THE EFFECTS OF CLOCK ERRORS ON TIMESCALE STABILITY

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Abstract

The weighting scheme for the cesium clocks and hydrogen masers constituting the USNO timing ensemble is reexamined from an empirical standpoint of maximizing both frequency accuracy and timescale uniformity. The utility of a sliding-weight relation between the masers and the cesiums is reaffirmed, but improvement is found if one incorporates inverse Allan variances for sampling times of 12 and 6 hours for the cesiums and masers, respectively, with some dependence on clock model.

INTRODUCTION

Maximum timescale stability and efficient use of resources require the proper relative weighting of data from atomic clocks. This paper represents a continuation in our quest for an optimal weighting scheme as the U.S. Naval Observatory (USNO) clock ensemble has changed, first with the addition of hydrogen masers to our cesium-beam frequency standards and then with the introduction of the new-model HP5071A cesium standards, which are phasing out our HP5061 standards. The previous study of our weights was based on data from HP5061 cesiums and a few masers[1]. The lower noise of the HP5071A cesiums justifies a reexamination of our weighting procedures.

Since timescale algorithms are generally designed to optimize frequency stability, clocks are commonly weighted according to their individual frequency stabilities, as measured by inverse Allan variances $1/\sigma_y^2$. A previous study, however, found no significant improvement in our timescale if inverse Allan variances were used rather than equal weights, so the latter have been retained[2]. The performance of each USNO clock is closely monitored and any change in its rate precipitates total deweighting until its behavior is again satifactory and its rate accurately redetermined. The deweighting is done automatically in the computation of the near-real-time mean timescale (done every hourly measurement) toward which the Master Clocks are steered; deweighting is done manually during the repeated postprocessing which ultimately results in the final timescale.

The incorporation of hydrogen masers, with their different noise characteristics, requires special treatment of their data. Some labs use a Kalman filter to handle data from such a heterogenous ensemble[3]. We have obtained good results from a sliding–weight relation between the masers and the cesiums that mirrors their respective class average sigma–tau plots, with the sampling time τ replaced by the time prior to the latest measurement[1]. This results in a time dependence of the weights, requiring recomputation of the entire timescale every hourly time step. The masers dominate the clock weights in the recent past, but are entirely phased out over a 75–day period, so only the last 75 days actually need to be recomputed. This zeroing of the maser weights prevents any drift of the timescale due to the masers, since even though the frequency drifts of all clocks are determined and removed from the data, some errors in these drift corrections might otherwise accumulate.

Still, as data collect, reconsideration of the sliding–weight relation might be worthwhile, as might that of assuming equal weights for the masers among themselves. Also, fitting average sigma–tau plots to a class of clocks is not a straightforward, and may be a questionable, procedure, so the approach taken here is to select clocks of homogeneous type for generation of test timescales whose sigma–tau plots may then be meaningfully compared.

THE HP5071A CESIUMS

The new–model HP5071A commercial cesium frequency standards exhibit a significant reduction in noise level over the older HP5061 models and other cesiums due to improvements in electronics and careful allowance for environmental effects[4]. USNO currently has 50 HP5071A cesiums in 13 vaults or environmental chambers available for timescale data acquisition. In fact, they have been used in the timescale computation since February 1992. A preliminary scheme weighted an HP5071A cesium equal to 1.5 times that of an HP5061 cesium.

In order to further investigate their weights, twelve of the best HP5071A cesiums were selected which displayed constant rate and negligible drift over an interval of 200 days (MJD 49137–49337, when the reference maser changed rate). Fig. 1 is a sigma–tau plot for the twelve HP5071A cesiums relative to the Sigma–Tau maser NAV5 (in all such plots, a frequency offset has been removed). Approximating their weights with inverse Allan variances at a sampling time of 30 days (around the minimum), we find that the weights range over a factor of 3.1.

However, how valid are Allan–variance–based weights for these clocks, and what sampling time should be used? Though the theoretical answer to the latter for our algorithm is one hour, the true answers to both questions are affected by noise and systematics. In particular, the noise of our time–interval–counter measurement system is significant at one hour. A proper gauge of a clock's contribution to a timescale is:

$$\frac{1}{\sigma^2_{c,i}(\tau)} = \frac{1}{\sigma^2_{\mathrm{twsi}}(\tau)} - \frac{1}{\sigma^2_{\mathrm{twso}}(\tau)} \tag{1}$$

where $\sigma^2_{c,i}(\tau)$ is the reduction in variance contributed by clock i, $\sigma^2_{\mathrm{twsi}}(\tau)$ is the Allan variance of the timescale computed including clock i, and $\sigma^2_{\mathrm{twso}}(\tau)$ is the Allan variance of the timescale computed without clock i.

This assumes that the clocks involved are not significantly correlated. This has been found to be the case for USNO clocks when the clocks are not disturbed by environmental and human influences[2,5], which are minimized by the enviromental control and maintenance procedures at USNO; data affected by such disturbances have been rejected from this study, as they are from the computation of UTC (USNO). While correlations may seem to be significant when clock frequency variations are intercompared[5], unpublished USNO results indicate that few of these cannot be explained by the use of a common reference, as has been found by others[6].

The intention was to use these clocks to generate test timescales, and twelve clocks were thought to be sufficient to produce a stable timescale, while still being few enough for such a timescale to show a measurable effect if one of the clocks was omitted. Test timescales were generated for all twelve clocks and every subset of eleven clocks, using equal weights; the clock contributions were then calculated via Eq. (1). An indication of the best Allan variance to weight by would be that which best predicts a clock's contribution to such a timescale. Unfortunately, a scatter plot showed only that an Allan variance for a sampling time of a few hours was better for weighting than one for a few days.

To better quantify this, a relative error parameter ϕ was defined such that:

$$\phi_i^2(\tau) = \frac{\left|\sigma_{c,i}^2(\tau) - \sigma_{y,i}^2(\tau)\right|}{\sigma_{c,i}^2(\tau)} \tag{2}$$

where $\sigma_{y,i}^2(\tau)$ is the Allan variance of clock i. Values of $\log\phi$ are plotted vs. $\log\tau$ for all the cesiums in Fig. 2. Some points are missing because ϕ was not available when the computed clock contribution was negative, as it occasionally was, due to noise. Averages were not very stable, but the median minimum relative error occurred for a sampling time of 12 hours.

As a check, test timescales were generated for the same interval and clocks, weighting the clocks by inverse Allan variances over a range of sampling times from 1 hour to 30 days. The resulting sigma–tau plots are given in Fig. 3. There is little difference between most of them, but the worst are the long sampling times, as one would expect. The best sampling time was around 12 hours. Variances computed for $\tau = 12$ hours would also reflect well the effects of any diurnal environmental perturbations. At $\tau = 12$ hours, $\sigma_{c,i}^2(\tau)$ varied over a factor of 2.8 and $\sigma_{y,i}^2(\tau)$ varied over a factor of 2.0. Consequently, inverse 12–hour Allan variances will be our choice for weighting the HP5071A cesiums.

THE HP5061 CESIUMS

At present, 14 HP5061A cesiums and two HP5061B cesiums in four vaults or environmental chambers are available for timescale data acquisition. The sigma–tau plots for ten HP5061A cesiums are given in Fig. 4 for from 80 to 169 days of data. A similar analysis was attempted of the clock contributions as was done for the HP5071A cesiums. Also, each HP5061A clock was substituted for a member of the HP5071A ensemble, and timescales were generated and analyzed for each. In both cases, the HP5061A data were too noisy to reach reliable conclusions.

Comparing the average 12–hour Allan deviations in Fig. 4 with those in Fig. 1 gives:

$$\langle \sigma_{5071}/\sigma_{5061} \rangle = 0.795$$

Comparing the median 12–hour Allan deviations gives:

$$\langle \sigma_{5071}/\sigma_{5061} \rangle = 0.785$$

Consequently, we will adopt a weight ratio of:

$$w_{5061}/w_{5071} = \sigma^2_{5071}/\sigma^2_{5061} = 0.62$$

for any HP5061 cesium relative to a typical HP5071A cesium.

As a check on whether equal weights should be retained for the HP5061A clocks, test timescales were generated for 104 days of data (MJD 49233–49337), weighting by inverse Allan variances for a range of sampling times. The results are presented in Fig. 5. While inverse 1-hour Allan variances make slightly better weights than those for somewhat longer sampling times, equal weights yielded significantly better stabilities than did any of the Allan–variance–based weights.

THE MASERS

USNO currently has three SAO masers and ten Sigma–Tau masers in seven vaults or environmental chambers available for timescale data acquisition. During a 222–day interval (MJD 49404–49626) of constant drift and variance, four SAO masers (one has since left) and five Sigma–Tau masers were selected for analysis. Some rate corrections and occasional outlier rejections were required, but this is done routinely by the timescale algorithm. Some of these masers were steered in frequency, so their data were mathematically desteered.

An n–cornered–hat analysis was performed to obtain their absolute Allan deviations, which are plotted in Figs. 6 (for the SAO masers) and 7 (for the Sigma–Tau masers). (The analytical method, which produces identical results as the method commonly in use[7], is described in the Appendix and is due, as far as we know, to Winkler[8].) The curves for the Sigma–Tau masers differ systematically from those of most of the SAO masers, as might be expected, since the former are auto–tuned. The average τ of minimum variance is 0.8 days for the SAO masers and 5.9 days for the Sigma–Tau masers. Approximating their weights with inverse Allan variances at the average τ of their minimum variances, we find that the weights range over a factor of 166 for the SAO masers and 14 for the Sigma–Tau masers. This indicates that the weights of the two types of masers should be derived separately and that an upper limit on the weight of a clock will be necessary (more on this later).

Meaningful computations of clock contributions, then, will require unequal weighting. In order to determine the τ of the Allan variances for such weights, test timescales were generated for the above interval and masers relative to the masers MC #1 and MC #2, weighting by inverse Allan variances over a range of sampling times. A three–cornered–hat analysis was then done between each timescale relative to Master Clock (MC) #1, each timescale relative to MC #2, and the difference MC #1 – MC #2 in order to determine the absolute Allan variances of each timescale. The sigma–tau curves of these timescales are displayed in Fig. 8. At smaller sampling times (where the stability of the masers is of most interest to us), 6–hour Allan–variance–based weights are best.

On that basis, 6–hour Allan–variance–weighted timescales were generated for all nine masers and for every subset of eight. Clock contributions were next computed as they were for the cesiums. The corresponding values of relative error ϕ are plotted in Figs. 9 (for the SAO masers) and 10 (for the Sigma–Tau masers). Again, some of the points are missing due to noise. The situation is less clear than for the HP5071A cesiums, but the sampling time of the median minimum relative error is also 6 hours. Hence, we will adopt 6–hour Allan–variance–based weights for the masers.

THE MASERS RELATIVE TO THE CESIUMS

The rationale behind the sliding–weight scheme relating the masers to the cesiums is that: (1) it combines the short–term stability of the masers with the long–term stability of the cesiums; and (2) it retains the systematic frequency accuracy of the cesiums as an anchor to the final timescale, while maximizing the relative frequency stability of the timescale in the recent past, where it used to steer the Master Clocks. A Kalman–filter–based timescale algorithm can provide (1), but not (2). As noted above, the method requires recomputation of the timescale every time step, with the consequences that: (1) our timescale only becomes final 75 days in the past; and (2) UTC (USNO) at any given time may change by a few nanosceonds during those 75 days. The latter is logical because, as data accumulate, clock rates and drifts become more accurately determined, improving one's knowledge of the timescale at any point in the past.

As mentioned, HP5071A cesiums have been used in the timescale computation since February 1992. A preliminary scheme weighted the HP5071A cesiums and the hydrogen masers as follows:

$$w_{5071}(t) = 1/[\text{antilog } (0.130x^2 - 0.137x - 13.959)]^2 \quad (3)$$

$$w_{hm}(t) = 1/[\text{antilog } (0.309z^2 - 0.037x - 14.239)]^2 \quad (4)$$

where $x = \log t - 5.9$, $z = \log t - 5.2$, and t is the time difference in seconds prior to the most recent measurement. At t = 0, the weight was arbitrarily set equal to t = 3600 (not t = -1, as misstated in [1], p. 299).

In order to redetermine these relations using sigma–tau plots of timescales rather than those of clocks, test timescales were generated for: (1) the four SAO masers; (2) the five Sigma–Tau masers; and (3) five of the above HP5071A cesiums, all for the above 222–day interval. The masers were weighted by inverse 6–hour Allan variances and the cesiums by inverse 12–hour Allan variances. An upper limit of 33% of the total weight was placed on the individual clock weights. Sigma–tau plots were computed for all three timescales. The ratio of the corresponding Allan variances for each maser timescale and the cesium timescale were taken and fitted with a second–order curve, as shown in Figs. 11 (for the SAO masers) and 12 (for the Sigma–Tau masers). The equations of these fits are:

$$w_{SAO/5071}(t) = [6.1 \pm 0.9](\log t)^2 - [79 \pm 9](\log t) + [257 \pm 22] \quad (5)$$

$$w_{ST/5071}(t) = [5.5 \pm 1.4](\log t)^2 - [76 \pm 14](\log t) + [261 \pm 34] \quad (6)$$

where t, the time in seconds prior to the latest measurement, has been substituted for τ. The weights at t = 0 are arbitrarily set equal to those at t = 3600. These relations reach minima at log t = 6.5 and 6.9, respectively, at which point they can be ramped down to zero by t = 75 days.

As a final test of the new weights, these sliding-weight relations, 6–hour maser weights, and 12–hour cesium weights were used for the same nine masers and nine of the HP5071A cesiums to generate timescales for the above 222–day interval. A single sigma–tau plot cannot properly characterize such a timescale because of the change in short–term stability relative to long–term stability with time. Since the cesiums dominate after 15 days in the past and it has been shown that the new weights provide some improvement over the old, the remaining question is whether the stability in the last 15 days has been enhanced. Accordingly, the interval was divided into fourteen 15–day segments and timescales were generated for each segment, with t reckoned from the end of each segment. The Allan variances of these timescales were then averaged and are presented in Fig. 13, where for comparison there have also been plotted the corresponding averages if one used the old weights[1] and the new weights but with no sliding relation. As can be seen, the new weights are a significant improvement on the short term over both the old and to not using the sliding relation at all.

SUMMARY

The proper choice of timescale algorithm and clock weighting scheme depends on the purpose to which the resulting timescale is to be put. One objective of the USNO timescale is systematic frequency accuracy of the final timescale coupled with optimal relative stability in the recent past for the purpose of steering the Master Clocks. Compromise between these two aims is avoided by use of the sliding-weight relations between the masers and the HP5071A cesiums given in Eqs. (5) and (6). Adoption of inverse 6–hour Allan–variance weights for the masers and similar 12–hour weights for the cesiums will further improve UTC (USNO) by introducing responsiveness of the timescale to the performance of individual clocks beyond that already provided by careful monitoring and deweighting.

The new weights for an HP5071A clock i (of n such clocks) and an HP5061 clock j are, respectively:

$$w_{5071,i}(t) = \left[\frac{\frac{1}{\sigma^2_{12,i}}}{\sum_{i=1}^{n} \frac{1}{\sigma^2_{12,i}}} \right] D_i(t) \quad (7)$$

and

$$w_{5061,j} = 0.62 \langle w_{5071} \rangle D_j(t) \quad (8)$$

where σ^2_{12} is the Allan variance for τ = 12 hours; D_i and D_j are deweighting factors in case of changes in performance, an uncertain rate, or an upper limit on the weight; and < > denotes an average over all clocks. The new weight for a maser k (of a total of m such) is:

$$w_{hm,k}(t) = \left[\frac{\sigma^2_{6,k}}{\sum_{k=1}^{m} \sigma^2_{6,k}} \right] D_k(t) w_{hm/5071}(t) \quad (9)$$

where $\sigma^2_6$is the Allan variance for τ = 6 hours, D_k is a deweighting factor, and $w_{hm/5071}$ is given by Eq. (5) or (6). A upper limit on the weight prevents one or more superior clocks from dominating the timescale, which might lead to jolts of the timescale in the case of clock failure. The imposition of such a limit detracts from optimality, but is a requirement for reliability, which is another objective of the USNO timescale.

If the weights were based on stability relative to the mean timescale, a correction factor would have to be added to Eqs. (7), (8), and (9) for the so-called clock-ensemble effect, which would otherwise bias the timescale toward the best-performing clocks[9]. One may also question variances based on reference to a timescale whose own stability changes with time. Both problems may be avoided by referring the clocks to an unweighted, unsteered (or desteered) maser, rather than to the mean timescale.

Whether the adoption of gradual (robust), rather than instantaneous, deweighting would be a significant improvement remains to be tested; our large number of clocks has not made this a priority. Our short-term measurement noise should be appreciably reduced when our experimental Erbtec, or its successor the Steintech, system is reliable and capacious enough to be implemented, at which time the above weighting scheme will need to be reexamined. Further automation of the postprocessing procedure and more statistically rigorous treatment of rate and drift determination and rate and drift change detection are planned.

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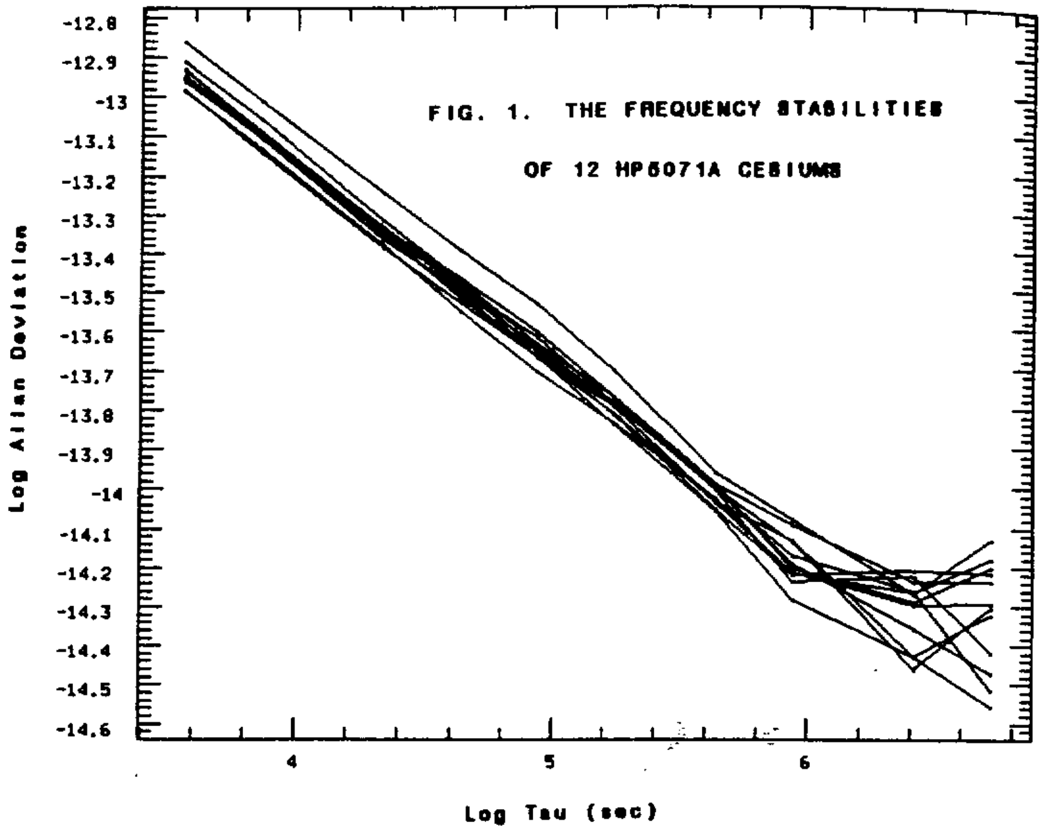

FIG. 1. THE FREQUENCY STABILITIES OF 12 HP5071A CESIUMS

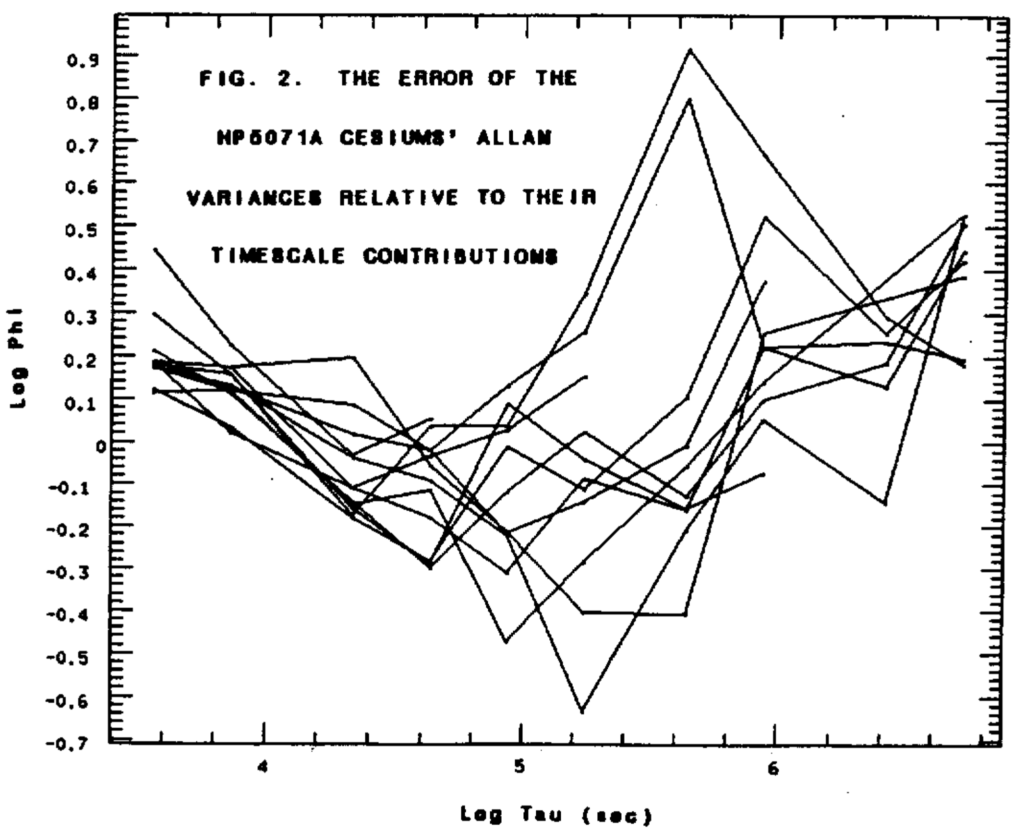

FIG. 2. THE ERROR OF THE HP5071A CESIUMS' ALLAN VARIANCES RELATIVE TO THEIR TIMESCALE CONTRIBUTIONS

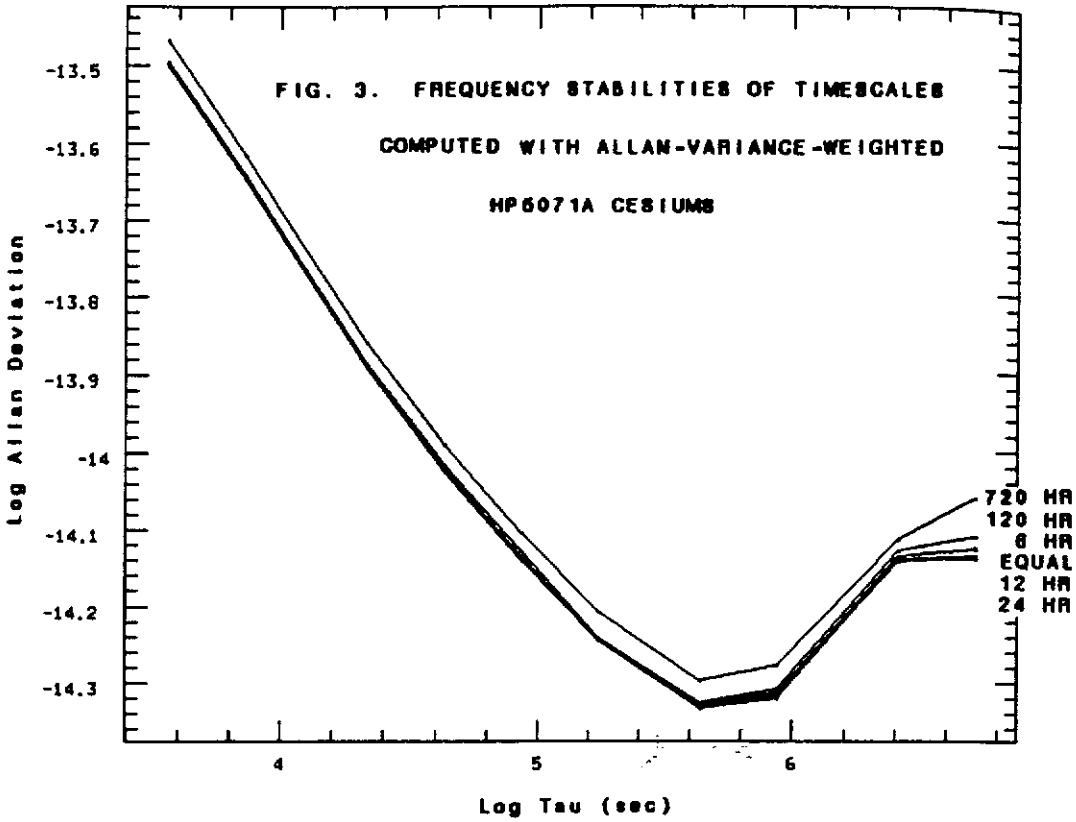

FIG. 3. FREQUENCY STABILITIES OF TIMESCALES COMPUTED WITH ALLAN-VARIANCE-WEIGHTED HP5071A CESIUMS

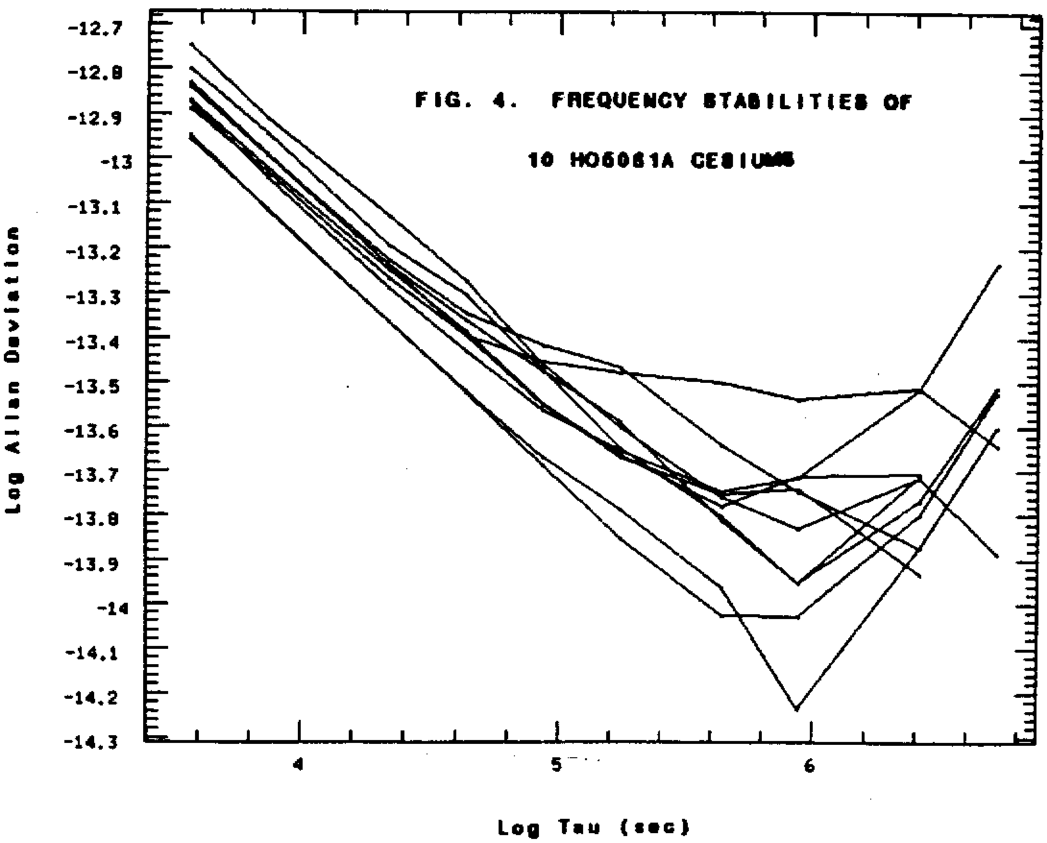

FIG. 4. FREQUENCY STABILITIES OF 10 HO5061A CESIUMS

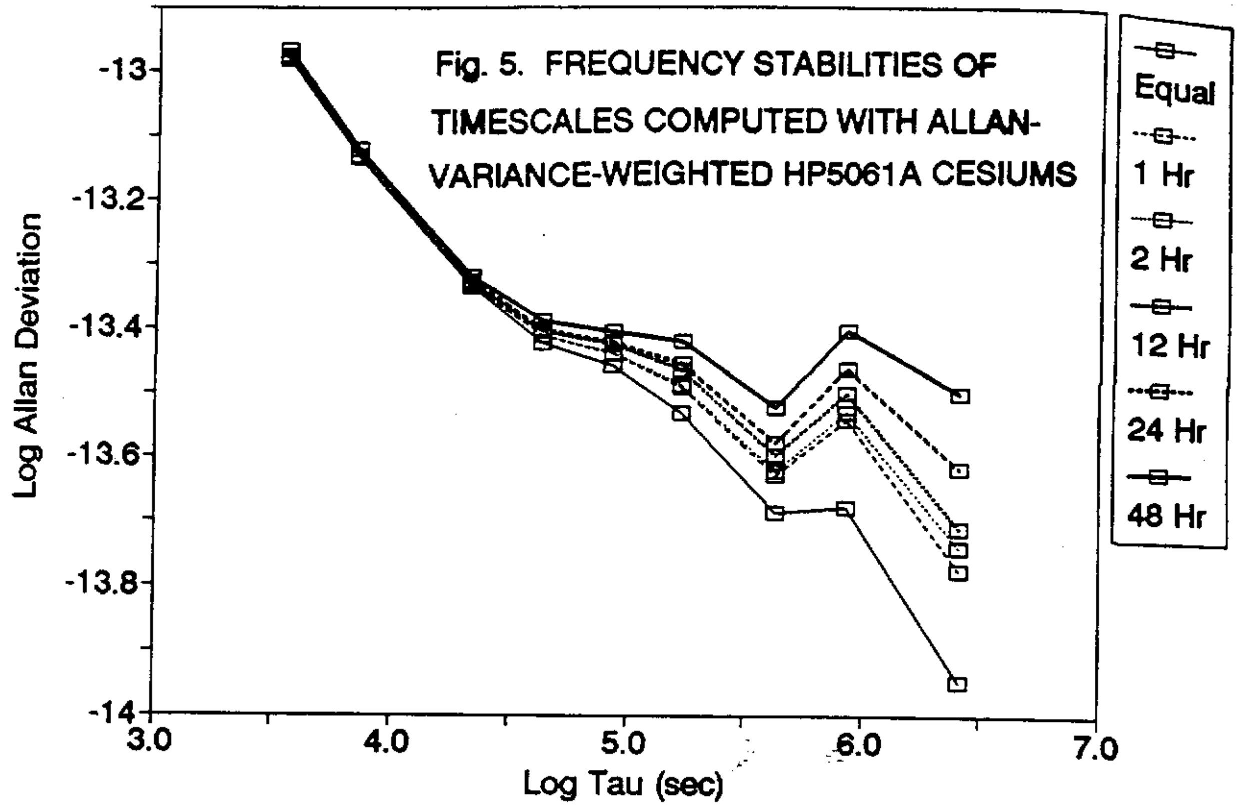
Fig. 5. FREQUENCY STABILITIES OF
TIMESCALES COMPUTED WITH ALLAN-
VARIANCE-WEIGHTED HP5061A CESIUMS
Log Allan Deviation
Log Tau (sec)
-13
-13.2
-13.4
-13.6
-13.8
-14
3.0
4.0
5.0
6.0
7.0
Equal
1 Hr
2 Hr
12 Hr
24 Hr
48 Hr

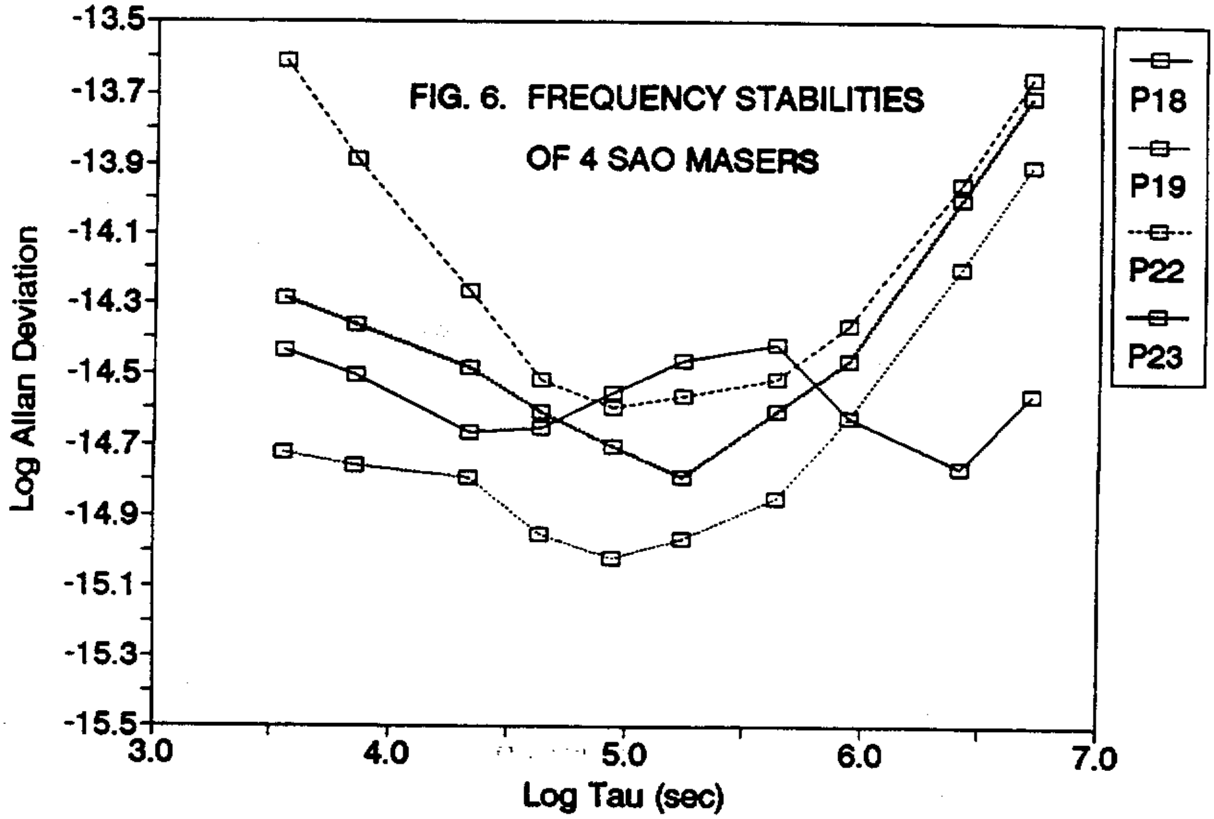
FIG. 6. FREQUENCY STABILITIES
OF 4 SAO MASERS
Log Allan Deviation
Log Tau (sec)
-13.5
-13.7
-13.9
-14.1
-14.3
-14.5
-14.7
-14.9
-15.1
-15.3
-15.5
3.0
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5.0
6.0
7.0
P18
P19
P22
P23

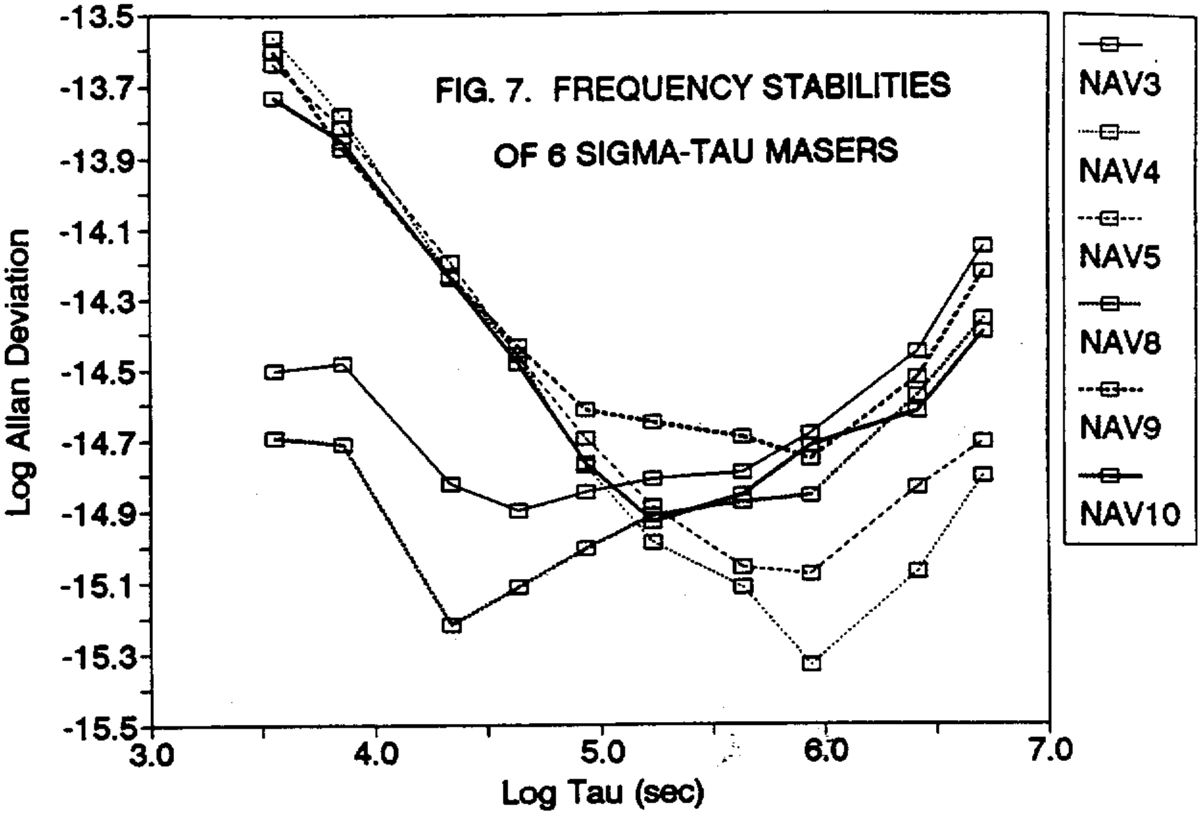
FIG. 7. FREQUENCY STABILITIES
OF 6 SIGMA-TAU MASERS
Log Allan Deviation
-13.5
-13.7
-13.9
-14.1
-14.3
-14.5
-14.7
-14.9
-15.1
-15.3
-15.5
3.0
4.0
5.0
6.0
7.0
Log Tau (sec)
NAV3
NAV4
NAV5
NAV8
NAV9
NAV10

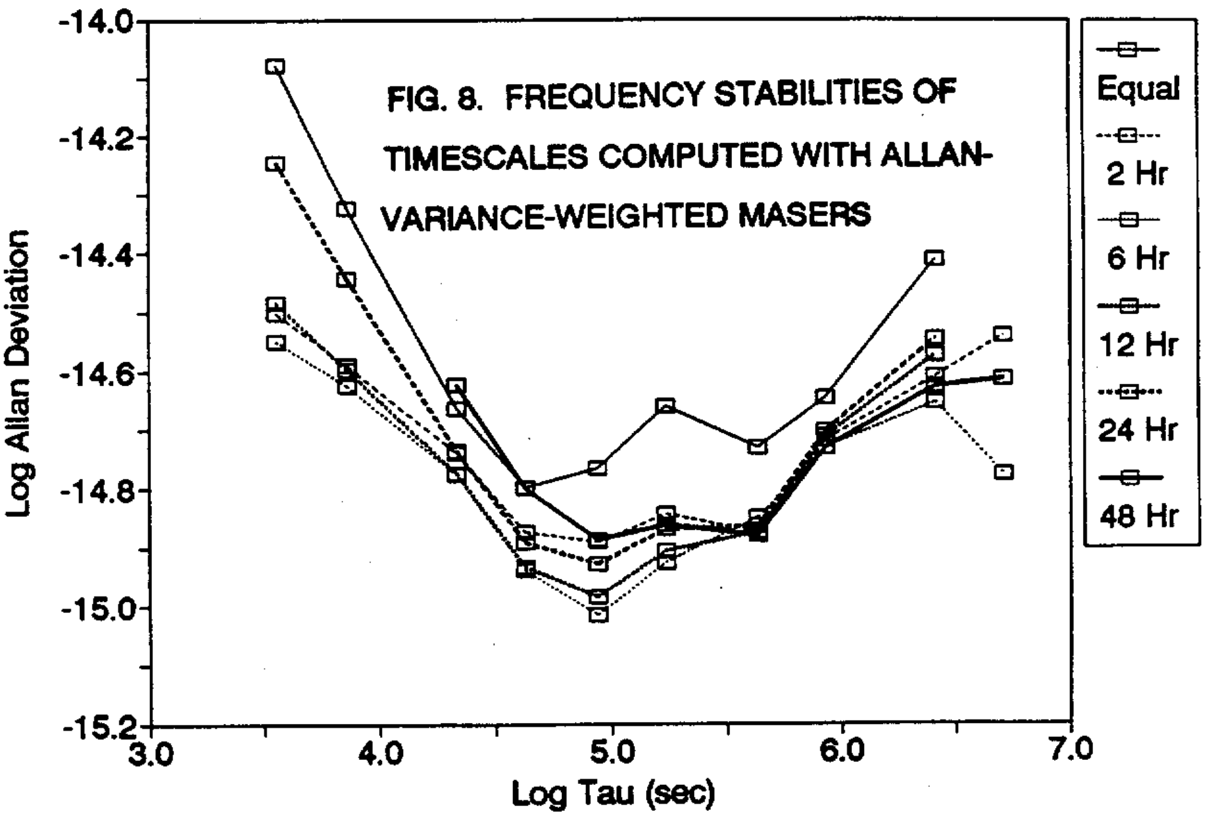
FIG. 8. FREQUENCY STABILITIES OF
TIMESCALES COMPUTED WITH ALLAN-
VARIANCE-WEIGHTED MASERS
Log Allan Deviation
-14.0
-14.2
-14.4
-14.6
-14.8
-15.0
-15.2
3.0
4.0
5.0
6.0
7.0
Log Tau (sec)
Equal
2 Hr
6 Hr
12 Hr
24 Hr
48 Hr

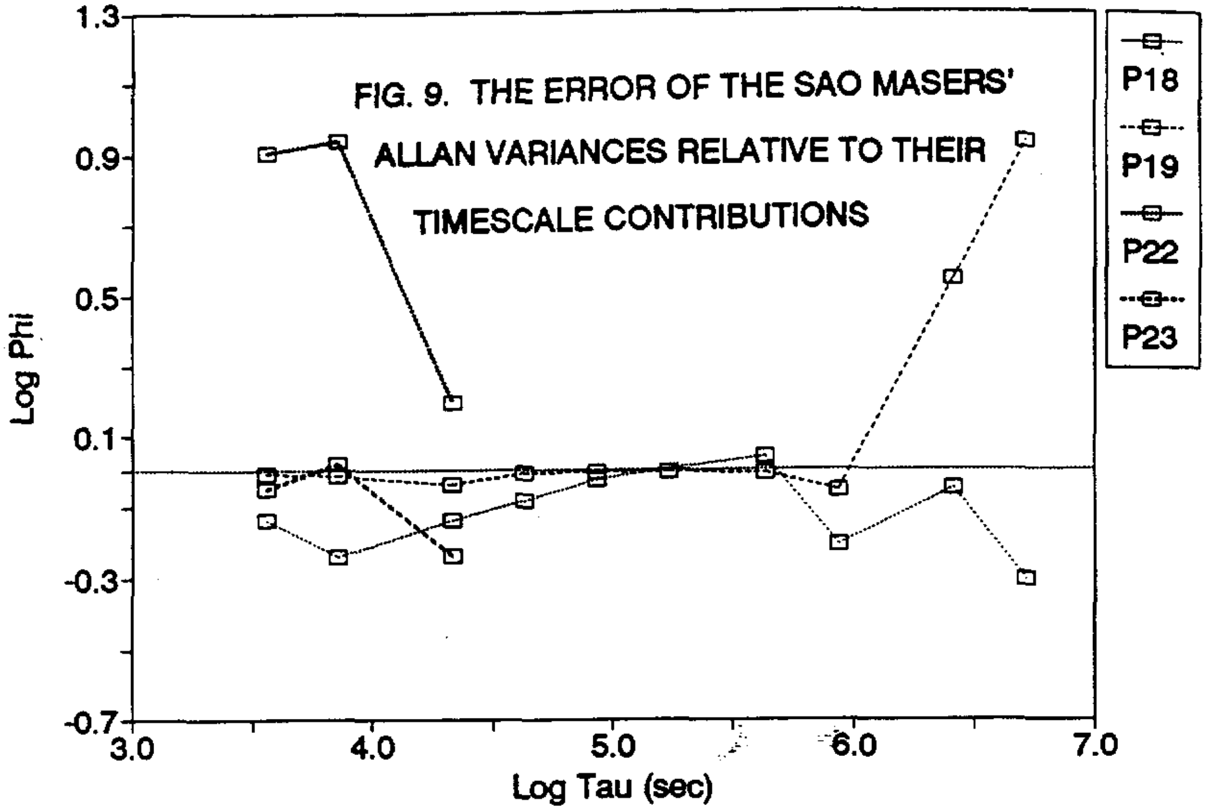

FIG. 9. THE ERROR OF THE SAO MASERS' ALLAN VARIANCES RELATIVE TO THEIR TIMESCALE CONTRIBUTIONS

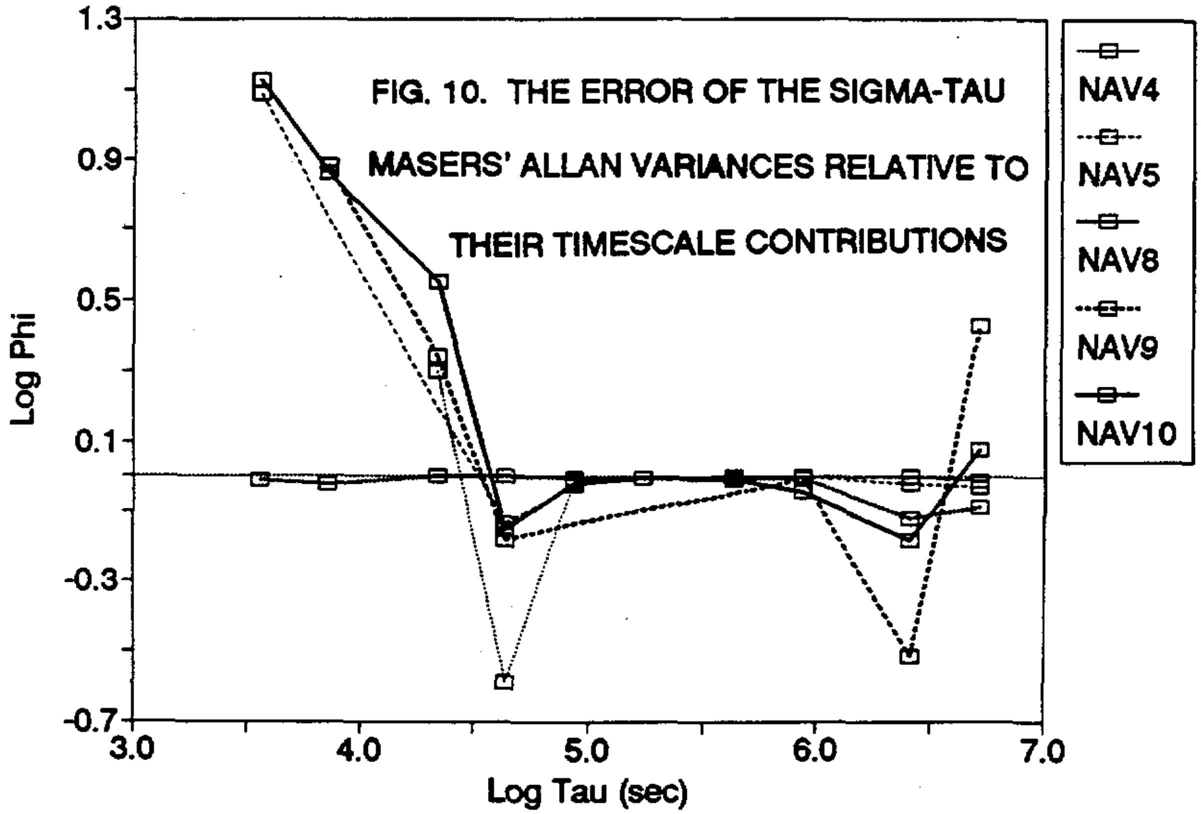

FIG. 10. THE ERROR OF THE SIGMA-TAU MASERS' ALLAN VARIANCES RELATIVE TO THEIR TIMESCALE CONTRIBUTIONS

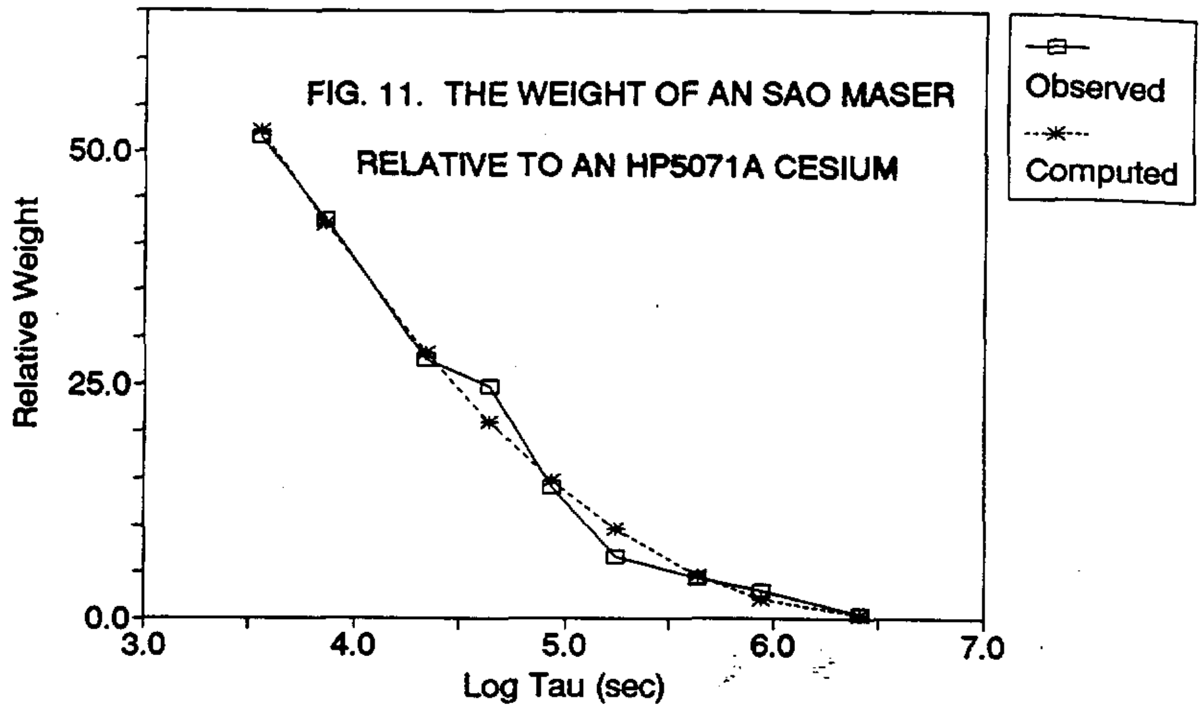

FIG. 11. THE WEIGHT OF AN SAO MASER RELATIVE TO AN HP5071A CESIUM

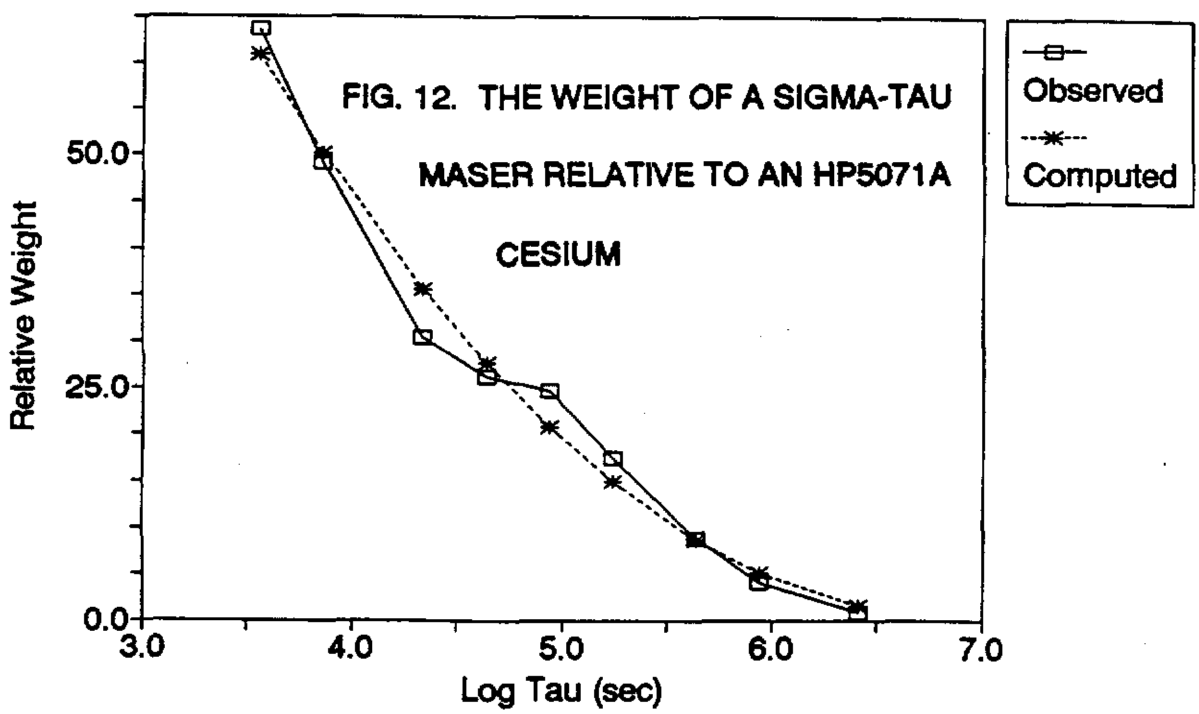

FIG. 12. THE WEIGHT OF A SIGMA-TAU MASER RELATIVE TO AN HP5071A CESIUM

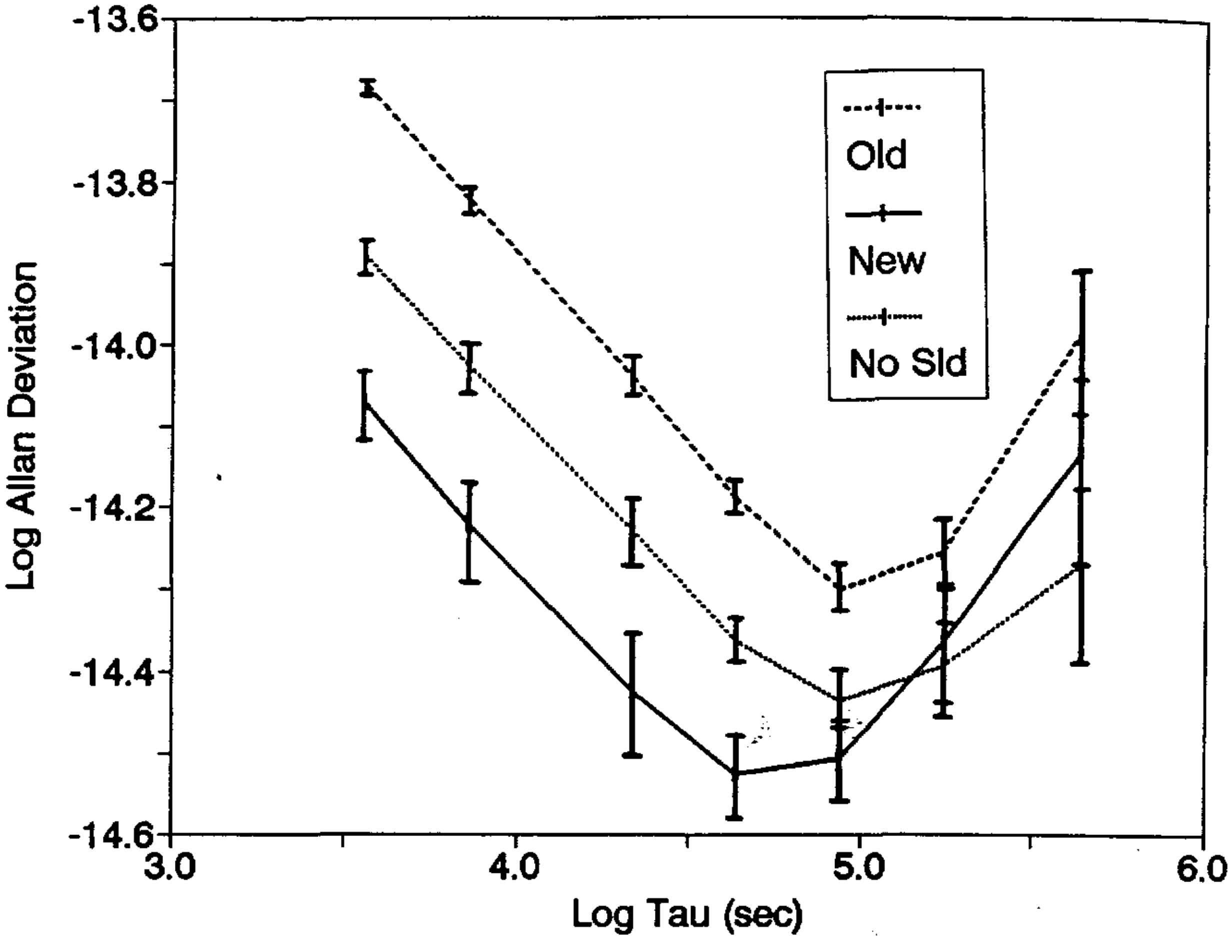

FIG. 13. FREQUENCY STABILITIES OF TIMESCALES COMPUTED WITH THE OLD WEIGHTS, THE NEW WEIGHTS, AND THE NEW WEIGHTS BUT WITHOUT THE SLIDING RELATION

APPENDIX

An n–cornered–hat analysis for the individual variances of a set of uncorrelated clocks may be performed by writing the variance of the difference between the measurements of clocks i and j as the sum of their individual variances:

$$\sigma_i^2 + \sigma_j^2 = \sigma_{ij}^2$$

for all possible pairs of n clocks and then solving these as a system of n (n - 1)/2 simultaneous linear equations. The matrix equation could be expressed as:

M X = Y

where, for four clocks:

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \sigma_3^2 \\ \sigma_4^2 \end{bmatrix} \quad \mathbf{Y} = \begin{bmatrix} \sigma_{12}^2 \\ \sigma_{13}^2 \\ \sigma_{14}^2 \\ \sigma_{23}^2 \\ \sigma_{24}^2 \\ \sigma_{34}^2 \end{bmatrix}$$

X may then be solved for by multiplying both sides by the Penrose pseudo–inverse of M, which here is:

$$\mathbf{M}^{-1} = \begin{bmatrix} 0.3\bar{3} & 0.3\bar{3} & 0.3\bar{3} & -0.1\bar{6} & -0.1\bar{6} & -0.1\bar{6} \\ 0.3\bar{3} & -0.1\bar{6} & -0.1\bar{6} & 0.3\bar{3} & 0.3\bar{3} & -0.1\bar{6} \\ -0.1\bar{6} & 0.3\bar{3} & -0.1\bar{6} & 0.3\bar{3} & -0.1\bar{6} & 0.3\bar{3} \\ -0.1\bar{6} & -0.1\bar{6} & 0.3\bar{3} & -0.1\bar{6} & 0.3\bar{3} & 0.3\bar{3} \end{bmatrix}$$

As with the standard n–cornered–hat method, the analysis fails if any of the variances solved for comes out negative. This generally occurs when the clocks are significantly intercorrelated, causing the variances to be underestimated.

QUESTIONS AND ANSWERS

MARC WEISS (NIST): The result of that equally–weighted scale looks as good as a differently–weighted scale. It was rather surprising to me because we found results that are rather different.

Also, in general, using weights by themselves is really dependent on what algorithm you're using. And the tau that's used for determining the Allan Variance, $\sigma_y(\tau)$, the tau should come out of the algorithm; the algorithm should dictate the tau you use to determine weights. And using that kind of analysis, we found quite a difference when we used scales that are weighted differently for different clocks.

LEE A. BREAKIRON (USNO): It depends on exactly how you intend to use the time scale that you generate. We would like the highest systematic accuracy to determine our final time scale. That's why we phase out the masers, and we want the highest relative accuracy in the short term to steer our masers too. Yes, it depends what you are going to put the time scale to as to what tau you would weight by and whether it would make any difference.

MARC A. WEISS (NIST): What do you mean by "accuracy" in that context?

LEE BREAKIRON (USNO): We want the systematic accuracy of the time scale to be maximized, based only on the cesiums. So that's why we phase out the masers.

MARC A. WEISS (NIST): By accuracy, do you mean "frequency accuracy" or accuracy of time relative to some other scale?

LEE BREAKIRON (USNO): Right. The systematic frequency accuracy.

JUDAH LEVINE (NIST): I would just like to ask you to clarify your point of your recomputation of the data already submitted to the BIPM. Does that mean that the data in Circular T are, in fact, amended after the fact?

LEE BREAKIRON (USNO): *Slightly.*

JUDAH LEVINE (NIST): Is that amendment published subsequently? What I am saying is that if I look at Circular T and I copy a number down –– I'm trying to understand how it works.

GERNOT M. WINKLER (USNO): Two things. The Circular T values are determined from the individual clock readings which we submit to the BIPM. They are not changed. These are measurements which are made against a physical signal, which is the Master Clock; which is the same which is used to link to other laboratories.

What we are talking about here is the internal time scale which is used to steer the Master Clock. There is a very long time constant. We have to have something to steer to us. BIPM, of course, in a last analysis, is to panel for that. But for the day– to–day performance, we have an internal time scale, and that is the one which changes. Now that is the internal time scale which, again, has to be differentiated from the coordinated scale. The coordinated scale has additional frequency changes imposed because of our efforts to stay as close as we can to BIPM.

So the result is, the bottom line is that the Circular T values for the UTC, USNO are not changed.

CLAUDINE THOMAS (BIPM): Of course, UTC minus UTC USNO Master Clock is not changed, for sure. But I think that Gernot is speaking about the second page of Circular T which gives DI minus the individual TA; while what is called TA USNO is A.1 mean from USNO and for which we do not have the definitive values, as far as I understood with this thing with Dr. Breakiron last Friday. We do not have the definitive values; the values published are not the definitive values simply because definitive values are obtained 70 days after the fact, while USND values are before 70 days.

But it is true, we have not the definitive values. Maybe we can change this for the annual report. But that is something that I didn't know before coming here. But maybe we can change this.

GERNOT M. WINKLER (USNO): There is another point. And that is that TAI is not based on the contributions of the time scales. It is based on the contribution of individual clocks.

CLAUDINE THOMAS (BIPM): Yes, yes, of course. The independent time scales are not weighted in TAI. What are weighted is are the independent clocks supposed to be free–running.

But it's true that we are also publishing DI minus independent local TA on the second page of Circular T. And I think that Gernot was alluding to that particular publication, which has nothing to do with the first page which is UTC minus local UTCs. Thank you.

LES BREAKIRON (USNO): I thought of a better answer to Dr. Weiss. You have to realize that this data have been chosen because they are the highest quality that we have. And all things like rate corrections have already been either corrected for or the clock is removed for that reason.

So when you're dealing with data of that quality, and it's essentially been combed through like that, I think you would find results closer to ours.

CLAUDINE THOMAS (BIPM): Excuse me. I just want to add a small comment or question. On one of your transparencies, there is a difference, something like 1 over variance equals 1 over variance of something else, minus 1 of a variance of something else.

LES BREAKIRON (USNO): Right.

CLAUDINE THOMAS (BIPM): I'm very suspicious of doing differences of variances. If you have differences of one of a variance, it means you are able to do the sum of one of a variance. And this is only possible, of course, if the clocks are independent and if there is no limit of weight. As soon as you have a limit weight in any time scale, it is no longer true. That's an argument against TAI, which is very often said and discussed. Thank you.

GERNOT M. WINKLER (USNO): I think, since that is a planning meeting and requirements of great interest, I would like to hear of anyone who would be affected by these changes of the internal time scale, after the fact. We have been under the firm impression that it doesn't affect anyone, because it does not affect TAI, it does not affect the actual differences which are reported on page one. It only affects the difference of DI minus internal time scale, which you published.

However I would like to hear from anyone who feels that this produces a difficulty.

CLAUDINE THOMAS (BIPM): Of course, the second page of Circular T, which gives DI minus TAK, is used only for laboratories which are giving TA. Of course, this has absolutely no impact on tau and users of UTC–K, which are published. The use is for you, in fact, to know it's doing your E.1 mean relative to TAI. Well, if anyone wants to make the same kind of thing, of course, if he has not the last updated values, he is mistaken, of course.

But the primary role of this answer is for the laboratories themselves.

GERNOT M. WINKLER (USNO): We have to ask ourselves.

CLAUDINE THOMAS (BIPM): Yes. All the users who would liked to be linked for some reason to A.1 mean. Of course, if it only takes Circular T values, they do not have the last of this. So they may feel mistaken or have a distrust about that.

JUDAH LEVINE (NIST): My comment is that the most important aspect of the Circular T data is that I understand what it means. Speaking as a user of page two, the most important characteristic to me is that I understand exactly how those numbers are calculated. And now that I understand it, I understand it; and before, I didn't understand. And I think that is the most difficult aspect, is to know what the number actually means or how it was calculated.

RANDY CLARKE (USNO): I'm the one that does it. So just to let you know what you're facing, I would say that it's probably only a few nss. Because, we send in the reports after 30 days, so the major processing has already been done. So it's very rare that it's over five. If you're interested in what it is, it's something like five ns.

CLAUDINE THOMAS (BIPM): This can be done so simply in the new report of the BIPM. Just send me the last values when you have them, and I will publish them in the annual report; so everyone can get the updates after the fact, and that's all.