STATISTICAL PROBLEMS IN THE ANALYSIS OF UNEQUALLY SPACED DATA

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Abstract

The statistical analysis of irregularly spaced data or the handling of series with missing data asks for particular care because results may be biased. In particular, in PTTI applications, two points seems to be addressed:

- ***the noise analysis in terms of Allan or related variances;***
- ***the estimation of a missing datum on a particular date.***

Both these issues are examined in the particular case of the current TWSTT measurement, by means of an analytical and theoretical approach that can be of more general interest. Some numerical estimates based on the TWSTT measures are eventually reported.

1 INTRODUCTION

Time and frequency comparisons repeated with an irregular periodicity are today the possible results of the new clock comparison technologies or the new working principles of atomic frequency standards. For example, the TWSTT (Two-Way Satellite Time Transfer) is currently operated on Mondays, Wednesdays, and Fridays, thus with a time interval between measures of 2, 2, and 3 days. But the problems of handling unevenly spaced data is of more general interest, for example in case of optical synchronization links, in case of inter-satellite links, and in case of the repeatability analysis of the new frequency standards which do not work continuously. The statistical treatment of unevenly spaced data deserves some attention. In particular, in this paper, the issues of noise analysis and of missing datum estimation are investigated.

As far as noise analysis is concerned, type and amount of noise are commonly estimated by the use of the Allan or a related variance. Such estimation requires a series of equally spaced data. When data are not equally spaced they are usually filled in by means of some reconstruction technique, e.g., linear interpolation. The reconstruction technique induces a modification on the true noise of the data series, but it should give the possibility to identify the real noise, at least in some spectral bandwidth. By the introduction of the Allan covariance matrices and by using the transformation laws of such matrices, the effect of interpolating missing data was inferred with an analytical treatment and its effect on typical clock noises was previously reported [1]. In this paper some other possibilities of estimating ADEV without or with minor data manipulation are also presented.

The second task of interpolating a missing measure on a particular date is here approached with the double aim of smoothing the noise added by the comparison link and of retaining the clock noise for a correct evaluation of the clock behavior. The uncertainty of the reconstructed data is also evaluated by means of the least square theory and the use of the Kalman filter.

Finally, the above procedures are applied to some TWSTT measures performed in the last months to provide an example of application and a preliminary evaluation of TWSTT data. This work was performed in the frame of a collaboration with the CCTF Study Group on TWSTT [2].

2. NOISE ANALYSIS

When dealing with time and frequency measure sequences, the statistical tools commonly used to characterize the noise are the Allan variance or the associated Time variance (TVAR). These tools must be applied on equally spaced series. In case of unevenly data series, first of all it is thus necessary to obtain some evenly spaced sequences. With the aim of developing a general treatment, an analytical formulation was preferred instead than a simulative approach. This gives the advantages of allowing an insight on those parameters that most affect the results and of examining with a minimal effort another measurement periodicity or another noise case or a different reconstruction technique. The analytical processing requires the introduction of covariance matrices and the noise properties are examined by the transformation laws of such covariance matrices. In particular, the Allan covariance matrix and the TVAR(τ) matrix are introduced. Mathematical details and discussion are reported in [1].

Three possible approaches are examined in the case of a statistical analysis of unevenly data sequence:

1) identifying some regularities in the irregular sequence;
2) considering the original data as equally spaced by a "fictitious" τ_0. In the particular case of the current TWSTT measures an equal separation with an average gap of τ_0=2.33 days between the data can be assumed;
3) reconstructing the missing data on Tuesdays, Thursdays, Saturdays and Sundays with some specific interpolation rules, in order to obtain a daily spaced series.

2.1 CHECKING FOR SOME REGULARITIES IN THE IRREGULAR SEQUENCE

This approach consists in the research of those possible regularities that are present in the unevenly series. The interest is to highlight in the sequence those particular triads of phase samples spaced in such a way as to permit the estimation of a single two-sample variance i.e. a single phase second difference. Let's remind ourselves, in fact, that the ADEV estimation is performed by averaging a certain number of phase second differences $\Delta_2 x$ as:

$$ADEV(\tau) = \sqrt{\frac{1}{2\tau^2}\left\langle \left(x(t) - 2x(t+\tau) + x(t+2\tau)\right)^2 \right\rangle} = \sqrt{\frac{1}{2\tau^2}\left\langle (\Delta_2 x)^2 \right\rangle}$$

The three phase measures $x(t)$, $x(t+\tau)$ and $x(t+2\tau)$ must be subsequent and spaced by the same τ interval. Other second differences $\Delta_2 x$ can be obtained from following triads of phase measures even if not subsequent to the first considered triad. The ADEV determination in fact asks for the estimation of the variation of the mean frequency averaged over two subsequent τ intervals. Then, other phase second differences $\Delta_2 x$ can be estimated over whichever 2τ interval provided that another series of three phase measures, evenly spaced by a τ interval, is available. As an example, the case of the integration time equal to 2 days can be considered (Figure 1). For each week, the estimation of two adjacent mean frequencies is possible (one averaged between Monday and Wednesday and one between Wednesday and Friday). This implies that for each week it is possible to estimate one second difference $\Delta_2 x$. The higher is the number of weeks in the sequence, the higher the number of the possible $\Delta_2 x$ estimations to determine the final estimation of the ADEV(2 days) and the better the degree of confidence.

Analogously this is possible for any $\tau = 2, 5, 7, 9, \ldots, 7n, 7n \pm 2$ days. The case of $\tau = 7, 14, \ldots 7n$ days is trivial because for example all the Monday measures form a sequence of weekly equally spaced data. In this case, moreover, the overlapping estimation technique can be applied and also the MDEV and TDEV can be estimated. In case of $\tau = 2, 5, 9, .. 7n \pm 2$ days, the triad of phase measures are not subsequent; therefore the overlapping ADEV and the classical estimation of MDEV and TDEV is not possible. In such cases, a *point estimate* of TDEV was proposed [2]. In case of a missing measure, the only consequence is that a particular triad of phase measures cannot be used and the number of possible estimates $\Delta_2 x$ is decreased, but if the number of measurement weeks is large, the confidence on the ADEV estimation can be assured. In conclusion, with an accurate analysis on the measurement date regularities, it is possible to find out specific values of τ for which ADEV estimation can be determined directly from real data, without any manipulation.

2.2 CONSIDERING DATA AS EQUALLY SPACED BY AN "AVERAGE" τ_0.

A second approach when dealing with irregularly spaced data series is based on the assumption that all the data are indeed equally spaced with an average gap. In the case of the current two-way measurements, an artificial separation of τ_0=2.33 days can be assumed for the values in the sequence. In each week, in fact, three measures are available and 7/3≅2.33 days.

The effects of this treatment on the noise recognition depend on the kind of noise and on the true separation between data. If WPM is considered, the noise analysis turns out to be always correct because data are completely uncorrelated and there is no difference in considering them at a certain date or another. Only in case of TDEV evaluation, attention is to be paid because TDEV values depend on the stated τ_0 value. With other noise types the assumption, instead, is nearly correct when the data aperiodicity is small. Otherwise the result may be highly biased. In the particular case of the TWSTT periodicity, it was found [2] that, in case of WFM and RWFM, such technique leads to a noise overestimation for small τ. Since the repetition rate of TWSTT has only a slight aperiodicity, the bias in the results is only a minor one, but other data sequences affected by different aperiodicities could present more critical problems if treated as equally spaced.

Despite the fact that the evaluation technique with an "average" τ_0 is quite simple and allows the use of the commonly made software that requires evenly spaced data, some underhand pitfalls can cause significant errors. For example, always in the case of the TWSTT periodicity, the average spacing of 2.33 days can be assumed when all the scheduled measures are performed, i.e. each Monday, Wednesday, and Friday. In reality it may happen that some measures fail or that entire measurement weeks are absent. The easiest procedure is to make a list of all the available measures regardless of their dates and to evaluate the ADEV with an overlapping procedure as if the data were equally spaced by an average τ_0 (estimated by total measurement period divided by the number of actual measurements). This can be very dangerous. In case of colored noises, the noise identification can be misleading, because data spaced for example by one or more weeks are treated as if they were spaced only by two or three days.

2.3 MISSING DATA RECONSTRUCTION

A third approach consists in the reconstruction of the missing data by using interpolation techniques. This solution was examined in [1], and also in [3]. It presents the disadvantage that results are biased because the interpolation acts as a filter. The results obtained in the case of WPM, WFM, and RWFM, considering the current TWSTT periodicity and a straight line or a 5^{th} order polynomial interpolation of the missing data, are illustrated in [1]. Here only the example of white PM is reported in Figure 2, from which it can be seen that the technique of a fictitious τ_0=2.33 days seems to lead to overestimation, while the interpolation leads to underestimation of the true noise, particularly for small values of the integration time. It is worthwhile to stress that the overestimation of the first method is due to the dependency of TDEV on τ_0 value, while with colored noises, the overestimation is effective.

3. THE ESTIMATE OF MISSING DATA AND UNCERTAINTY EVALUATION

Particular applications need the knowledge of the clock comparison exactly at a certain date which is not in the measurement schedule. The complete evaluation of the missing data asks for its estimate and also for the uncertainty of such estimate.

To this aim, it is necessary to know:

1. the uncertainty on the measured values and the kind of noise due to the comparison technique;

2. the dynamical model describing the evolution in time of the clock difference and the noise affecting the clocks over the observation intervals of interest.

Let's consider, for example, in the typical TWSTT sequence, measurements performed on Monday and Wednesday, but the necessity of estimating the clock difference on Tuesday. As a first step let's consider the simple case of measurements affected by negligible uncertainty. Two hypothesis are then assumed:

1. measurements with negligible uncertainty;
2. clocks affected by phase random walk (i.e. white FM) and clock difference (e.g. UTC[i]-UTC[j]) described by the following equation:

$$x(t+\tau) = x(t) + y \cdot \tau + \varepsilon(t)$$

where τ represents the interval of one day, $x(t)$ the clock difference, y the relative frequency deviation, and $\varepsilon(t)$ a random Gaussian noise yielding the white FM and thus the random walk PM. The latter assumption is reasonable in case of cesium clocks compared daily. From one day to the following one the dominant noise driving the clock behavior is a white FM, resulting in a phase random walk. It is also assumed that the relative frequency deviation y is known. That may not always be true because the y also has to be estimated from the measures. The estimation of y can follow different ways (long or short observation interval, different number of measures...) and we will not discuss here this topic. If comparison measures are available for a certain period of time (e.g. months), it is reasonable to assume that the frequency deviation can be easily estimated and thus, for the Tuesday interpolation, it may be supposed known. The situation is thus depicted in Figure 3 where the knowledge on Monday and Wednesday is "perfect," but in the middle the random behavior of the clocks can follow different paths. The estimate of the clock state and its uncertainty follows from the theory of random walks.

A random walk is a process defined by the accumulation of independent random steps and it is a particular Markov process [4], whose peculiarities is that the knowledge of the future state depends only on the present state and not on the past. In case of clocks, it means that the "position" on Tuesday depends only on the position on Monday and not on the previous behavior. The knowledge of the Monday value is then sufficient to estimate the possible state on Tuesday. Since the state on Wednesday is also known by measurement, estimation on Tuesday can also be seen as a "backward" estimation problem. From the theory of least squares and its dynamical version (Kalman filter) it can be demonstrated that the best estimate $\hat{x}_{Tue}$ of the Tuesday value, knowing Monday and Wednesday measures and with the assumptions above, is the average value (corresponding to a straight line interpolation) between the measures of Monday and Wednesday. Such estimate is described by a Gaussian probability density centered on the average value $\hat{x}_{Tue}=(x_{Mon}+x_{Wed})/2$ and its uncertainty is given by $u_{Tue} = \sqrt{\frac{\sigma^2_{r.w.}}{2}}$, where $\sigma^2_{r.w.}$ is the "diffusion coefficient" describing the daily random walk and that can be easily estimated by observing [5] that $\sigma^2_{rw} = AVAR(\tau)\cdot\tau$, with dimensions of $[\sigma^2_{rw}] = \text{ns}^2/\text{day}$, when x is measured in ns and τ in days. The estimation of the Tuesday value is thus obtained as depicted in Figure 4.

Let's now consider a more realistic case with the following assumptions:

1. measurements with an uncertainty due to the comparison system, corresponding to a white PM (thus uncorrelated from one measure to the following and uncorrelated from clock noise) with zero average and variance σ^2_{ms};
2. (as before) clocks affected by phase random walk (i.e. white FM) and clock difference (e.g. UTC(i)-UTC(j)) described by the following equation:

$$x(t+\tau) = x(t) + y\cdot\tau + \varepsilon(t)$$

In this case, the measurements are executed with an uncertainty σ_{ms}; therefore the knowledge of the clock state on Monday and Wednesday is not perfect. From Monday to Wednesday, the evolution of the clock state is always described by random walk in phase. The best estimate is also in this case obtained by the average of the Monday and Wednesday values, but the uncertainty of this estimate contains also the contribution of the measurement uncertainty, leading to a final Tuesday uncertainty u_{Tue} equal to:

$$u_{Tue} = \sqrt{\frac{(\sigma^2_{rw} + \sigma^2_{ms})}{2}}$$

In this situation also the previous and following measures become useful. By the knowledge of clock behavior and the characteristics of the involved noises, the measures performed before that Monday can be inserted in a Kalman filter and the estimation of the clock state on that Monday can improve. That means reducing the uncertainty on the knowledge of the clock state on Monday, i.e. reducing the σ_{ms} by the use of previous measurements. Let's indicate by $\sigma_{b.e}$ i.e. "best estimate," the resulting uncertainty on the knowledge of the clock state on such particular Monday. If the model is correct the following relationship holds:

$$\sigma_{b.e} < \sigma_{ms}$$

The same can be done backwards, for improving the knowledge of the Wednesday state by filtering the measures at disposal after that particular Wednesday. We are now in the same situation of the beginning but with the estimates on Monday and Wednesday affected by a minor uncertainty $\sigma_{b.e}$. Nevertheless the clock random walk between Monday and Wednesday is unchanged and unaffected by the knowledge of previous and following data. The previous and the following data can be used to reduce the uncertainty of the Monday and Wednesday estimates, but nothing can be done to reduce the noise contribution σ_{rw}. Therefore the best estimate of the Tuesday datum is always obtained by the average $\hat{x}_{Tue} = [(x_{Mon}+x_{Wed})/2]$ with uncertainty given by $\sqrt{(\sigma^2_{rw} + \sigma^2_{be})/2}$ as depicted in Figure 5.

Let's now briefly examine the case of a reconstruction on Saturday or Sunday datum based on the Friday and Monday measures. In the case of measures without uncertainty, the best estimate of the Saturday value is obtained from the linear interpolation of the Friday and Monday values, but, if the measures are affected by an uncertainty σ_{ms}, the estimate on Saturday is not directly the linear interpolation of Friday and Monday data, but it has a term depending on the σ_{ms} and the frequency deviation y also. The best estimate $\hat{x}_{Sat}$ of the Saturday value can be written as:

$$\hat{x}_{Sat} = x_{Fri}\frac{(\sigma_{ms}^2 + 2\sigma_{rw}^2)}{2\sigma_{ms}^2 + 3\sigma_{rw}^2} - y\frac{\sigma_{ms}^2}{2\sigma_{ms}^2 + 3\sigma_{rw}^2} + x_{Mon}\frac{(\sigma_{ms}^2 + \sigma_{rw}^2)}{2\sigma_{ms}^2 + 3\sigma_{rw}^2}$$

and it can be seen that, in case σ_{ms}=0, the estimate reduces to the linear interpolation given by :

$$\hat{x}_{Sat} = \frac{2}{3}x_{Fri} + \frac{1}{3}x_{Mon}$$

The uncertainty u_{Sat} on such best estimate can be written as:

$$u_{Sat} = \sqrt{\frac{\sigma_{ms}^4 + 3\sigma_{rw}^2 \cdot \sigma_{ms}^2 + 2\sigma_{rw}^4}{2\sigma_{ms}^2 + 3\sigma_{rw}^2}}$$

the same expressions are valid for the Sunday reconstruction, by interchanging the role of Friday and Monday.

The assumptions herewith considered can be quite realistic in the case of TWSTT comparing two high performance Cs clocks. Actually, the model of the clock noise could be incomplete because a pure random walk was considered with frequency deviation y assumed to be known and fed as an external input. In case the frequency offset had to be estimated inside the same estimation process, the estimate would depend on the technique chosen for the estimation of the frequency deviation. In this case a complete Kalman filter has to be examined and some runs can give an estimate on the uncertainty of reconstructed values [2].

If the frequency offset is not known, the Kalman filter has to estimate both frequency and phase offsets between clocks, so some uncertainty is added on the Tuesday estimate because there is one more state element to be estimate and thus introducing uncertainty. Therefore, by adding in the model an unknown frequency deviation, the result is that the Tuesday estimate is obtained with larger uncertainty than the case here evaluated. The cases presented above can then be considered as the case leading to the minor uncertainty on the Tuesday reconstruction. Let's recall that the estimate is optimal only if the model, concerning clock dynamics and noise as well as measurement noise, is correct.

4. RESULTS OBTAINED USING THE TWSTT MEASURES

The developed theory was applied to TWSTT experimental measures supplied by the TWSTT Study Group, through the BIPM, and concerning three different data sequences as summarized in the following table

UTC(i)-UTC(j)	First Datum	Last datum	Days in the period	Weeks in the period
PTB-NIST	01-08-97	08-05-98	281	40
TUG-NIST	01-08-97	08-05-98	281	40
TUG-PTB	21-02-97	06-05-98	440	62

The results on experimental data are to be considered examples of how the theory can be applied and which can be the consequent estimates in the frame of particular assumptions, with the aim of helping the following development and understanding of particular aspects of TWSTT and the use of their measures.

4.1 NOISE ANALYSIS

In order to evaluate the noise affecting these series, the three different approaches described in Sec. 2 were followed. In the case of ADEV evaluation, with the method referred as "true ADEV," i.e. the first of the outlined methods (Sec. 2.1), only the measures at disposal were used without any prior manipulation. The other two methods need some kind of missing data reconstruction, in fact also by using the method of a fictitious τ_0=2.33 days, the missing scheduled measures (i.e. on Monday, Wednesday, and Friday) need to be reconstructed to preserve the spacing of 2-2-3 days between measures. This was done by linear interpolation, because it seemed safer than leaving the "holes" and evaluating a new average τ_0. The new sequences obtained are affected only by the typical uneven periodicity of the TWSTT. The last method (Sec. 2.3) asks for a complete reconstruction (i.e. also of Tuesday, Thursday, Saturday, and Sunday) in order to obtain a daily sampled data series. ADEV and TDEV were estimated according to the three procedures [2].

Let's here examine only a particular interesting result concerning only the "true ADEV." From an inspection of Figure 6 some important hints can be obtained. First, the ADEV behaviors are very similar to the high performance HP clock stability, indicating that, at least on such observation intervals, the noise added by the TWSTT link is negligible and clock instabilities are dominating. This is particularly true for the sequences TUG-PTB and TUG-NIST, where a first part due to white FM is recognizable. In the second part of the plot something similar to a flicker FM appears and that could be perhaps due to the combined effect of true clock noise together with the steering or clock correction effect. If it is assumed that the ADEV of the sequences TUG-PTB and TUG-NIST represent clock noise, it is possible to trace the slope corresponding to white FM and estimating the white FM level of clock noise over τ=1 day. This leads to the estimation of $\mathrm{ADEV}(1day) \approx 3 \cdot 10^{-14}$.

On the other hand, the ADEV corresponding to the sequence PTB-NIST shows the typical clock instability behavior except maybe for the first point on τ=2 days, which seems a bit higher than it should be if belonging to a white FM slope. Since we don't have here other information on the measurement system noise, let's assume that this first point belongs to a white PM slope representing the noise added by the comparison link. This is a conservative estimate of the synchronization noise; the actual noise could be lower. Also, this is an hypothesis on the TWSTT noise, and other hypotheses would be possible. The white PM assumption is certainly the simplest and often it is reasonable, but the final statement concerning TWSTT system noise come from experimental evidences and not from assumptions. Since we are here interested in the evaluation of missing data, such hypotheses are necessary for the following treatment, but results are valid only as long as the assumptions of the theoretical model can reasonably represent reality.

If a white PM is assumed, by tracing the corresponding slope, a value of $\mathrm{ADEV}(1day) \approx 3 \cdot 10^{-14}$ is obtained, representing the noise added by the measurement system. Since, in the following evaluations, the classical variance σ^2_{ms} of the measurement system is needed, it can be evaluated by remembering that in case of white PM the classical variance is equal to TVAR(τ_0) and that, when $\tau=\tau_0$=1 day, ADEV(τ_0)=MDEV(τ_0). Therefore,

$$\sigma^2_{ms} = TVAR(\tau_0) = AVAR(\tau_0) \cdot \frac{\tau_0}{3} \qquad \Rightarrow \qquad \sigma_{ms} \approx 1.7 \quad \mathrm{ns}$$

4.2 MISSING DATA EVALUATIONS

The second problem of estimating UTC(i)-UTC(j) values on a certain date is also addressed in the frame of the following working assumptions:

1. measurements with an uncertainty due to the comparison system, corresponding to a white PM (thus uncorrelated from one measure to the following and uncorrelated from clock noise) with zero average and variance σ^2_{ms};
2. clocks affected by phase random walk (i.e. white FM) characterized by a "diffusion coefficient" σ^2_{rw} and known relative frequency deviation y.

From the noise analysis illustrated in the previous section and in particular from the discussion concerning Figure 6, some estimates of the noise are assumed, with the aim of providing an example on how the estimation of the missing data can be performed. Therefore, the following numbers are not to be considered definitive, but only an illustrative example.

As far as the noise of the comparison link is concerned, it was estimated that:

$$\sigma^2_{ms} = TVAR(\tau_0) = AVAR(\tau_0) \cdot \frac{\tau_0}{3} \qquad \Rightarrow \qquad \sigma_{ms} \approx 1.7 \quad \mathrm{ns}$$

As far as the clock noise σ^2_{rw} is concerned, it was estimated that $\mathrm{ADEV}(1day) \approx 3 \cdot 10^{-14}$. By recalling the relationship, $\sigma^2_{rw} = AVAR(\tau) \cdot \tau$, with dimensions of $[\sigma^2_{rw}] = \mathrm{ns}^2/\mathrm{day}$, it can be estimated:

$$\sigma^2_{rw} = AVAR(1day) \cdot 1day = 9 \cdot 10^{-28} \cdot 86400\,\mathrm{s} = 9\left(\frac{\mathrm{ns}^2}{day}\right)\left(\frac{86400\,\mathrm{s}}{10^5}\right)^2 \approx 7.8\,\frac{\mathrm{ns}^2}{day}$$

With these noise values, it appears that the best estimate of Tuesday value, when measurements are performed on Monday and Wednesday, is the average value of the Monday and Wednesday measures with uncertainty u_{Tue}:

$$u_{Tue} = \sqrt{(\sigma^2_{rw} + \sigma^2_{ms})/2} \approx 2.3\,\mathrm{ns}$$

while the best estimate of the Saturday value, when measures are performed on Friday and Monday depends on the value of the relative frequency deviation y, and its uncertainty is:

$$\mathrm{u}_{Sat} = \sqrt{\frac{\sigma^4_{ms} + 3\sigma^2_{rw} \cdot \sigma^2_{ms} + 2\sigma^4_{rw}}{2\sigma^2_{ms} + 3\sigma^2_{rw}}} \approx 2.6\,\mathrm{ns}$$

5. CONCLUSION

The statistical problems that arise in the treatment of irregularly sampled data were investigated and some possible procedure to overcome such problems were proposed. Firstly, the ADEV and TDEV analysis can be performed by following three approaches:

1) finding some regularities in the irregularly sampled sequence. For example, in case of TWSTT, for τ=2, 5, 7, 9, 12, 14 ... days the ADEV can be evaluated without any manipulation of data;
2) proceedings as if the data were equally spaced of a fictitious τ_0=2.33 days. Particular care is to be taken: results are correct only in case of pure white PM, if not the noise is overestimated;
3) interpolating missing measures with the aim of obtaining a daily spaced sequence. Noise results depend on the real noise and on the data reconstruction which filters the faster noise frequencies. Therefore, for small values of τ, ADEV and TDEV are underestimated.

The second aim is the best estimate of the clock difference on a certain date when TWSTT measurements are not available and the uncertainty of such estimate. By using the theory of least squares and the Kalman filter it was possible to evaluate the best estimate and its uncertainty, which depends on the noise of the TWSTT measure as well as on the random noise of the clocks and on the clock model. To provide an example, the following working assumptions were formulated:

- TWSTT comparison noise corresponding to a WPM;
- clock noise over one day corresponding to a WFM;
- relative frequency deviation of the compared clocks known with negligible uncertainty.

In this frame the best estimates of the missing data and the uncertainties were inferred.

It is worthwhile to remember that different level of noise or different models would lead to different estimates, therefore each particular situation has to be suitably evaluated, accordingly to the main lines here developed. In particular, the final uncertainty added by the measurement system should be derived from experimental tests and not only be based on assumptions. As a last remark, if the measurement was performed on the requested Tuesday or Saturday, the only uncertainty would be due to the measurement system.

6 REFERENCES

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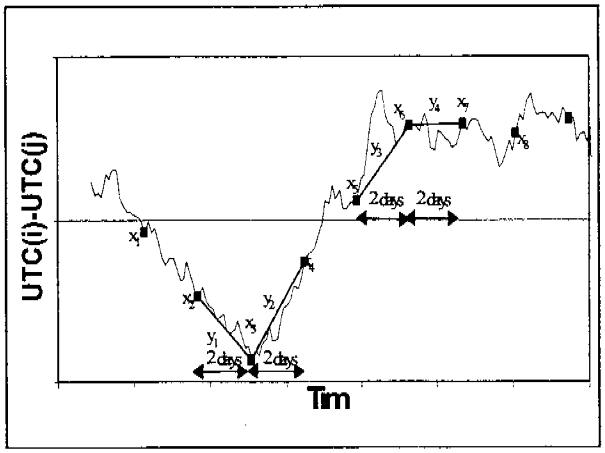


Figure 1. *Estimation of ADEV(2 Days). Measures are executed on Monday, Wednesday and Friday. For any $\Delta_2 x$ estimation, the three consecutive measures are considered for each week.*

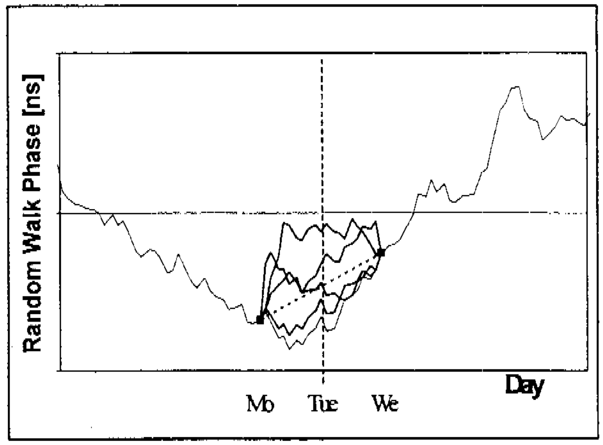


Figure 3. *Random walk behavior between Monday and Wednesday.*

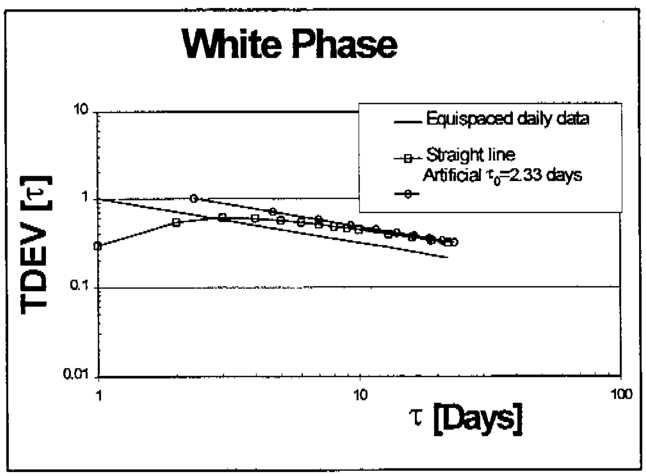
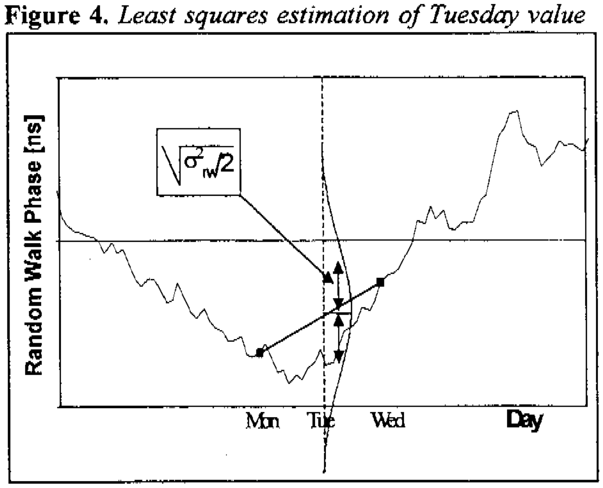


Figure 2. *Theoretical behavior in case of white PM as estimated by: a) true daily equally spaced series, b) TWSTT spaced series considered as equally spaced by $\tau_0=2.33$, c) daily series reconstructed by a straight line interpolation.*

Figure 4. *Least squares estimation of Tuesday value*



in case of negligible measurement uncertainty

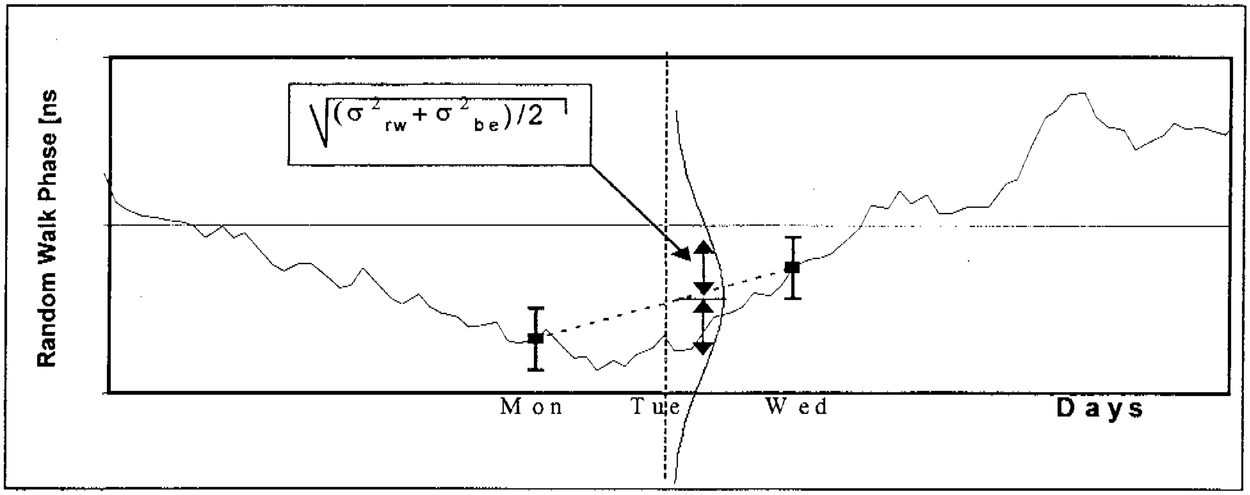


Figure 5. *Least square estimate of Tuesday value in case of measurement uncertainty.*

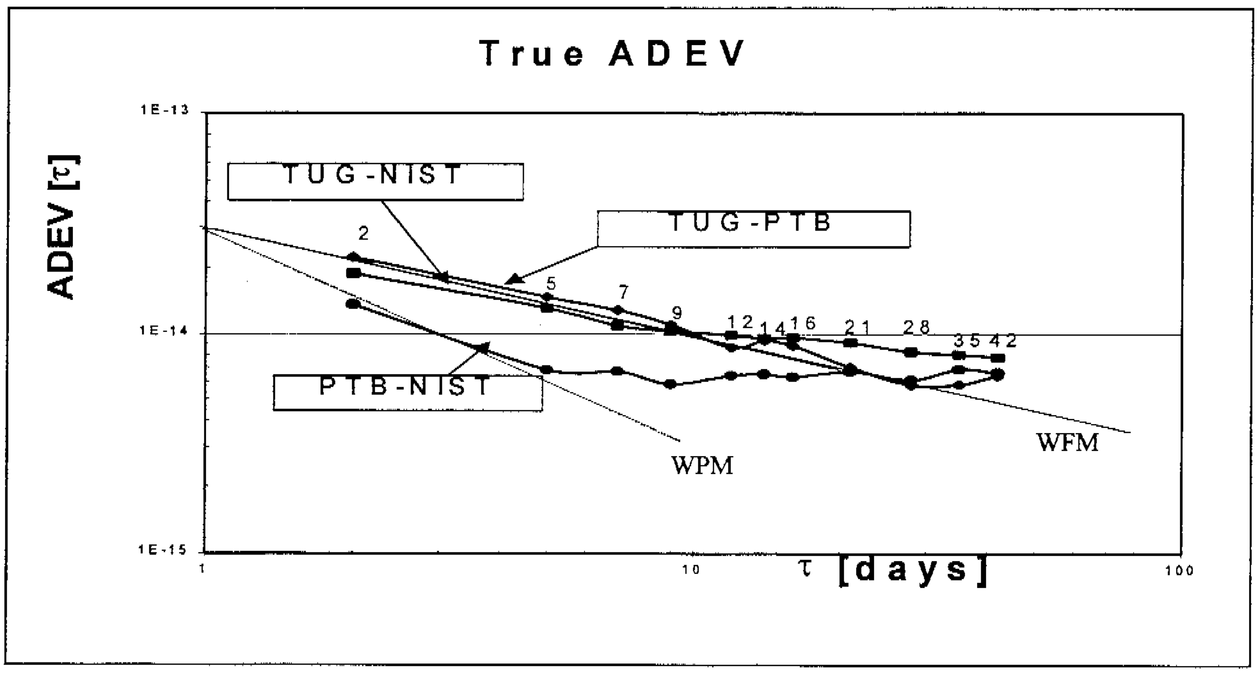


Figure 6. *True ADEV estimates for the three different links with the slope of the white PM and white FM.*

Questions and Answers

ROBERT DOUGLAS (NRC): For the uncertainty in the hydrogen maser comparison, were you including the flicker floor noise?

PATRIZIA TAVELLA (IEN):: Actually, I used white frequency and a drift, because I had some drift specifications on the hydrogen maser, which I used. I spoke with persons making the measurements, and I asked for the actual measurement, which seems to have a flicker. Since all these evaluations are done in an analytical way, the flicker is difficult to be treated analytically. I suppose that even if we consider something that is worse than flicker, for example, random walk frequency, it will be at such a lower level that in any case it will be negligible. The random part of the hydrogen maser will be negligible, I guess.