NARROW-BAND SEARCHES FOR GRAVITATIONAL RADIATION WITH SPACECRAFT DOPPLER TRACKING

Massimo Tinto and J. W. Armstrong
Jet Propulsion Laboratory, California Institute of Technology
Pasadena, California 91109

Abstract

We discuss a filtering technique for reducing the two-way Doppler frequency fluctuations of noise sources localized in space (like the troposphere, or the master clock) that affect the sensitivity of spacecraft Doppler tracking searches for gravitational radiation. This method takes advantage of the sinusoidal behavior of the transfer function to the Doppler observable of these noise sources, which displays sharp nulls at selected Fourier components.

The non-zero gravitational wave signal remaining at these frequencies makes this Doppler tracking technique the equivalent of a series of narrow-band detectors of gravitational radiation[1]*, distributed across the low-frequency band. Estimates for the sensitivities achievable with the future Cassini Doppler tracking experiments are presented in the context of broad-band gravitational wave bursts, monochromatic signals, and a stochastic background of gravitational radiation*[2]*.*

INTRODUCTION

Doppler tracking of interplanetary spacecraft is the only existing technique that allows searches for gravitational radiation in the millihertz frequency region.[3] The frequency fluctuations induced by the intervening media have severely limited the sensitivities of these experiments. Among all the propagation noise sources (ionosphere, troposphere, interplanetary plasma) observed at high radio frequencies, the troposphere is the largest and the hardest to calibrate to a reasonably low level. The frequency fluctuations due this noise source have been observed with water vapor radiometers to be a few parts in 10^{-14} at 1000 seconds integration time[4].

Mechanical vibrations of the ground antenna also introduce frequency fluctuations in the Doppler data, and in some occasions represent the limits in the sensitivity of these gravitational wave experiments. Recent frequency stability measurements performed on the antenna of the Deep Space Network (DSN) that will track the Cassini spacecraft at Ka-Band

(32 GHz), indicate that the frequency fluctuations it introduces are as large as 3.0×10^{-15} at 1000 seconds integration time.[5] This number is comparable to the requirement in frequency stability requested by the experimenters for the overall performance of the DSN during the Cassini gravitational wave experiments at Ka-Band. Although work is in progress to identify the causes of this performance, it might turn out to be intrinsically impossible to compensate or calibrate the mechanical vibrations of the antenna.

In order to minimize the effects of the troposphere, ionosphere, and of the ground antenna in the Doppler data, it was first pointed out by Estabrook[6] that the explicit transfer function to the Doppler observable of, for example, tropospheric and antenna mechanical noise are sinusoidal, and therefore their magnitudes can be reduced at selected Fourier frequencies. In this paper we show how this modulation of the noise spectrum can be exploited in a very robust way when these noise sources are the limiting ones. As a specific example we will estimate the sensitivity to gravitational radiation that Doppler tracking data from Cassini will be able to identify by using this narrow-banding technique. Cassini will take advantage of a multilink radio system for removing the frequency fluctuations introduced by the interplanetary plasma in the Doppler data, and its Doppler sensitivity to gravitational radiation could be limited by the frequency fluctuations introduced by the ground antenna.

THE TWO-WAY DOPPLER RESPONSE

In the Doppler tracking technique, the Earth and an interplanetary spacecraft act as free test masses. The Doppler tracking system continuously measures their relative dimensionless velocity, $\Delta v/c = \Delta\nu/\nu_0$ (here Δv is the relative velocity, $\Delta\nu$ is the Doppler frequency fluctuation, and ν_0 is the nominal frequency of the microwave link). A gravitational wave of strain amplitude $h(t)$ incident on the system causes small perturbations in the Doppler time series of $\Delta\nu(t)/\nu_0$. These perturbations are of order h and are replicated three times in the tracking record with a characteristic pattern that depends on the source-Earth-spacecraft angle[3]. These three events in the time series can be thought of as due to the gravitational wave buffeting the Earth, the spacecraft, and the original Earth buffeting event transponded back to the Earth at a time $2L/c$ later, where we have denoted with L the distance between the Earth and the spacecraft, and c is the speed of light. The sum of these three Doppler perturbations is zero. This overlap of the three events, and partial cancellation, occurs for gravitational wave pulses having widths larger than about L/c.

Detection of gravitational wave signals in this millihertz band is complicated by various noises. The principal noise sources are[4,5,7]: ground electronics noise (including frequency standard and frequency distribution noise), antenna mechanical noise (unmodeled motion of the antenna), thermal noise (finite signal-to-noise ratio on the radio links), spacecraft noise (electronics and unmodeled motion of the spacecraft), and propagation noise (phase scintillation as the radio beams pass through irregularities in the troposphere, ionosphere, and the solar wind). Electronic and thermal noises can be made very small, and propagation noises can be mitigated through use of higher or multiple radio frequencies to suppress or remove entirely charged particle scintillation, and by employing water vapor radiometers to measure tropospheric scintillation. Residual uncalibrated troposphere and antenna mechanical noise are expected to be leading noise sources in the future Cassini gravitational wave experiment[9].

If we introduce a set of Cartesian orthogonal coordinates (X, Y, Z) in which the wave is propagating along the Z-axis and (X, Y) are two orthogonal axes in the plane of the wave, then the Doppler response at time t can be written in the following form[1,3,4]

$$\begin{aligned} y(t) \equiv \left(\frac{\Delta\nu(t)}{\nu_0}\right) &= -\frac{(1-\mu)}{2}\, h(t) \;-\; \mu\, h(t-(1+\mu)L) + \frac{(1+\mu)}{2}\, h(t-2L) \\ &+\; C(t-2L) \;-\; C(t) \;+\; 2B(t-L) \;+\; T(t-2L) \;+\; T(t) \\ &+\; A_E(t-2L) \;+\; A_{sc}(t-L) \;+\; TR(t-L) \;+\; EL(t) \;+\; P(t)\;, \quad (1) \end{aligned}$$

where $h(t)$ is equal to

$$h(t) = h_+(t)\cos(2\phi) + h_\times(t)\sin(2\phi)\;. \quad (2)$$

Here $h_+(t)$, $h_\times(t)$ are the wave's two amplitudes with respect to the (X, Y) axis, (θ, ϕ) are the polar angles describing the location of the spacecraft with respect to the (X, Y, Z) coordinates, μ is equal to $\cos\theta$, and L is the distance to the spacecraft (units in which the speed of light $c = 1$).

We have denoted by $C(t)$ the random process associated with the frequency fluctuations of the clock on the Earth, $B(t)$ the joint effect of the noise from buffeting of the probe by non–gravitational forces and from the antenna of the spacecraft, $T(t)$ the joint frequency fluctuations due to the troposphere, ionosphere and ground antenna, $A_E(t)$ the noise of the radio transmitter on the ground, $A_{sc}(t)$ the noise of the radio transmitter on board, $TR(t)$ the noise due to the spacecraft transponder, $EL(t)$ the noise from the electronics on the ground, and $P(t)$ the fluctuations due to the interplanetary plasma. Since the frequency fluctuations induced by the plasma are, to first order, inversely proportional to the square of the radio frequency, by using high frequency radio signals or by monitoring two different radio frequencies transmitted to the spacecraft and coherently transmitted back to Earth, this noise source can be suppressed to very low levels or entirely removed from the data respectively.[10]

From Eq. (1) we deduce that gravitational wave pulses of duration longer than the round-trip light time $2L$ give a Doppler response $y(t)$ that, to first order, tends to zero. The tracking system essentially acts as a pass-band device, in which the low-frequency limit f_l is roughly equal to $(2L)^{-1}$ Hz, and the high-frequency limit f_H is set by the thermal noise in the receiver. Since the reference clock and some electronic components are most stable at integration times around 1000 seconds, Doppler tracking experiments are performed when the distance to the spacecraft is of the order of a few Astronomical Units. This sets the value of f_l for a typical experiment to about 10^{-4} Hz, while the thermal noise gives an f_H of about 3×10^{-2} Hz.

It is important to note the characteristic time signatures of the clock noise $C(t)$, of the probe antenna and buffeting noise $B(t)$, of the troposphere, ionosphere, and ground antenna noise $T(t)$, and the transmitters $A_E(t)$, $A_{sc}(t)$. The time signature of the clock noise can be understood by observing that the frequency of the signal received at time t contains clock fluctuations transmitted $2L$ seconds earlier. By subtracting from the received frequency the frequency of the radio signal transmitted at time t, we also subtract clock frequency fluctuations[1,3,4] with the net result shown in Eq. (1).

As far as the fluctuations due to the troposphere, ionosphere, and ground antenna are concerned, the frequency of the received signal is affected at the moment of reception,as well as $2L$ seconds earlier. Since the frequency of the signal generated at time t does not contain yet any of these fluctuations, we conclude that $T(t)$ is positive-correlated at the round trip light time $2L$[1,4]. The time signature of the noises $B(t)$, $A_E(t)$, $A_{sc}(t)$, and $TR_{sc}(t)$ in Eq. (1) can be understood through similar considerations.

If we denote with $\widetilde{y}(f)$ the Fourier transform of the Doppler response $y(t)$

$$\widetilde{y}(f) = \int_{-\infty}^{\infty} y(t)\, e^{2\pi i f t}\, dt \ , \tag{3}$$

we can rewrite Eq. (1) in the Fourier domain as follows

$$\begin{aligned}\widetilde{y}(f) \;=\; & \left[-\frac{(1-\mu)}{2} - \mu\, e^{2\pi i f(1+\mu)L} + \frac{(1+\mu)}{2}\, e^{4\pi i f L}\right] \widetilde{h}(f) \;+\; \widetilde{C}(f)\, \left[e^{4\pi i f L} - 1\right] \\ & + \; \widetilde{T}(f)\, \left[e^{4\pi i f L} + 1\right] + 2\widetilde{B}(f)\, e^{2\pi i f L} + \left[\widetilde{A_{sc}}(f) + \widetilde{TR}(f)\right] e^{2\pi i f L} \\ & + \; \widetilde{A_E}(f)\, e^{4\pi i f L} + \widetilde{EL}(f) + \widetilde{P}(f)\ .\end{aligned} \tag{4}$$

If the noise due to the troposphere, ionosphere, and mechanical vibrations, T, will dominate over the remaining noise sources, as it might be the case during the Cassini experiments, then the spectra of the noise will appear modulated, and will display minima at the following frequencies

$$f_k = \frac{(2k-1)}{4L} \qquad ; \qquad k = 1, 2, 3, \dots\ . \tag{5}$$

At these frequencies Eq. (4) can be rewritten in the following approximate form[1,2]

$$\begin{aligned}\widetilde{y}(f_k) \;\approx\; & \left[-1 + i\,(-1)^k\, \mu\, e^{\frac{\pi}{2}\, i\, (2k-1)\, \mu}\right] \widetilde{h}(f_k) \;-\; 2\,\widetilde{C}(f_k) \;+\; \widetilde{T}(f_k)(\pi i L \Delta f) \;+ \\ & + \; i\,(-1)^{k+1} \left[2\widetilde{B}(f_k) + \widetilde{TR}(f_k) + \widetilde{A_{sc}}(f_k)\right] - \widetilde{A_E}(f_k) + \widetilde{EL}(f_k) + \widetilde{P}(f_k)\ ,\end{aligned} \tag{6}$$

where Δf is the frequency resolution in the Fourier domain.

For current generation precision Doppler experiments, utilizing approximately 8 GHz radio links, observations in the anti-solar hemisphere have significant contributions from tropospheric and extended solar wind scintillation, while ionospheric, frequency standard, and antenna mechanical noise are secondary disturbances in the Doppler link.[4] As an example, in Figure 1 we show the temporal autocorrelation function of 10-second time resolution Mars Global Surveyor *(MGS)* data taken on April 17, 1997, when the two-way light time $2L/c$ was equal to 504 seconds. The clear positive correlation at time lag of 504 seconds indicates that tropospheric scintillation and fluctuations induced by mechanical vibrations of the ground antenna dominate the noise on this data set.

The inset plot in Figure 1 shows the power spectrum of the *MGS* data, with frequency scale marked in units of $1/2L$. No smoothing of the spectrum has been done and much of its spikiness is due to estimation error.[11] The cosine-squared modulation is however evident, showing that there are nulls at odd multiples of $1/(4L)$. At these frequencies, fluctuations from other noise sources will dominate the power spectrum. If the spectral level of these secondary noises is low, there is a potentially large improvement in SNR for gravitational wave signals having Fourier power at the nulls of the troposphere/antenna mechanical transfer function. In its simplest form, filtering the data to pass a comb of narrow bands centered on the nulls of the cosine-squared transfer function blocks the troposphere/antenna mechanical noise while passing gravity wave power at these frequencies. This is robust, in that nothing needs to be known about the signal except that it (by hypothesis) has power at odd multiples of $1/(4L)$.

In Figure 2 we plot the estimated one-sided power spectrum of the noise that will affect the Cassini Doppler data in its frequency band of observation. The lower curve represents the configuration in which eighty percent of the noise due to the troposphere is calibrated out by means of water vapor radiometry, while the upper curve corresponds to the configuration without calibration of the troposphere. Note that the minima of both curves coincide at the nulls of the transfer function of the random process T.

We should point out that, in order to derive the Fourier transform of the Doppler response in Eq. (6), we have assumed the distance to the spacecraft to be constant during the duration of the experiment. In the case of the *MGS* experiment we have found that we could integrate coherently over a time scale equal to about eight hours, before the frequencies f_k would change by an amount larger than the frequency resolution Δf. For the Cassini trajectory instead the integration time can be extended to the entire duration (forty days) of the experiments. This can be seen by considering the time dependence of the ranging to Cassini, $L(t)$, during the gravitational wave experiments. For the three experiment opportunities, $L(t)$ can be represented quite accurately by a quadratic expression[12]

$$
\begin{aligned}
L(t) &= a + bt + ct^2 \\
a &= 2.9 \times 10^3 \ \text{sec} \\
b &= -1.4 \times 10^{-5} \\
c &= 1.1 \times 10^{-11} \ \text{Hz}
\end{aligned} \tag{7}
$$

where the numerical values of the coefficients a, b, c correspond to the first Cassini gravitational wave experiment. Since the variation δf of the frequencies f_k is determined by the time derivative $\dot{L}(t)$, from Eq. (7) we find that

$$
\dot{L}(\tau) = \frac{\delta f}{f_1} = 6.3 \ \times \ 10^{-5} < \frac{1}{(\tau f_1)} = \frac{\Delta f}{f_1} = 4.8 \ \times \ 10^{-3} \ , \tag{8}
$$

where τ is the integration time, which we have assumed to be equal to about 14 days. We have calculated the right-hand side of Eq. (8) also for the remaining two Cassini oppositions and we have found that the maximum variation of the frequencies f_k due to the change of the spacecraft distance is smaller than the frequency resolution Δf.

SENSITIVITIES WITH CASSINI

From the plot given in Figure 2 of the estimated one-sided power spectral density of the noise affecting the Cassini Doppler data, we can calculate the root-mean-squared (r.m.s.) noise level $\sigma(f_k)$ of the frequency fluctuations in the bins of width Δf, around the frequencies f_k $(k = 1, 2, 3, ...)$. This is given by the following expression

$$
\sigma(f_k) = [S_y(f_k) \ \Delta f]^{1/2} \ , \ k = 1, 2, 3, ... \ , \tag{9}
$$

where $S_y(f_k)$ is the one-sided power spectral density of the noise sources in the Doppler response $y(t)$ at the frequency f_k as given in Figure 2.

This measure of the Doppler sensitivity is appropriate for sinusoidal or stochastic gravitational wave signals, while it probably overestimates the sensitivity to bursts. A detailed and quantitative analysis for various burst waveforms will be investigated in a forthcoming paper. Here we will rely on the quantitative results implied by the formula given in Eq. (9), keeping in mind the considerations made.

In Figure 3 we plot the sensitivity curve achievable with this technique when applied to the Cassini Doppler data during the first solar opposition, when the distance to the spacecraft is about $L = 5.5$ AU. The corresponding fundamental frequency f_0 is equal to $4.5 \ \times 10^{-5}$ Hz. We also assume an integration time of about 14 days, since only one of the three DSN complexes will have the capability of supporting simultaneous transmission and reception of

X and K_a band signals. In the assumption of calibrating entirely the plasma noise with the use of dual frequencies, at $f_0 = 4.5 \times 10^{-5}$ we find a sensitivity equal to about 2.0×10^{-16}. The best sensitivity is however achieved at about 2.0×10^{-3}, at a level of about 5.0×10^{-17}.

CONCLUSIONS

The main result of our analysis shows that, during searches for gravitational radiation, it is possible to reduce the effects of the troposphere, ionosphere, and mechanical vibrations of the ground antenna at selected Fourier components of the power spectrum of two-way Doppler tracking data. A sensitivity equal to about 5.0×10^{-17} at a frequency of 2.0×10^{-3} has been estimated for the future Cassini gravitational wave experiments, which will first be performed in December 2001.

The experimental technique presented in this paper can be extended to a configuration with two spacecraft tracking each other through microwave or laser links. Future space-based laser interferometric detectors of gravitational waves[13], for instance, could implement this technique as a backup option, if failure of some of their components would make the normal interferometric operation impossible.

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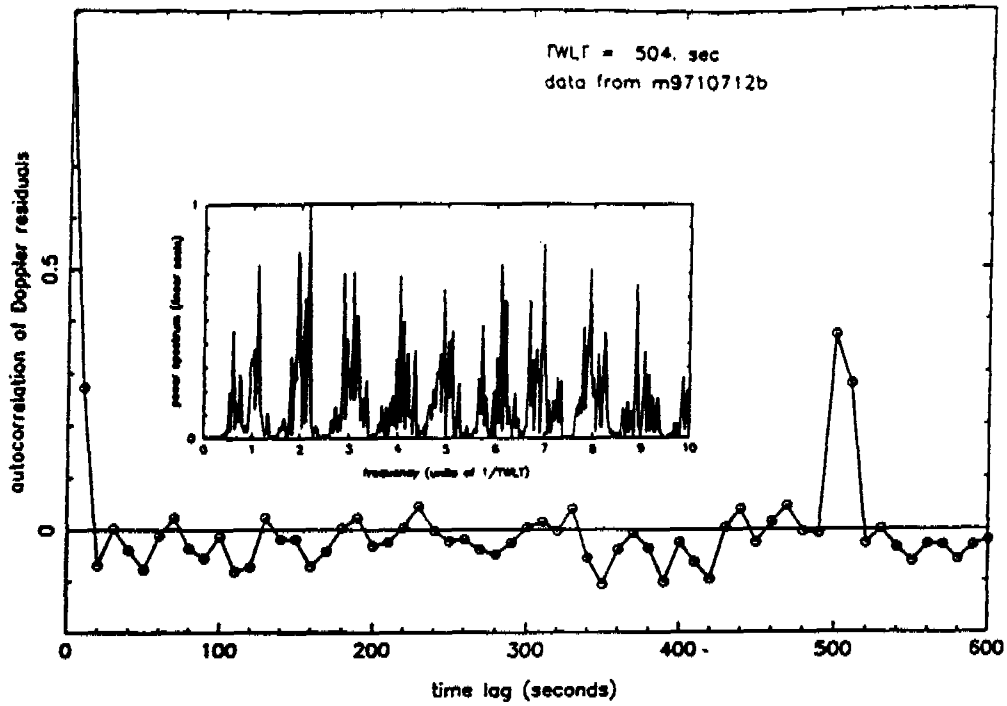

Figure 1: The temporal autocorrelation function of 10-second time resolution Mars Global Surveyor *(MGS)* data taken on April 17, 1997, when the two-way light time $2L/c$ was equal to 504 seconds. The inset plot shows the power spectrum, with frequency scale marked in units of $1/2L$.

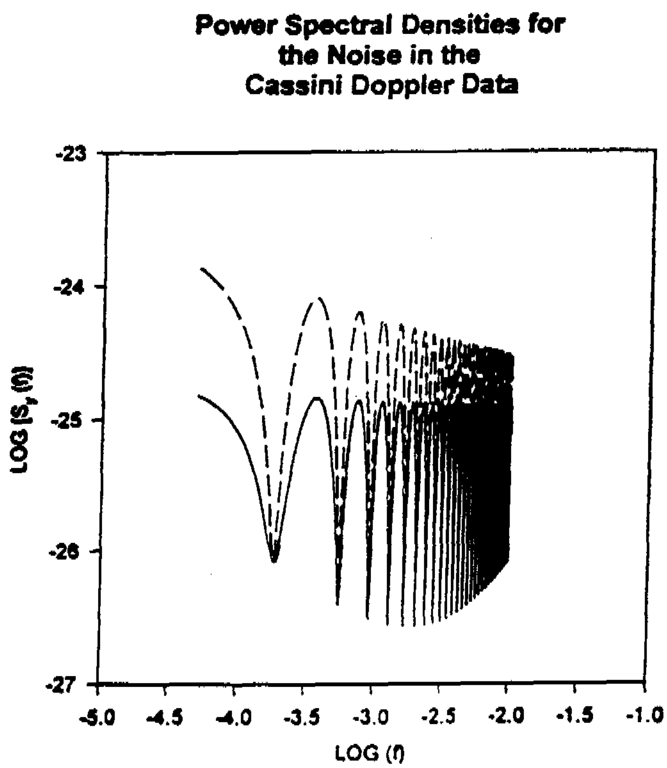

Figure 2: The estimated one-sided power spectrum of the noise that will affect the Cassini Doppler data in its frequency band of observation. The lower curve represents the configuration in which eighty percent of the noise due to the troposphere is calibrated out by means of water vapor radiometry. The upper curve corresponds to the configuration without calibration of the troposphere.

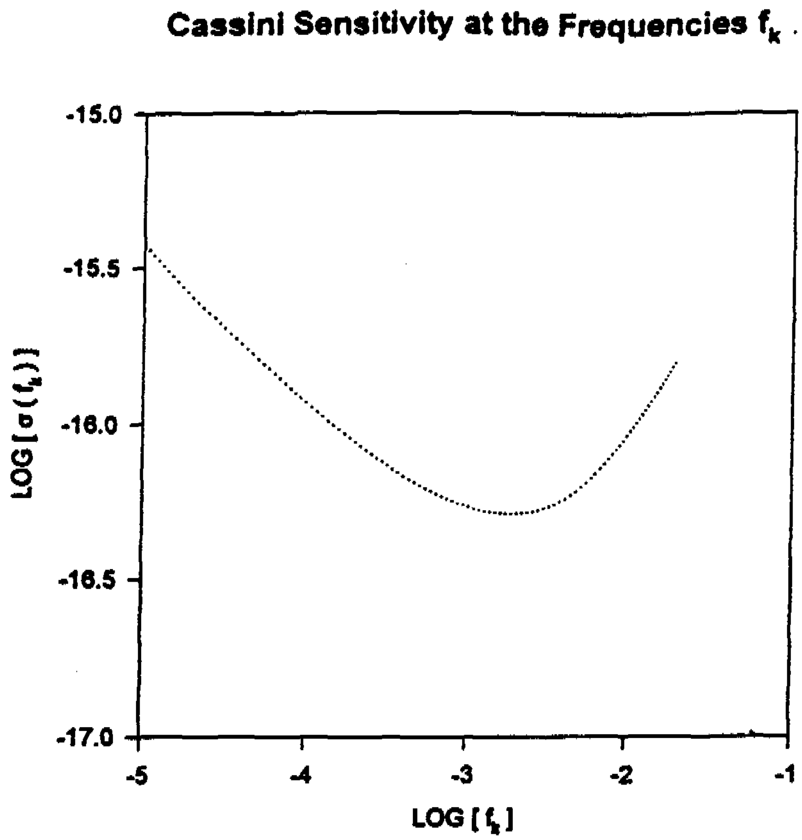

Figure 3: The sensitivity curve achievable with the Cassini Doppler data during the first solar opposition. The distance to the spacecraft is about $L = 5.5$ AU, and the corresponding fundamental frequency f_0 is equal to 4.5×10^{-5} Hz.