PULSAR-APPROPRIATE CLOCK STATISTICS

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Abstract

Because pulsar phase, spin rate, and spin-down rate cannot be determined a priori, ***a pulsar's possible contribution to a time scale is best measured using a statistic insensitive to these quantities. One such statistic is $\sigma_z(\tau)$ (Matsakis, Taylor, and Eubanks 1996, hereafter*** [1]), ***which is based upon the third-order polynomial variations of the clock phases and named in analogy to $\sigma_y(\tau)$. It is similar to measures based upon the variance of the third differences, but more robust in the presence of data gaps. About 6 years of data, from MJD 47752 to 50343 (18 Aug 89-17 Sep 96), were used to determine the $\sigma_z(\tau)$ of cesium and maser standards in a variety of imperfect but probably acceptable ways; directions for improved reductions are also indicated. Because of the considerably improved stability of cavity-tuned masers and Hewlett-Packard Model 5071 cesium standards, the best results use only the data taken since MJD 49400 (17 Feb 94), by which time most of the new standards had been acquired. An upper limit to the time variation of the fine structure constant is also given.***

INTRODUCTION

The possibility of using the exceptional rotational stability of millisecond pulsars to generate a time scale has long been of interest.[2-7] Although the phase, rate, and spin-down of their observed radio pulsations are physically meaningful (see [8]), they are not predictable on the basis of measurable quantities, so that their contribution to any state-of-the-art terrestrial time scale can only be to constrain derivatives of higher order than the frequency drift. While this precludes their use as absolute frequency standards, incorporating pulsar data in the time scale is still a highly worthy goal.

Clock data are commonly analyzed using a statistic called σ_y, the square root of the "Allan variance," which is proportional to the mean square of the second differences of a series of clock offset measurements. Second differences are used because they are insensitive to the clock phase offset and frequency bias, and because the frequency noise of these standards is often white over time scales of less than a few days or weeks. In the presence of arbitrary frequency drifts and the "redder" noise components which dominate these time series on longer time scales, it is a natural extension to consider statistics such as the Hadamard variance, which is based upon the third differences.[9] Due to the irregular spacing of pulsar data, Taylor[4] suggested use of a statistic analogous to the Modified Hadamard Variance, which he termed σ_z, which is related to a third-order fitted polynomial of the timing residuals; the definition of σ_z was slightly altered by Matsakis, Taylor, and Eubanks.[1] This statistic describes the lowest-order deviations remaining in a pulsar time series after the phase, frequency, spin-down rate, and astrometric parameters have been determined by comparison with terrestrial time, and their effects removed.

Since σ_z is inherently insensitive to the pulsar properties which are observationally arbitrary, it is ideally suited for comparing pulsar stabilities with those of other time scales. Such comparisons are made in [1] and elsewhere; this paper presents an analysis of the σ_z characteristics of the individual frequency standards kept at the U.S. Naval Observatory (USNO).

THE USNO CLOCK DATA

The USNO clock data were taken using the Digital Acquisition System (DAS), and consist of hourly timing differences between the "zero crossings" of the 5 MHz signal output by individual frequency standards and the USNO Master Clock #2 (MC2), which is referred to as UTC(USNO) by the International Bureau of Weights and Measures (BIPM), and in this time period was the output of a maser steered daily toward the weighted mean of the USNO ensemble, which itself is steered toward TAI. Data from 70 Hewlett-Packard Model 5071 cesium standards and 12 cavity-tuned Sigma-Tau masers were used; data from older-style frequency standards were ignored. The data measurement precision was less than 35 picoseconds rms (Breakiron, private communication).

Individual clock differences with MC2 were converted to frequencies and edited for outliers. Four maser and five cesium clock time series with jumps were subdivided into two series each so as to avoid imposing large rate or drift jumps in the data. Data from one cesium clock were subdivided into three series. For four masers and ten cesium clocks, the need to break a time series up was not clear, and so both the original series and the "optional" subsets were retained. The number of standards plotted is, therefore, slightly larger than the number of physical clocks we had, and about double the number of standards currently used by the USNO to compute its time scales.

PAIRWISE COMPUTATION OF σ_z

Once the editing was complete, the σ_z statistic was generated for the difference series generated by subtracting each clock's frequency data from other clock data, from MC2, from the USNO mean, from the free-running USNO time scale A.1, from the spline-interpolated MC2-TAI, and from the spline-interpolated TT(96) time series, which is generated by the BIPM particularly for pulsar work (see [10]).

To compute (and define) σ_z using frequency data instead of timing residual data, the recipe given in [10] was adapted as follows:

1. For each clock, phase data pairs separated by 1 day were differenced to generate the frequencies $y(t_i)$.

2. The data were divided, according to time of measurement, into subsequences defined by continuous intervals of length τ.

3. By minimizing the weighted sum of squared differences, $[(y(t_i) - Y(t_i))]^2$, the data in each subsequence were modelled to the derivative of the cubic function used by [1]:

$$Y(t) = c_1 + 2 \times c_2(t_i - t_0) + 3 \times c_3(t_i - t_0)^2 \quad (1)$$

The weights were chosen to be unity. All overlapping subsequences were included if they contained four or more measurements and the interval between first and last measurement was at least $\tau/\sqrt{2}$.

3. It follows that

$$\sigma_z(\tau) = \frac{\tau^2}{2\sqrt{5}} \left\langle c_3^2 \right\rangle^{1/2} , \tag{2}$$

where angle brackets denote averaging over the subsequences, weighted by the inverse squares of the formal errors in c_3.

A comparison with analyses based upon the Hadamard variance, such as Hutsell[11], Hutsell et al.[12], and Walter[13,14], can be made by noting that the "square root of the modified Hadamard variance" has a τ-dependence similar to that of σ_z, after allowance is made for the factor of $\sqrt{3/10}$ normalization difference and for the fact that the τ used here is three times larger than the τ used to define the Hadamard variance.

ESTIMATION OF INDIVIDUAL AND AVERAGE σ_z

If one is willing to make the questionable assumption, which is discussed in the next section, that the clock errors are independent and uncorrelated, then the σ_z^2 associated with each clock difference is equal to the sum of the "intrinsic" σ_z^2 variances of each clock. It then becomes mathematically possible to derive the individual variances from a least-squares fit to all pairs via an N-cornered-hat analysis, which is a weighted least-squares solution to find those values for individual clock σ_z^2's which best fit the statistics of the clock pair-difference σ_z^2's.

In the solutions individual clock pairs can be weighted in several ways. Solutions were generated by weighting each pair identically, by using the statistical weights derived from variances near τ of a month when compared to MC2, by downweighting or deweighting masers or cesiums, by excluding all but the best third of any type (as measured by their deviations from MC2 at intermediate periods), and by combinations of these. While there was not a large difference between the results of these weighting schemes for any given clock, the lowest individual variances usually were those inferred from the formal errors generated by assuming an identical "white" error dominating all measurements, so that the results depend only on the irregularities in data spacing.

Different solutions were found using subsets of the data. Lower variances were found if the data were restricted to be after MJD 49400, and still lower were found by excluding clocks which did not completely fill the interval from MJD 49400 to 50343.

In Figures 1 a-d the data for the individual cesium standards are plotted, using only data after MJD 49400 and weighting each clock pair by the formal error with MC2. Figure 2 is a similar plot for the masers. Note that individual standards can differ by an order of magnitude.

In Figures 3 and 4, various averages of the (non-imaginary) derived values of σ_z are plotted. Most of the curves are from estimates derived with different weighting schemes. Denoted with a "1" is the average when the standards are compared to MC2, while that with a "2" is the average comparing the standards to TAI. The curves are presented as an indicator of robustness. The limits to the technique are shown by the fact that the curves in Figure 3, which incorporate all the data, are higher than those in Figure 4, which are based on only data taken more recently than MJD 49400. The estimates in Figures 3 and 4 are not corrected for the bias due to the fact that the average measured value of σ_z^2 has a chi-squared distribution and for this distribution the median-centered logarithm of σ_z is lower than the average value by .017/n, where n is the number of degrees of freedom.[1] Applying this correction yields median-centered curves lower than the plotted figures. The difference, in logarithmic units, is .17 for the longest τ (7.9), and half as much for τ = 7.6.

ROBUSTNESS OF THE SOLUTIONS

The N-cornered-hat method will not reveal any "external" error which is correlated with the ensemble average, and it is sensitive to the fact that the observed variances are themselves random variables (see [15]). It also can give highly erroneous results in the presence of large measurement noise, data gaps, nonstationarity, and nonzero correlations between clock outputs. Such problems often lead to the derivation of negative estimates for individual clock σ_z^2s (which were discarded in this work). The derivation of negative variances increases with τ because the clock correlations fail to average to zero as the number of independent points decreases. Some benefit is gained from the stochastic character added to the covariance terms by data gaps and other irregularities.

In any event, the solution results do not appear to vary significantly when different weighting functions are used in the fit, and they are consistent with comparisons between each clock and TAI or the various USNO time series which are more accurate and stable than their components. Not surprisingly, the scatter between the different methods increases at the longest intervals, which is the regime where the approximations are most questionable.

It is possible to generate improved clock variances by making a definable and explorable set of assumptions about the characteristics of just one reference clock[16-19] or through use of a reference time scale much more precise than the individual clocks. Using these variances, it is possible to estimate the characteristics of the noise. Although the presence of non-white noise is obvious in all the figures, we have not attempted to quantify its characteristics. This is best done by a multivariance approach[13], which is based upon the use of complementary statistical measures, such as σ_y. We intend to address these problems in subsequent papers.

AN ASIDE ON COSMOLOGICAL IMPLICATIONS

It has been pointed out[20] that variations in the fine structure constant, such as those predicted by recent grand unified theories and Dirac's Large Numbers Hypothesis, would lead to a differential drift in frequency standards which are based upon different atomic transitions. The frequency drifts of best-measured individual standards in this analysis range over 3×10^{-13}/year peak to peak, and we find no significant difference between the two types of standards to a 1-sigma upper limit of 2.5×10^{-14}/year. According to the formulas of [20], this corresponds to a limit of 3.2×10^{-14}/year in the fractional variation of the fine structure constant, and is similar to a limit they derive by comparing mercury- and hydrogen-based standards. It is possible that a large variation in the fine structure constant is being fortuitously masked in our data by some equally large (differential) instrumental effect common to one type of standard as maintained at the USNO. We consider this to be extremely unlikely because, if it were true, a different differential instrumental effect (of a specific magnitude) would also have to be present in the data analyzed by [20], which involved different atomic elements and different institutions–the mercury-based clocks were built by and maintained at the Jet Propulsion Laboratory and the primary cesium standards were built and maintained by the Physikalisch-Technische Bundesanstalt (PTB).[21]

COMPARISONS WITH PULSAR DATA

The purpose of this paper is to present the observed clock statistics, but no discussion of those statistics would be complete without a comparison with pulsar data. In the last figure, data from Figures 3 and 4 of [1] are combined. The σ_z statistic of the timing difference between two pulsars and TAI are presented, as is the comparison between the free-running terrestrial time scales of the USNO (A.1) and the PTB, to which the BIPM steered TAI over the time range of the pulsar data. To generate these curves, data from MJD 44979 to 50079 were used (more recent data were not used because no high-quality pulsar data are available to us due to the Arecibo telescope upgrade work). Although one would infer from those curves that pulsar data can contribute meaningfully to time scales over periods exceeding a few years, the recent incorporation of HP 5071 cesiums and cavity-tuned masers into the terrestrial time scales results in none of the comparisons being stationary in a statistical sense. The lowest-lying curve shows the difference between the two terrestrial time scales when only the last 2 years of data are used, which corresponds to the introduction of the new standards. Since the pulsar data analysis necessarily depends upon comparison with the older time scales, an adequate pulsar database for comparison will not be available for several more years.

ACKNOWLEDGMENTS

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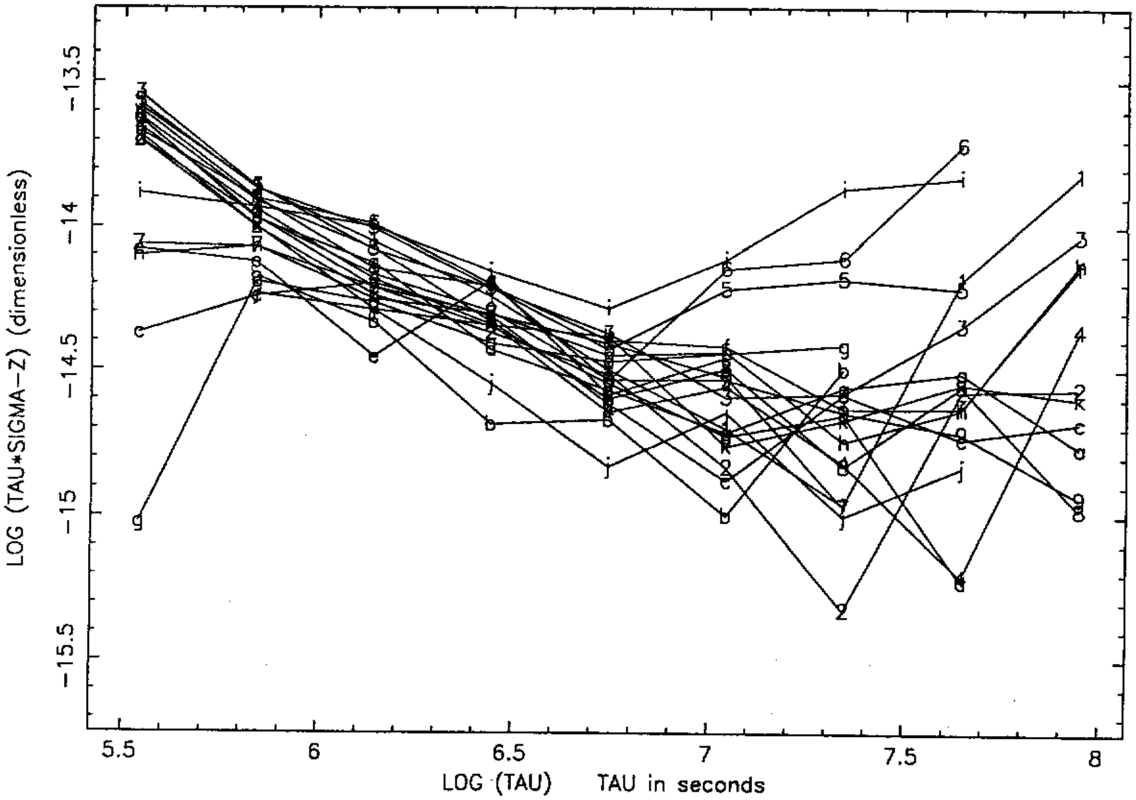

Figure 1 a. Twenty individual cesium 5071s, MJD 49400-50343

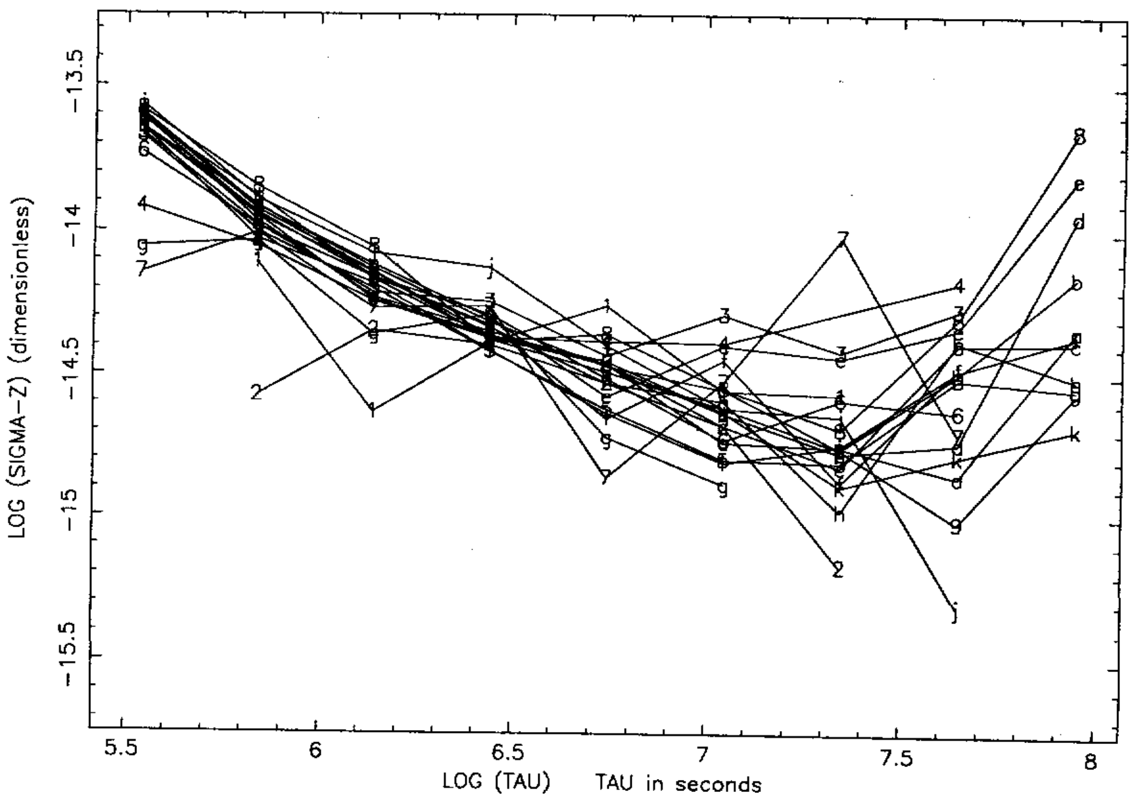

Figure 1 b. Twenty more individual cesium 5071s, MJD 49400-50343

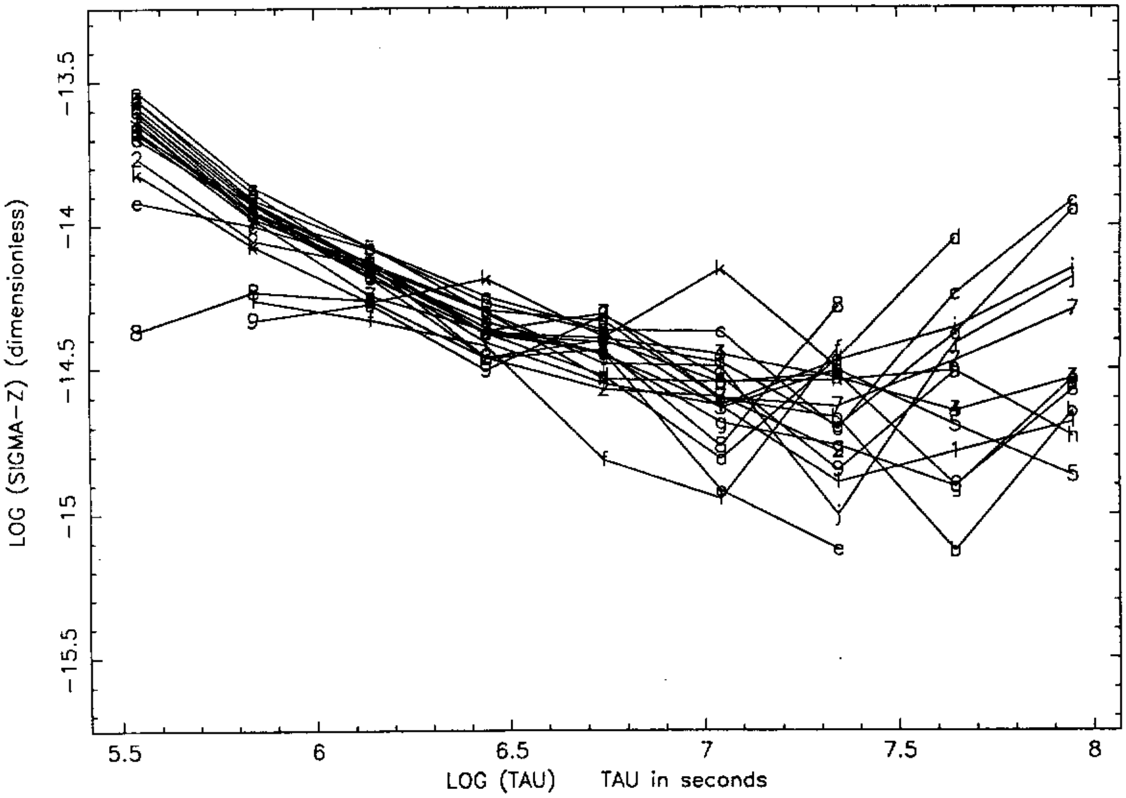

Figure 1 c. Another twenty individual cesium 5071s, MJD 49400-50343

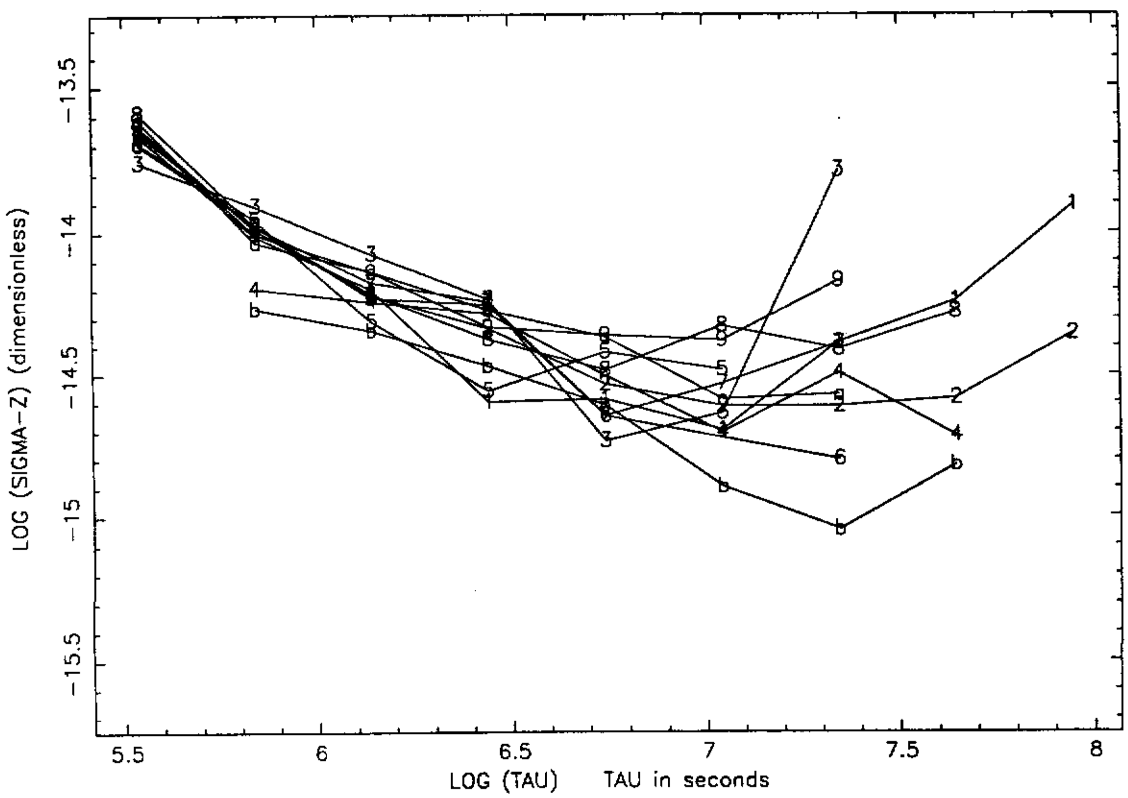

Figure 1 d. Ten individual cesium 5071s, MJD 49400-50343

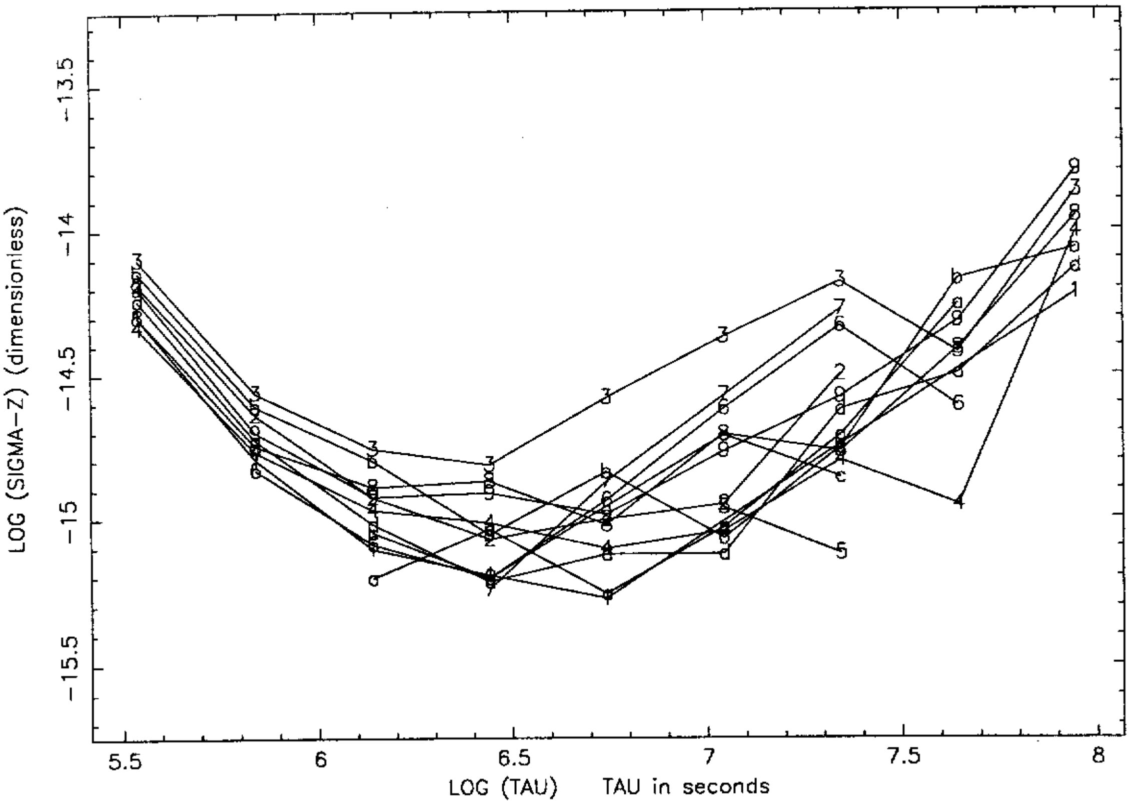

Figure 2. Individual Sigma-Tau hydrogen masers, MJD 49400-50343

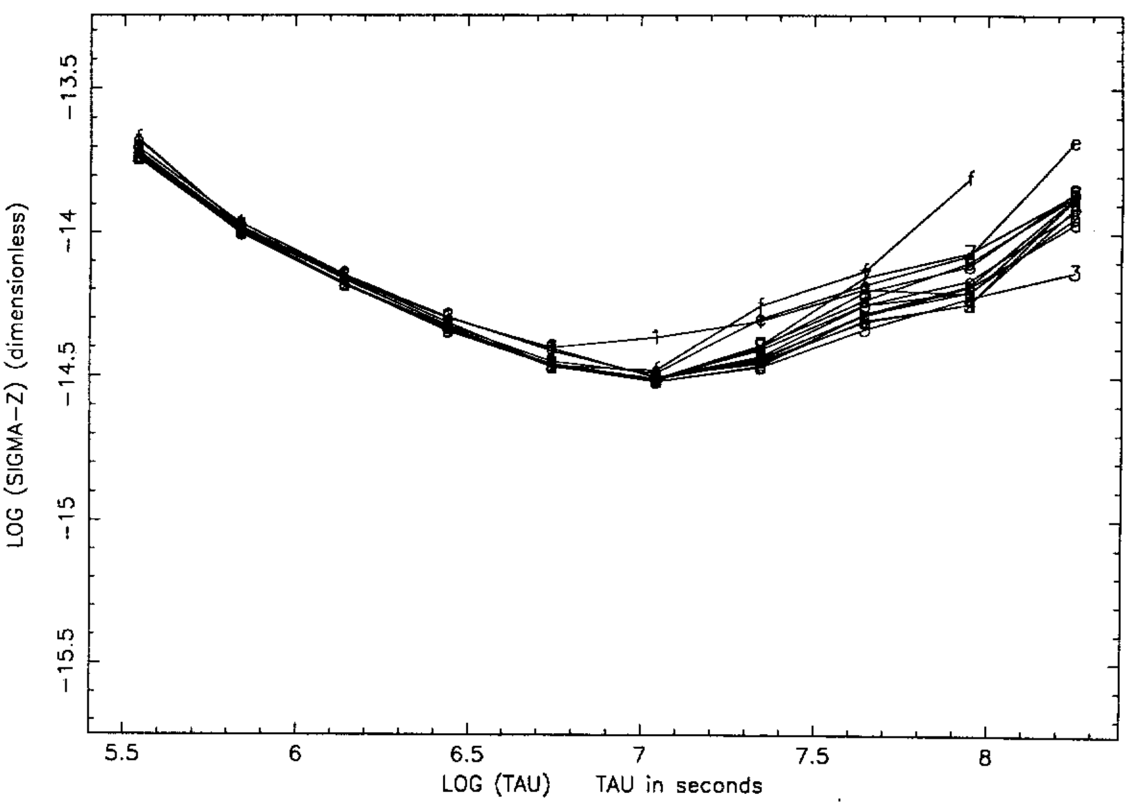

Figure 3 a. Averages of cesium 5071 standards, MJD 47752-50343

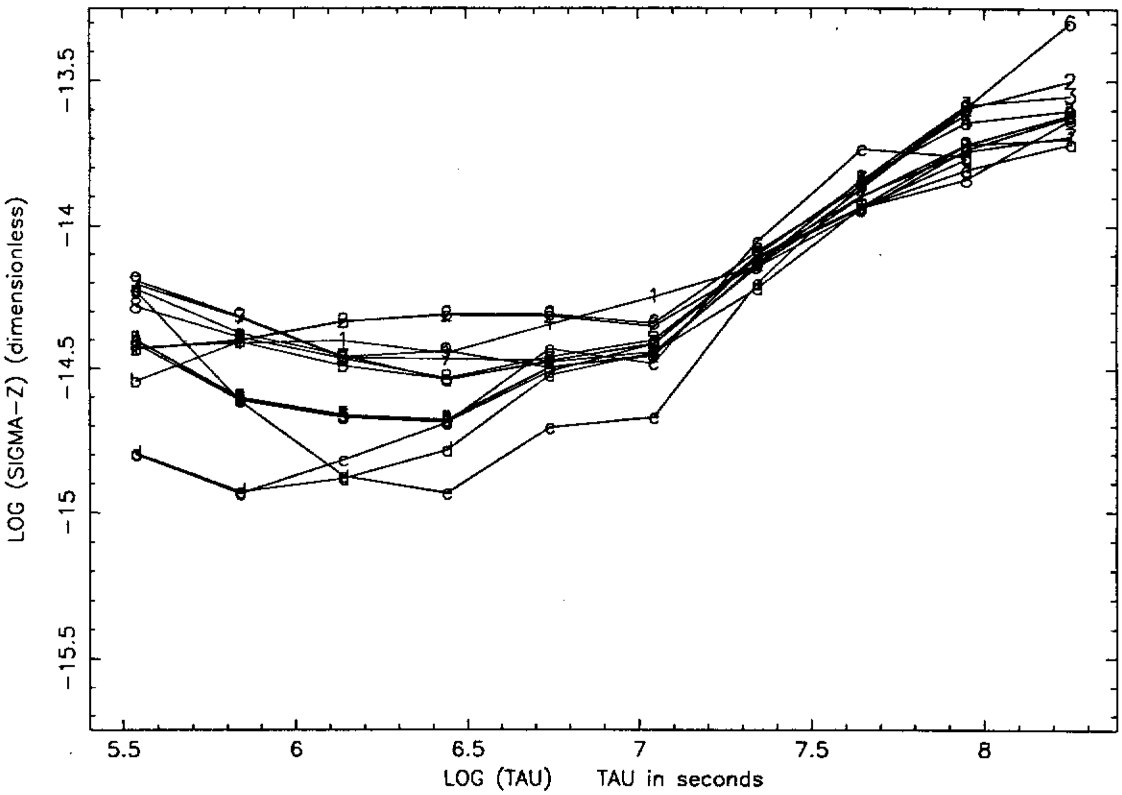

Figure 3 b. Averages of Sigma-Tau masers, MJD 47752-50343

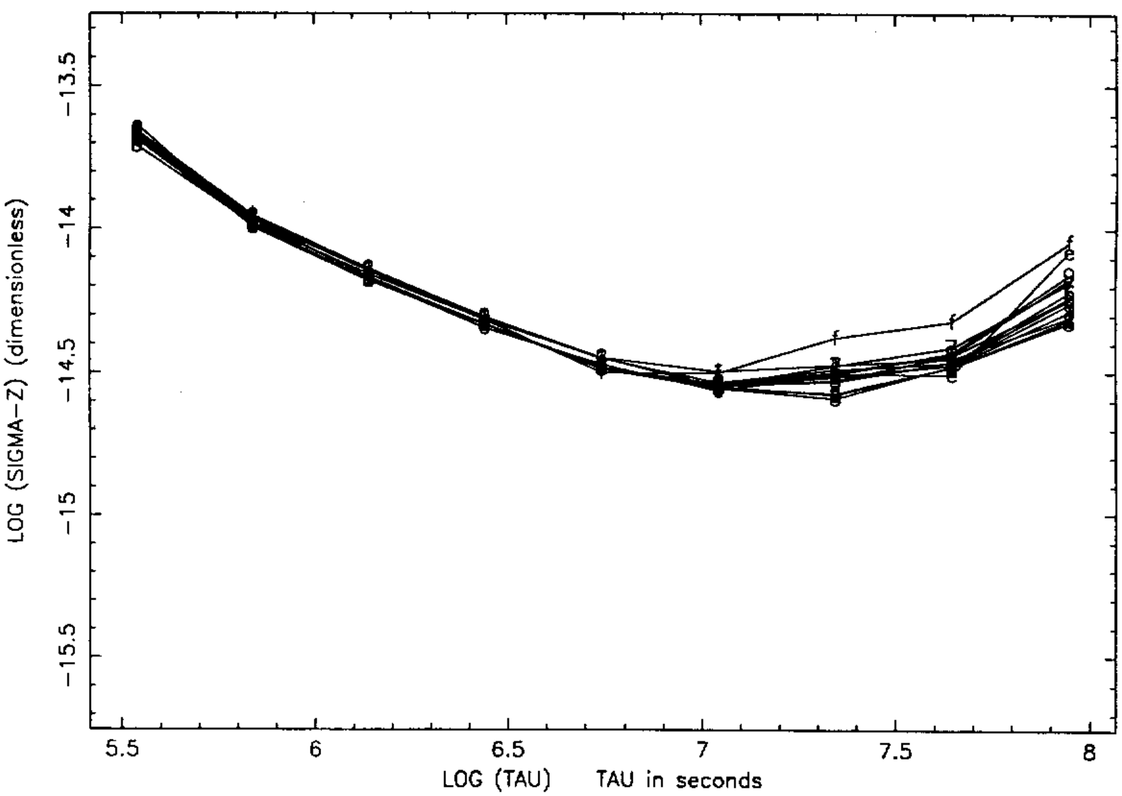

Figure 4 a. Averages of cesium 5071 standards, MJD 49400-50343

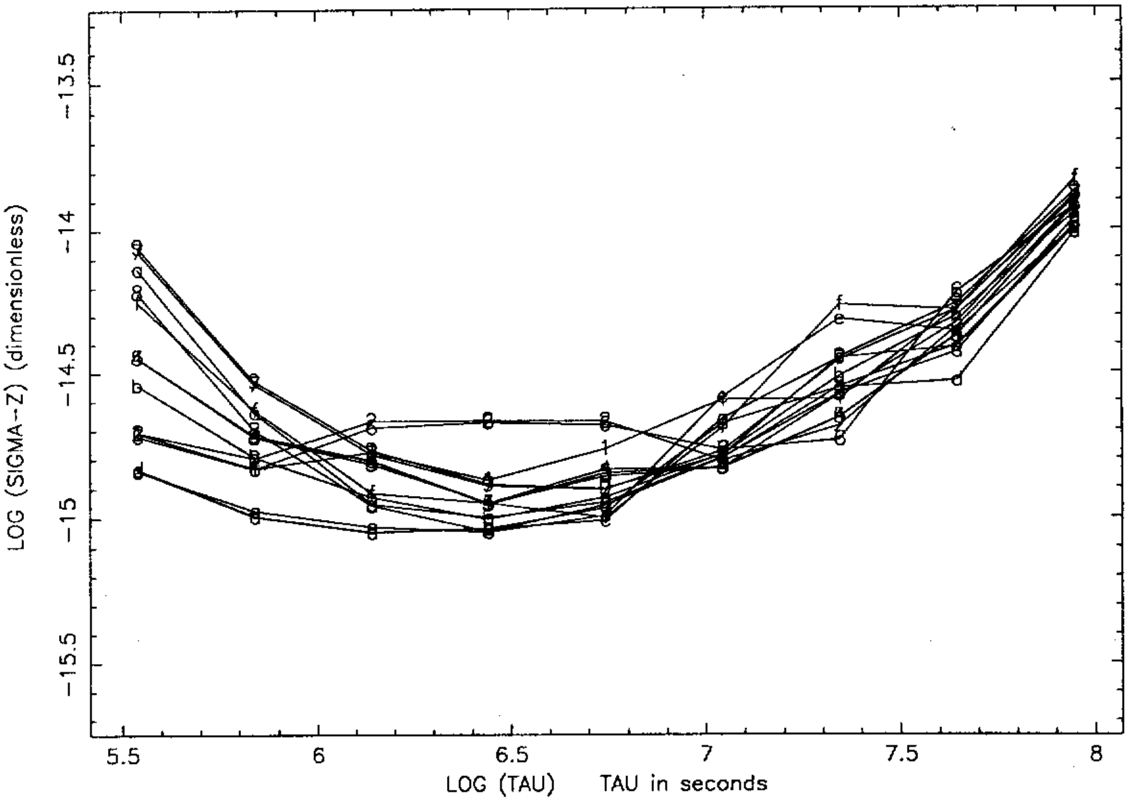

Figure 4 b. Averages of Sigma-Tau masers, MJD 49400-50343

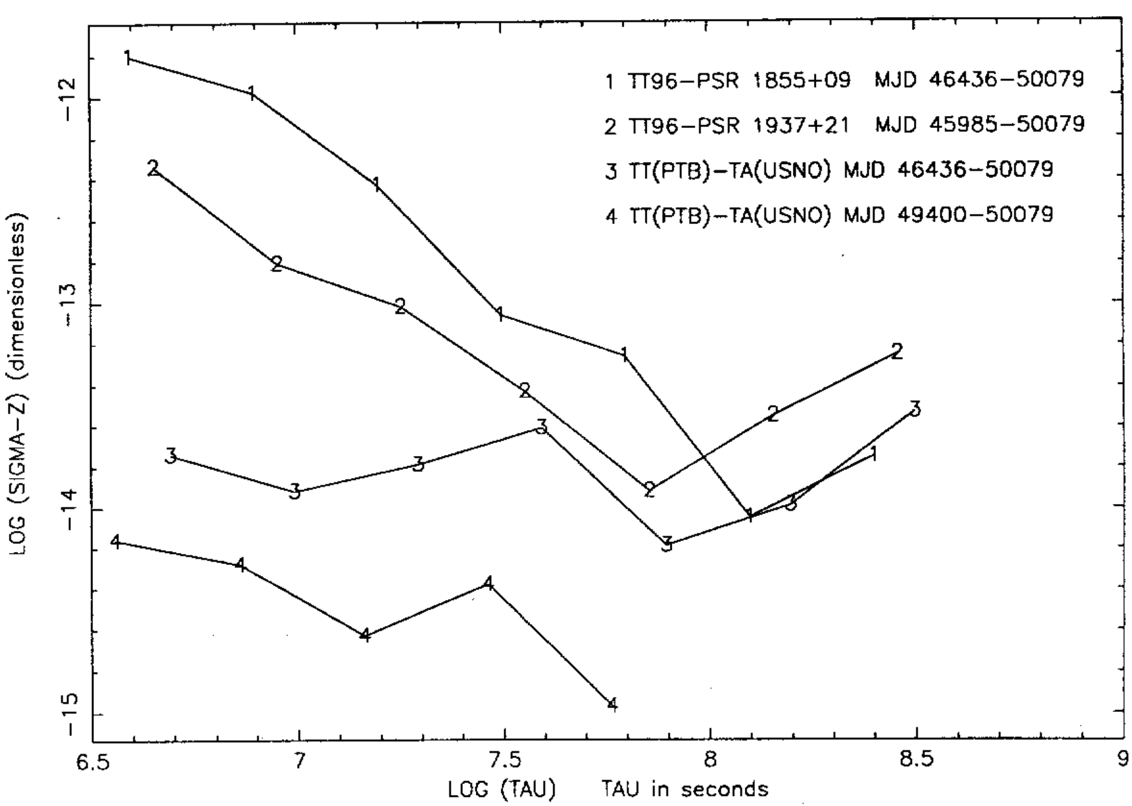

Figure 5. Pulsar and terrestrial time scale stabilities