A COMPARATIVE STUDY OF CLOCK RATE AND DRIFT ESTIMATION

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ABSTRACT

Five different methods of drift determination and four different methods of rate determination were compared using months of hourly phase and frequency data from a sample of cesium clocks and active hydrogen masers. Linear least squares on frequency is selected as the optimal method of determining both drift and rate, more on the basis of parameter parsimony and confidence measures than on random and systematic errors.

INTRODUCTION

In the presence of random, time-independent errors, the mean of a series of measurements is an unbiassed estimator of the expectation value of a single variable, and least squares provides an optimal solution for unbiassed estimates of parameters which are a function of observed variables. In the case of time-dependent errors, these same estimators are valid if the errors (in this case called noise) are uncorrelated with frequency, i.e. their spectrum is white. The output of clocks (phase or its derivative, frequency) is typically afflicted by a mixture of different types of (generally power-law) noises which can bias the statistical estimation of such parameters as mean frequency (rate) or change in frequency (drift).

For example, in the presence of white FM noise, the rate of a clock could be accurately measured by either averaging successive first differences (differences between successive time-difference readings between a clock and another time reference) or by solving by least squares for the slope of the line relating phase and time. White FM noise corresponds to a noise process in phase called random walk. Similarly, random walk FM corresponds to a process in its derivative in what might be termed "white drift noise." In the presence of white drift noise, the drift of a clock could be accurately measured by either averaging second differences (differences between successive first differences) or by solving by least squares for the slope of the line relating frequency and time.

The type of power-law noise process (white PM or flicker PM or white FM or flicker FM or random walk FM) that predominates depends mainly on the sampling time and the type of clock. White FM noise predominates in cesium frequency standards from 10 seconds to days and in active hydrogen masers from 100 seconds to a few hours; random walk FM prevails in both types of clocks over periods of weeks or more [1].

If averages are taken or least-squares solutions made over sampling times outside the white-noise region for a particular type of measurement, the resulting estimates will be nonoptimally noisy and systematically unreliable. The choice of proper sampling time is complicated if different types of clocks are combined in an ensemble for the purpose of generating a mean timescale.

The current USNO timescale algorithm averages the rates of cesiums and masers determined from five-day, linear least-squares solutions on hourly phase measurements. The five-day rates are further averaged, unless drift is evident, in which case the drift is solved for by linear least squares on frequency over periods of 90 days or more and then is allowed for. The weighting scheme takes account of the different, time-varying weight of the masers relative to the cesiums [2].

Least-squares solutions on phase for frequency are optimal only for white PM noise, which applies to sampling times much shorter than an hour for both cesiums and masers. Averaging hourly first differences might yield superior rates. The current separate solutions for rate and drift also risk parameter incompatibilities and error underestimation. Perhaps rate and drift should be solved for simultaneously by least squares on frequency.

Solution for drift by averaging second differences should give valid results only when conducted in the random walk FM region. Indeed, Weiss et al. [3] analyzed simulated data and showed that an overall second difference spanning the entire data set yielded more efficient results than averaging successive second differences if white FM noise was present in addition to random walk FM. It would be of interest to repeat this test on real clock data and compare the drifts to those obtained by least squares.

DATA

The data consisted of hourly time-interval-counter measurements for 28 HP5071A cesiums, 5 SAO masers, and 4 Sigma Tau masers, all referred to the same maser (the very stable Sigma Tau maser "NAV5"; see Table 1). In comparison, HP5061 cesium data proved too noisy to use. Data segments of apparently constant drift and (aside from that) constant rate at least 90 days (and up to 565 days) in length were selected. Being primary frequency standards, stable cesium clocks should possess no intrinsic drifts. Accordingly, their average drift should be the negative of the drift of the reference maser; any cesium drifts > 3.0 times the rms of this average were rejected. Individual phase, frequency, or drift residuals (depending on method, as described below) > 3.0 rms were also rejected.

DRIFT DETERMINATION

Five possible methods of drift determination were selected for testing:

METHOD #1	Solve for drift and rate by linear least squares on frequency (specifically, first differences of phase)
METHOD #2	Solve for drift and rate by quadratic least squares on phase
METHOD #3	Solve for rate by least squares on 5-day bins of phase and then solve for drift by least squares on the rates
METHOD #4	Solve for drift by averaging frequency changes (specifically, second differences of phase)
METHOD #5	Solve for drift by computing the overall second difference (i.e. from the initial, mid, and final phases of the data segment)

Some methods may be more susceptible to nonwhite noise than others, producing systematic errors between them. Method #2 should yield inferior results to Method #1 because the former solves for one more least-squares parameter, decreasing the accuracy of all parameters obtained. Whether such deficiencies are statistically significant remains to be seen. Method #5 permits solution for the drift in the white drift noise region, and Method #3 nearly does as well, unlike Methods #1, #2, and #4.

The Cesiums

How should the methods be compared? Formal errors cannot be directly compared because they depend on how the drifts were derived. For example, Method #2 drifts have a much smaller rms error on average than the other methods because most of their error is absorbed into the least-squares constant term. Method #5 does not even have an rms.

Since the cesiums should not have intrinsic drifts, the methods were compared by computing the standard error (s.e.) of the average drift over all cesiums. This average drift should be the negative of the drift of the reference maser, assuming we have successively excluded cesiums with their own drifts. The best method should be the one giving the most consistent results, i.e. the smallest s.e. for this average drift. Since the rms of any drift determination decreases with the square of the time interval spanned, the clocks were weighted by the fourth power of the data length. The results were:

Method	Mean Drift	s.e.	Mean Drift Error	s.e.	
#1	+0.009	±0.013	±0.01530	±0.00022	parts in 10^{15} /day
#2	+0.009	0.013	0.000491	0.000007	
#3	+0.009	0.013	0.01814	0.00021	
#4	+0.8	1.2	43.59	0.14	
#5	+0.011	0.013			

From the mean drift and its s.e., we conclude:

- Method #4 is rejected out of hand. This is hardly surprising in view of the presence of noises other than random walk FM in the hourly data.

- The s.e.s of the other methods are identical, so these methods are equally good.

- The mean drifts of all methods agree within their s.e.s, so there are no systematic errors between methods.

- The drift of the reference maser is -0.009 ± 0.013 parts in 10 to the 15th/day.

As a check on the systematic errors, an average drift was computed across the methods (weighting them equally, except #4, which was weighted zero) for each clock and then the resulting residuals were averaged across the clocks (again weighting by the fourth power of the data length) for each method. The systematic errors thus obtained are strictly relative. The results were:

Method	Mean Sys. Error	s.e.	
#1	+0.0002	±0.0014	parts in 10^{15} /day
#2	+0.0002	0.0021	
#3	-0.0004	0.0011	
#4	+0.8	1.2	
#5	+0.0013	0.0020	

Again, no systematic error is significant.

The drifts obtained by Method #1 are given in Table 1. Unlike the drifts above, these have been referred to TAI by the addition of the drift (+0.007 ± 0.010 parts in 10 to the 15th/day) for the reference maser, as determined from 460 days (MJD 48769-49229) of data by G. M. R. Winkler (USNO; private communication). This maser has displayed the same drift for at least the 565 days spanned by our data, but solutions outside of this time span are degraded by variations in TAI.

Some 38% of the available cesiums displayed significant drifts of their own at one time or another, at which time they were excluded from our data. Most such cases were probably due to random walk FM, but it was decided to err on the side of caution, since the objective was to compare different methods using the same data, rather than to determine the absolute accuracy of any particular kind of drift.

The Masers

The drifts of the masers cannot be averaged as were those of the cesiums because they differ from maser to maser, but one can perform the same final check on the systematic errors as was done above on the cesiums. The results did not depend on the type of maser, though the Sigma Tau masers had smaller average drifts (see Table 1):

Method	Mean Sys. Error	s.e.	
#1	+0.0023	±0.0019	parts in 10^{15}/day
#2	-0.0017	0.0034	
#3	-0.0005	0.0017	
#4	-0.08	0.93	
#5	-0.0019	0.0012	

We conclude:

- Because the s.e. of the s.e.s themselves is about ±0.0005 parts in 10 to the 15th/day, the s.e.s of Methods #1, #3, and #5 are statistically identical, so these methods are equally good. Method #2 is slightly worse and Method #4 is rejected.

- The systematic errors agree within their s.e.s (allowing for the s.e.s of the s.e.s), so no difference between them is significant.

All the masers displayed significant changes in drift even in the data selected as apparently free of such changes. Maser NAV4 may have an annual variation (see Figure 1); more data are needed to be certain. The others showed changes in drift with coefficients ranging from -0.0012 ± 0.0001 to +0.0044 ± 0.0007 parts in 10 to the 15th/day^2, but these also change with time, often quite suddenly (see Figures 2 and 3). The figures plot one-day moving averages.

Choice of Method

We favor Method #1 because it involves solution for one less parameter than Method #2, and Method #2 is somewhat inferior for masers. Both, being simultaneous solutions, would probably yield slightly more compatible results for drift and rate than Method #3. Method #5 would not be robust against spontaneous or deterministic rate changes, all such being excluded from our data. It also lacks an rms as a measure of confidence, so practical use would require thorough filtering that subverts any savings of computational effort inherent in its simplicity.

RATE DETERMINATION

Four possible methods of rate determination were selected for testing:

METHOD #1	Solve for rate and drift by linear least squares on frequency (specifically, first differences of phase; same as Drift Method #1)
METHOD #2	Solve for rate and drift by quadratic least squares on phase (same as Drift Method #2)
METHOD #3	Solve for rate by linear least squares on phase, assuming a drift value
METHOD #4	Solve for rate by averaging first differences of phase, assuming a drift value

The drift values assumed were those found by Drift Method #1 above.

The Cesiums

The rates of the cesiums cannot be averaged as were their drifts because they of course differ from cesium to cesium, but one can compute sytematic errors and compare the s.e.s, as was done above for the drifts. We found:

Method	Mean Sys. Err.	s.e.	Mean Rate Error	s.e.	
#1	+0.12	±0.11	±0.0329	±0.0034	parts in 10^{15}
#2	-0.12	0.11	0.00092	0.00012	
#3	-0.11	0.11	0.0220	0.0016	
#4	+0.11	0.11	1.632	0.054	

We conclude:

- No method has a significant systematic error.
- All s.e.s of the systematic errors are identical, so the methods are equally good.

The Masers

Computing systematic errors as above, we found (unlike for the drifts) that the results differed significantly between the SAO and Sigma Tau masers. For the SAO masers:

Method	Mean Sys. Err.	s.e.	Mean Rate Error	s.e.	
#1	+0.07	±0.57	±0.0075	±0.0021	parts in 10^{15}
#2	-0.16	0.55	0.00069	0.00012	
#3	-0.16	0.55	0.0249	0.0045	
#4	+0.25	0.53	0.407	0.029	

For the Sigma Tau masers:

Method	Mean Sys. Err.	s.e.	Mean Rate Error	s.e.	
#1	+0.23	±0.12	±0.0154	±0.0025	parts in 10^{15}
#2	-0.26	0.11	0.00058	0.00012	
#3	-0.27	0.11	0.0108	0.0023	
#4	+0.30	0.11	0.543	0.056	

We conclude:

- No method has a significant systematic error for the SAO masers.

- All methods have significant systematic errors for the Sigma Tau errors. Methods #1 and #4 are statistically identical, as are Methods #2 and #3, but the two pairs differ significantly. This may only be evident because the s.e.s of the Sigma Tau masers are significantly smaller than those of the SAO masers. On the other hand, the sample of four masers may simply be too small to accurately gauge systematic errors. In view of the noises present, results from Methods #1 and #4 would seem to be preferable to those of Methods #2 and #3.

- All s.e.s for the systematic error of each type of maser are statistically identical, so no method can be preferred on the basis of random errors.

We favor Method #1 because it involves solution for one less parameter than Method #2, does not require a separate solution for drift like Methods #3 and #4, and is identical with Drift Method #1 that we preferred above.

Method #1 rates are given in Table 1, referred to TAI by the addition of the rate (+360.90 ± 1.3 parts in 10 to the 15th) of the reference maser, as determined by Winkler (private communication).

SUMMARY

Of the different methods of drift and rate determination studied, for hourly data:

- All are equally good when judged on the basis of their random errors, except averaging second differences, which is by far the worst method of drift determination.

- None has significant systematic errors, except perhaps among the rate determination methods for Sigma Tau masers.

- Solution by linear least squares on frequency is preferred on the basis of parsimony of parameters. Other studies concur that this the optimal method for estimating drift in the presence of white FM noise [1]. Also, compared to the overall second difference, it is a more robust method of determining drift. Accordingly, the former method will be tested for incorporation into the USNO mean timescale algorithm.

- All masers display significant changes in drift.

ACKNOWLEDGMENTS

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REFERENCES

[1] Allan, D. W., "Time and frequency (time-domain) characterization, estimation, and prediction of precision clocks and oscillators," IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, vol. UFFC-34, 1987, pp. 647-654 - NIST Technical Note 1337, pp. TN121-TN128.

[2] Breakiron, L. A., "Timescale algorithms combining cesium clocks and hydrogen masers," Proceedings of the 23rd Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, 3-5 December 1991, Pasadena, California, pp. 297-305.

[3] Weiss, M. A., Allan, D. W., and Howe, D. A., "Confidence on the second difference estimations of frequency drift," Proceedings of the 1992 IEEE Frequency Control Symposium, 27-29 May 1992, Hershey, Pennsylvania, pp. 300-305.

CORRIGENDA TO PREVIOUS PAPER

Because of typesetter errors, the following corrections should be made in [2]:

p. 298, eq. (5), for "z_{t_T}" read "z_{t-T}"

and for "x_{t_T}" read "x_{t-T}"

p. 300, l. 11, for "300 ns" read "300 ps"

FIGURE 1. SIGMA TAU MASER NAV4

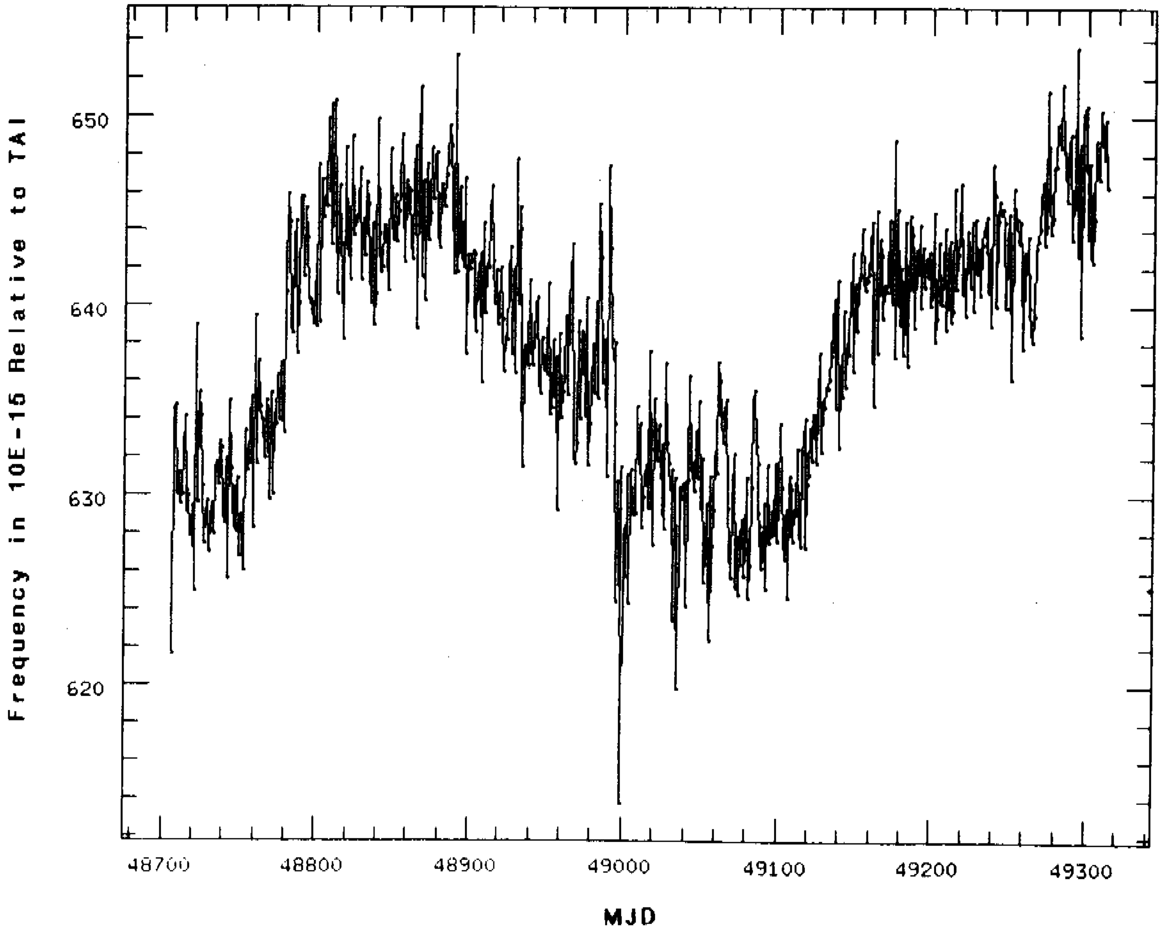

FIGURE 2. OTHER SIGMA TAU MASERS

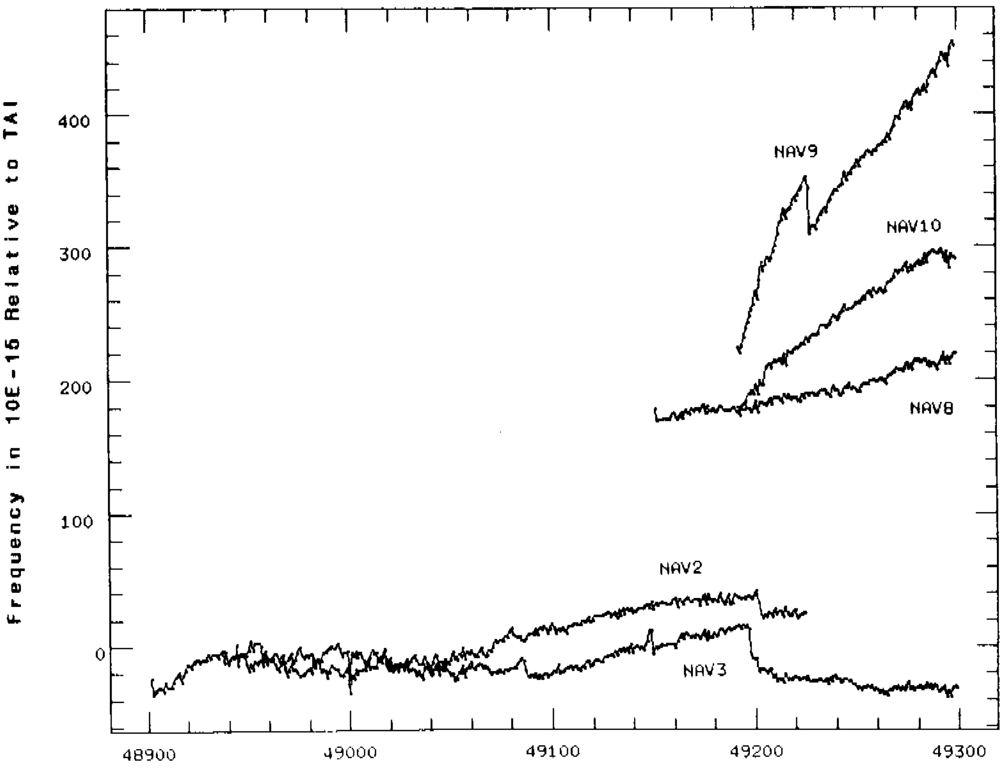

FIGURE 3. SAO MASERS

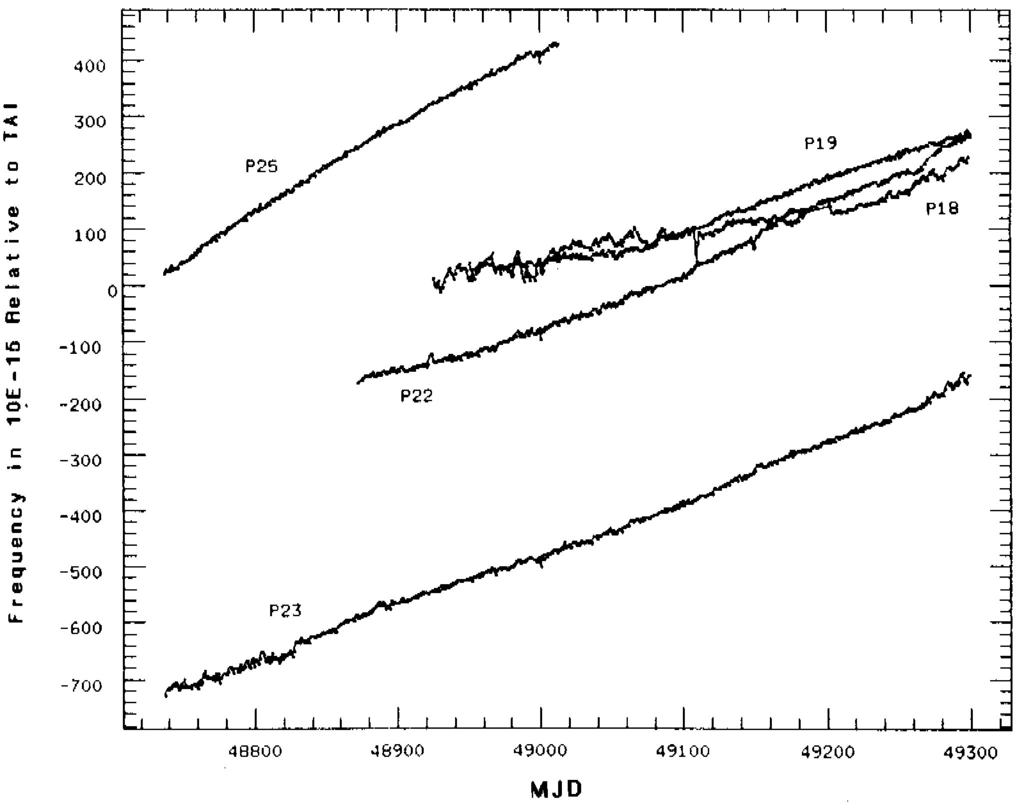

TABLE 1. Data Sources

Clock Type	Serial #	MJD Range	Drift Rel. to TAI (parts in 10^{15} /d)		Rate Rel. to TAI (parts in 10^{15})	
HP5071A Cs	0114	49094-49310	+0.068 ±	0.024	-184.022 ±	0.024
	0142	48922-49110	-0.002	0.029	+11.927	0.029
	0145	48912-49310	-0.098	0.011	+33.304	0.011
	0146	48932-49310	+0.101	0.010	-17.159	0.010
	0148	48912-49310	+0.015	0.009	+184.786	0.009
	0150	49105-49310	-0.003	0.026	-248.073	0.026
	0153	49038-49310	-0.030	0.016	-210.341	0.016
	0156	49100-49310	+0.045	0.027	-68.490	0.027
	0161	49027-49310	+0.046	0.018	-38.154	0.018
	0164	49028-49310	+0.026	0.016	-67.367	0.016
	0165	49041-49310	+0.027	0.017	-227.714	0.018
	0166	49047-49310	+0.049	0.019	+42.830	0.019
	0167	49028-49310	+0.058	0.016	-131.464	0.016
	0169	49042-49238	+0.045	0.031	+89.774	0.031
	0171	49027-49310	+0.073	0.017	-149.196	0.017
	0213	49126-49310	+0.014	0.032	+126.499	0.032
	0217	49139-49310	+0.069	0.035	+84.277	0.035
	0225	49134-49258	-0.080	0.055	-102.865	0.055
	0226	49196-49310	+0.076	0.059	+15.955	0.060
	0231	49134-49310	-0.084	0.037	+305.064	0.037
	0233	49145-49310	+0.094	0.035	+2.703	0.035
	0242	49140-49310	+0.135	0.038	-154.508	0.038
	0249	49160-49278	+0.043	0.065	+71.013	0.065
	0253	49140-49310	+0.012	0.033	+112.216	0.033
	0254	49140-49310	+0.060	0.034	+9.369	0.034
	0255	49189-49310	+0.055	0.059	+91.221	0.059
	0268	49160-49310	+0.009	0.039	-35.203	0.039
	0270	49179-49273	+0.096	0.084	-23.177	0.084
SAO masers	P18	48924-49181	+0.451	0.006	+71.163	0.006
		49202-49310	+1.044	0.016	+175.284	0.016
	P19	48949-49069	+0.289	0.015	+46.291	0.015
		49069-49310	+0.888	0.005	+177.147	0.005
	P22	48872-49270	+1.030	0.003	+6.372	0.003
	P23	48745-49310	+0.985	0.002	-444.601	0.002
	P25	48745-49013	+1.500	0.006	+251.615	0.006
Sigma Tau masers	NAV2	48943-49068	-0.029	0.014	-8.027	0.014
	NAV3	49087-49197	+0.368	0.016	-3.270	0.016
		49201-49310	-0.127	0.015	-28.837	0.016
	NAV4	48890-49106	-0.066	0.010	+634.392	0.010
		49160-49263	+0.035	0.027	+642.043	0.027
	NAV8	49150-49310	+0.341	0.009	+192.309	0.009