KALMAN FILTERING USNO's GPS OBSERVATIONS FOR IMPROVED TIME TRANSFER PREDICTIONS

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Abstract

The GPS Master Control Station (MCS) performs the UTC time transfer mission by uploading and broadcasting predictions of the GPS-UTC offset in subframe 4 of the GPS navigation message. These predictions are based on only two successive daily data points obtained from USNO. USNO produces these daily smoothed data points by performing a least-squares fit on roughly 38 hours worth of data from roughly 160 successive 13-minute tracks of GPS satellites. Though sufficient for helping to maintain a time transfer error specification of 28 ns (1 Sigma), the MCS's prediction algorithm does not make the best use of the available data from USNO, and produces data that can degrade quickly over extended prediction spans.

This paper investigates how, by applying Kalman Filtering to the same available tracking data, the MCS could improve its estimate of GPS-UTC, and in particular, the GPS-UTC A_1 term. By refining the A_1 (frequency) estimate for GPS-UTC predictions, error in GPS time transfer could drop significantly. Additionally, the risk of future spikes in GPS's time transfer error could similarly be minimized, by employing robust Kalman Filtering for GPS-UTC predictions.

INTRODUCTION

The UTC time transfer mission of GPS depends on daily connectivity with the official Department of Defense (DoD) agency for PTTI, the United States Naval Observatory (USNO) [4]. Currently, the GPS Master Control Station (MCS) downloads a daily file from USNO (called FALCON), containing smoothed GPS tracking data, estimates of GPS-UTC, and time transfer performance metrics. Every morning, the on-duty operations crew at the MCS enters the daily estimate of the GPS-UTC phase offset (called the daily UTCBIAS value), based on USNO's least-squares fit of approximately 38-hours worth of the smoothed GPS tracking data. This tracking data consists of a series of measurements, each smoothed over a 13-minute tracking interval.

Current MCS software calculates the GPS-UTC frequency (slope) based on a linear fit of only two successive daily GPS-UTC phase (bias) values. Calculating a slope based on only two data points assumes that GPS-UTC predictions are optimal when using only a 24 hour span of data points. Intuitively, since most of the frequency standards within the GPS reference time scale (the GPS Composite Clock [1]) are Cesium clocks, the ensemble time should theoretically behave as a paper clock with noise characteristics roughly proportional to, and a fraction of, that of a single Cesium clock. Since white FM is the predominant noise type for $\tau \leq$ several days on most GPS Cesium clocks, intuitively, white FM should dominate for GPS time itself. Hence, a GPS-UTC slope calculated from a data span of only 24 hours contradicts these intuitions.

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This paper presents the results of testing an off-line computer program that a) estimates GPS-UTC using a Kalman Filter optimized for the empirical noise characteristics of GPS time, and b) applies this Filter onto the same smoothed data currently used for calculating the least-squares fit estimate of the GPS-UTC phase offset.

A GPS-UTC KALMAN FILTER

The following is a description of a PC-based computer program written in Quick Basic, designed for applying Kalman Filtering to USNO's smoothed 13-minute measurements. The program is similar in design to the MCS's miniature Kalman Filter designed to estimate the states of the backup vs. operational frequency standards at the GPS monitor stations (MSs). This GPS-UTC Filter reads successive text files that the MCS downloads daily from USNO, and calculates a smoothed estimate of the GPS-UTC phase *and* frequency, via a modified two-state Kalman Filter.

Equations

The GPS-UTC Kalman Filter employs the following equations, similar to those of many small Kalman Filters [5]:

Notes: $^\wedge = Aposteriori$

$^\sim = Apriori$

$t = Kalman\ Time$

$\tau = Time\ update\ prediction\ span$

(1) State Vector:

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$x_1(t)$ is the bias (phase) estimate (s)

$x_2(t)$ is the drift (frequency) estimate (s/s)

$x_3(t)$ is slaved to the time steering drift rate (s/s^2), currently $\pm$ 1.0 E-19 s/s^2

(2) State Covariance Matrix:

$$P(t) = \begin{bmatrix} p_{11}(t) & p_{12}(t) & 0 \\ p_{21}(t) & p_{22}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Units: $p_{11}(t)$ s^2

$p_{12}(t), p_{21}(t)$ s^2/s

$p_{22}(t)$ s^2/s^2

(3) Process noise values:

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

q_1 = The bias (phase) q (set to 1.11 E-23 s^2/s)
q_2 = The drift (frequency) q (set to 2.22 E-33 s^2/s^3)
$q_3 = 0$

(4) Noise addition matrix:

$$N(\tau) = \begin{bmatrix} q_1\tau + q_2(\tau^3/3) & q_2(\tau^2/2) & 0 \\ q_2(\tau^2/2) & q_2\tau & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(5) The measurement:

$$Z_{ns} = \begin{bmatrix} z_{ns} \end{bmatrix} \ (ns)$$

(6) The measurement transformation

$$Z = \begin{bmatrix} z_{ns} \end{bmatrix} \cdot \begin{bmatrix} 1.0E-9 \end{bmatrix} \ (s)$$

(7) Measurement noise:

$$R = \begin{bmatrix} r \end{bmatrix} \ (set\ to\ 3.6\ E-16\ s^2)$$

(8) Transition matrix:

$$\Phi(\tau) = \begin{bmatrix} 1 & \tau & \frac{1}{2}\tau^2 \\ 0 & 1 & \tau \\ 0 & 0 & 1 \end{bmatrix}$$

(9) Identity matrix:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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(10) Unity vector:

$$H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

(11) The Time Update:

$$\tilde{X}(t_k) = \Phi(\tau) \cdot \hat{X}(t_{k-1})$$

$$\tilde{P}(t_k) = \Phi(\tau) \cdot \hat{P}(t_{k-1}) \cdot \Phi^T(\tau) + N(\tau)$$

(12) The Kalman Gain equation:

$$K(t_k) = \tilde{P}(t_k) \cdot H^T \cdot \left(H \cdot \tilde{P}(t_k) \cdot H^T + R \right)^{-1}$$

(13) Measurement Acceptance:

$$\left(Z(t_k) - H \cdot \tilde{X}(t_k) \right) < Tolerance$$

(14) The Measurement Update:

$$\hat{X}(t_k) = \tilde{X}(t_k) + K(t_k) \cdot \left(Z(t_k) - H \cdot \tilde{X}(t_k) \right)$$

$$\hat{P}(t_k) = \left(I - K(t_k) \cdot H \right) \cdot \tilde{P}(t_k)$$

Data Base Values

The GPS-UTC Kalman Filter uses several important data base values:

(a) The process noise values, q_1 = 1.11 E-23 s^2/s, and q_2 = 2.22 E-33 s^2/s^3, are based on the stability performance of GPS-UTC, as visualized in figure 1, and the following equation [1]:

$$\sigma_y^2(\tau) = (q_1)/\tau + (q_2)\tau/3$$

(b) The measurement noise value, R = 3.6 E-16 s^2, is based on a User Range Accuracy (URA) index of 3, equivalent to 5.67 meters, or ~ 18.9 ns. $(18.9 \text{ ns})^2 \cong 3.6$ E-16 s^2.

(c) The rejection tolerance, 4.0 E-08 s, is based on a maximum estimated range deviation (ERD) of 12 meters.

(d) The MCS currently steers GPS time at a maximum value of 1.0 E-19 s/s^2 [3].

Filter Processing

The following is a synopsis of the functionality of the GPS-UTC Filter program. The program:

(a) Inputs values from *PC data base files*.

(b) Initializes the Filter using the most recent state estimates, which reside in the PC state files.

(c) Inputs the daily USNO file, or if requested, an archived USNO file. If the operator requests an archived file, he/she may enter successive files during the same execution of the GPS-UTC Filter program.

(d) Cycles through the following steps (for each measurement):

- Reads columns 2 and 4, from the DFR24 subfile of USNO's FALCON file. (These are the time tag and the corresponding single-satellite GPS-UTC estimate, respectively).

- Accounts for the current time steering sign and magnitude.

- Performs a time update of the current phase and frequency estimates.

- Performs a residual check on the measurement. If the measurement is rejected, the program reads the next measurement, and continues.

- Performs a measurement update if the measurement is accepted.

- Displays the current state estimates and variances on the PC screen.

As a side note, if GPS-UTC experiences a substantial excursion in bias and/or drift, due to, for instance, an undetected single clock frequency jump, the GPS-UTC Kalman Filter may be manually re-initialized, by re-setting the covariance matrix elements to default values (below) and by restarting the Filter:

$$
P_I(t) = \begin{bmatrix} 1.0 & E-15 & s^2 & 0 & 0 \\ 0 & & & 1.0 & E-25 & s^2 & 0 \\ 0 & & & 0 & & 0 \end{bmatrix}
$$

TESTING THE FILTER

The author processed the GPS-UTC Kalman Filter against two months' worth of daily USNO files. The author chose this period primarily because of the relatively erratic performance of GPS time, caused, in part, by a frequency jump at the Colorado Springs monitor station, at ~ 0200z, 21 Dec 94 [2].

Test Plan

As stated earlier, the Kalman Filter produces daily estimates of GPS-UTC phase and frequency. The test compared the current linear model's 24-hour prediction error against that of the Kalman Filter. The test:

(a) Produced daily Kalman Filter estimates of the GPS-UTC phase and frequency: $X_{aposteriori}(t)$.

(b) Propagated the daily Filtered estimates 24 hours in the future: $X_{apriori}(t + 1\text{Day})$.

(c) Compared the predicted estimates against the next day's Filtered estimates:

$$\text{Filter Error} = X_{apriori}(t + 1\text{Day}) - X_{aposteriori}(t + 1\text{Day})$$

(d) Compared the Filter error to the daily operational average time transfer error.

The test included a re-initialization of the Kalman Filter on 22 Dec 94, to compensate for the known GPS-UTC frequency excursion.

Test Results

Figure 2 compares the GPS-UTC phase estimate produced by the least-squares fit, to that produced by the Kalman Filter. The test indicated only a *small* improvement in GPS-UTC prediction. Figure 3 shows a plot of the actual GPS time transfer average error (using the current linear model) for 1 Dec 94 - 31 Jan 95, and the simulated systematic (average) time transfer error (based on the Kalman Filter) for the same period.

The linear model's RMS of daily average error was 4.81 ns, and its overall average was -2.13 ns. By comparison, the Kalman Filter model's RMS of average error was 4.04 ns, and its overall average was 0.22 ns.

Test Findings

No test can completely simulate reality, and hence, the test results may paint a picture nicer than reality.

(a) The Kalman Filter allows the operator to generate GPS-UTC predictions based on optimal estimation, with data base values tailored towards the true (empirical) noise characteristics of GPS time. This optimal estimation could theoretically produce a small reduction in time transfer error.

(b) The theoretical reduction in time transfer error may not be significant enough to justify the extra operational burden at the MCS, at the present time. The computer setup that the MCS currently employs to download daily GPS-UTC information already has some operational problems, including numerous communication drop-outs, and occasional errors caused by the manual processing of data. The extra hassle imposed on operations crews, caused by new, intricate software, may very well *degrade* GPS-UTC predictions, if the number of operator-related errors were to increase. The undetected corruption of one PC file could theoretically prove disastrous for time transfer.

(c) The GPS-UTC Kalman Filter was unable to detect (and hence, reconcile) the -22 ns/day jump in GPS time that occurred on 21 Dec 94. The above test results were based on the assumption that the operator would detect and manually re-initialize the Filter to compensate for the frequency step. If the

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operator couldn't have detected the jump, the Filtered GPS-UTC prediction, based on a theoretical frequency predictability longer than 24 hours, could have been unacceptably late in converging on a new frequency estimate, and hence, could have degraded time transfer performance.

CONCLUSION

This test of the PC-based GPS-UTC Kalman Filter produces several recommendations:

(a) For the time being, continue with the current operational approach for GPS-UTC prediction.

(b) Pursue a study on a more refined Kalman Filter. This PC-based filter processed *raw* measurements, based on the broadcasted navigation message of 25 GPS satellites. Since the MCS typically updates the navigation message only once every 24 hours, these GPS measurements can, and will, have up to 40 ns of noise. This high measurement error causes the Filter to produce very coarse estimates of GPS-UTC, which are only predictable to about 4-5 ns over one day.

(c) Incorporate the USNO Download into MCS mainframe architecture. By introducing solid configuration management into the data connectivity between USNO and the MCS, the MCS could a) receive more timely measurements from USNO, for improved accuracy and integrity monitoring, b) refine these USNO measurements by subtracting known observables (ERDs), and c) employ a more reliable setup for the MCS to perform its critical role in GPS's time transfer mission.

ACKNOWLEDGMENTS

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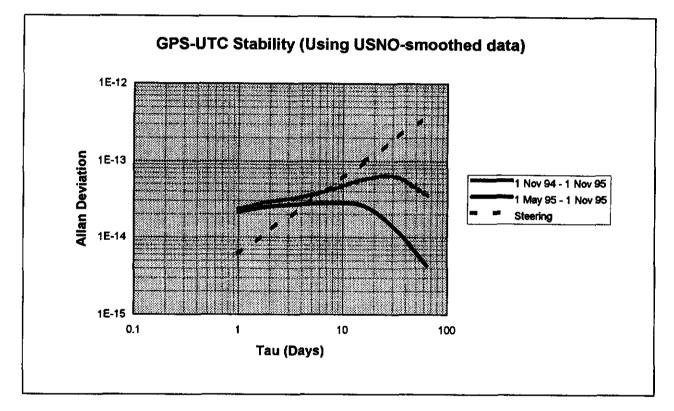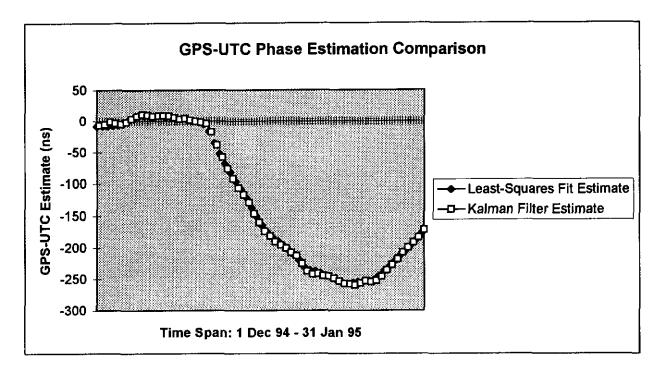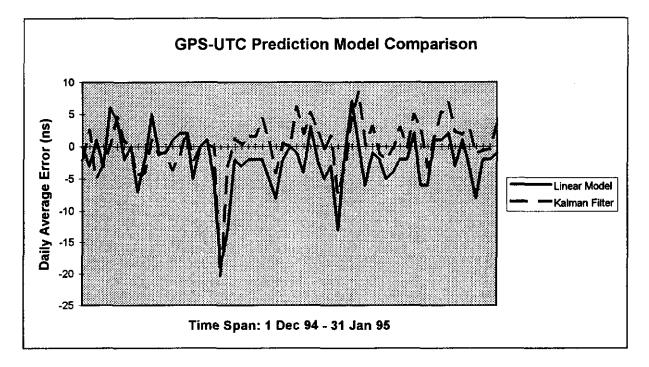[5] Maybeck, Peter S., *Stochastic Models, Estimation and Control*, 1979

Figure 1

GPS-UTC Phase Estimation Comparison

Time Span: 1 Dec 94 - 31 Jan 95

Figure 2

GPS-UTC Prediction Model Comparison

Time Span: 1 Dec 94 - 31 Jan 95

Figure 3