLODZIMIERZ LEWANDOWSKI (BIPM, France): Just a comment on what was said. What Jacques is suggesting, if I understand, it will not increase the number of masers used, but it will take into account the best masers. So there will be a big increase of the use of masers, but it will be an increase of the best, taking advantage of the best masers. There will be a better discrimination between masers, but not an increase of the number of masers used for TAI. The idea is a better discrimination between the best clocks and not to increase. So the best masers will have bigger impact on TAI. There will be no more masers.

AZOUBIB: You can see on this scale that the maximum weight will have 10 hydrogen masers, exactly. It is 36% of the number of hydrogen masers. In EAL with omega maximum at 0.7%, we have 60% of the total number of hydrogen masers. That means that here only the very, very best hydrogen masers are selected.

DEMETRIOS MATSAKIS (USNO): I'll just mention my prediction which I made in print, and you've heard before. I wonder if you would comment on my comment. I made a prediction that 10 years from now the BIPM will be using the improved time transfer, probably two-way, but maybe GPS carrier-phase, to view EAL much more closely spaced, and they will be using masers completely for the short term; and they'll be steering it to cesiums in the long term. Which would answer Judah's comment. And I see this as a step in that direction.

AZOUBIB: Yes, we are working in that direction.

MATSAKIS: Let me ask you a question also. I just wondered when you computed your sigmas, you did it by N-cornered hats and, you know, that they are very—

AZOUBIB: Yes, of course, and I used the best time scales in the world, but not USNO's. Not because it's a bad time scale, but it is too much correlated with the scales. USNO has too many clocks. I used NIST, the AMC, and LPTF.

MATSAKIS: And they all agreed.

AZOUBIB: Yes, because the correlation is very small. For instance, for NIST, we have 9% percent of the total weight, so the correlation is small enough to use it.

MATSAKIS: You could also make a prediction based on elementary Gaussian statistics.

AZOUBIB: Yes.

MATSAKIS: Did that agree with the other thing?

AZOUBIB: Our way to work with this is, for each computation make a diagram and to fix sigma squared so that one clock of the three reaches the maximum weight. But it is far too complicated to do now. We work step by step, you know.

MATSAKIS: You need Judah's computers.