SPACECRAFT-SPACECRAFT DOPPLER TRACKING AS A XYLOPHONE INTERFEROMETER DETECTOR OF GRAVITATIONAL RADIATION

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Abstract

We discuss spacecraft-spacecraft Doppler tracking as a detector of gravitational radiation, in which one-way and two-way Doppler data recorded onboard the two spacecraft are time-tagged and telemetered back to Earth. By linearly combining the four Doppler data sets[1]*, we derive a method for reducing by several orders of magnitude, at selected Fourier components, the frequency fluctuation due to the clocks onboard the spacecraft. The nonzero gravitational wave signal remaining at these frequencies makes this spacecraft-spacecraft Doppler tracking technique the equivalent of a xylophone interferometer detector of gravitational radiation.*[2,3] *In the assumption of calibrating the frequency fluctuations induced by the interplanetary plasma, a strain sensitivity of* 3.7×10^{-19} *at* 10^{-3} *Hz is estimated.*

Experiments of this kind could be performed by future interplanetary multi-spacecraft missions planned by the National Aeronautics and Space Administration (NASA).

MOTIVATIONS

- The New Millenium Program is planning to launch multi-spacecraft missions.
- Doppler/ranging measurements between spacecraft can potentially decrease reliance on ground-based tracking.[4]
- Possible backup option for space-based laser interferometers in case of failure.

WHAT IS A XYLOPHONE?

- A detector displaying a significantly enhanced sensitivity at selected Fourier components!
- How can we make a "narrowband" detector out of Doppler tracking?[5]

TWO-WAY DOPPLER RESPONSE

$$\frac{\Delta\nu}{\nu_0}(t) = \frac{(\mu-1)}{2}\, h(t) \;-\; \mu\, h(t-(1+\mu)L) \;+\; \frac{(1+\mu)}{2}\, h(t-2L)$$

$$+C_E(t-2L) \;-\; C_E(t) \;+\; T(t-2L) \;+\; T(t) \;+\; 2B(t-L)$$

$$+A_E(t-2L) \;+\; A_{sc}(t-L) \;+\; TR_{sc}(t-L) \;+\; EL_{E_2}(t) \;+P_{E_2}(t)$$

where:

$$\mu = \cos(\theta) \qquad ; \qquad h(t) = h_+(t)\,\cos(2\phi) \;+\; h_\times(t)\,\sin(2\phi)$$

(Estabrook and Wahlquist[6]).

$$\frac{\widetilde{\Delta\nu}}{\nu_0}(f) = \left[\frac{(\mu-1)}{2} \;-\; \mu\, e^{2\pi i f(1+\mu)L} \;+\; \frac{(1+\mu)}{2}\, e^{4\pi i f L}\right] \tilde{h}(f)$$

$$+\widetilde{C_E}(f)\;\left[e^{4\pi i f L}-1\right] \;+\; \tilde{T}(f)\;\left[e^{4\pi i f L}+1\right] \;+\; 2\,\tilde{B}(f)\, e^{2\pi i f L}$$

$$+\widetilde{A_E}(f)\, e^{4\pi i f L} \;+\; \widetilde{A_{sc}}(f)\, e^{2\pi i f L} \;+\; \widetilde{TR_{sc}}(f)\, e^{2\pi i f L} \;+\widetilde{EL_{E_2}}(f) \;+\; \widetilde{P_{E_2}}(f)$$

(Armstrong[7]).

XYLOPHONE INTERFEROMETER

The two one-way Doppler data are linearly combined to minimize the rms noise level.[5]

$$y_{1a}(t) = y^0_{1a}(t) + C_b(t-L) - C_a(t) + B_b(t-L) + B_a(t) + A_b(t-L) + EL_{1a}(t)$$

$$y_{1b}(t) = y^0_{1b}(t) + C_a(t-L) - C_b(t) + B_a(t-L) + B_b(t) + A_a(t-L) + EL_{1b}(t)$$

By taking the Fourier transform of the following two linear combinations:

$$y_+(t) = (y_{1a}(t) + y_{1b}(t))/2$$

$$y_-(t) = (y_{1a}(t) - y_{1b}(t))/2$$

we derive the following expressions in the Fourier domain:

$$\widetilde{y_+}(f) = \frac{1}{4}\left[(\mu-1)\,(1-e^{2\pi i(\mu+1)fL}) + (\mu+1)\,(1-e^{2\pi i(\mu-1)fL})e^{2\pi ifL}\right]\widetilde{h}(f)$$

$$+\frac{1}{2}\left[\widetilde{C_a}(f)+\widetilde{C_b}(f)\right](e^{2\pi ifL}-1) + \frac{1}{2}\left[\widetilde{B_a}(f)+\widetilde{B_b}(f)\right](e^{2\pi ifL}+1)$$

$$+\frac{1}{2}\left[\widetilde{A_a}(f)+\widetilde{A_b}(f)\right]e^{2\pi ifL} + \frac{1}{2}\left[\widetilde{EL_{1a}}(f)+\widetilde{EL_{1b}}(f)\right]$$

$$\widetilde{y_-}(f) = \frac{1}{4}\left[(\mu-1)\,(1-e^{2\pi i(\mu+1)fL}) - (\mu+1)\,(1-e^{2\pi i(\mu-1)fL})e^{2\pi ifL}\right]\widetilde{h}(f)$$

$$+\frac{1}{2}\left[\widetilde{C_a}(f)-\widetilde{C_b}(f)\right](e^{2\pi ifL}+1) + \frac{1}{2}\left[\widetilde{B_a}(f)-\widetilde{B_b}(f)\right](e^{2\pi ifL}-1)$$

$$+\frac{1}{2}\left[\widetilde{A_b}(f)-\widetilde{A_a}(f)\right]e^{2\pi ifL} + \frac{1}{2}\left[\widetilde{EL_{1a}}(f)-\widetilde{EL_{1b}}(f)\right]$$

$\widetilde{y_+}(f)$, $\widetilde{y_-}(f)$ assume the following form at the Xylophone frequencies f_{2k}, f_{2k-1}

$$f_{2k} = \frac{2k}{2L} \pm \frac{\Delta f}{2}$$

$$f_{2k-1} = \frac{2k-1}{2L} \pm \frac{\Delta f}{2}\ ; \qquad k = 1,2,3,\ldots.$$

$$\widetilde{y_+}(f_{2k}) = \frac{1}{2}\,\mu\,\left[1-e^{2\pi ik\mu}\right]\widetilde{h}(f_{2k}) \pm \frac{1}{2}\left[\widetilde{C_a}(f_{2k})+\widetilde{C_b}(f_{2k})\right](\pi i\Delta fL)$$

$$+\left[\widetilde{B_a}(f_{2k})+\widetilde{B_b}(f_{2k})\right] + \frac{1}{2}\left[\widetilde{A_a}(f_{2k})+\widetilde{A_b}(f_{2k})\right] + \frac{1}{2}\left[\widetilde{EL_{1a}}(f_{2k})+\widetilde{EL_{1b}}(f_{2k})\right]$$

$$\widetilde{y_-}(f_{2k-1}) = \frac{1}{2}\,\mu\,\left[1+e^{\pi i(2k-1)\mu}\right]\widetilde{h}(f_{2k-1}) + \left[\widetilde{B_b}(f_{2k-1})-\widetilde{B_a}(f_{2k-1})\right]$$

$$+\frac{1}{2}\left[\widetilde{A_a}(f_{2k-1})-\widetilde{A_b}(f_{2k-1})\right]\frac{1}{2}\left[\widetilde{EL_{1a}}(f_{2k-1})-\widetilde{EL_{1b}}(f_{2k-1})\right]$$

$$\pm\frac{1}{2}\left[\widetilde{C_a}(f_{2k-1})-\widetilde{C_b}(f_{2k-1})\right](\pi i\Delta fL)$$

If $L = 1$ AU, and $\Delta f = 3 \times 10^{-7}$ Hz

$$\frac{\pi \Delta f L}{c} = 4.7 \times 10^{-4}$$

$L = 1$ AU, $f = 5 \times 10^{-4}$ Hz, and $\Delta f = 3 \times 10^{-7}$ Hz, imply:

$$\Delta L = 1.0 \times 10^{5} \text{ km}$$

$$\sigma(f_k) = \sqrt{S_y(f_k)\ \Delta f}$$

$S_y(f_k)$ = One-sided Power Spectral Density of the remaining noise sources at the Xylophone frequencies f_k.

CONCLUSIONS

- We have shown how frequency fluctuations due to the onboard clocks can be reduced by several orders of magnitude at selected Fourier components (*Xylophone Interferometer*)
- We will investigate how this technique can be extended to other tests of relativistic gravity.

REFERENCES

[1] R.F.C. Vessot, and M.W. Levine, 1979, **General Relativity and Gravitation, 10**, 181.

[2] M. Tinto 1996, **Physical Review, D53**, 5354.

[3] M. Tinto, **Physical Review, D**, submitted.

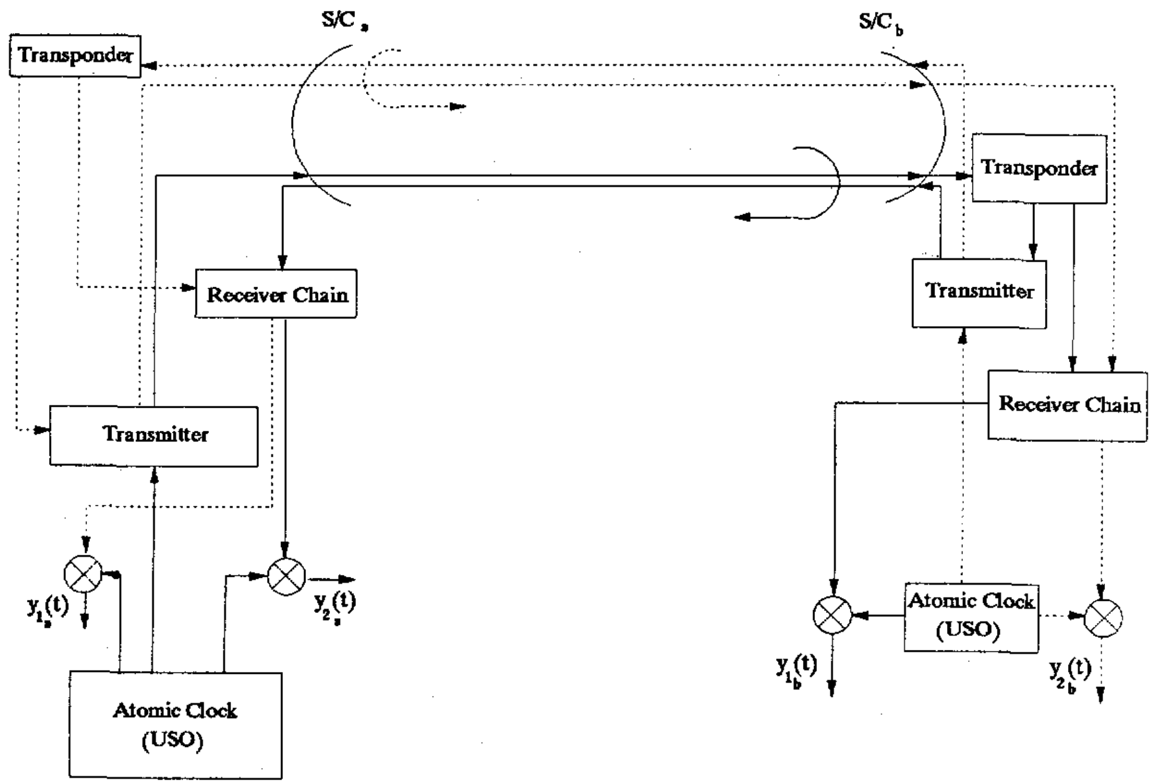
[4] R.D. Kahn, S. Thurman, and C. Edwards 1994, *"TDA Progress Report 42-17,"* 15 May 1994.

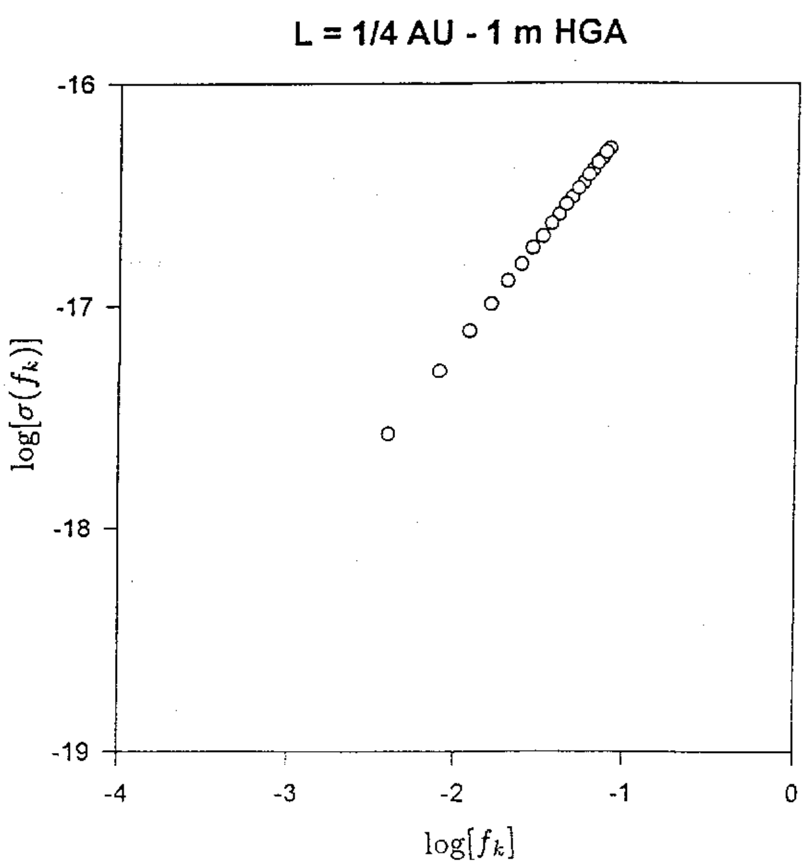
[5] M. Tinto 1996, *"Spacecraft Doppler tracking as a xylophone detector,"* Proceedings of the 27th Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, 29 November-1 December 1995, San Diego, California, USA (NASA CP-3334), pp. 467-478.

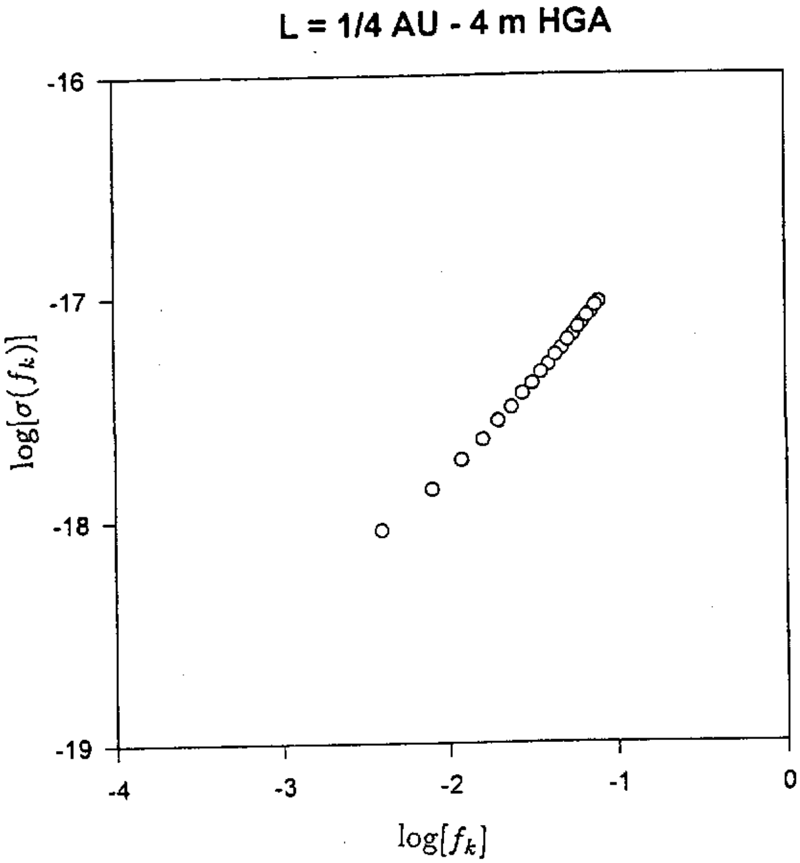
[6] F.B. Estabrook, and H.D.Wahlquist 1975, **General Relativity and Gravitation, 6**, 439.

[7] J.W. Armstrong 1987, **Gravitational Wave Data Analysis**, ed. B.F. Schutz (Kluwer, Dordrecht, Netherlands).

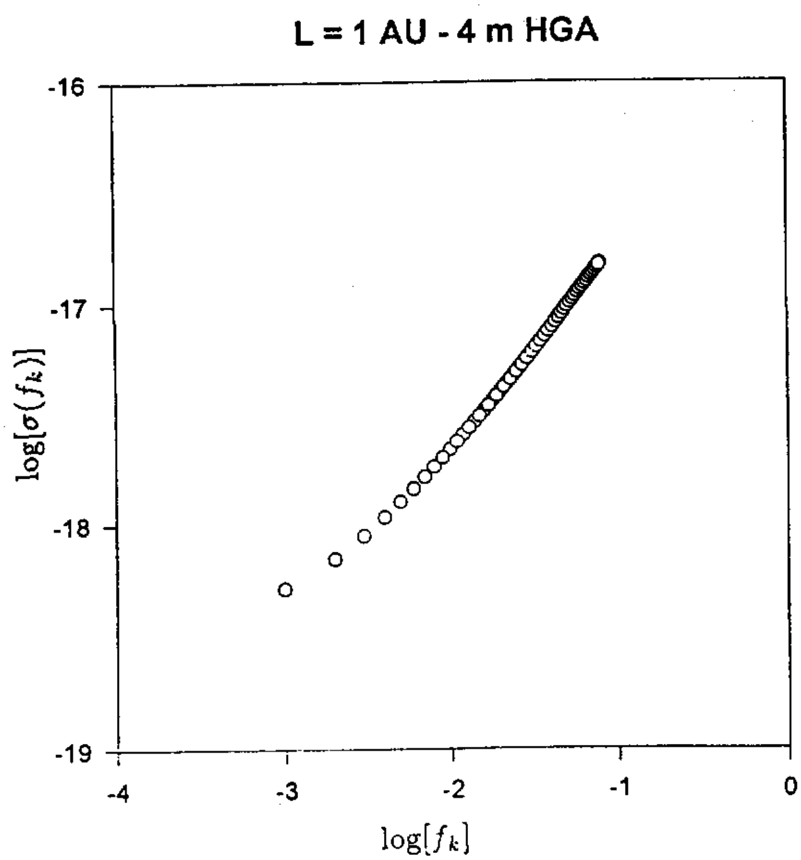
4-Links (Vessot's) Method







L = 1/4 AU - 4 m HGA
-16
-17
-18
-19
-4
-3
-2
-1
0
$\log[\sigma(f_k)]$
$\log[f_k]$



L = 1 AU - 4 m HGA
-16
-17
-18
-19
-4
-3
-2
-1
0
$\log[\sigma(f_k)]$
$\log[f_k]$