NEW WAYS TO VISUALIZE TIME AND FREQUENCY DATA

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Abstract

Some of the standards of numerical analysis in time and frequency are the formation of the various square roots of variances, such as Time Deviation (TDEV), Allan Deviation (ADEV), and Total Deviation (TotDev), among others. As time and frequency measurements and transfer becomes better and better, especially at smaller sampling intervals, transient disturbances from such things as environmental perturbations become more and more important to characterize, locate, and understand. While developing software tools to more fully analyze, visualize, and model time and frequency data, especially time transfer data, several "new" ways of looking at the data were tested for usefulness. One new way of looking at time-series data was first reported in 1987 and is called visual recurrence plots or analysis (VRA) [1]. VRA, the auto-correlation function (ACF), power spectral density by the Barnes' Digital Spectrum Analyzer method [2] (PSD), periodogram using phase-dispersion-minimization techniques (Jurkevich[7]), phase plane visualization (PPV), time-frequency analysis (TFA), and even 1-D wavelet decomposition of a time-series signal are being tested. This paper will show some recent results that show that all these numerical tools are useful. Tests will be run on both real and synthetic data.

INTRODUCTION

Over the years the author has been developing and locating useful software tools which lend themselves to useful analysis and visualization of time-series data, both equally spaced and non-equally spaced data, both in real-time and after the fact, and are generally platform–independent. Recent data obtained between USNO(MC2) and USNO AMC(MC1) via TWSTT and a recent article on real-time data analysis [3] brought this subject back to the author's attention. The goal of this paper is to encourage researchers and anyone with time-series data to fully investigate their data and not just stop with the standard TDEV, ADEV, or TotDev variance analysis.

VISUAL RECURRENCE ANALYSIS (VRA)

The author was surfing the Web recently looking for software to compute a Henon mapping [4], phase-plane visualization, and other chaos analysis tools for analysis of the output of his Burlirsch-Stoer method of integrating the solar-system and minor planets. One of the search engine hits produced an interesting link to Visual Recurrence Analysis (VRA) software [5] which was apparently developed for analysis and prediction of the stock markets! The software was downloaded and found to be quite useful and fully functional in that it allows the user to import his/her own data files.

It was learned under the VRA help pages that a new method of visualizing equally spaced time series data called recurrence plots was reported in 1987 and was developed to aid researchers in analysis of dynamical systems [1]. The method produces 2-D plots or images which show large scale structure the authors call typology and small-scale structure they call texture.

The formulation to make a 2-D recurrence image (plot) from a time series [5] is

$$y(i) = (x(i), x(i+d), x(i+2d), ..., x(i+(m-1)d) \qquad 1)$$

where

i is the time index of the original signal X(t),
m is the embedding dimension and,
d is the time delay.

A series of vectors are produced [5]

$$Y=\{y(1), y(2), y(3), ..., y(N-(m-1)d)), \qquad 2)$$

where

N is the length of the original time series.

The result may be a color-coded matrix (image) of the Euclidean distances between all pairs of vectors.

For random signals a uniform pattern over the entire image is seen, while the more deterministic the input signal, the more structure is seen in the image [5].

Also available in the VRA software is a single value estimate of the structure in the VRA image called the Spatial-Temporal Entropy (STE) [5], but no details are given as to how it is computed. 100% entropy indicates pure randomness (unpredictability) in the VRA, while 0% entropy indicates perfect structure (predictability).

PHASE-PLANE VISUALIZATION (PPV)

Phase-plane views of time series are produced from the original time-series by plotting y from the original (x,y) pairs with the first difference of the original y pairs to form essentially (y, y'). In our examples below this will be a time difference between clocks and their rates.

EXAMPLES OF VRA VIEWS OF WHITE NOISE

One of the common power-law noise processes seen in time and frequency keeping is white PM. A synthetic White PM time series was constructed containing 1,000 samples. The time series looks white and normally distributed (Figure 1). The AC shows no correlation at any lag (Figure 2). The PSD is flat (Figure 3). The TDEV shows a slope of -1/2 a characteristic of white PM (Figure 4). The PPV shows a symmetrical distribution along the diagonal (Figure 5). The VRA of white PM shows an overall flat and uniform distribution with little if any significant structure (Figure 6). The STE for this data set is 95%.

EXAMPLES OF VIEWS OF WHITE FM IN THE TIME DOMAIN

Another common power-law noise process often seen is white FM in the time domain. A synthetic time series was again produced and contains 1,000 samples. The times series looks standard for this noise process (Figure 7). The AC shows correlation with lag (Figure 8). The PSD is well behaved and shows a slope of -1.752 (Figure 9). The TDEV shows an overall slope of +½ which is a characteristic of white FM in the time domain (Figure 10). The PPV shows a nearly white frequency distribution across the y-coordinate (frequency) coordinate, but an extension in the x-coordinate (time) which comes from the slight overall slope as seen in the original times series (Figure 11). The VRA image starts to show typological, large-scale structure and shows the small slope as a gradient towards the upper-left and lower-right corners (Figure 12). The STE is 89%.

USNO(MC2) - USNO AMC(MC1) 1-DAY AVERAGES VIA TWSTT

USNO(MC2) is an auxiliary output generator (AOG) whose source is a hydrogen maser. USNO(MC2) is steered once per day using classical methods towards a steered version of our USNO(A.1), which is called USNO(Mean). USNO(Mean) is an on-time and on-rate real-time estimate of UTC(BIPM). The USNO AMC(MC1) is also an AOG with a hydrogen maser source. The USNO AMC(MC1) output is steered nominally once per hour to USNO(MC2) via a Kalman filter.

The time series shows similar structure to white FM in the time domain with some amplitude dampening after about MJD 50750 (Figure 13). The AC shows slight correlation with very small lags (Figure 14). The PSD has a shallow minimum or flattening between frequency 0.001 and 0.03, but otherwise looks much like the

PSD of White FM in the time domain (Figure 15). The spectrum slope is -1.599. The TDEV shows a strong, but small amplitude periodic component with a peak deviation at about 15 days (Figure 16). The TDEV plot should show a peak deviation at 1/2P, which should be one-half of ~30d if it is likely related to the monthly rate changes in the USNO(Mean) as we steer it and the USNO(MCs) to UTC(BIPM). This peak might be related to our steering of this system towards UTC(BIPM), which is updated monthly. The PPV is fairly symmetrical in both coordinates and shows a nearly optimally controlled system in both time and frequency (Figure 17). The VRA (Figure 18) shows a fairly flat image with an intermediate structure between the white PM and white FM examples given above. A strong sharp-edged peak around 420 is a known short-term environmental perturbation in this system which is not easily seen in the original time series. This is an example of a detection of a short-term transient behavior in the system, which shows one of the uses of visualizing data with the VRA. The STE is 93% and is intermediate between the white PM and white FM examples.

A ONE-DAY PERIODIC IN UNEQUALLY SPACED TWSTT DATA

Several tools are available from the astronomical community to generate periodograms from unequally spaced data. The methods for unequally spaced data analysis are typically brute force methods. For example, to generate a periodogram the time series is phase-folded using trial periods and then a statistic is generated to estimate the correlation of data across the phased data that has been sub-binned and a scatter estimate formed for each bin. Phase-folded data that contains no structure has variance statistics that are uniform and large across the phased and sub-binned data. If the data contain structure in phase across the phased and sub-binned data, then the variance statistic is small for each bin and the overall statistic for that trial period is small, often near zero, indicating correlation.

Plotting the overall variance statistic with each trial period generates the periodogram. The method used here is a modified Jurkevich method [7], which gives as the statistic the square of the ½ amplitude of the variation. As an example, a sample unequally spaced data set was generated by the convolution of a 1 cycle per day and 3.754 cycle per day sine waves using the exact same sampling rate as the real TWSTT data to be shown below, i.e. unequally spaced, but well sampled. Periodograms were generated for every month from March 1996 to March 1998. The individual periodograms were then combined to form a 2-dimensional image (Figure 19). The time-series periodogram shows the two signals very well with both showing low-amplitude variation in the 1/2P locations. When data are missing the noise floor is increased, but the synthetic period always have very high signal-to-noise ratios.

In the real nominally hourly sampled TWSTT data at certain times of the year, a small amplitude periodic was seen in the measured time differences with a period of 1 day. It was decided that the data could be broken up into monthly sections and a periodogram generated for each month using the modified Jurkevich method [7] and the unequally spaced data. A 2-D digital image was then generated by merging each monthly periodogram into a matrix and displayed in the form of an image (Figure 20). The strongest signal indicated is the peak at 1 day and it can be easily seen that its amplitude varies over the course of a year. The highest signal level was at the 250 to 300 ps level maximum amplitudes and was seen in the summer. No signal is seen during the winter months. This would imply a seasonal temperature related periodic signal contribution. Several sources could be contributing to this variation in the time-transfer link that will be isolated in the near future.

CONCLUSIONS

All of these methods of analysis including routine inspections of ACFs, PSDs, and all the standard variances have become integral parts of the author's software tool chest. Phase-plane views (PPVs) are easy to produce and provide a quick and useful visualization of the state of a controlled clock in both the time and frequency. PPVs also allow a quick view of how well a controlled system is behaving and so are quite useful in monitoring of steered time and frequency systems. VRA is a useful tool, among others, in visualizing and detecting uncharacteristic short-time-scale impulse behavior in time and frequency systems.

REFERENCES

[1] J.-P. Eckmann, S. O. Kamphorst, and D. Ruelle, 1987, "Recurrence Plots of Dynamical Systems," Europhys. Lett., 4, 973-977.

[2] J. A. Barnes, 1993, "A Digital Equivalent of an Analog Spectrum Analyzer," Proceedings of the 1993 IEEE International Frequency Control Symposium, 2-4 June 1993, Salt Lake City, Utah, USA, pp.270-281.

[3] S. Schneider, "Visualizing Real-Time Data," Performance Computing, December 1998, pp. 33-41.

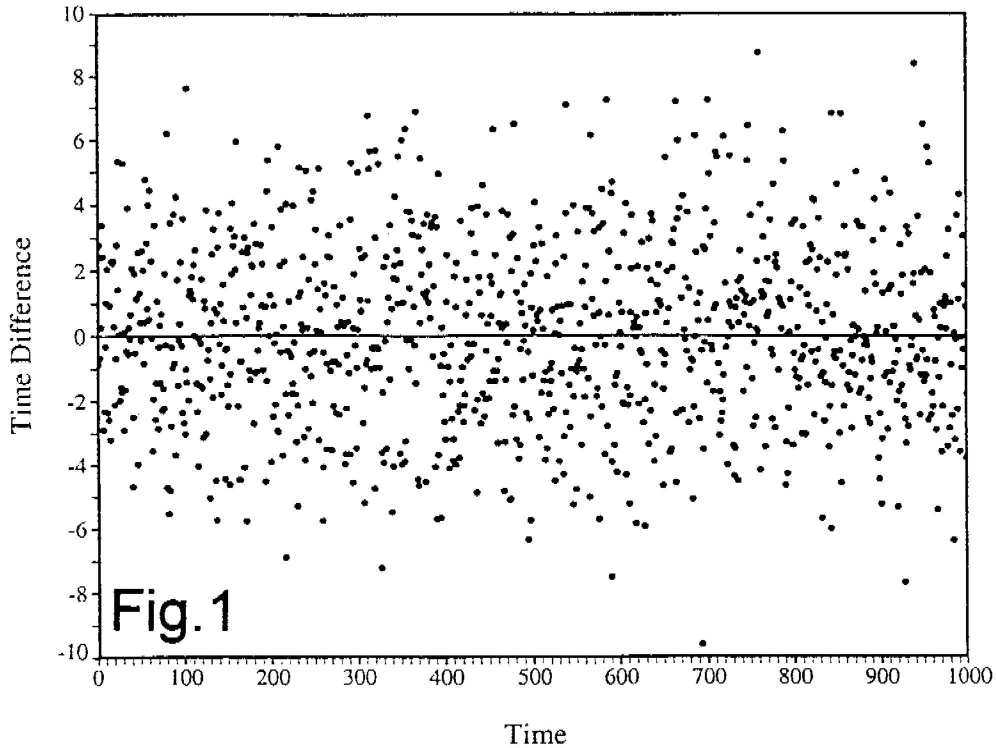
[4] M. Henon,1969, "Numerical Study of Quadratic Area-Preserving Mappings," Quarterly of Applied Mathematics, 27, 291-312.

[5] E. Kononov, Software VRA Ver. 2.5, June 1998, http://pw1.netcom.com/newugenek/index.html and attached Web pages.

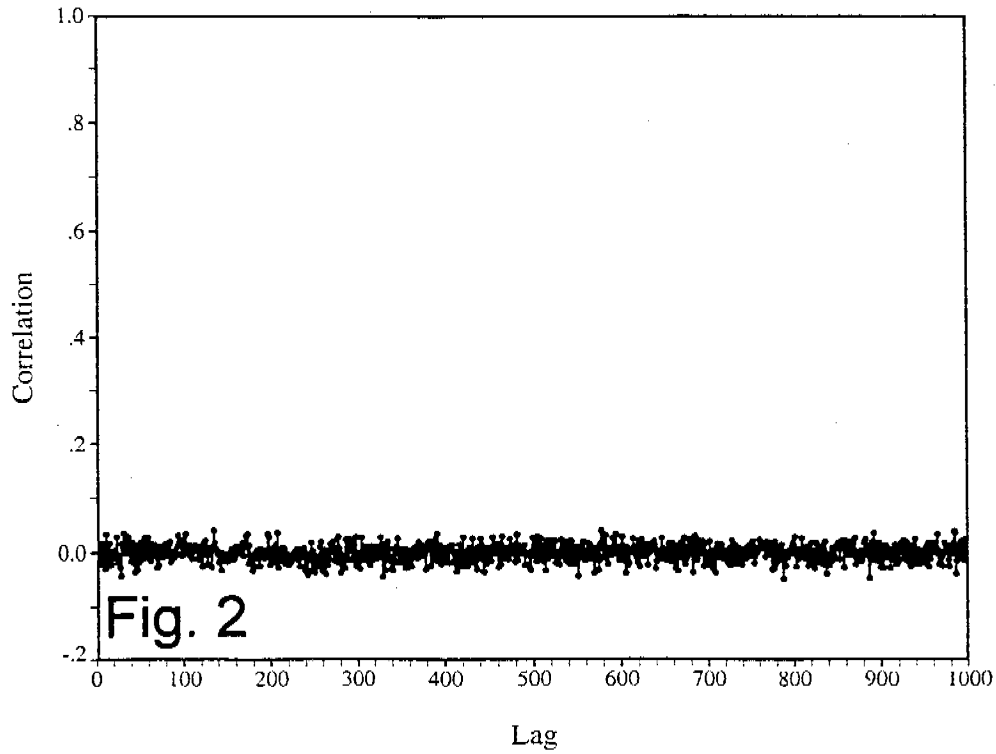
[6] Stable 32 Software Version 1.0, Hamilton Technical services, 195 Woodbury Street, S. Hamilton, MA 01982 USA.

[7] S. Morris and D. L. DuPuy, 1980, "A Photoelectric Study of Three Southern Delta Scuti Stars," Publ. Astron. Soc. Pacific, 92, 303-314.

Time-Domain White PM

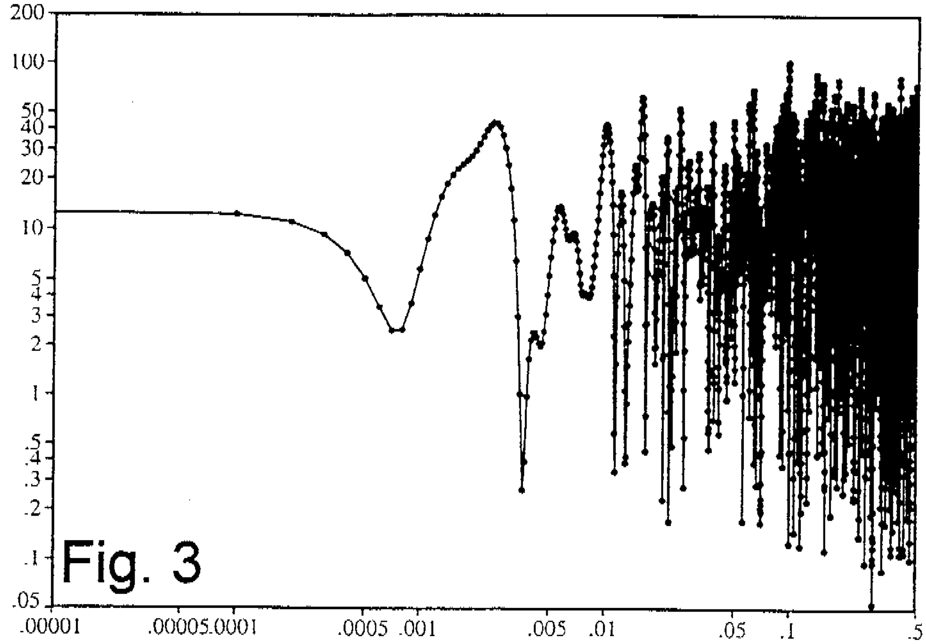


Auto-correlation Function of White PM



Power Spectral Density of White PM

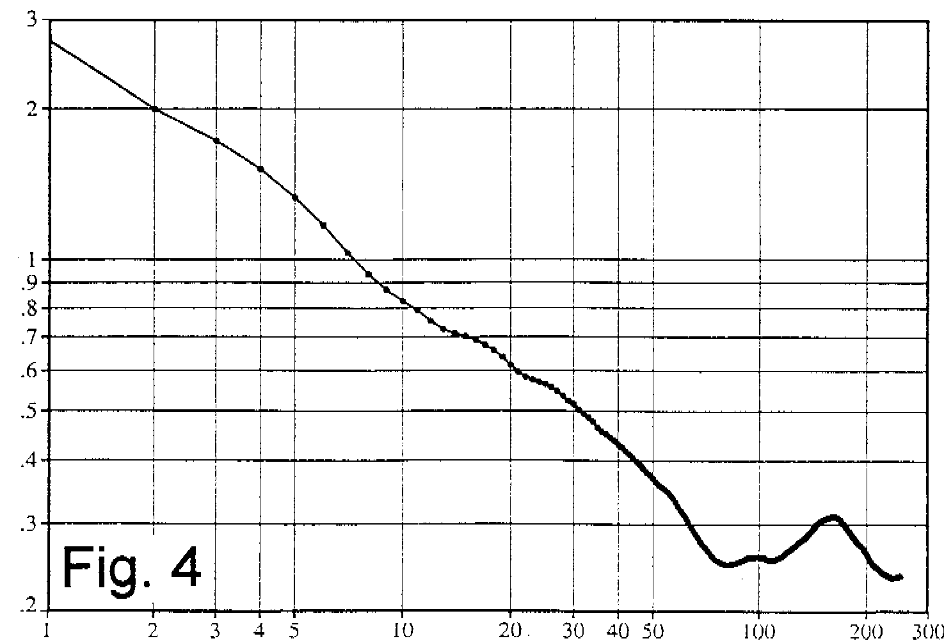
from the Digital-Spectrum-Analyzer Method



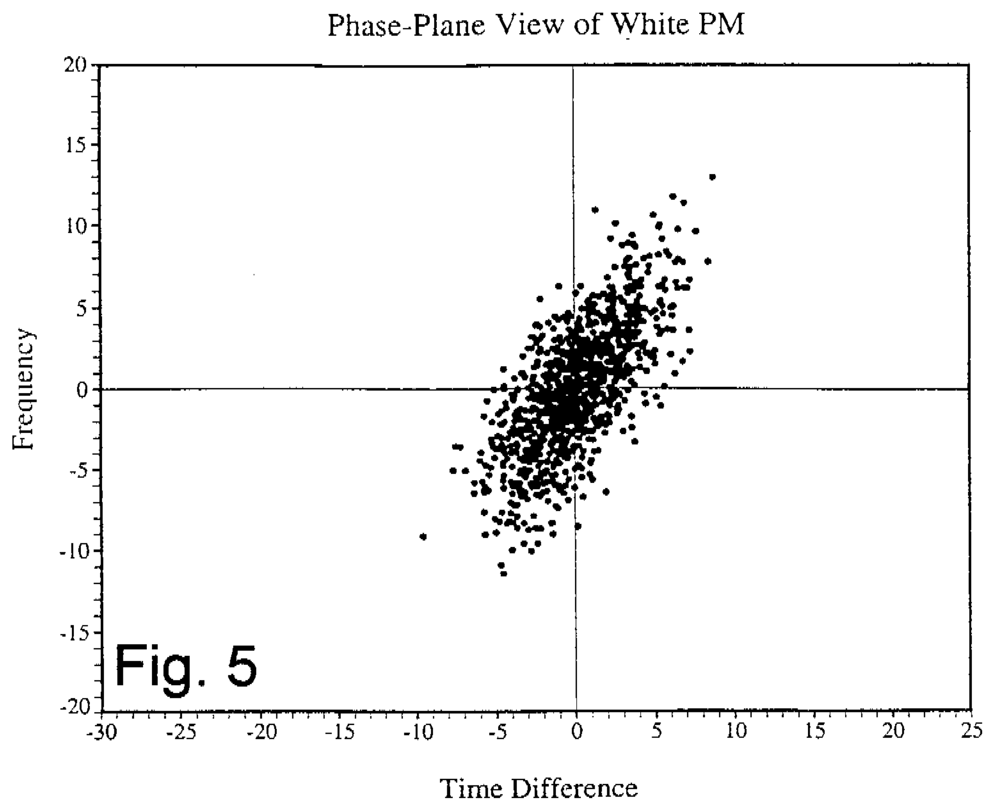
Frequency

Time Deviation (TDEV) of White PM

For All Tau from Stable Software

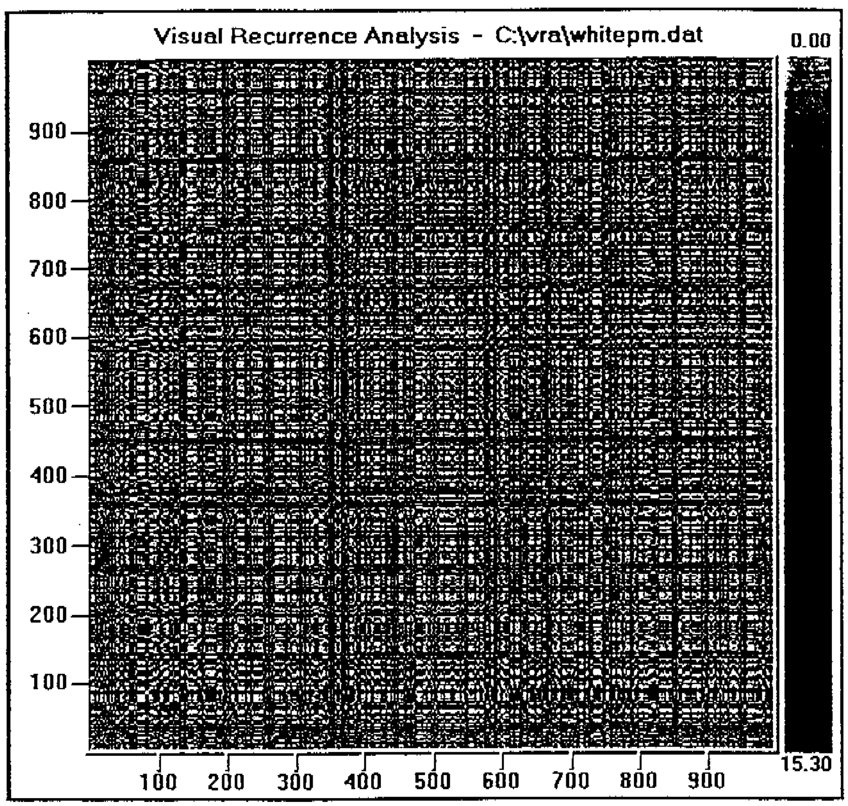


Tau



Phase-Plane View of White PM
Frequency
Time Difference
20
15
10
5
0
-5
-10
-15
-20
-30
-25
-20
-15
-10
-5
0
5
10
15
20
25

Fig. 5



Visual Recurrence Analysis - C:\vra\whitepm.dat
0.00
15.30
900
800
700
600
500
400
300
200
100
100
200
300
400
500
600
700
800
900

Fig. 6

Time-Domain White FM

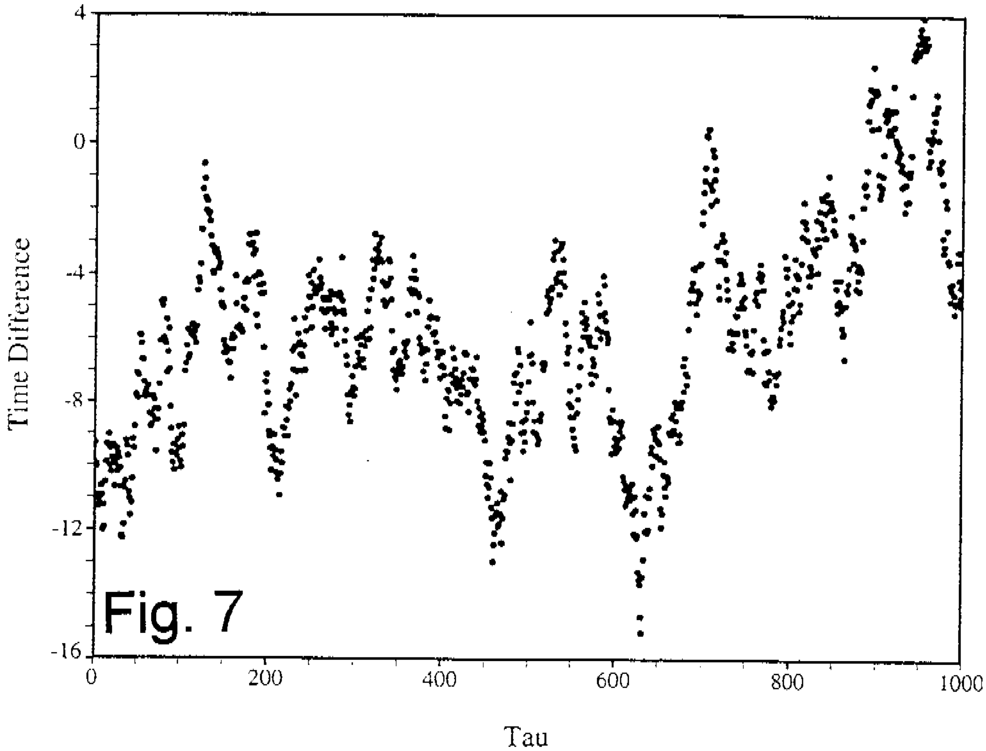


Fig. 7

Autocorrelation Function of White FM

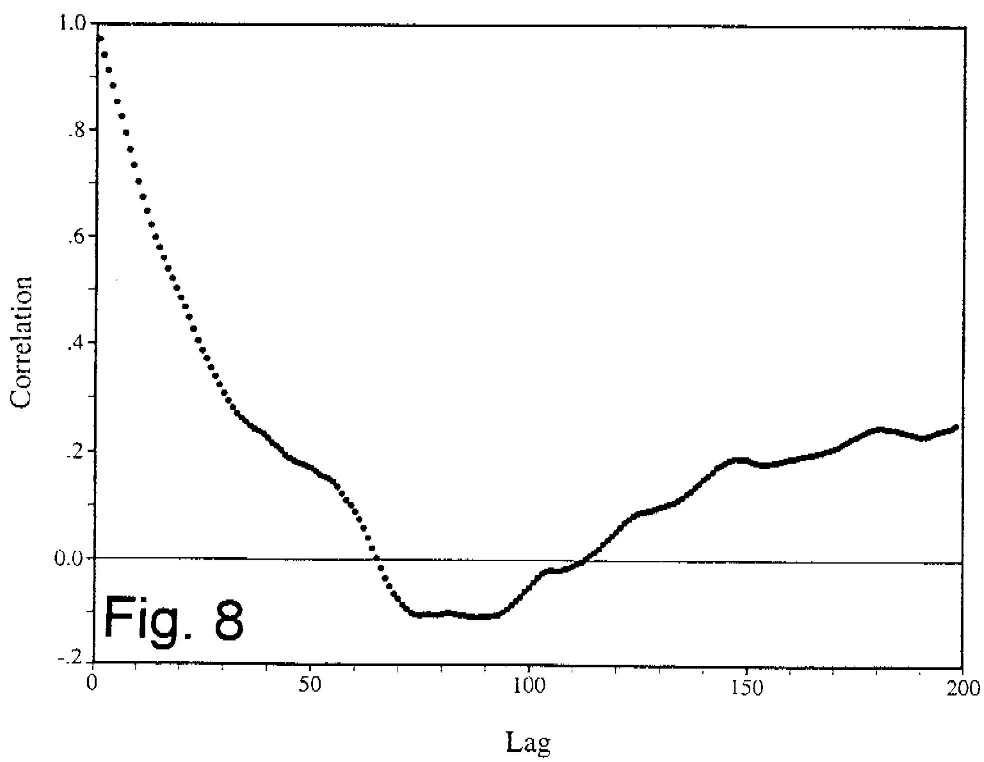
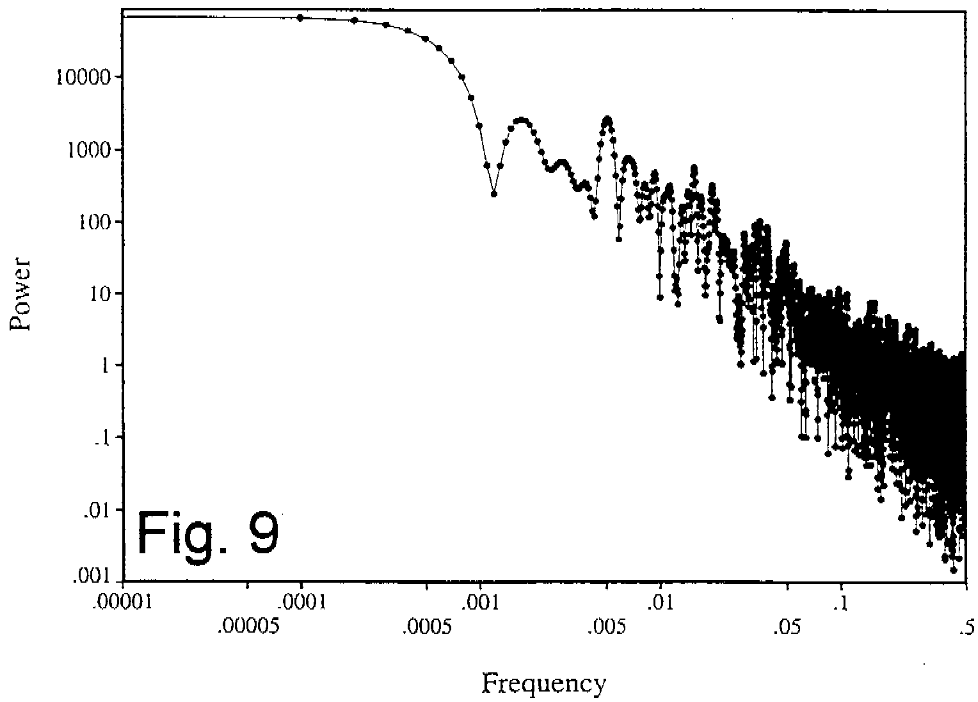


Fig. 8

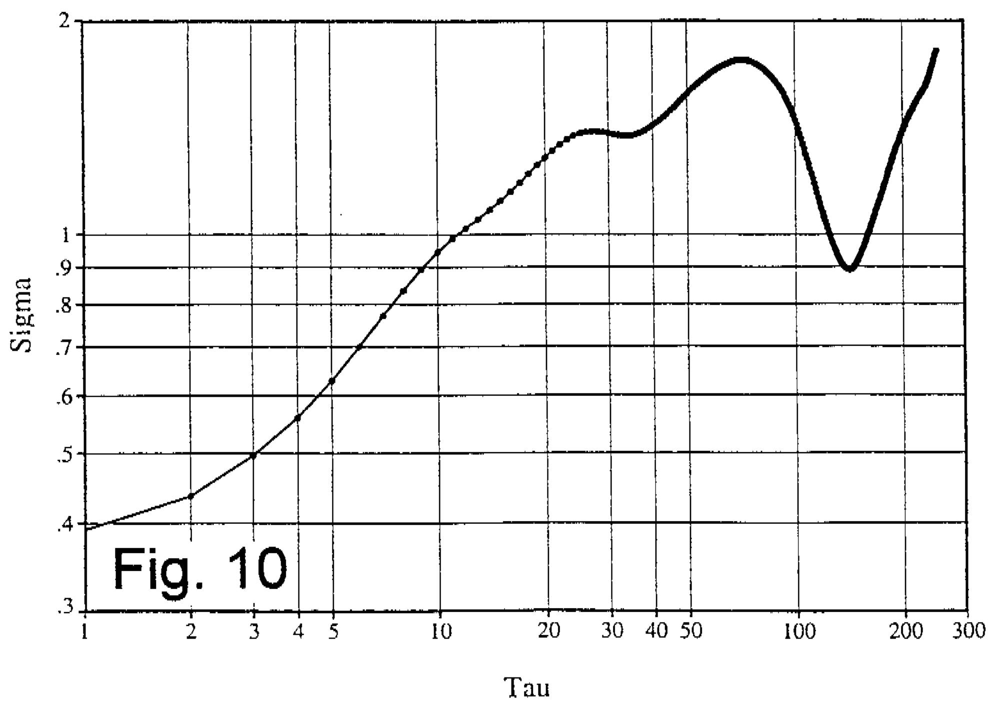
Power Spectral Density of White FM

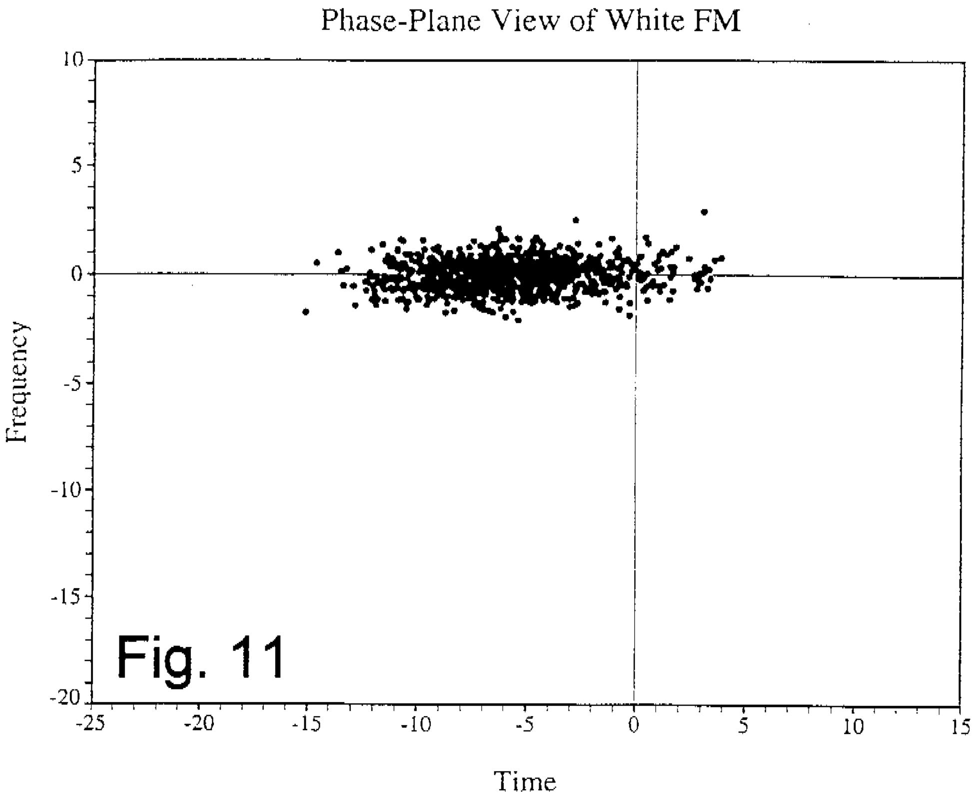
from the Digital-Spectrum-Analyzer Method



Time Deviation (TDEV) of White FM

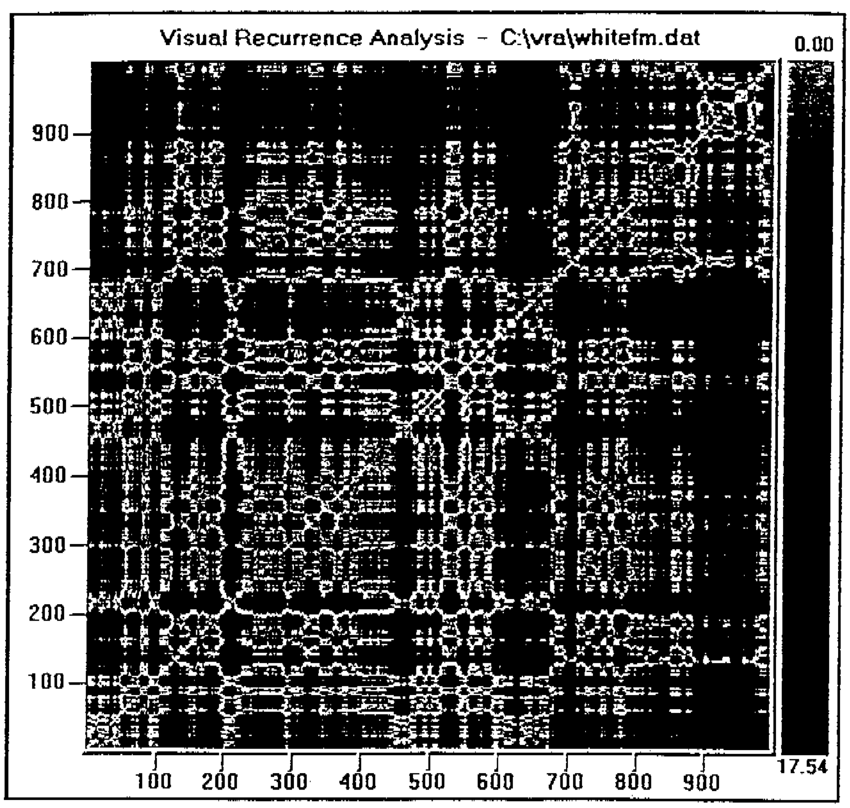
For All Tau from Stable Software





Phase-Plane View of White FM
Frequency
Time
10
5
0
-5
-10
-15
-20
-25
-20
-15
-10
-5
0
5
10
15

Fig. 11



Visual Recurrence Analysis - C:\vra\whitefm.dat
0.00
17.54
900
800
700
600
500
400
300
200
100
100 200 300 400 500 600 700 800 900

Fig. 12

USNO(MC2) - USNO AMC(MC1) via TWSTT

Both are Steered Systems

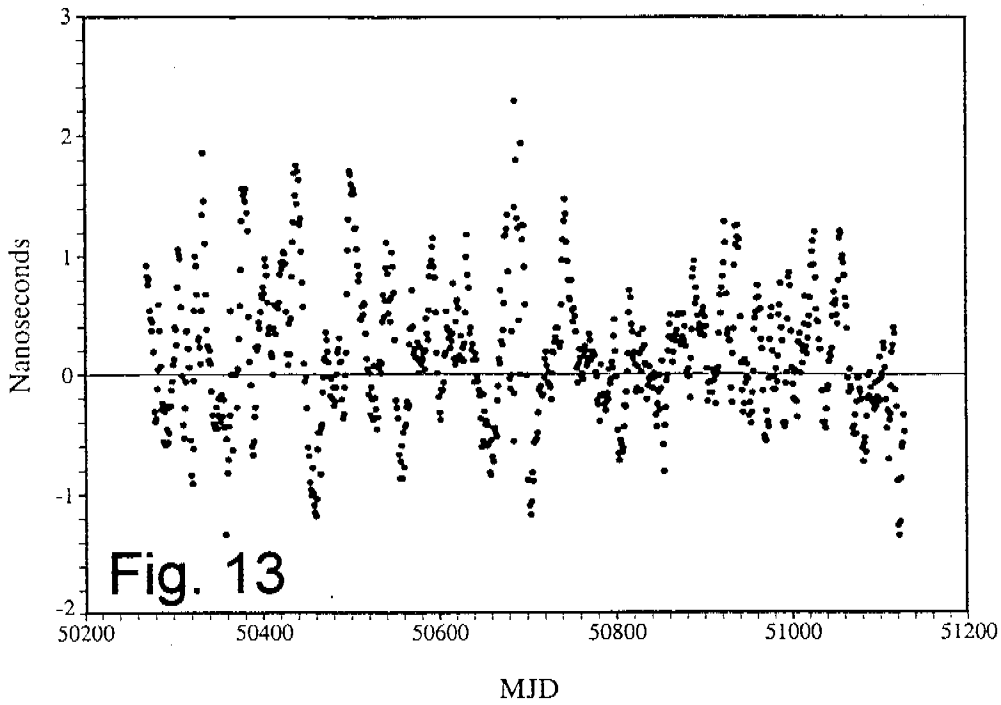


Fig. 13

Autocorrelation Function of

USNO(MC2) - USNO AMC(MC1) via TWSTT

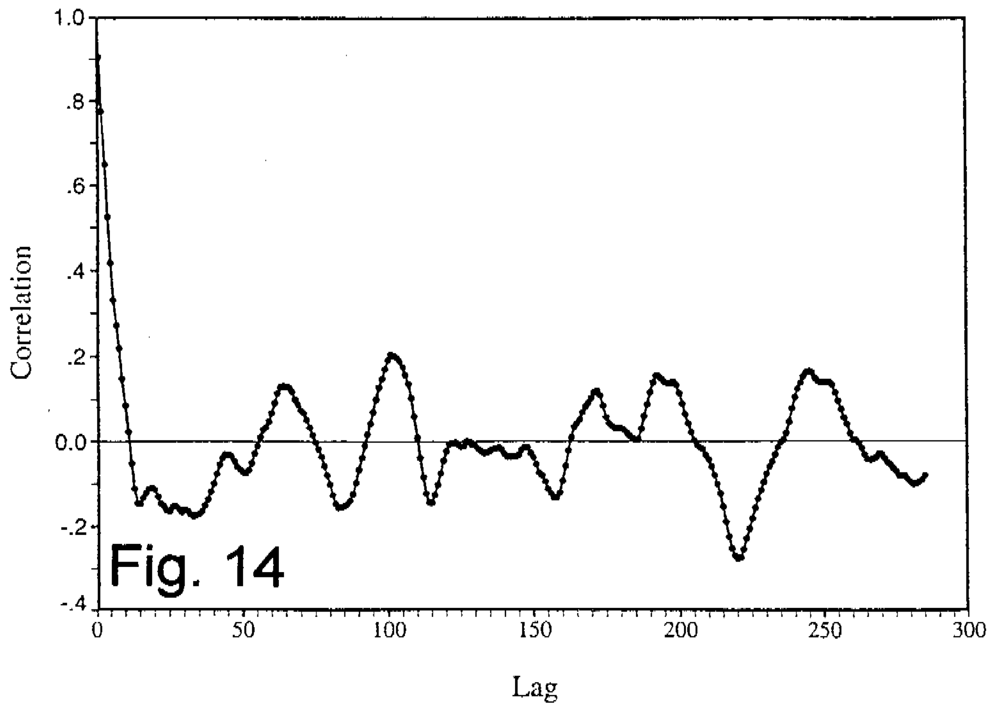
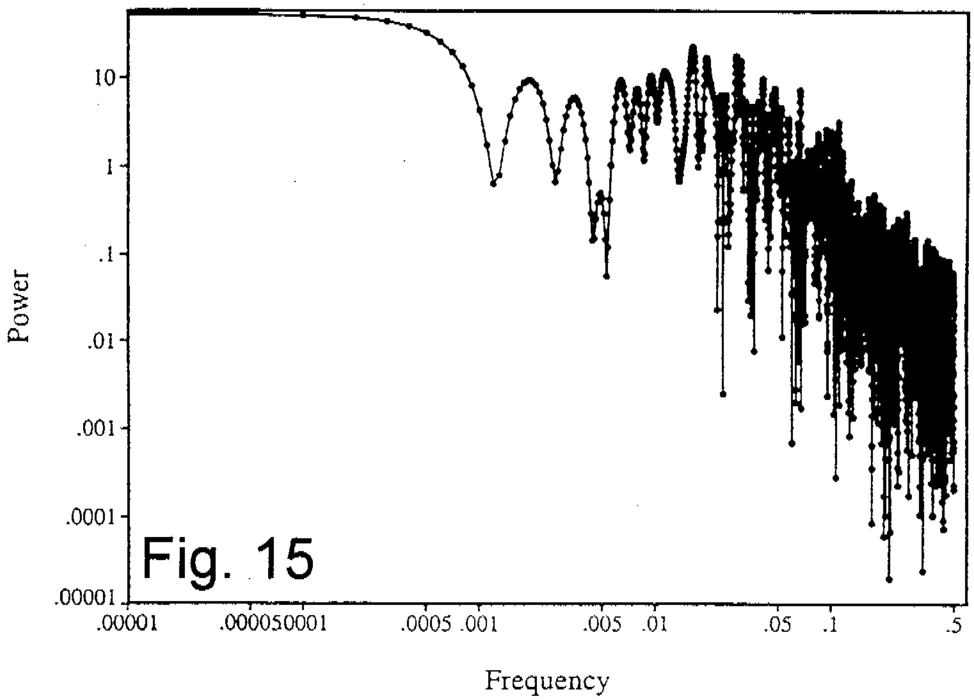
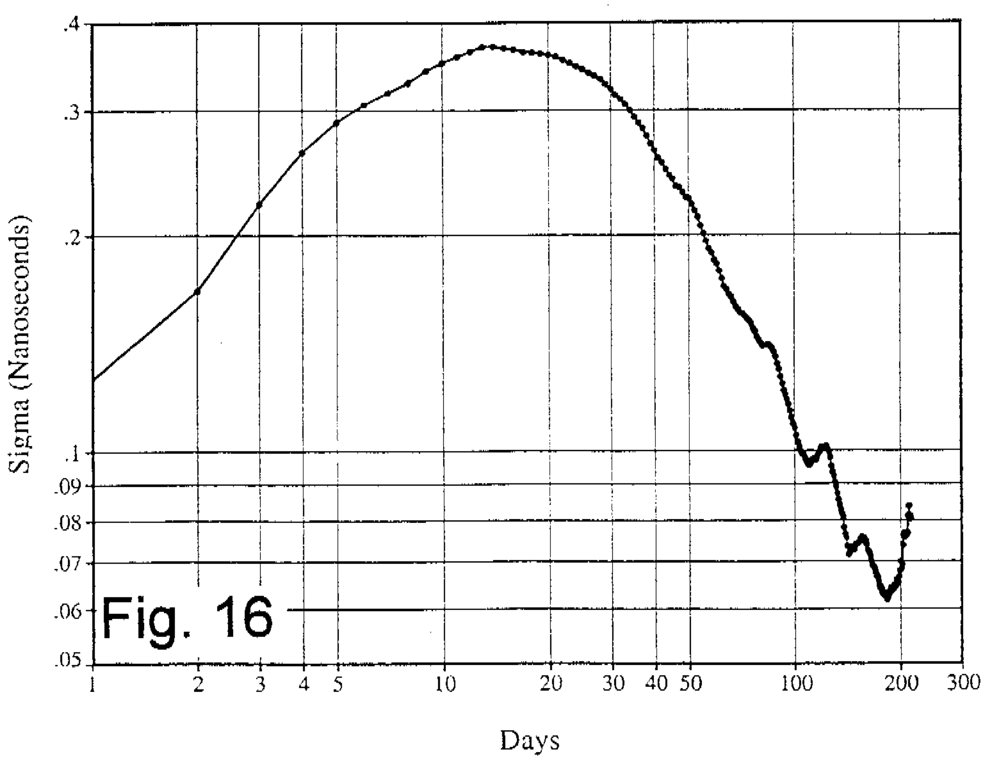


Fig. 14

Power Spectral Density of TWSTT
from the Digital-Spectrum-Analyzer Method



Time Deviation (TDEV) of TWSTT



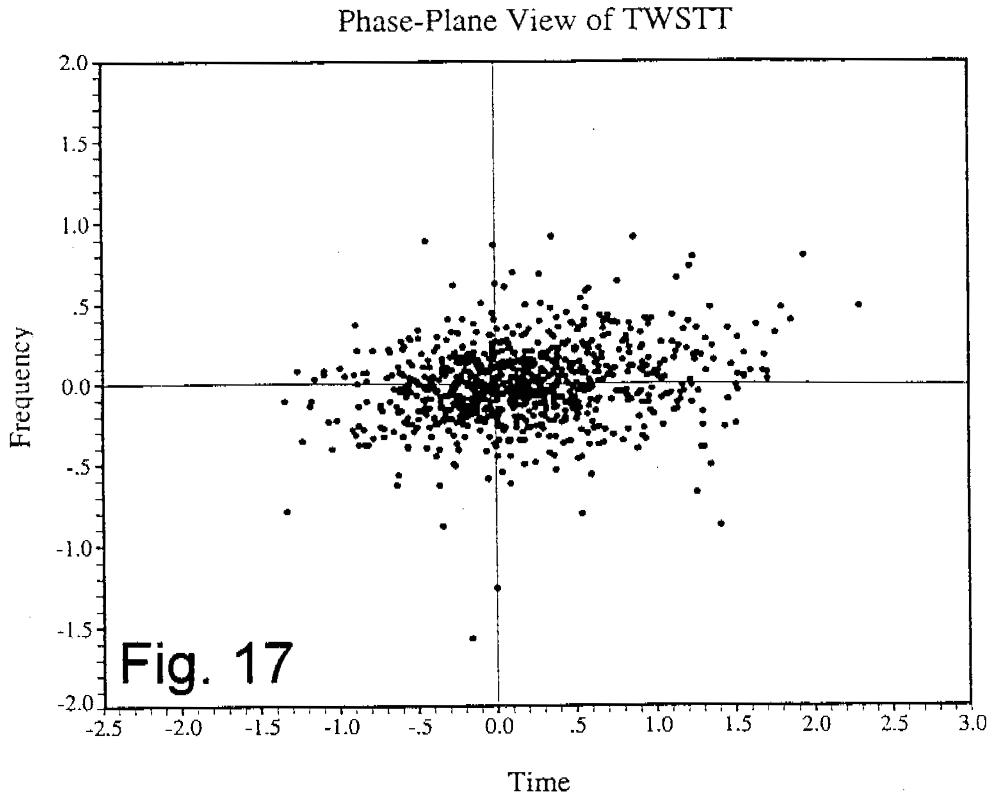

Phase-Plane View of TWSTT
Frequency
Time
2.0
1.5
1.0
.5
0.0
-.5
-1.0
-1.5
-2.0
-2.5
-2.0
-1.5
-1.0
-.5
0.0
.5
1.0
1.5
2.0
2.5
3.0

Fig. 17

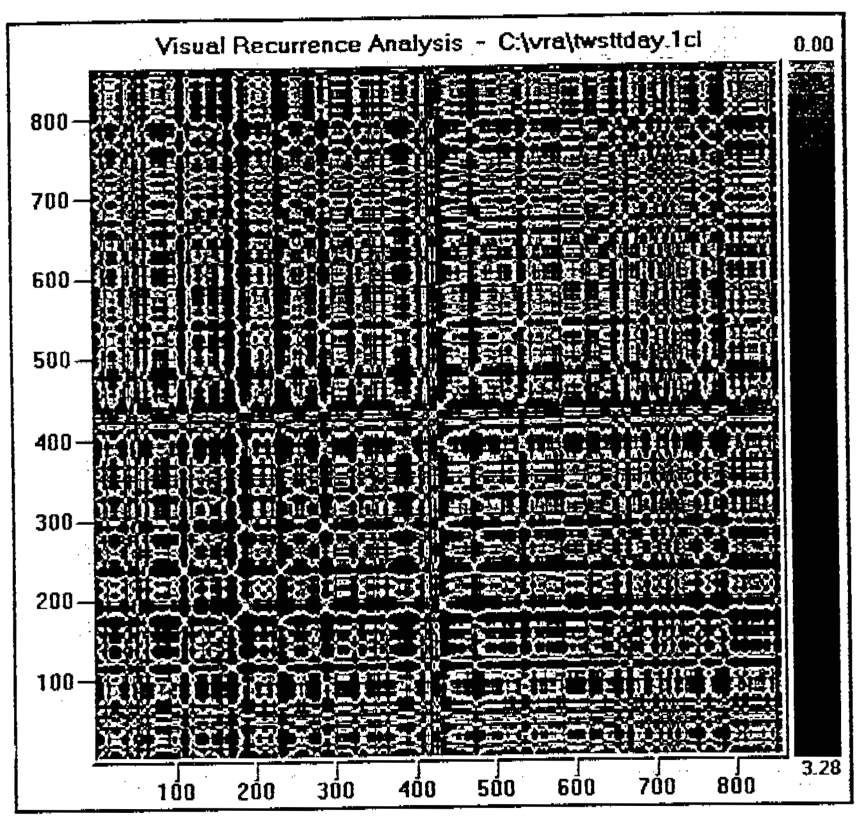

Visual Recurrence Analysis - C:\vra\twsttday.1cl
0.00
3.28
800
700
600
500
400
300
200
100
100
200
300
400
500
600
700
800

Fig. 18

Time-Frequency Analysis
Synthetic Unequally Spaced Data
Two Signals @ 1cpd & 3.754cpd

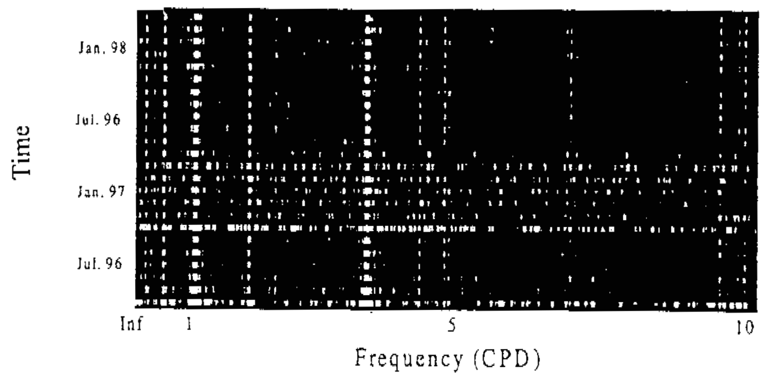


Fig. 19

Scale: Dark Red ---> Red ---> Yellow ---> White
Low signal --------------------> High Signal

Time-Frequency Analysis
of USNO-AMC TWSTT

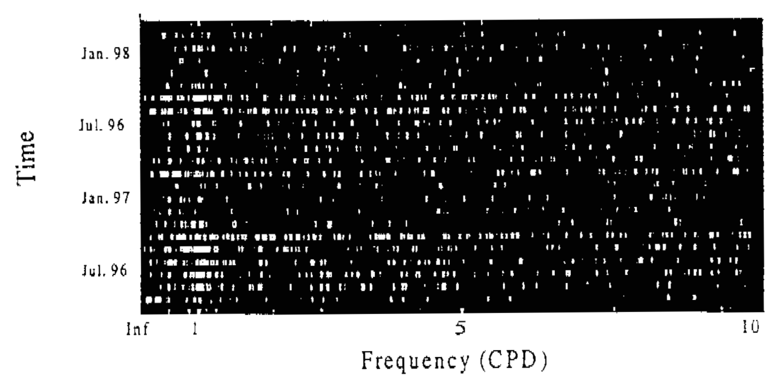


Fig. 20

Scale: Blue ---> Red ---> Yellow ---> White
Low signal --------------> High Signal