

# CONTINUOUS OBSERVATION OF NAVSTAR CLOCK OFFSET FROM THE DOD MASTER CLOCK USING LINKED COMMON-VIEW TIME TRANSFER

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## Abstract

*Analysis of the on-orbit Navstar clocks and of the Global Positioning System (GPS) and National Imaging and Mapping Agency (NIMA) monitor station reference clocks is performed by the Naval Research Laboratory (NRL) using both broadcast and postprocessed precise ephemerides. The precise ephemerides are produced by NIMA for each of the GPS space vehicles from pseudorange measurements collected at the five GPS and seven NIMA monitor stations spaced around the world. That the time reference for the NIMA Washington, D.C., monitor station is the DoD Master Clock has enabled synchronized time transfer every 15 minutes via Linked Common-View Time Transfer from the DoD Master Clock to the other eleven monitor stations. Summing the offset of a space vehicle clock from a monitor station time reference with the offset of the monitor station time reference from the DoD Master Clock yields the offset of the space vehicle clock from the DoD Master Clock for the period during which the space vehicle was in view of the monitor station. Repeating this procedure for each of the monitor stations produces continuous overlapping observations of the offset of the Navstar clock from the DoD Master Clock. Following this procedure for the Navstar 29 cesium clock for 118 days during which there were no anomalies in either the space vehicle clock or the Washington, D.C., monitor station time reference yielded a measurement noise with a standard deviation of 1.1 nanoseconds. This was reduced to an estimated measurement precision of 641 picoseconds by averaging overlapping measurements from multiple monitor stations at each observation time. Analysis of the low-noise clock offset from the DoD Master Clock yields not only the bias in the time of the space vehicle clock, but focuses attention on structure in the behavior of the space vehicle clock not previously observable. Furthermore, the uniformly sampled database of 15-minute measurements makes possible for the first time the exhaustive computation of the frequency stability of the space vehicle clocks.*

## INTRODUCTION

Historically, the data available to NRL for analyzing the behavior of the GPS clocks—both space vehicle and monitor station—have been of two types. The first type consisted of measurements of the offset of the space vehicle clocks from the DoD Master Clock obtained by the U.S. Naval Observatory (USNO) from a linear least-squares fit to 13 minutes of 6-second measurements. These measurements were nominally timed according to a schedule issued by the Bureau International des Poids et Mesures (BIPM) for establishment of International

Atomic Time (TAI). Although this schedule was adequate for the purpose intended, the number of measurements was sparse compared to those taken by the GPS and NIMA monitor stations. Moreover, the 13-minute measurements utilized the broadcast ephemeris, and the receiver operated for some period of time at a single frequency requiring use of the ionospheric model transmitted in the navigation message. The second type of data—measurements made by the GPS and NIMA monitor stations—were synchronized to GPS system time and were scheduled on the hour and every 15 minutes thereafter during the time that the space vehicle was in view of the monitor station. The receivers, which operated at two L-band frequencies, measured the ionospheric delay. The phase offset was obtained using the post-fit precise ephemerides supplied by NIMA.

The first step in the analysis of any clock is to identify in the data all discontinuities—either spontaneous or those caused deliberately by the action of the Master Control Station—and to remove them. This was a time consuming task, since any correction attributed to a space vehicle clock had to be consistent with observations of that clock from the 13 monitor stations represented in the database. Similarly, any correction attributed to a monitor station clock had to be consistent with the data from the 24 space vehicle clocks that were observed by that monitor station. Correction of the discontinuities in the data for the 13 monitor stations and for the 24 active space vehicle clocks required the examination of no fewer than 312 files. After all corrections were made, the choice of a file for analysis was often predicated on which showed the lowest noise or which contained observations during the occurrence of an anomaly that was under investigation.

The next logical step in the time transfer work reported previously<sup>[1]</sup> was to obtain a uniformly sampled database of 15-minute measurements of the offset of the space vehicle clocks from the DoD Master Clock using Linked Common-View Time Transfer from the DoD Master Clock to each of the remaining monitor stations. This has been accomplished by combining the files containing measurements of the phase offset of a space vehicle clock from each of the monitor stations and the files obtained from Linked Common-View Time Transfer, which contain a uniformly sampled database of 15-minute measurements of the offset of the monitor station clocks from the DoD Master Clock.

## METHODOLOGY

If the uniformly sampled 15-minute measurements of the offset of a monitor station clock from the DoD Master Clock—obtained by Linked Common-View Time Transfer—are added to the 15-minute measurements of the offset of a space vehicle clock from the clock at that monitor station, the monitor station clock cancels out, leaving the 15-minute measurements of the offset of the space vehicle clock from the DoD Master Clock for the period of time that the space vehicle was in view of that particular monitor station. If the process is then repeated for each of the remaining monitor stations and the measurements of the offset of the space vehicle clock from the DoD Master Clock for the period of observation from each of the monitor stations are summed, then continuous coverage results. This process is described schematically in Table 1 for the Navstar 29 clock.

In Table 1 the three-byte numeric code 294 refers to the cesium clock in position #4 on the Navstar 29 space vehicle. The three-byte alphabetic codes refer to the monitor stations and are defined in Table 2. The entries in column #1 of Table 1 refer to the measurements of the offset of the space vehicle clock from the reference clock at the indicated monitor station. The entries in column #2, shown being added to the entries in column #1, refer to the uniformly sampled 15-minute measurements of the offset of the monitor station reference clock from the

DoD Master Clock obtained by Linked Common-View Time Transfer. Adding the entries in columns #1 and #2 yields the offset of the space vehicle clock from the DoD Master Clock for the period of observation of the space vehicle by the indicated monitor station. Accumulating the entries in column #3 yields continuous overlapping observations of the space vehicle clock from the DoD Master Clock.

*Table 1*

**PROCESSING TO OBTAIN CONTINUOUS PHASE OFFSET  
OF THE NAVSTAR 29 SPACE VEHICLE CLOCK  
FROM THE DOD MASTER CLOCK**

(294-ASC)	+	(ASC-WAS)	=	(294-WAS)	(ASC Observations)
(294-ARG)	+	(ARG-WAS)	=	(294-WAS)	(ARG Observations)
(294-BAH)	+	(BAH-WAS)	=	(294-WAS)	(BAH Observations)
(294-BEI)	+	(BEI-WAS)	=	(294-WAS)	(BEI Observations)
(294-CSP)	+	(CSP-WAS)	=	(294-WAS)	(CSP Observations)
(294-DGI)	+	(DGI-WAS)	=	(294-WAS)	(DGI Observations)
(294-ENG)	+	(ENG-WAS)	=	(294-WAS)	(ENG Observations)
(294-HAW)	+	(HAW-WAS)	=	(294-WAS)	(HAW Observations)
(294-KWJ)	+	(KWJ-WAS)	=	(294-WAS)	(KWJ Observations)
(294-QUI)	+	(QUI-WAS)	=	(294-WAS)	(QUI Observations)
(294-SMF)	+	(SMF-WAS)	=	(294-WAS)	(SMF Observations)
(294-WAS)				(294-WAS)	(WAS Observations)
				<hr/>	
				(294-WAS)	(All Observations)

The overlapping measurements from multiple monitor stations at each observation time are then averaged to get a single estimate of the offset of the space vehicle clock from the DoD Master Clock at each of the observation times. Because the averaging of multiple measurements, the measurement noise is correspondingly reduced.

*Table 2*

**MONITOR STATION CODES**

ASC	Ascension Island
ARG	Argentina
BAH	Bahrain
BEI	Beijing, China
CSP	Colorado Springs
DGI	Diego Garcia Island
ENG	England
HAW	Hawaii
KWJ	Kwajalein Island
QUI	Quito, Ecuador
SMF	Smithfield, Australia
WAS	Washington, D.C.

## FEASIBILITY TEST

An algorithm was derived to manipulate the data as shown in Table 1 to test the feasibility of the process. Figure 1 is the offset of the Navstar 29 clock from the Washington, D.C. time reference and shows that, during the 118 days from 10 October 1995 to 5 February 1996, there were no anomalies in either the Navstar 29 clock or in the Washington, D.C., time reference<sup>1</sup>. Superimposed in the figure is a plot of the residuals of a linear fit to the phase offset on an expanded scale which enables examination of the fine structure. Hence, in each line of Table 1, except for the measurement times which will differ, the data resulting from the sum should lie along the curve shown in Figure 1.

The offset of the Navstar 29 clock from the Colorado Springs time reference shown in Figure 2 reveals a number of discontinuities all of which occurred in the monitor station time reference. Figure 3, which is a plot of the offset of the Colorado Springs time reference from the Washington, D.C. time reference obtained through common-view time transfer, shows the same discontinuities in the Colorado Springs time reference, but with opposite sign since the Colorado Springs time reference is now the remote clock. Figure 4 is the result of adding the data in Figures 2 and 3 and corresponds to line 5 of Table 1. It can be seen that the behavior of the Colorado Springs time reference with all of its discontinuities drops out of the sum and that the resulting data does, indeed, lie along the same curve as the data in Figure 1.

This process was repeated for the other ten monitor stations, and each yielded data that were without discontinuities and that lay along the same curve as in Figure 1. Adding the measurements of the offset of the Navstar 29 clock from the Washington, D.C., time reference corresponding to the observations from each of the monitor stations (column #3 of Table 1) resulted in the data in Figure 5 which—except for short gaps in the data due to communications outages, power outages, receiver malfunctions, clock switches, etc.—filled in the measurement times completely with overlapping measurements, confirming the feasibility of the process. The raw overlapping measurements at each measurement time were then averaged to yield the smoothed 15-minute continuous observations of the Navstar 29 cesium clock from the Washington, D.C., time reference. In Figure 6 is plotted the residuals of a linear fit to the smoothed data, showing in detail the behavior of the phase every 15 minutes except, as noted, for the few areas where data were missing for the causes mentioned. This provides, then, the most definitive estimate of the behavior of the space vehicle clock.

## MEASUREMENT STATISTICS

If the smoothed measurements are subtracted from the raw measurements the measurement noise shown in Figure 7 results. The histogram in Figure 8 shows the empirical probability distribution of the measurement noise to be decidedly normal, which is a consequence of the numerous additions inherent in the processing, implying multiple convolutions of individual density functions. By the Central Limit Theorem<sup>[2]</sup> these convolutions tend to normality. In Figure 9 the normalized integrated periodogram of the measurement noise lies well within the 75% Kolmogoroff-Smirnov confidence interval, supporting the hypothesis that the noise is white, or uncorrelated. That the noise measurements are normally distributed and uncorrelated implies that they are also independent.

To estimate the precision of the smoothed measurement requires computation of the standard

<sup>1</sup>Although the time reference for the NIMA Washington, D.C., monitor station is nominally the DoD Master Clock, which is free of anomalies, whenever the receiver loses power the phase registration with the DoD Master Clock is lost, resulting in a discontinuity in the phase of the time reference.

deviation of the distribution of the sample mean—the sample being the overlapping measurements from multiple monitor stations made during the same 15-minute interval. Since the measurements in the sample are independent and identically distributed random variables, the standard deviation of the sample mean will be the standard deviation of the population from which the sample was taken reduced by  $1/\sqrt{n}$ , where  $n$  is the number of measurements in the sample. The number of noise measurements ( $n_{noise}$ ) in Figure 7 was 28,653. The number of estimates ( $n_{mean}$ ) of the offset obtained by taking the mean of each sample of raw measurements at the measurement times was 9,735. Dividing the two yields  $n_{sample} = 2.94$  for the average size of the samples for which the mean was found. With the standard deviation of the measurement noise  $\sigma_{noise} = 1.10$  nanoseconds from Figure 7, the standard deviation of the sample mean is

$$\sigma_{mean} = \frac{\sigma_{noise}}{\sqrt{n_{sample}}} = 641 \text{ ps}$$

## FREQUENCY OFFSET

The one-day average frequency offset determined from the 15-minute data in Figure 6 is shown in Figure 10. Figure 11 is a plot of the same data seen through a 6-hour moving average filter used to reduce the white frequency noise which was determined from the frequency-stability profile in Figure 12 to be dominant at that sample time. Superimposed on the continuous coverage is the 1-day average frequency offset determined by direct measurement once per day between the space vehicle clock and the DoD Master Clock at the Naval Observatory. The direct measurements were the type of data routinely analyzed prior to institution of Linked Common-View Time Transfer. That the measurements made once per day by the Naval Observatory fall atop the smooth curve corresponding to continuous coverage suggests that what might have been interpreted as white frequency noise in the 1-day data is, in fact, undersampling of a process with possibly meaningful structure. Hence, the uniformly sampled database of 15-minute measurements can be seen to provide much higher resolution for analysis of the behavior of the space vehicle clocks.

## FREQUENCY STABILITY

The uniformly sampled database of 15-minute measurements of the offset of the space vehicle clocks from the DoD Master Clock resulting from continuous coverage makes possible for the first time the direct and exhaustive computation of the frequency stability of the space vehicle clocks. Figure 12 is a plot of the frequency-stability profile for the Navstar 29 cesium clock for sample times of 15 minutes to 30 days. By exhaustive calculation is meant that the frequency stability is calculated for every multiple of the basic sample interval of 15 minutes up to the maximum sample time.

The stability for a sample time of one day was estimated from the continuous coverage to be 7.3 parts in  $10^{14}$ , whereas the values estimated from the 15-minute data for each of the 12 monitor stations separately, i.e. the data represented by the entries in the first column of Table 1, ranged from 6.6 parts in  $10^{14}$  for Bahrain to 1.18 parts in  $10^{13}$  for Diego Garcia Island, which has been noisy for some time. That the estimate obtained from the uniformly sampled database falls within the range of values estimated from the observations made by the individual monitor stations is not surprising, since the uniformly sampled database was derived from observations made by all of the monitor stations. Where certain monitor stations are

known to be noisy, these might be omitted from the computation of the continuous coverage to preclude their degrading the estimate of the stability of the space vehicle clock.

The stability for a sample time of one day of 7.3 parts in  $10^{14}$  is well within the GPS system specification of 2.0 parts in  $10^{13}$  for cesium clocks. A flicker floor of 2 parts in  $10^{14}$  appears to have been temporarily reached at about 8 days, but after about 12 days the profile turns sharply downward again. It appears that no confidence limits have been placed on the stability estimates, but the limits were, in fact, very small because of the very large number of phase triplets that entered into the calculation of the stability at each sample time.

## CONCLUSIONS

Institution of the continuous coverage method, which makes use of Linked Common-View Time Transfer from the DoD Master Clock to the remaining monitor stations represented in the database, produces for each of the Navstar space vehicle clocks a uniformly sampled database of 15-minute measurements having low-noise, e.g. a 641-picosecond measurement precision for the Navstar 29 cesium clock. The new database makes the following contributions: (1) It provides much higher resolution for analysis of the behavior of the space vehicle clocks. (2) It makes possible for the first time the exhaustive computation of the frequency stability of the space vehicle clocks. And (3) the manual labor involved in the break correction process is substantially reduced by the analysis of  $N + M = 37$  files in lieu of  $N \times M = 312$  files, where  $N = 24$  is the number of Navstar space vehicles currently active and  $M = 13$  is the number of monitor stations represented in the database.

## REFERENCES

- [1] W.G. Reid, T.B. McCaskill, O.J. Oaks, Laboratory, J.A. Buisson, and H.E. Warren 1996, "*Common-view time transfer using worldwide GPS and DMA monitor stations,*" Proceedings of the 27th Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, 29 November-1 December 1995, San Diego, California, USA (NASA CP-3334), pp. 145-158.
- [2] A. Papoulis 1965, *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill Book Company, New York, New York, USA, p. 266ff.

PHASE OFFSET OF NAVSTAR 29 CESIUM CLOCK FROM  
Washington, D.C. Time Reference

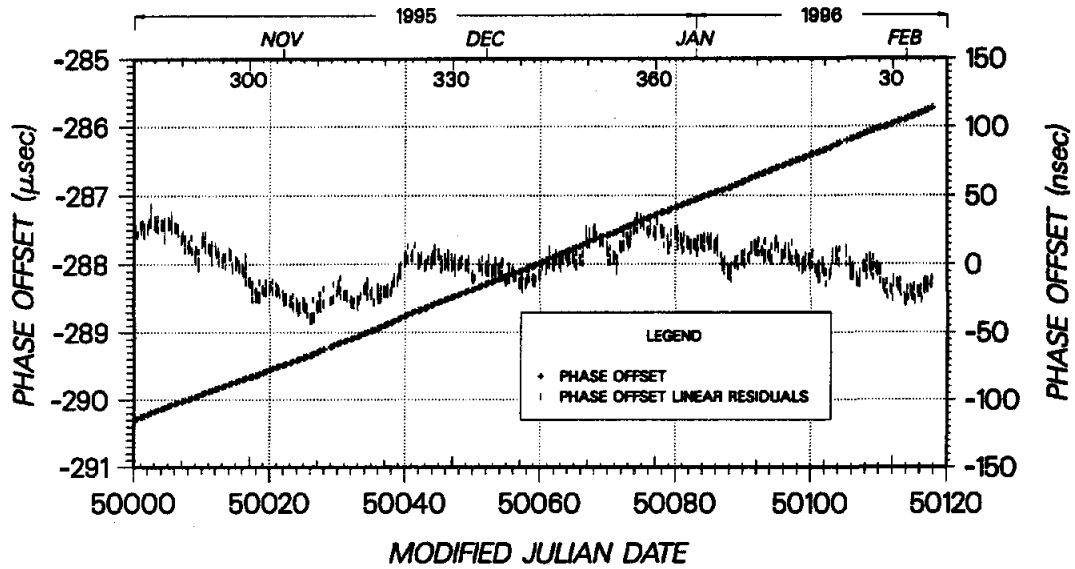


Figure 1

PHASE OFFSET OF NAVSTAR 29 CESIUM CLOCK FROM  
Colorado Springs Time Reference

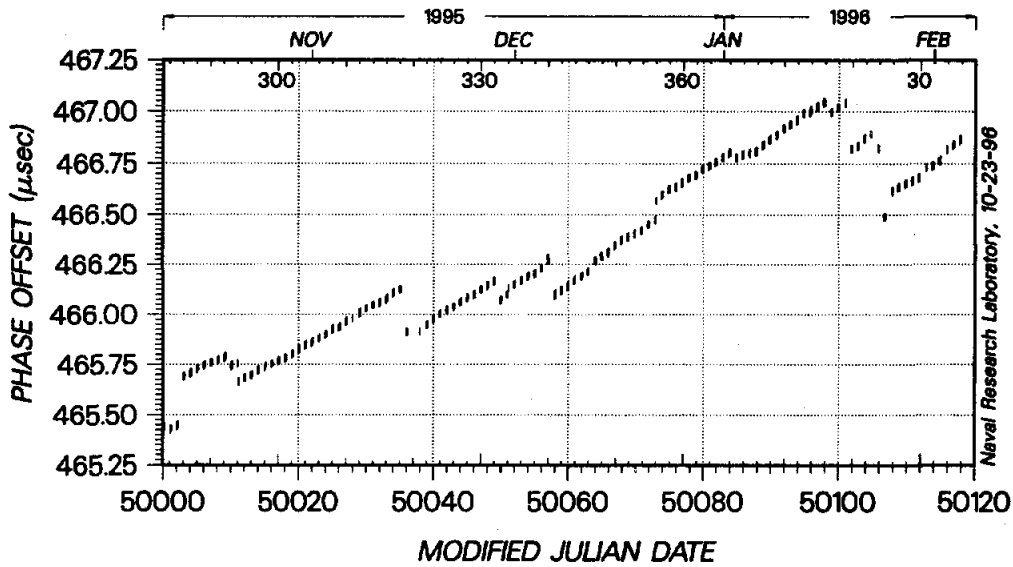


Figure 2

PHASE OFFSET OF COLORADO SPRINGS TIME REFERENCE FROM  
Washington, D.C. Time Reference Using  
Common-View Time Transfer

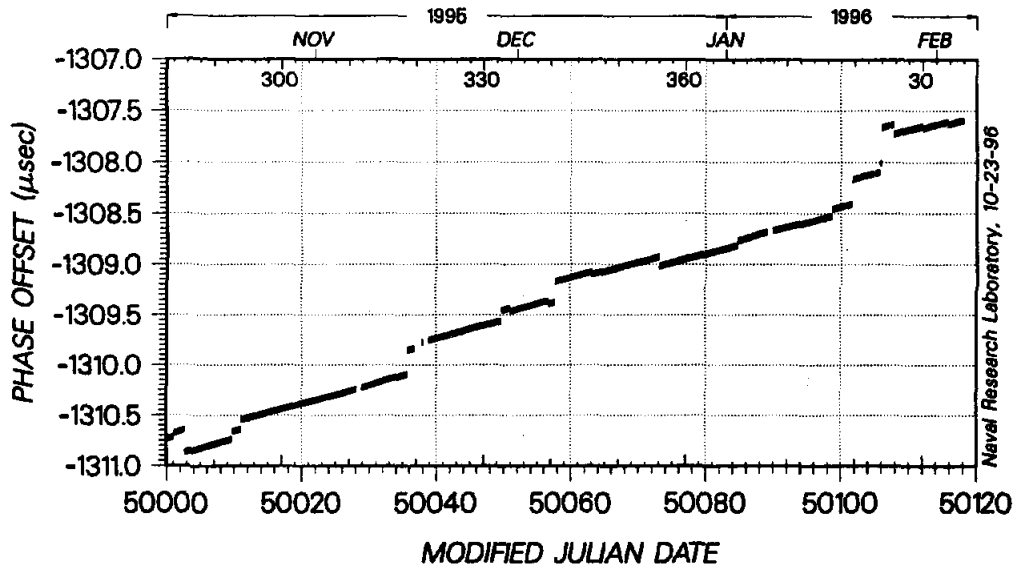


Figure 3

PHASE OFFSET OF NAVSTAR 29 CESIUM CLOCK FROM  
Washington, D.C. Time Reference Using  
Colorado Springs Observations

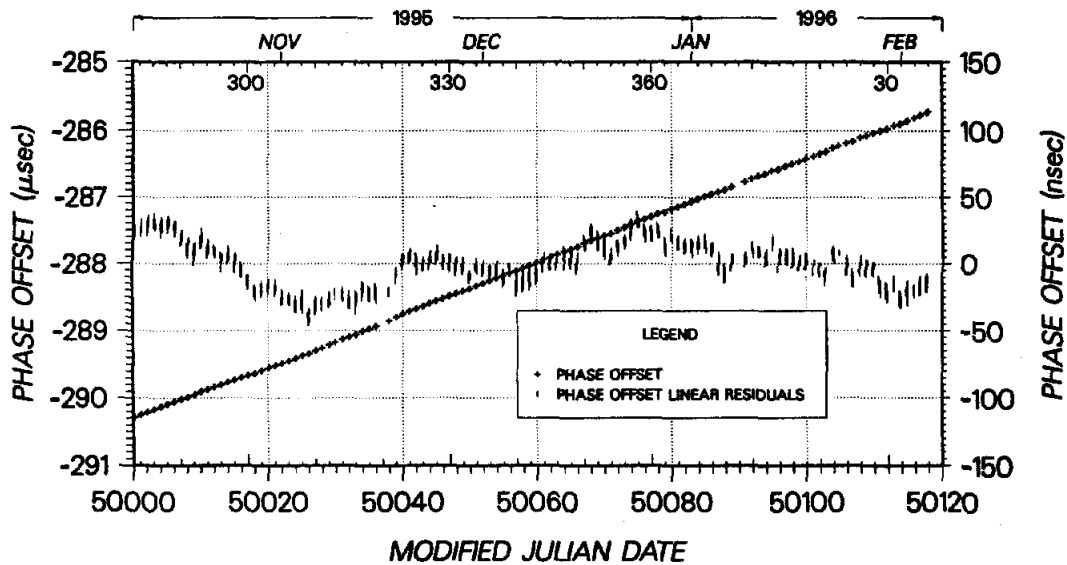


Figure 4



PHASE OFFSET OF NAVSTAR 29 CESIUM CLOCK FROM  
 Washington, D.C. Time Reference Using  
 Twelve Monitor Stations Observations  
 Raw Measurements

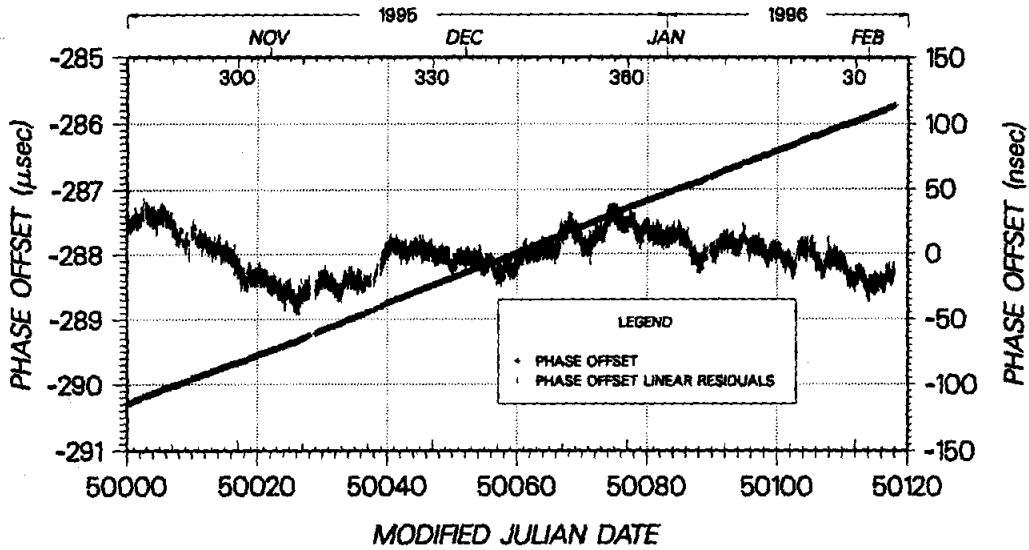


Figure 5

PHASE OFFSET OF NAVSTAR 29 CESIUM CLOCK FROM  
 Washington, D.C. Time Reference Using  
 Twelve Monitor Stations Observations  
 Smoothed Measurements

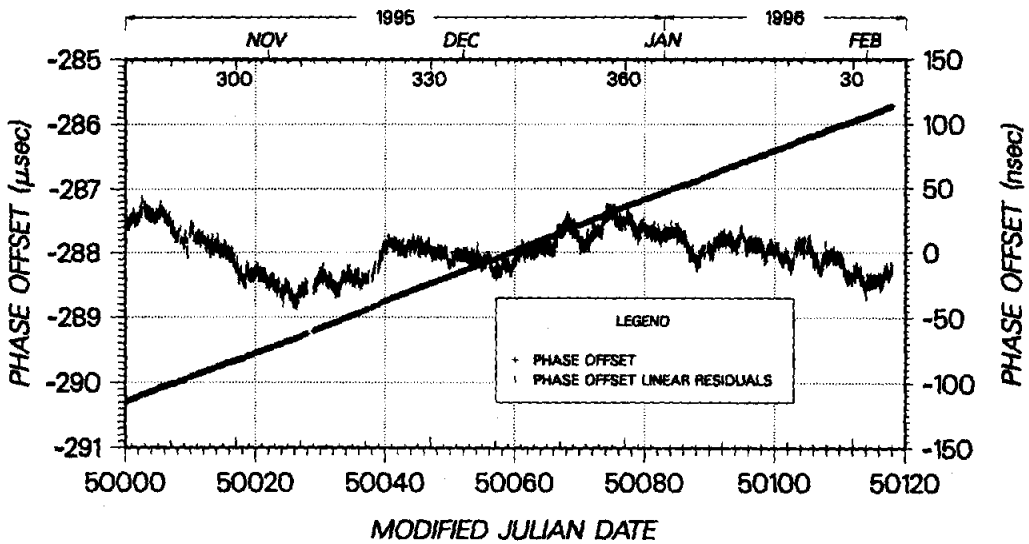


Figure 6

MEASUREMENT NOISE OF NAVSTAR 29 PHASE OFFSET FROM  
Washington, D.C. Time Reference Using  
Twelve Monitor Stations Observations

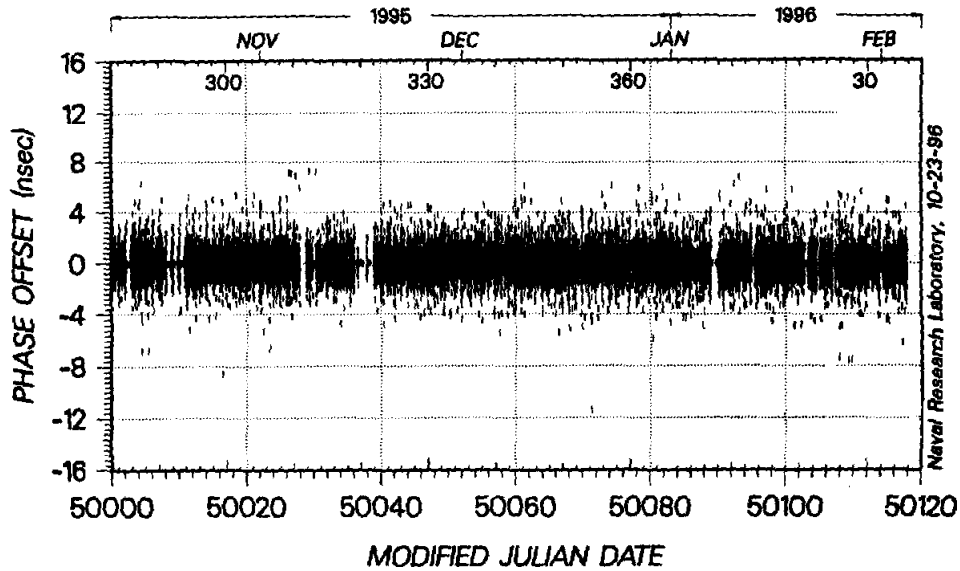


Figure 7

EMPIRICAL PROBABILITY DENSITY FUNCTION OF  
MEASUREMENT NOISE OF NAVSTAR 29 CLOCK OFFSET FROM  
Washington, D.C. Time Reference

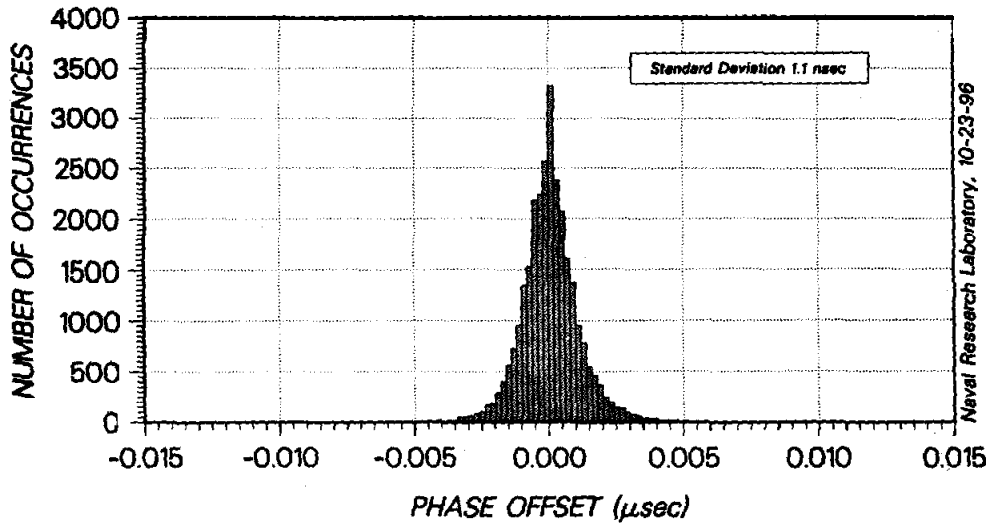


Figure 8

NORMALIZED INTEGRATED PERIODOGRAM OF  
NAVSTAR 29 CONTINUOUS COVERAGE MEASUREMENT NOISE

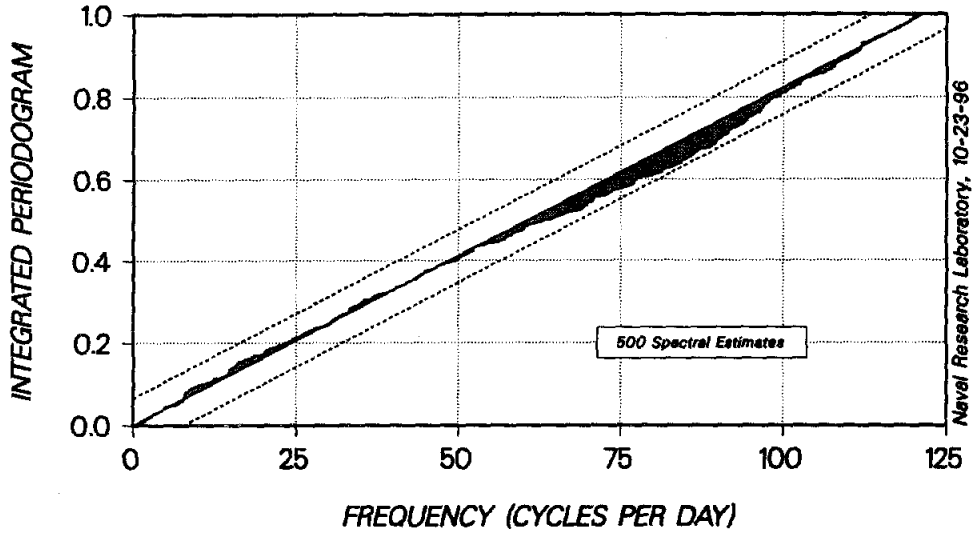


Figure 9

FREQUENCY OFFSET OF NAVSTAR 29 CESIUM CLOCK FROM  
Washington, D.C. Time Reference Using  
Twelve Monitor Stations Observations

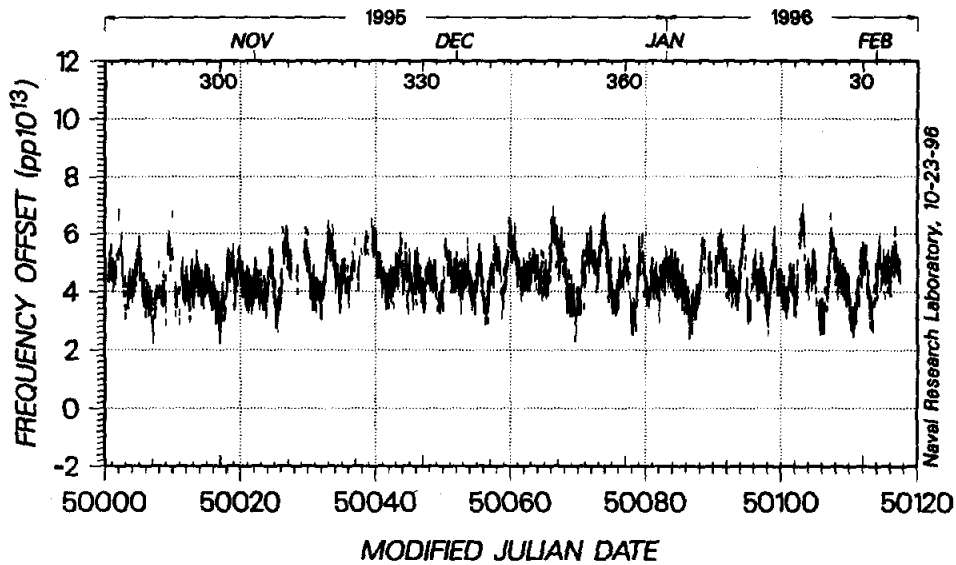


Figure 10

FREQUENCY OFFSET OF NAVSTAR 29 CESIUM CLOCK  
 Comparison of Continuous Coverage and  
 Direct Measurements

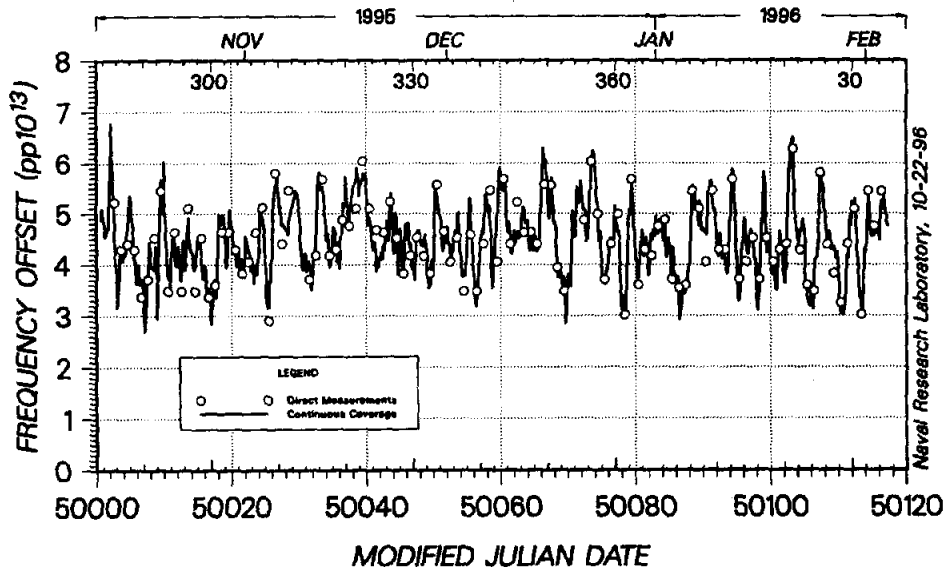


Figure 11

FREQUENCY STABILITY OF NAVSTAR 29 CLOCK OFFSET FROM  
 Washington, D.C. Time Reference Using  
 Continuous Coverage  
 10-OCT-95 to 5-FEB-96

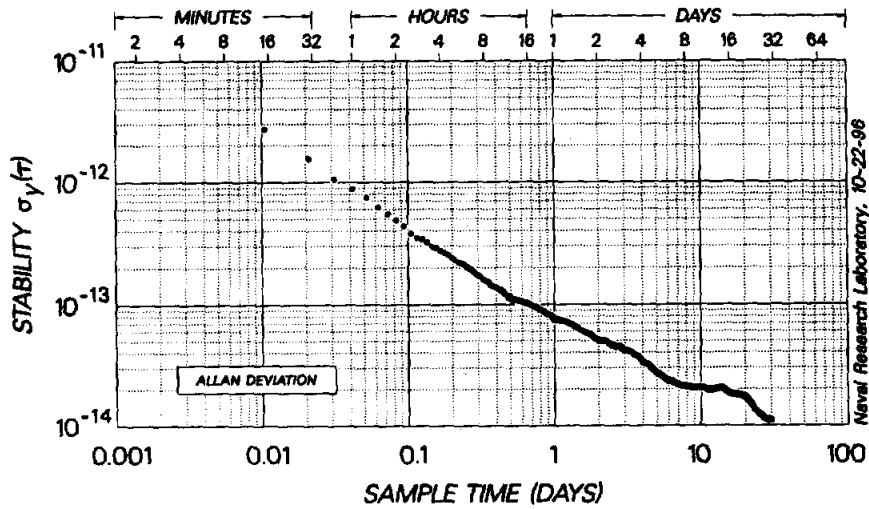


Figure 12