Noninertial Coordinate Time: A New Concept Affecting Time Standards, Time Transfers and Clock Synchronization

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Abstract

Relativity compensations must be made in precise and accurate measurements whenever an observer is accelerated. Although many believe the Earth-centered frame is sufficiently inertial, accelerations of the Earth, as evidenced by the tides, prove that it is technically a noninertial system for even an Earth-based observer. Dr. Einstein introduced the concept that time was essentially a fourth component that could be added to any three-dimensional position. Using the constant speed of light, a set of fixed remote clocks in an inertial frame can be synchronized to a fixed master clock transmitting its time in that frame. The time on the remote clock defines the coordinate time at that coordinate position. However, the synchronization procedure for an accelerated frame is affected, because the distance between the master and remote clocks is altered due to the acceleration of the remote clock toward or away from the master clock during the transmission interval.

An exact metric that converts observations from noninertial frames to inertial frames was recently derived. Using this metric with other physical relationships, a new concept of noninertial coordinate time is defined. This noninertial coordinate time includes all relativity compensations. This new definition raises several timekeeping issues, such as proper time standards, time transfer processes, and clock synchronizations, all in the noninertial frame such as Earth.

Background

Relativity compensations must be made in precise and accurate measurements whenever an observer is accelerated. Noninertial reference frames are ones that experience accelerations, which include rotations. A reference frame centered on the Earth would appear to be inertial, but the observation of the tides demonstrates the existence of a force acting on the oceans. This force is the product of mass and acceleration, which proves that the mass of the Earth is being accelerated. The existence of the tides proves that any Earth-centered frame is not sufficiently inertial.

Dr. Albert Einstein accurately assumed that the speed of light (i.e. any electromagnetic radiation) in a vacuum is always the same constant for all inertial frames. He accurately predicted that a moving clock would appear to run slower than an identical, but stationary, clock. Dr. Einstein developed the concept that time was a relative quantity that essentially is a fourth coordinate associated with any three-dimensional position of a chosen reference frame. This resulted in the definition of coordinate time unique to every reference frame.

Conversion of position and time coordinates between inertial frames was accomplished by Dr. Einstein through the Lorentz transformation. The current practice in relativity science is

to use comoving inertial reference frames to match a noninertial reference frame and then apply the Lorentz transformation to convert observations from one inertial frame to another.

Only recently, an *exact* transformation which converts observations from noninertial frames to inertial frames was derived[1]. This was developed by Dr. Robert Nelson, and this transformation will be designated as the Nelson transformation to eliminate confusion. The metric for an accelerated, rotating frame has been derived as:

$$g_{00} = -\left[1 + \frac{\bar{A} \cdot \bar{\rho}}{c^2}\right]^2 + \left[\frac{\bar{\omega} \times \bar{\rho}}{c^2}\right]^2$$

$$g_{0j} = \frac{1}{c}(\bar{\omega} \times \bar{\rho})_j)$$

$$g_{ij} = \delta_{ij}$$

Definition of Noninertial Coordinate Time: A New Concept

The Nelson metric was modified in the g_{oo} term to include gravity effects[2] from the post-Newtonian approximation as follows:

$$g_{00} = -1 + \overset{2}{g}_{00} + \overset{4}{g}_{00} + \ldots = -1 - 2\Phi + \ldots \tag{1}$$

One fundamental property that remains invariant in relativity regardless of the reference frame is "proper time," denoted here as τ. The proper time of an object is defined as the time measured by an ideal clock attached to the moving object[3]. An invariant equation relates coordinate time (t) and coordinate position ($\bar{X}$) with proper time (τ).

$$(c \, d\tau)^2 = (c \, dt)^2 - dx^2 - dy^2 - dz^2 \tag{2}$$

To facilitate the use of the relativity equations, the modified Nelson metric and the proper time used in Equation 2 have been converted from Einstein's repeating Roman index notation to the more familiar vector notation. The modified Nelson metric (Equation 1) was inserted into the invariant equation (Equation 2). Equation 3 was completely derived[4] using Equations 1 and 2.

$$d\tau = \sqrt{\left[1 + \frac{\bar{A} \cdot \bar{\rho}}{c^2}\right]^2 + \frac{2\Phi}{c^2} - \left[\frac{\bar{v} - \bar{V}}{c}\right]^2} \; dt \tag{3}$$

where

Φ = the sum total of each gravitational potential at the remote clock's location as contributed by each measurable mass source. For locations near or on the Earth's surface, $\Phi = g(\phi)h$ as defined below.

τ = the proper time of the noninertial Earth at the geoid.

$\bar{A}$ = the acceleration vector of the remote clock in the chosen inertial reference frame.

$\bar{V}$ = the velocity vector of the remote clock in the chosen inertial reference frame.

$\bar{v}$ = the velocity vector of the master clock in the chosen inertial reference frame.

$$\phi \;=\; \text{the remote clock's geodetic latitude relative to Earth's geoid if that clock is near or on the Earth's surface.}$$

$$g(\phi) \;=\; \text{the perpendicular gravity constant at the remote clock at the Earth's surface, which is a function of the geodetic latitude } \phi \text{ of the remote clock's location.}$$

$$h \;=\; \text{the altitude of the remote clock above the Earth's geoid for applications when that clock is on or near the Earth's surface.}$$

$$c \;=\; \text{the speed of light (i.e. any electromagnetic radiation) in a vacuum of an inertial frame.}$$

$$\rho \;=\; \text{the range vector from the remote clock to the master clock in the inertial reference frame.}$$

$$t \;=\; \text{the noninertial coordinate time of the remote clock at reception.}$$

Noninertial coordinate time is therefore defined as a function of proper time of the remote clock in the noninertial frame. The square root term in Equation 3 includes the relativity contributions for nongravitational accelerations $\frac{\bar{A}\cdot\bar{\rho}}{c^2}$, gravity $\frac{2\Phi}{c^2}$, and velocity $\left[\frac{\bar{v}-V}{c}\right]$. This square root term is the time dilation factor that will always exceed the value of one for a noninertial frame. So, division of this factor into proper time yields the noninertial coordinate time interval, which is always smaller than the proper time interval.

Noninertial coordinate time is the time given by a fixed remote clock in a noninertial reference frame synchronized to a fixed master clock in that frame, which includes all relativity compensations. Even the theorization of all the relativity compensations in a noninertial frame was not possible before the advent of the Nelson metric, and only assumptions and approximations for these relativity compensations have been previously available.

Conclusions

Based on the new definition of noninertial coordinate time, a reexamination of several timekeeping issues is warranted. A few of these issues include the proper time standard, the time transfer process and the clock synchronization procedure, all in a noninertial frame (e.g. the Earth).

Inertial coordinate time standards (e.g. TAI), which are based on time calibrations in an inertial frame, beat faster than a moving proper time standard, which undergoes time dilations in its noninertial reference frame. Theoretically, the leap second between TAI and UT1 standards may be the result of this difference. Work is ongoing to quantify what portion of the leap second is due to differences between inertial and noninertial coordinate times. It is recommended that a study be initiated to determine whether the current atomic time standard, which is correctly defined for an inertial reference frame, is appropriate in Earth's noninertial frame.

Time transfers are currently done between two remote precise time stations that simultaneously observe a satellite time transmission. Time transfers determine the time differences between stations A and B without having to transport physical clocks for comparison. Global Positioning System (GPS) time transfers use a GPS time receiver to get a coordinate time at reception. The time transfer equation is $[A\text{-}t_A] - [B\text{-}t_B] = A - B$ when $t_A = t_B$ at equivalent time marks. The local proper times of the atomic clocks are A and B, respectively, and t_A and t_B are the noninertial coordinate reception times from GPS receivers.

Time transfers are also affected by Earth's rotation. The Earth's geoid is a theoretical construct where all ideal clocks will beat at the same rate. However, even on the geoid, the nongravitational

relativity effects have first and second order dependence on the velocity of the local clocks in the noninertial local frame. The time transfer relationship between a satellite clock (e.g. GPS) and the fixed local clocks not on the geoid of the rotating Earth, has been derived. The Earth's gravity, the rotational acceleration and tangential velocity were inserted into Equation 3 to yield:

$$t_{GPS} = \left[1 + \frac{g(\phi)h}{c^2} + \frac{(\bar{\omega} \times \bar{R})^2 - (\bar{\omega} \times \bar{R}_{\text{geoid}})^2}{2c^2} \right] t + \frac{\bar{\omega} \times \bar{R} \cdot \bar{\rho}}{c^2} \qquad (4)$$

where $\bar{R}_{\text{geoid}}$ is the position vector where receiver would be on the geoid if the receiver had no altitude.

Equation 4 is used to compute the noninertial coordinate times t for the two remote stations for t_A and t_B. The transmission time from the GPS satellite is t_{GPS}, and t is the noninertial coordinate time at reception for the local clock. When the noninertial coordinate time t_A equals t_B, the time transfer algorithm correctly gives the difference in proper times of A and B of the two clocks.

The last term in Equation 4 is equivalent to the Sagnac effect, which corrects for the first order change in the geometric range as the clock moves toward or away from the satellite during the time interval of transmission. Two new relativity compensations in Equation 4, which were not previously included in GPS time transfers, affect the noninertial coordinate time t. The gravitational effect, $\frac{g(\phi)h}{c^2}$, is due to the additional change in gravity due to the altitude h as compared to the expected gravity in GPS at the Earth's geoid. The nongravitational effect, $\frac{1}{2} \left[\frac{\bar{\omega} \times \bar{R}}{c} \right]^2 - \frac{1}{2} \left[\frac{\bar{\omega} \times \bar{R}_{\text{geoid}}}{c} \right]^2$, is difference in the expected tangential velocity due to Earth's rotation as compared to the expected tangential velocity in GPS at the Earth's geoid..

It is assumed that the current GPS receivers correct for the geometric range, which is the last term in Equation 4. The additional gravitational effect for an atomic clock 2000 meters above the Earth's geoid, would result in a drift rate of 2.18 x 10^{-13} s/s or 18.8 ns/day. The nongravitational drift rate for an atomic clock affected by Earth's rotation when elevated 2000 meters above the geoid at the equator would be 7.55 x 10^{-16} s/s or 0.06 ns/day. Such offsets in frequency contributions may currently be attributed to mechanical errors in the clocks rather than these uncompensated relativity effects.

Clock synchronization is simple to perform in an inertial frame, and all stationary clocks will beat the same for both proper and coordinate time. Clock synchronization in an inertial frame is simply accomplished by :

$$t_{\text{remote}} = t_{\text{transmitted master time}} + \frac{\text{distance between remote and master}}{\text{speed of light}}$$

However, with a noninertial frame, clock synchronization between a master clock and a remote clock at rest must be accomplished differently. The distance that the master clock transmission must travel to the remote clock varies, because the remote clock can be accelerated toward or away from the master clock during the transmission interval. In general, the noninertial master clock beats will fluctuate differently from the noninertial remote clock rate, compared to the steady beat of any synchronized inertial clock. To perform clock synchronizations in a noninertial frame, Equation 3 must be used to convert proper time of a remote noninertial clock to its noninertial coordinate time. Only then will the remote noninertial clocks be synchronized to the noninertial master clock in that frame.

In summary, noninertial coordinate time includes all the relativity compensations required with a noninertial reference frame. Since the Earth-centered frame is not sufficiently inertial, the potential applications for noninertial coordinate time are far-ranging. Timekeepers concerned with optimium accuracies would achieve substantial improvements by using this concept.

References

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[3] Arthur I. Miller, **Albert Einstein's Special Theory of Relativity**, Addison-Wesley, 1981, p. 271.

[4] Steven D. Deines, *"Missing Relativity Terms in GPS,"* Navigation, Spring 1992.