UNEQUAL-ARMS MICHELSON INTERFEROMETERS

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Abstract

Michelson interferometers allow phase measurements many orders of magnitude below the phase stability of the laser light injected into their two almost equal-length arms. If, however, the two arms are unequal, the laser fluctuations can not be removed by simply recombining the two beams. This is because the laser jitters experience different time delays in the two arms, and therefore can not cancel at the photo detector. We present here a method for achieving exact laser noise cancellation, even in an unequal-arm interferometer. The method presented in this paper requires a separate readout of the relative phase in each arm, made by interfering the returning beam in each arm with a fraction of the outgoing beam[1]. *By linearly combining the two data sets with themselves, after they have been properly time-shifted* [2], *we show that it is possible to construct a new data set that is free of laser fluctuations.*

An application of this technique to future planned space-based laser interferometer detectors of gravitational radiation[3] *is discussed.*

I. INTRODUCTION

Michelson interferometers, experimental devices used in a large variety of Earth- and space-based, high-precision experiments, rely on a coherent train of electromagnetic waves of nominal frequency ν_0. The injected beam is typically folded into several beams, and at one or more points where these intersect, relative fluctuations of frequency or phase are monitored (homodyne detection). The observed low frequency variations of the fringes are due to frequency fluctuations of the source of the electromagnetic signal about ν_0, to relative motions of the source and the mirrors that do the folding, to temporal variations of the index of refraction along the beams, and to any time-variable field present the experimenter is trying to measure. To perform an experiment in this way it is thus necessary to control, or monitor, the other sources of relative frequency fluctuations, and, in the data analysis, to use optimal algorithms based on the different characteristic interferometer responses to the signal, and to the other sources (the noise). By comparing phases of split beams propagated along equal-length arms, frequency fluctuations of the laser can be removed and signals at levels many orders of magnitude lower can be measured.

A space-based experiment for detecting gravitational radiation, using Michelson interferometry, has been proposed [3]. Since the frequency stability of the lasers it will use will be at best of a few parts in 10^{-13} in the millihertz frequency band, it will be essential for this experiment to be able to remove these fluctuations when searching for gravitational waves of dimensionless amplitudes less than 10^{-20} in the millihertz band[3]. Since the armlengths of this space-based interferometer can be different by several percent, the direct recombination of the two beams at a photo detector will not effectively remove the laser noise. This is because the frequency fluctuations of the laser will be delayed by a different amount of time inside the two different-length arms.

In order to solve this problem, a technique involving heterodyne interferometry with unequal arm lengths and independent phase-difference readouts in each arm has been identified [2], which yields data from which source frequency fluctuations can be removed exactly. This is achieved by taking a suitable linear combination of the two Doppler time series after having time-shifted them properly. This direct method achieves the exact cancellation of the laser frequency fluctuations. An outline of the paper is presented below.

In Sec. II we state the problem, and derive the two Doppler responses, from the two unequal arms of a space-based interferometer, to a gravitational wave signal. The difference between the armlengths implies that the frequency fluctuations of the laser can not be removed by direct differencing of the two data sets. In Sec. III we present our technique for *synthesizing* an unequal-arm interferometer detector of gravitational waves. Our method is implemented in the time domain, and relies on a properly chosen linear combination of the two Doppler data. Our comments and conclusions are finally outlined in Sec. IV.

II. STATEMENT OF THE PROBLEM

Let us consider three spacecraft flying in an equilateral triangle-like formation, each acting as a free falling test particle, and continuously tracking each other via coherent laser light. One spacecraft, which we will refer to as spacecraft a, transmits a laser beam of nominal frequency ν_0 to the other spacecraft (spacecraft b and c at distances L_1 and L_2, respectively). The phase of the light received at spacecraft b and c is used by lasers on board spacecraft b and c for coherent transmission back to spacecraft a. The relative two *two-way* frequency (or phase) changes as functions of time are then independently measured at two photo detectors on board spacecraft a (Figure 1). When a gravitational wave crossing the solar system propagates through these electromagnetic links, it causes small perturbations in frequency (or phase), which are replicated three times in each arm's data[4].

To determine the response of an unequal arm interferometer to a gravitational wave pulse, let us introduce a set of Cartesian orthogonal coordinates (X, Y, Z) centered on spacecraft a (see Figure 2). The X axis is assumed to be oriented along the bisector of the angle enclosed between the two arms, Y is orthogonal to it in the plane containing the three spacecraft, and the Z axis is chosen in such a way to form with (X, Y) a right-handed, orthogonal triad of axes. In this coordinate system we can write the two two-way Doppler responses, measured by spacecraft a at time t, as follows[5,6] (units in which the speed of light $c = 1$).

$$\left(\frac{\Delta\nu(t)}{\nu_0}\right)_1 \equiv y_1(t) = h_1(t) + C(t - 2L_1(t)) \; - \; C(t) \; + \; n_1(t) \; , \tag{1}$$

$$\left(\frac{\Delta\nu(t)}{\nu_0}\right)_2 \equiv y_2(t) = h_2(t) + C(t - 2L_2(t)) \; - \; C(t) \; + \; n_2(t) \; , \tag{2}$$

where $h_1(t)$, $h_2(t)$ are the gravitational wave signals in the two arms[5,6], and we have denoted by $C(t)$ the random process associated with the frequency fluctuations of the master laser on board spacecraft a; $n_1(t)$, $n_2(t)$ are the remaining noise sources affecting the Doppler responses $y_1(t)$, $y_2(t)$ respectively.

From equations (1, 2) it is important to note the characteristic time signature of the random process $C(t)$ in the Doppler responses y_1, y_2. The time signature of the noise $C(t)$ in $y_1(t)$ for instance, can be understood by observing that the frequency of the signal received at time t contains laser frequency fluctuations transmitted $2L_1$ seconds earlier. By subtracting from the frequency of the received signal the frequency of the signal transmitted at time t, we also subtract the frequency fluctuations $C(t)$ with the net result shown in equation (1).

Among all the noise sources included in equation (1), the frequency fluctuations due to the laser are expected to be by far the largest. A space-qualified single-mode laser, such as a diode-pumped Nd:YAG ring laser of frequency $\nu_0 = 3.0 \times 10^{14}$ Hz and phase-locked to a Fabry-Perot optical cavity, is expected to have a spectral level of frequency fluctuations equal to about $1.0 \times 10^{-13}/\sqrt{Hz}$ in the millihertz band[3]. Laser noise is to be compared with, e.g., the expected secondary noises which will be 10^7 or more times smaller.

If the armlengths are unequal by an amount $\Delta L = L_2 - L_1 \equiv \epsilon L_1$ (with $\epsilon \simeq 3 \times 10^{-2}$ for a space-based interferometer[3]), the simple subtraction of the two Doppler data $y_1(t)$, $y_2(t)$ (which would be appropriate for a conventional equal-arm interferometer) gives a new data set that is still affected by the laser fluctuations by an amount equal to

$$C(t - 2L_1) - C(t - 2L_2) \simeq 2\dot{C}(t - 2L_1)\epsilon L_1 \; . \tag{3}$$

As a numerical example of equation (3) we find that, at a frequency of 10^{-3} Hz and by using a laser of frequency stability equal to about $10^{-13}/\sqrt{Hz}$, the residual laser frequency fluctuations are equal to about $10^{-16}/\sqrt{Hz}$. Since the goal of proposed space-based interferometers[3] is to observe gravitational radiation at levels of $10^{-20}/\sqrt{Hz}$ or lower, it is crucial for the success of these missions to cancel laser frequency fluctuations by many more orders of magnitude.

III. UNEQUAL-ARMS INTERFEROMETERS

In what follows we will show that there exists an algorithm in the time domain for removing the frequency fluctuations of the laser from the two Doppler data $y_1(t)$, $y_2(t)$ at any time t. This method relies only on a properly chosen linear combination of the two Doppler data in the time domain. In order to show how this technique works, we will assume the two armlengths L_1, L_2 to be constant and known exactly. The interested reader is referred to [2] for a detailed analysis covering the most general configuration.

From equations (1, 2) we may notice that, by taking the difference of the two Doppler data $y_1(t)$, $y_2(t)$, the frequency fluctuations of the laser now enter into this new data set in the following way

$$\begin{aligned}\Lambda_1(t) \equiv y_1(t) - y_2(t) &= h_1(t) - h_2(t) + C(t - 2L_1) - C(t - 2L_2) \\ &+ n_1(t) - n_2(t) \ . \end{aligned} \tag{4}$$

If we now compare how the laser frequency fluctuations enter into equation (4) against how they appear into equations (1, 2), we can further make the following observation. If we time-shift the data $y_1(t)$ by the round trip light time in arm 2, $y_1(t - 2L_2)$, and subtract from it the data $y_2(t)$ after it has been time-shifted by the round trip light time in arm 1, $y_2(t - 2L_1)$, we obtain the following data set

$$\begin{aligned}\Lambda_2(t) \equiv y_1(t - 2L_2) - y_2(t - 2L_1) &= h_1(t - 2L_2) - h_2(t - 2L_1) + C(t - 2L_1) \\ &- C(t - 2L_2) + n_1(t - 2L_2) - n_2(t - 2L_1) \ . \end{aligned} \tag{5}$$

In other words, the laser frequency fluctuations enter into $\Lambda_1(t)$, and $\Lambda_2(t)$ with the same time-structure. This implies that, by subtracting equation (4) from equation (5), we can generate a new data set that does not contain the laser frequency fluctuations $C(t)$

$$\begin{aligned}\Sigma(t) \equiv \Lambda_2(t) - \Lambda_1(t) &= h_1(t - 2L_2) - h_1(t) - h_2(t - 2L_1) + h_2(t) \\ &+ n_1(t - 2L_2) - n_1(t) - n_2(t - 2L_1) + n_2(t) \ . \end{aligned} \tag{6}$$

From the expression of $\Lambda_2(t)$ given in equation (5), it is easy to see that the new data set $\Sigma(t)$ should be set to zero for the initial $MAX[2L_1, 2L_2]$ seconds. This is because some of the data from y_1 and y_2 entering into $\Lambda_2(t)$ "do not yet exist" during this time interval. Since the typical round trip light time for proposed space-based laser interferometer detectors of gravitational waves will never be greater than about 33 seconds[3], we conclude that the amount of data lost in the implementation of our method is negligible.

We have simulated the procedure (equation 6) using realistic laser and shot noise spectra[3], known arm lengths (differing by about 3 percent), and a simulated monochromatic gravitational wave incident normal to the plane of the interferometer. The results of the simulation are shown in Figure 3. Plotted are spectral densities of the raw laser noise, the raw shot noise, and the canceled time series, $\Sigma(t)$ (equation 6). This illustrates cancellation of the laser noise and modulation of the residual secondary noises[2].

IV. CONCLUSIONS

We presented a time-domain procedure for accurately cancelling laser noise fluctuations in an unequal-arm one-bounce Michelson interferometer relevant to space-borne gravitational wave detectors. The method involves separately measuring the phase of the returning light relative to the phase of the transmitted light in each arm. By suitable offsetting and differencing of these two time series, the common laser noise is cancelled exactly (equation 6).

The technique presented in this paper is rather general, in such that it can be implemented with any (Earth- as well as space-based) unequal-arms Michelson interferometers.

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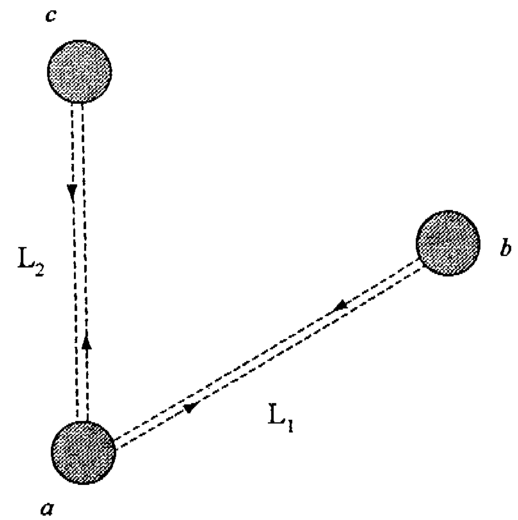

Figure 1. Typical configuration of a space-based, unequal-arm interferometer detector of gravitational waves. The corner spacecraft *a* transmits coherent laser light to the other spacecraft, *b* and *c*. They coherently retransmit back to spacecraft *a*, where coherent two-way phase (or frequency) changes in each arm are then measured. The two arm lengths are denoted with L_1, and L_2.

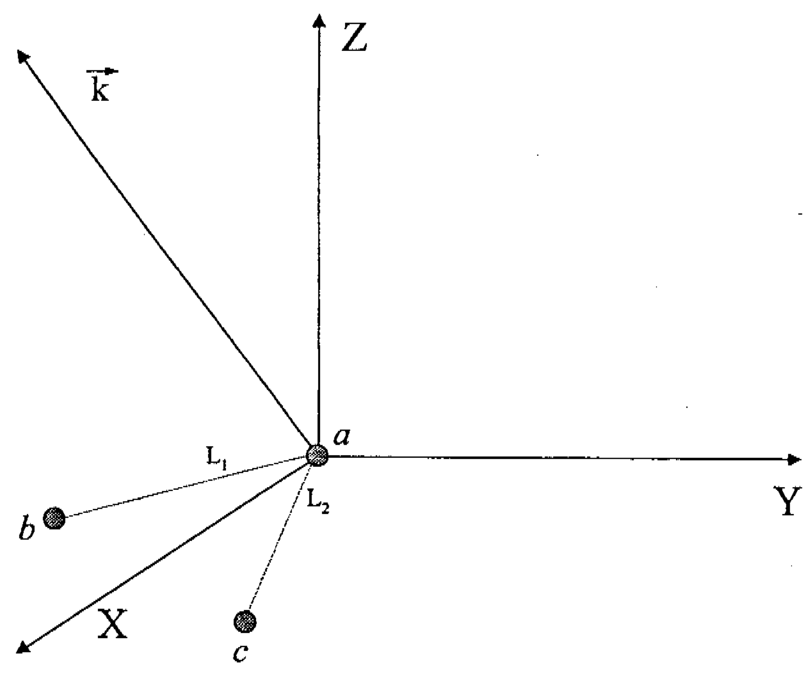

Figure 2. Coherent laser light is transmitted simultaneously from spacecraft *a* to spacecraft *b* and *c*, and coherently transponded back to *a*. The **X** axis is along the bisector of the angle enclosed between the two arms of the interferometer. The **Y** axis is orthogonal to the **X** axis in the plane of the interferometer, and the **Z** axis is chosen in such a way to form together with (**X**, **Y**) a right-handed set of axes. The gravitational wave train propagates along the **k** direction.

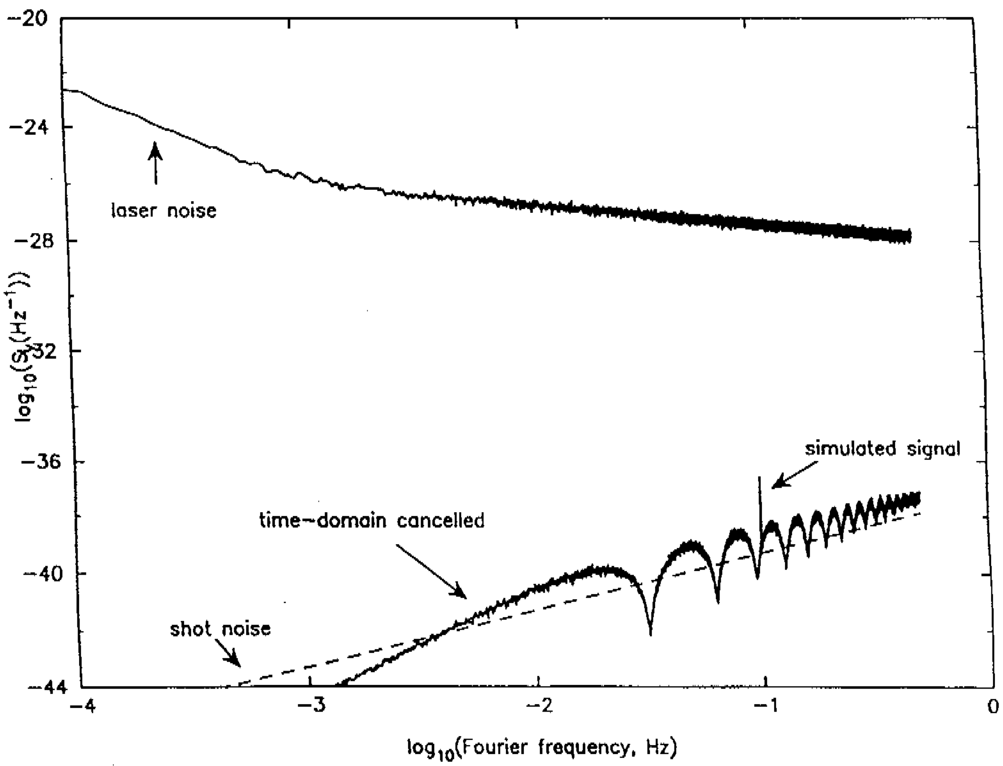

Figure 3. Simulation of the time-domain laser noise cancellation procedure for unequal-arm interferometers described in the text. Fractional frequency fluctuation spectra, $S_y(f)$, are plotted versus Fourier frequency for: (upper curve) raw laser noise having spectral density 10^{-28} (f/1 Hz)$^{-2/3}$ + 6.3 x 10^{-37} (f/1 Hz)$^{-3.4}$ Hz^{-1}, and (lower curve) residual noise after time-domain cancellation procedure. Dashed curve shows the level of shot noise added to each arm (spectral density \$5.3 x 10^{-38} (f/1 Hz)^2 Hz^{-1}, independent in each arm) and dot-dashed curve showing the predicted modulation of the shot noise spectrum due to our laser noise cancellation is also plotted. Other parameters are 2 L_1 = 32 sec, 2 L_2 = 31 sec, and transform length \$$2^{15}$ sec. In addition to shot noise, a simulated sinusoidal gravitational wave with amplitude $h_0 = 10^{-20}$ and f_0 = 0.1 Hz incident normal to the plane of the interferometer was added. The time-domain procedure, using the known arm lengths, cancels the laser noise exactly making the simulated signal clearly visible above the (now modulated) shot noise spectrum.