# PHYSICAL ORIGIN OF THE FREQUENCY SHIFTS IN CESIUM BEAM FREQUENCY STANDARDS: RELATED ENVIRONMENTAL SENSITIVITY

C. AUDOIN, N. DIMARCQ, V. GIORDANO and J. VIENNET
Laboratoire de l'Horloge Atomique
Unité Propre de Recherche du CNRS
associée à l'Université Paris–Sud
Bât. 221 – Université Paris–Sud
91405 ORSAY Cedex – France

*Abstract*

***When observed in a cesium beam frequency standard, the hyperfine transition frequency of the atoms differs slightly from the invariant transition frequency of the unperturbed atoms at rest. The various physical and technical origins of the frequency offsets are stated. They relate to fundamental physical effects, to the method of probing the atomic resonance and to the frequency control of the slaved oscillator. The variation of the frequency offsets under a change of the value of the internal operating characteristics is considered. The sensitivity to a change of the magnetic induction, the microwave power and the temperature is given. A comparison is made of the sensitivity of cesium beam frequency standards of the commercially available type, making use of magnetic state selection, and of devices under study, in which the state preparation and detection is accomplished optically. The pathways between the external stimuli and the physical origin of the frequency offsets are specified.***

## 1. INTRODUCTION

According to a basic postulate of physics, the properties of atoms do not change with space and time (except for known relativistic effects). Therefore, the frequency delivered by atomic frequency standards should not vary with time and should not depend on perturbations applied such as constraints, change of temperature and humidity, etc.

However, although very slightly, this frequency varies during the device lifetime and depends on external factors. To understand this undesirable fact, one must not forget that the postulate mentioned implies that the atoms are assumed at rest, isolated and in free space. This is obviously not the case in actual devices: the atoms are moving at thermal velocities; although in a beam they are not isolated and, moreover, the two energy levels of interest pertain to manifolds; the atoms are subjected to fields. Furthermore, accessories are necessary to observe the microscopic properties of the cesium atoms. They are: an oven; state selectors; a microwave cavity; a set–up to generate the C–field; a detector; and electronic systems to probe the transition, to process the beam tube response and to frequency lock a quartz crystal oscillator.

These departures from the ideal conditions and some of these accessories can be sources of frequency offsets, the magnitudes of which depends on the technical choices made.

These frequency offsets couple the observed resonance frequency to the macroscopic world. Since macroscopic objects are usually sensitive to external factors such as temperature, the frequency offsets depend on environmental conditions and, consequently, the frequency of the standard does also. Frequency changes can thus be induced by a variation of the temperature, the humidity, the constraints, the fields, etc... applied to the device.

Although the frequency offsets are rather small, they are numerous in the cesium beam frequency standard. However we shall ignore the gravitational frequency offset, whose value is easily predictable and the frequency offsets whose change is negligible in the usual range of variation of the external factors.

The frequency offsets can be sorted out, more or less arbitrarily, into three categories depending on the principal origin of the offset. They are related to fundamental physical effects, to the resonance interrogation method and to possible imperfections in the electronic system. We shall give their order of magnitude and the possible cause of their change.

We shall consider commercially available cesium beam frequency standards, without reference to a specific model, but with plausible values of the characteristic parameters. In these devices, the static and the microwave magnetic fields are applied perpendicularly to the atomic beam axis. The length of each interaction region, $l$, is equal to about 1 cm. The separation, L, between the two oscillatory fields is of the order of 15 cm. State selection is presently accomplished by deflecting the atom trajectories in magnets. The velocity distribution function of the atoms detected depends on the design and it is not an universal function. We will therefore introduce a crude, but representative distribution of the interaction times $\tau$ in each oscillatory field region. It has the shape of a triangle shown in Figure 1a. The maximum of the distribution arises at $\tau_1 = 71$ $\mu$s, corresponding to a velocity $v_1$ of 140 $\mathrm{ms}^{-1}$[1]. For the purpose of comparison, we shall also consider the distribution of interaction times which is encountered in the simplest configuration of an optically pumped cesium beam frequency standard, using a single diode laser for the state preparation and the state detection. In that case, the velocity distribution is that of an effusive beam[2]. The related distribution of the interaction time is shown in Figure 1b. It peaks at $\tau_0 = 46.5$ $\mu$s, assuming an oven temperature of 100°C. $\tau_0 = l/\alpha$ is the interaction time for atoms having the most probable velocity,$\alpha$, in the oven.

We shall give the order of magnitude of the frequency offsets and the possible cause of their change. For that purpose, we shall rely mainly on results published in references 3 through 8, where credit is given to prior work. Frequency offsets whose value depend on the velocity distribution have been computed from equations derived in [8], assuming slow square wave frequency modulation, for simplicity. However, the main conclusions of this paper are valid for other types of modulation.

In the following, the frequency offsets are expressed in Hertz, the static magnetic induction is expressed in gauss ($1\ G = 10^{-4}\ T$) and $\nu_0$ is the frequency of the unperturbed hyperfine transition of the cesium atom.

# 2. FREQUENCY OFFSETS DUE TO FUNDAMENTAL PHYSICAL EFFECTS

## 2.1. Second order Zeeman frequency offset

In the C–field region, a static magnetic induction is applied parallel to the main component of the microwave magnetic induction sustained in the interaction regions. It raises the degeneracy of the cesium atom hyperfine levels in the ground state and it enables $\Delta m_F = 0$ transitions to be induced. Its values is usually set around 60 mG, to separate sufficiently the second order field dependent $(F = 3,\ m_F = 0) \leftrightarrow (F = 4,\ m_F = 0)$ clock transition from the six other neighbouring, but first order field dependent, $\Delta m_F = 0$ transitions (see Figure 2).

### a) Magnetic induction assumed uniform

The second order Zeeman offset of the frequency of the clock transition is given by:

$$\Delta\nu_B = 427.45\ B_0^2. \tag{1}$$

Assuming $B_0 = 6 \times 10^{-2}\ G$, the frequency offset amounts to about 1.54 Hz, or $1.67 \times 10^{-10}$ in relative value. This is the largest frequency offset occuring in cesium beam frequency standards.

A change, $dB_0$, of the magnetic induction gives a change $d(\Delta\nu_B)$ of the frequency offset. We have:

$$d(\Delta\nu_B) = 855\ B_0\ dB_0. \tag{2}$$

For $B_0 = 6 \times 10^{-2}\ G$, $dB_0/B_0 = 3 \times 10^{-5}$ ($dB_0 = 1.8\ \mu G$), we have $d(\Delta\nu_B)/\Delta\nu_B = 10^{-14}$. This means that the current source needed to generate the static magnetic induction must be of a sufficient quality, with a small sensitivity to temperature changes.

A transverse shielding factor of about $5 \times 10^4$ gives a sensitivity, equal to $10^{-13}$ per Gauss, to a change of the magnetic induction applied externally and orthogonally to the beam tube. However, one should be cautious in assuming that the frequency shift is proportional to the variation of the external induction since, at least in some models, non–linearities have been observed, as well as a lack of reproducibility of the effect of the variation.

A sensitivity to alternating magnetic fields has been observed. The DC frequency offset produced might be the result of some rectification effect related to non linearities occuring either in the shielding material or in electronic components used in the associated electronic sub–assemblies.

### b) Effect of magnetic field inhomogeneities

Actually, the static magnetic induction is not uniform in the C–field region. The main reasons of this imperfection are the following. The permeability of the magnetic material is finite. There are holes in the magnetic shield (the largest one for the input waveguide). The static field created may simply be not uniform, at least in some tube designs.

The related frequency offset can be separated into two parts. The first one is associated to the field inhomogeneity in the space between the interaction regions. Let $B_0+\Delta B(z)$ be the magnetic induction along the beam path and $\Delta B(z)$ its fluctuation with quadratic mean value $\overline{\Delta B(z)^2}$. The frequency offset is equal to:

$$\Delta\nu'_B = 427\ \overline{\Delta B(z)^2} \tag{3}$$

Assuming a standard deviation of $2\times10^{-3}/G$, we have $\Delta\nu'_B/\nu_0 = 1.9\times10^{-13}$. The second source of frequency offset is the difference between the magnetic induction in the interval between the two interaction regions and the induction in these regions. We have:

$$\Delta\nu''_B = 214\frac{l}{L}(B_1^2 - B_2^2 - 2B_0^2)\ , \tag{4}$$

where $B_1$, $B_2$ and $B_0$ are the inductions in the first interaction region, in the second one and in the interval between them, respectively. The values of $\Delta\nu'\times B$ depends, but only slightly, on the microwave power and on the frequency modulation parameters. Assuming $B_1 = B_2 = 62\ mG$, $B_0 = 60\ mG$ and $l/L = 1/15$, we have $\Delta\nu''/\nu_0 = 7.6\times10^{-13}$, which is a significant frequency offset.

The frequency offset associated with the magnetic field inhomogeneities can change if the field distribution is modified by an externally applied magnetic field.

### c) Effect of a magnetic field applied parallel to the beam tube

An external magnetic field applied parallel to the beam axis should not give a frequency shift. However, it has been observed that such an effect exists and that it is even larger than when the field is directed transverse to the tube[9,10].

A magnetic induction is added parallel to the beam axis in the C–field region (the shielding factor is smaller in the elongated direction). It is thus perpendicular to the main component of the microwave induction and it can induce neighbouring $\Delta m_F = \pm1$ transitions, which may give rise to a frequency shift (see Section 2.2b). Furthermore, a distortion of the magnetic lines of force by the magnetic shields may create a small transverse component, $dB_0$, changing the value of the frequency offset $\Delta\nu_B$. Moreover, such a distortion may change the magnetic field inhomogeneities.

## 2.2. Neighbouring transitions

### a) $\Delta m_F = 0$ transitions

Six intense, field dependent, $\Delta F = \pm1$, $\Delta m_F = 0$ transitions are also present in the microwave spectrum of the cesium atom, as shown in Figure 2. The width of their pedestal is equal to about 15 to 20 kHz and their separation is approximately equal to 42 kHz when $B_0$ is close to 60 mG. The aisles of the pedestal of the $(F=4,\ m_F=1)\leftrightarrow(F=3,\ m_F=1)$ and of the $(F=4,\ m_F=-1)\leftrightarrow(F=3,\ m_F=-1)$ transitions overlap under the line of the $(F=3,\ m_F=0)\leftrightarrow(F=4,\ m_F=0)$ transition. In tubes where the state selection is accomplished magnetically, the field dependent lines, symmetrically placed around the central one, have significantly different amplitudes, as shown in Figure 2a. It follows that the shape of the central line is distorted by an added slanted base line, and that a frequency offset arises. The properties of the latter depends closely on that of the aisles of the Rabi pedestal. Their amplitude is proportional to the microwave power and it is an oscillating function of their distance to the line center. Consequently, the frequency offset depends on the microwave power and on the value of the static magnetic induction. Furthermore, it depends on the population and of the velocity distribution of the atoms involved in the neighbouring transitions. The offset is also a function of the modulation parameters (*cf* Appendix). The Rabi pulling effect has been studied in detail by de Marchi[3,5]. He has found a frequency offset varying according to the previous statements in Cs beam frequency standards produced by a given manufacturer, as shown in Figure 3. Results obtained with standards of a different supply, using tubes of a different design and in which the frequency modulation scheme is different, were not has clear as that shown in Figure 3[11].

The relative frequency offset may amount to several $10^{12}$ and its change may be several $10^{-13}$ per dB of microwave power variation. When the influence of the neighbouring Rabi pedestals is clearly identified, there are magnetic field settings for which the frequency offset goes to zero as well as the sensitivity to a microwave power variation. In that case, it has been experimentally verified that Cs beam tubes tuned at one of these points show an improved long term stability[5,7]. However, most of the manufactured tubes are still operating at a different point. Then, the Rabi pulling effect contributes to their sensitivity to a change of their microwave power. A frequency shift may also occur if the beam composition varies (change of the beam optics and of the beam deflexion angle, Majorana transitions) and thus the asymmetry of the microwave spectrum. Similarly, a change of the velocity distribution of atoms in the sublevels $m_F = \pm 1$ of the ground states $F = 3$ and $F = 4$ can produce a frequency shift.

As shown in Figure 3, the Rabi pulling effect introduces a sensitivity to a change of the value of the C–field. However, this sensitivity is smaller than that related to the second order Zeeman effect and it can be neglected.

The Rabi pulling effect is specific to beam tubes with magnetic state selection. The related frequency offset and its sensitivity to the microwave power and to other factors should be absent in cesium beam tubes optically pumped with a linearly polarized light[12]. In that case, the microwave spectrum of the cesium atom is highly symmetrical around the central $(F = 3,\ m_F = 0) \leftrightarrow (F = 4,\ m_F = 0)$ line, as shown in Figure 2b. Furthermore, this property gives the opportunity to decrease the value of the C–field and thus the sensitivity to magnetic field changes.

### b) $\Delta m_F = \pm 1$ transitions

A spurious component of the static magnetic field parallel to the beam tube axis, and therefore perpendicular to the main component of the microwave magnetic field can induce $\Delta m_F = \pm 1$ transitions. This component can be produced in the tube itself if the C–field region is poorly designed, or it can be the result of an externally applied magnetic field. The shape of the weak resonance features is similar to that of the $\Delta m_F = 0$ main lines. Such small lines can be seen in Figure 2a, in the dips between the $\Delta m_F = 0$ resonance patterns. Although weaker, they are closer than the neighbouring $\Delta m_F = 0$ lines. They may produce a frequency offset of the $\Delta m_F = 0$ clock transition when the populations of the $m_F = \pm 1$ sublevels are not identical. It depends on the microwave power level.

$\Delta m_F = \pm 1$ resonance features can take a different shape, as shown in Figure 2b at mid-distance between the $\Delta m_F = 0$ lines. This record has been obtained with a tube in which the apertures in the Ramsey cavity are larger than in conventional ones[12]. Moreover, the value of the C–field, and thus the separation between the lines, has been increased for the purpose of illustration. Although weak, they are very likely present in all beam tubes, but they may be masked by the aisles of the intense $\Delta m_F = 0$ lines at the usual values of the C–field. Furthermore, they may not emerge from the noise.

The $\Delta m_F = \pm 1$ transitions shown in Figure 2b are likely due to the existence of components of the microwave magnetic field which are perpendicular to the static field, but have opposite directions at points symmetrically disposed around the waveguide axis[13]. In the E–plane microwave cavities considered here, these components are likely related to the distortion of the microwave magnetic lines of force, which is created by the holes.

Like the $\Delta m_F = 0$ ones, these transitions can have different intensities in cesium tubes with magnet state selectors. Thus the closest to the central line may induce a frequency offset. The effect of the $\Delta m_F = \pm 1$ transitions is being studied in detail[14].

## 2.3. Second order Doppler effect

The second order Doppler frequency offset is directly related to the time dilatation effect of the special theory of relativity. In the presence of a velocity distribution, the value of the relative frequency offset is given by:

$$\frac{\Delta\nu_D}{\nu_0} = -\frac{v_i^2}{2c^2}\Delta_D \ , \qquad (5)$$

where $v_i$ is equal to $v_1$ or $\alpha$ for cesium beam tubes with magnetic state selection or optical pumping, respectively. $\Delta_D$ depends on the parameters mentioned in the Appendix. Assuming slow square wave modulation, Figure 4 shows its variation versus the amplitude of the microwave induction for different values of the depth of the frequency modulation.

For the values of the operating parameters given in the Appendix, the relative frequency offset is equal to $-1.1 \times 10^{-13}$ and to $-3.8 \times 10^{-13}$ for the tubes with magnet state selection and optical pumping, respectively. A 1 dB change of the microwave power yields a small variation of the frequency offset, equal to $0.5 \times 10^{-14}$ and $3.2 \times 10^{-14}$, respectively.

## 2.4 Spin exchange

Collisions between alkali atoms cause a relaxation of the population of the hyperfine energy levels and, also, a shift of the hyperfine transition frequency.

Collisions are present in a thermal beam. Their rate is approximately 1/3 that occuring in a vapor showing the same atom density. Furthermore, a background pressure of cesium vapor exists in a beam tube, mainly at the end of its life, when the getters get saturated.

However, the value of the spin exchange frequency shift cros–section of the cesium atom is not known at present. Work in this respect is desirable. It is thus not possible to give a quantitative estimate of the related frequency offset. However, it would be of interest to know whether spin–exchange can cause an ageing effect through a change of the background vapor pressure, or not.

### 2.5. Light shift

The frequency offset called the light shift is of course specific to optically pumped cesium beam tubes. It is due to a displacement of the hyperfine levels in the ground state, which is induced by transitions between the ground state and an excited state.

In a cesium beam tube, the photons emitted by the fluorescing atoms, and which propagate along the beam path, reach the microwave interaction region and perturb the transition frequency. Photons scattered from the laser pumping light can be efficiently eliminated by proper design of the optical part of the device.

An estimate of the magnitude of the light shift has been calculated by several authors[15→17]. For a small tube of the commercial type, it is predicted to be smaller than $10^{-13}$, in relative value, using the $D_2$ line at 852 nm for the atom preparation and detection[17], and one order of magnitude smaller with the $D_1$ line at 894 nm[16]. No experimental result is available yet.

This frequency offset depends on the number of fluorescing atoms and thus on the atomic beam intensity. The latter can easily be maintained sufficiently stable. Moreover, the number of fluorescing atoms is not much sensitive to the intensity of the laser light beam when the optical pumping is almost complete in the two optical interaction regions. Thus, the stability of this intensity should not be critical.

## 3. FREQUENCY OFFSETS DUE TO THE INTERROGATION METHOD

A microwave signal is necessary to probe the atomic resonance, since the Cs beam frequency standard is a passive device. Frequency offsets are related to the implementation of a microwave cavity, with its two interaction regions, and to the presence of spurious spectral components in the interrogation signal.

### 3.1. Mean phase shift between the two oscillatory fields

A small amplitude travelling wave is superimposed to the cavity standing wave. It carries the energy lost in the waveguide walls and in the two terminations. This travelling wave couples any dissymmetry in the electrical properties of the two arms (unequal length, unequal losses, unequal reflections, asymmetry of the feeding junction) into a small phase difference between the two oscillatory fields. The resonance line is then distorted by an additional component which is an odd function of $(\nu - \nu_0)$. A frequency offset follows which can be thought as a residual first order Doppler effect.

Assuming a phase difference, $\phi$, between the antinodes of the magnetic field in the two interaction regions, the frequency offset is given by:

$$\Delta\nu_\phi = -\frac{\phi}{2\pi T_i}\Delta_\phi \tag{6}$$

where $T_i$ is a characteristic value of the time of flight of atoms between the two interaction regions.

We have chosen $T_i = T_1 = L/v_1$ for the tube with magnetic state selection and $T_1 = T_0 = L/\alpha$ for the tube with optical pumping. $\Delta_\phi$ depends on the parameters listed in the Appendix.

In a short Ramsey cavity ($L = 15$ cm), a difference between the electrical length of the two arms equal to approximately $3 \times 10^{-5}$ m gives $\phi = 10\ \mu$rad. With the values of the operating parameters given in the Appendix, the relative frequency offset amounts to $1.5 \times 10^{-13}$ and $2.9 \times 10^{-13}$ for the tubes with magnetic state selection and optical pumping, respectively. A 1 dB change of the microwave power yields a relative frequency variation of $0.4 \times 10^{-14}$ and $1.3 \times 10^{-14}$, respectively.

A differential dilatation of the length of the cavity arms, corresponding to a temperature unbalance of 1 K gives a change of the phase difference of 0.6 $\mu$rad and thus a negligible frequency offset of the order of $1 \times 10^{-14}$.

## 3.2. Distributed phase shift

In each interaction region, there exists a space dependent phase shift of the microwave magnetic field. It is due to the travelling wave component transporting the energy lost in the cavity walls. The distributed phase shift is present even if the phase difference between the centers of the interaction regions is equal to zero. It has a component in the direction transverse to the beam. A longitudinal component likely exists also. It is related to the presence of the holes which enable the atoms to cross the oscillatory field region.

Actually, the order of magnitude of the frequency offset associated with the transverse phase shift is the same as that due to the phase difference between the two cavity ends[18]. The longitudinal phase shift gives a frequency offset only if the two apertures and the two cut–off sections attached to the main waveguide show an asymmetry[8].

### 3.3. Cavity pulling

If the cavity is not exactly tuned at the atomic transition frequency, then the amplitude of the microwave field does not vary symmetrically when the atomic resonance is scanned. A frequency offset $\Delta\nu_r$ of the resonance frequency follows. It is given by:

$$\frac{\Delta\nu_r}{\nu_0} = \frac{T_c^2}{T_i^2}\Delta_c\frac{\Delta\nu_c}{\nu_0}\ , \tag{7}$$

where $\Delta\nu_c$ is the cavity mistuning. $T_c$ is the cavity response time, equal to $Q_c/\pi\nu_0$ being the cavity quality factor. $T_i$ has the same meaning as in Equation 6. $\Delta_c$ depends on the operating conditions, as described in the Appendix. The quantity $(T_c/T_i)^2$ is of the order of magnitude of $(Q_c/Q_l)^2$, where $Q_l$ is the line quality factor. For slow square wave modulation, figure 5 shows the variation of $\Delta_c$ versus the amplitude of the microwave field, for different values of the depth of the frequency modulation. Assuming the operating conditions introduced previously, $Q_c = 5000$, $L = 15$ cm and a cavity mistuning of 0.1 MHz, the relative frequency offsets amounts to $1.1 \times 10^{-13}$ for the classical tube and to $1.4 \times 10^{-12}$ for the tube with optical pumping. The variation of the cavity pulling frequency offset, for a 1 dB change of the microwave power, is equal to $2.3 \times 10^{-13}$ and $3.6 \times 10^{-13}$ for the classical tube and the optically pumped tube, respectively.

Assuming $b$ constant, the frequency shift related to a change $\Delta T$ of the cavity temperature is given by:

$$\frac{\Delta\nu_r}{\nu_0} = -\Delta_c \frac{T_c}{T_i} \alpha \Delta T \tag{8}$$

where $\alpha$ is the linear coefficient of expansion of the cavity material. With $\alpha = 1.6 \times 10^{-5}$ and $\Delta T = 1\text{K}$, we have: $\Delta\nu_r/\nu_0 = 1.6 \times 10^{-13}$ and $2.0 \times 10^{-12}$ for the tube with magnetic state selection and optical pumping, respectively. Moreover, the cavity detuning changes the microwave power applied to the atoms, which must be taken into account[6]. Furthermore, the temperature change will vary the microwave power.

The cavity pulling frequency offset, and its variations, are larger for the optically pumped tube than for the traditional one. However, it can be reduced by decreasing the value of the cavity quality factor, since the frequency offset, and its changes, varies as $Q_c^2$. Similarly, it would be appropriate to increase the value of $b\tau_0$ towards the point where $\Delta_c$ goes to zero. It can be shown that, at that point, the DC content of the beam tube response shows a maximum, in the case of slow square wave modulation. Another, but more involved possibility consists in using two or three lasers instead of one. Implementation of detection *via* a cycling transition, and/or of pumping with two small power lasers, favors slow atoms[19,20]. This increases the mean time of flight, with a subsequent decrease of the related frequency shifts[21].

### 3.4. Majorana transitions

If the magnetic field varies along their path, the travelling atoms are subjected to a time dependent perturbation. It may have spectral components causing transitions between the $m_F$ sublevels of the $F = 3$ and $F = 4$ manifolds. They are called Majorana transitions. They are usually avoided by trimming properly the magnetic field along the atom path.

Should they occur, the effect of these transitions is not completely understood yet[22]. However, they may give indirect frequency shifts, as briefly explained now. In practical magnetic state selectors, the deflection of a trajectory depends on the atom $m_F$ value, besides the velocity. It follows that the occurence of Majorana transitions changes the trajectory of the atoms which are able to hit the hot wire detector and, therefore, their velocity distribution. The effect of these transitions is thus i) to modify the signal to noise ratio of the resonance detection, ii) to modify the amplitude of the neighbouring transitions and the related frequency offset, iii) to change the frequency offsets depending on the trajectory (such as that related to the transverse phase shift) and iv) to vary the frequency offsets depending on the velocity distribution.

In optically pumped tubes, Majorana transitions can be more easily avoided. Since, moreover, the atom deflection by light is very small, their effect should be negligible.

### 3.5. Spectral purity of the interrogation signal

The signal at 9.192 631... GHz, which is necessary to observe the resonance of the Cs passive frequency standard, is generated by the methods of frequency synthesis, starting from the output of a quartz crystal oscillator. Therefore, any spurious spectral component in the signal of this oscillator is enhanced by the process of frequency multiplication. Furthermore, frequency mixing is necessary to create the frequency required and, usually, the final signal contains unwanted spectral components.

External factors can also be the source of spectral impurities. Sidebands at 50 or 60 Hz from the carrier come from the power supply. Other spectral components can be coupled to the interrogation signal in the presence of external signal generators. Vibrations perturb the quartz crystal resonator and are another source of spurious sidebands.

The additional spectral components give rise to virtual transitions which shift the energy levels of the $F = 3$, $m_F = 0$ and $F = 4$, $m_F = 0$ states. The complete analytical expression of this frequency offset is complicated in the case of a cesium beam tube[23]. We shall only consider here the effect of an additional spectral component whose frequency $\nu'$ is such that we have $|\nu' - \nu_0| \lesssim 20$ kHz. This means that this component is present outside the Rabi pedestal bandwidth. The frequency offset is then given by:

$$\Delta\nu_S^{(f)} = \frac{l b_1^2}{4\pi^2 L(\nu' - \nu_0)} \Delta_S^{(f)} \tag{9}$$

where $b_1$ is a measure of the amplitude of the spurious microwave induction. Its definition is similar to that of $b$ (see Appendix). The quantity $\Delta_S^{(f)}$ is approximately equal to 0.65 for the operating conditions considered here and for both types of tube. Thus the frequency offset is proportional to the power of the added signal and inversely proportional to its separation from the carrier.

In many cesium beam frequency standards, a residual sideband is present at 12.6 MHz from the carrier. Taking into account the filtering effect of the cavity, and assuming $Q_c = 2000$, its power is approximately 20 dB below that of the carrier. We thus have $b_1 \cong 2 \times 10^3$ rad $s^{-1}$. With $l/L = 1/15$, the relative frequency offset is small, equal to $0.4 \times 10^{-13}$. It changes if the power of the side-band varies. It also depends on the power of the carrier in as much as the frequency mixing process binds the power of the side-band to that of the carrier. It is worth mentioning that the effect of a spurious side-band may be large for $|\nu' - \nu_0|$ comprised between approximately the half-width at half-maximum (HWHM) of the central fringe of the Ramsey pattern and the HWHM of the Rabi pedestal.

# 4. FREQUENCY OFFSETS DUE TO THE ELECTRONIC SYSTEM

## 4.1. Modulation and demodulation related frequency offsets

It can be shown that a false error signal is produced and therefore a frequency offset of the slaved quartz crystal oscillator results from the presence of the following imperfections[8]: i) the spectrum of the modulation waveform is distorted and comprises spectral components at frequencies $2p\nu_M$, $p$ being an integer and $\nu_M$ the frequency of the modulation, ii) the amplitude of the microwave field applied to the atoms is not a constant and has spectral components at frequencies $(2p+1)\nu_M$ and iii) the spectrum of the demodulation waveform contains even harmonics of the modulation frequency $\nu_M$ whereas the even harmonics of the beam tube response are not sufficiently rejected.

Results have been given in the case of a sinusoidal modulation waveform[24]. For instance, a second harmonic distortion ratio of $10^{-5}$ gives a relative frequency offset of about $10^{-13}$. We shall consider a slow square wave frequency modulation to exemplify the effect of a modulation of the amplitude of the microwave field. If $b + \Delta b$ and $b - \Delta b$ are the values of that amplitude during the first and the second half of the modulation period, respectively, then the frequency offset is given by:

$$\Delta\nu_m = \frac{1}{T_i}\frac{\Delta b}{b}\Delta_m \tag{10}$$

Assuming $\Delta b/b = 10^{-6}$, which is a severe requirement, and the value of the operating parameters given in the Appendix, the relative frequency offset is equal to $0.3 \times 10^{-14}$ and $2.1 \times 10^{-13}$ for the traditional tube and the optically pumped tube, respectively. The sensitivity to a microwave power change is approximately $4 \times 10^{-14}$ per dB for both types of tube. The quantities $\Delta_m$ and $\Delta_C$ are equal to zero for the same values of the operating parameters.

Alternating magnetic fields applied externally to the frequency standard, having a frequency equal to $\nu_M$ or its multiples, can add detrimental components to the beam tube response and/or perturb greatly the modulation and demodulation processes. They can be the cause of large frequency offsets.

### 4.2. Frequency offsets arising in the frequency control loop

Until now, analog electronics is implemented in most of the commercially available units to filter the beam tube response, to obtain the error signal and to realize the proper loop transfer function. However, operational amplifiers have a finite DC gain and show current and voltage offsets. The first of these imperfections translate a frequency offset of the free running quartz crystal oscillator (*e.g.* $10^{-7}$) in relative value) into a frequency offset at the output of the standard (*e.g.* $10^{-13}$). The latter is inversely proportional to the gain of the electron multiplier, which is known to change with time. The second imperfection, the voltage offset (*e.g.* 10 $\mu$volts) also gives a frequency offset at the output (*e.g.* $10^{-13}$) which depends on temperature, like the voltage offset. Such defects can be completely eliminated using a digital frequency control loop[25,26].

A drift of the quartz crystal oscillator, at a rate of $10^{-9}$ per day when it is free–running, gives a relative frequency offset equal to $10^{-14}$ assuming a first order loop with a time constant of 1 s. This effect disappears with a second order loop.

## 5. INFLUENCE OF THE ADJUSTEMENT OF THE INTERNAL PARAMETERS

Although we have given a plausible magnitude of the values of the various frequency offsets and of their possible change, the actual values depend on the particular adjustement of a given cesium beam frequency standard. This refers to the value of the C–field and of its inhomogeneities, the microwave power, the cavity mistuning, the cavity phase shift, the velocity distribution (*via* geometrical alignment), the level of the spectral impurities, the spurious modulation of the amplitude of the microwave induction, etc. Even the sign of the frequency offsets related to the cavity tuning and to the cavity phase shift may vary from unit to unit. The variability of the sensitivity to external factors[27→29], is therefore not surprising. We have noticed that optically pumped tubes should not be affected by the presence of neighbouring transitions, thanks to the symmetry of the microwave spectrum which can be achieved in that case. This symmetry has the additional advantage to allow one to decrease the value of the C–field and, consequently, to decrease the sensitivity to its change.

However, the sensitivity to the adjustment of other internal characteristics, such as the cavity tuning, is potentially larger than in traditional tubes. This is primarily due to the larger velocity of the atoms detected. The general cure is a better control of the misadjustments. In the particular case of the cavity pulling effect, a reduction of the value of $Q_c$ will contribute to solve the difficulty. Nevertheless, one should be careful to improve the spectral purity of the signal at the output of the final frequency multiplier or mixer, since the filtering effect of the cavity will be decreased accordingly.

# 6. CONCLUSION—SENSITIVITY TO EXTERNAL FACTORS

Figure 6 shows the most efficient pathways from the external variables tothe physical origins of the largest change of the output frequency in the cesium beam frequency standard[6].

The effect of external DC and AC magnetic fields has been described in Section 2.1. A variation of the temperature can change, for instance: i) the current creating the static magnetic field and the properties of the shielding material, ii) the characteristics of the electronic components used in the generation of the microwave power and thus the level of this power, iii) the dimensions of the microwave cavity and thus its resonant frequency and iv) the voltage offset of operational amplifiers in the frequency control loop. Humidity can change the value of capacitances used in the tuning of the various stages of frequency synthesis chain and, consequently, the microwave power. It can also change leakage currents in the frequency control loop. A constant acceleration varies the position of the atomic trajectories with respect to the beam tube structure and, thus, the velocity distribution. However, its main effect is to change the free-runnning frequency of the quartz crystal oscillator. Vibrations create sidebands in the quartz crystal oscillator output. Most of the external variables are not independant ones[30]. A change of the humidity, of the atmospheric pressure, of the orientation of the unit in the earth gravity field and of the supply voltage will modify the internal temperature and/or temperature gradient and thus the temperature of the various sub-assemblies of the device.

# APPENDIX

In the two separated oscillatory field method, the probability that a transition occurs depends on the amplitude of the microwave field and on the velocity distribution function of the atoms detected. It follows that most of the properties of a cesium beam frequency standard depends on these two parameters. Furthermore, the medium term frequency stability and a number of frequency offsets depend on the way the line shape is explored (frequency or phase modulation waveform, depth of the frequency or phase deviation and modulation frequency). These effects are considered in references [4,8,31].

It is convenient to characterize the amplitude of the microwave magnetic induction applied in the interaction regions by the quantity $b$, which has the dimension of rad $s^{-1}$. It is given by $b = \mu_B B/\hbar$, where $B$ is the actual value of the amplitude of the microwave magnetic induction, $\mu_B$ is Bohr magneton and $\hbar$ is Planck constant divided by $2\pi$.

Optimum operating conditions exist, depending on the criterion considered[8]. Considering the optimum amplitudes of the microwave field, they are all close to values such that we have $b\tau_1$ and $b\tau_0$ equal to 1.5, where $\tau_1$ and $\tau_0$ are defined in Section 1.

The amplitude of the frequency modulation which is needed to probe the atomic resonance is usually set close to its half width at half maximum. We have thus chosen $\omega_m T_1 = \omega_m T_0 = 1.5$, where $\omega/2\pi$ is the modulation depth. $T_1$ and $T_0$ are defined in Section 3.1.

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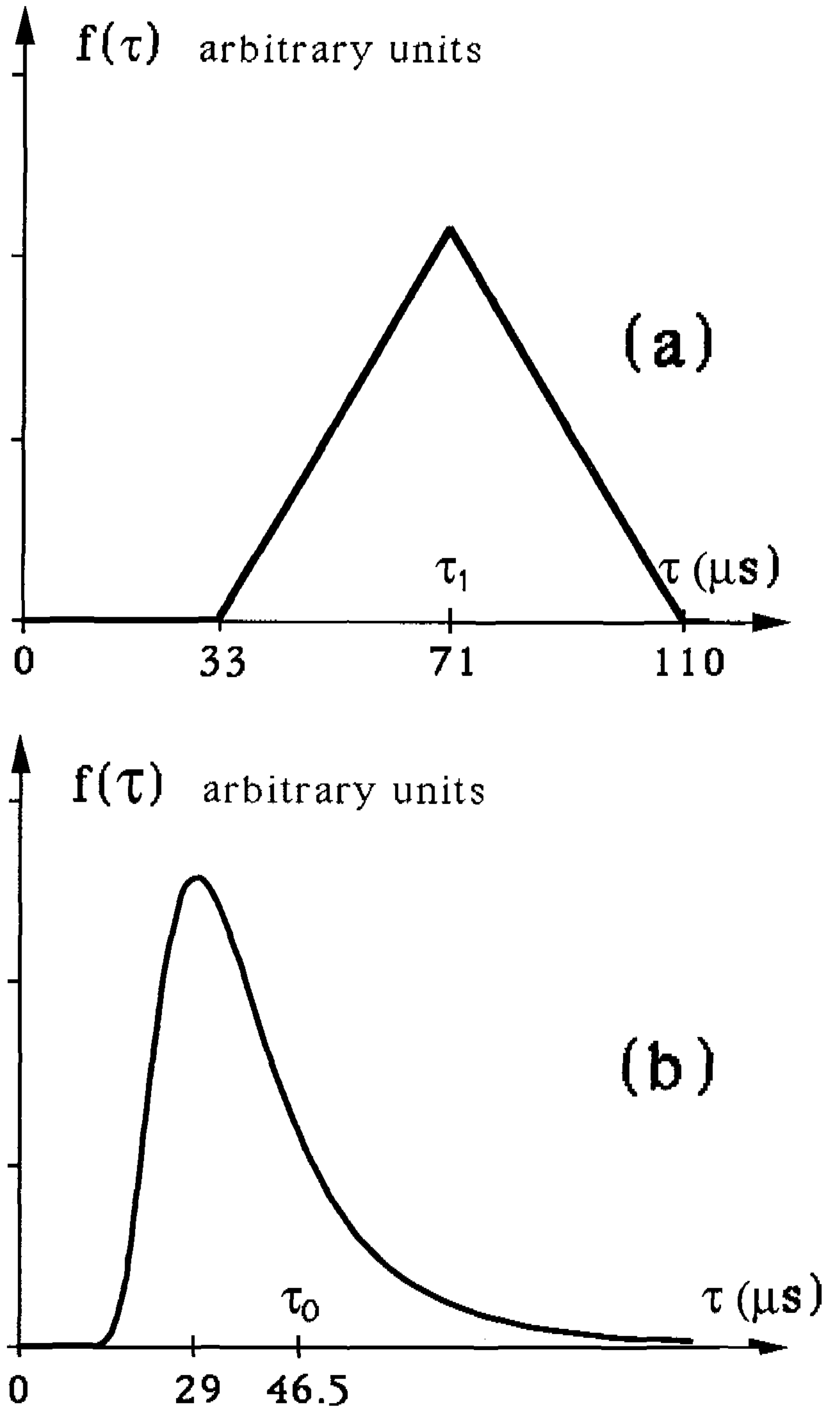


Figure 1. Distribution of interaction times
a) approximate distribution in a tube with magnet state selectors
b) distribution in a tube using a single laser for state selection and detection.

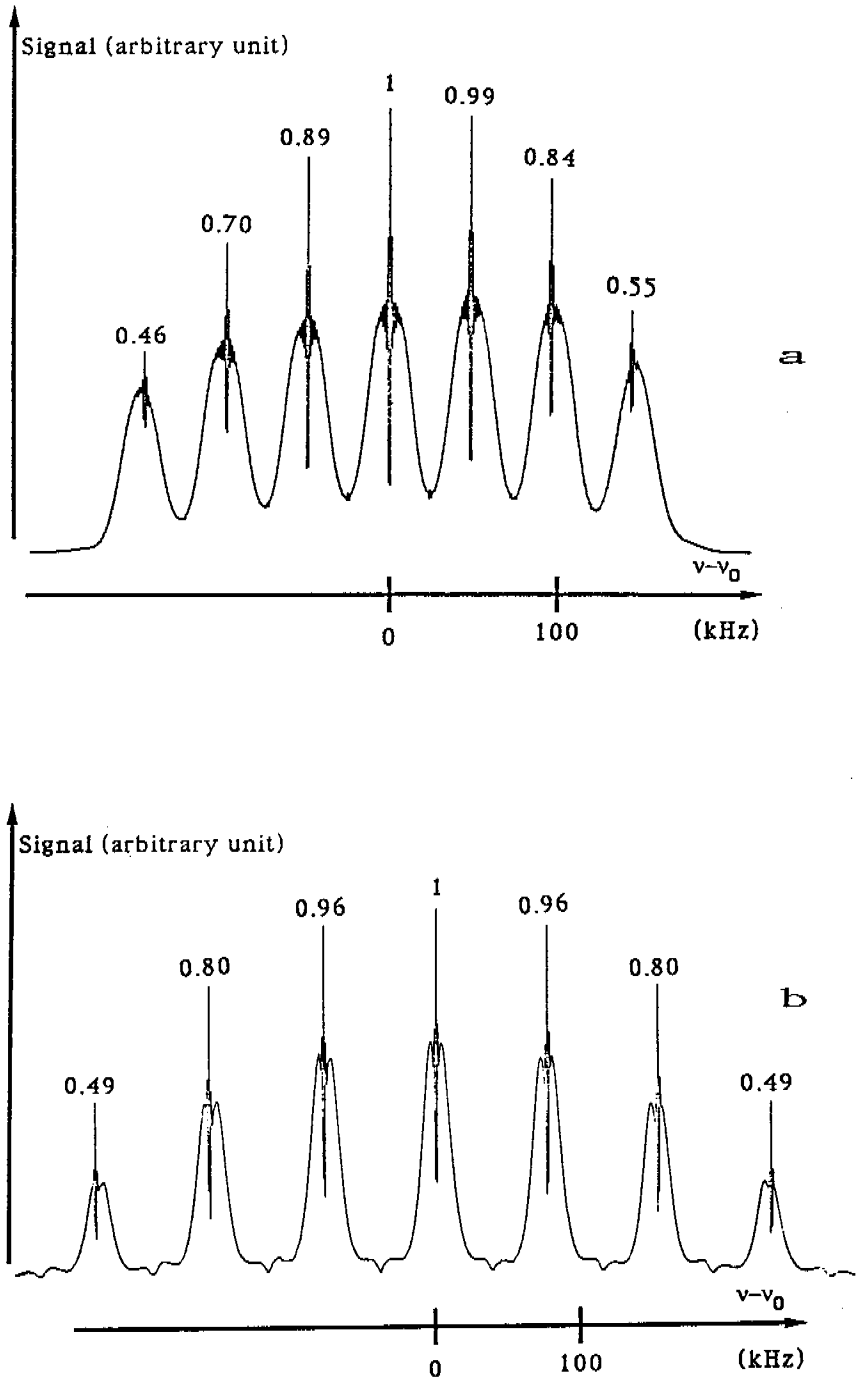


Figure 2. Microwave spectrum of cesium atoms in the ground state. The relative amplitude of the seven lines is shown

a) in a tube with magnet state selectors. $B_o$= 70 mG

b) in a tube using a single laser for state selection and detection. $B_o$ = 110 mG.

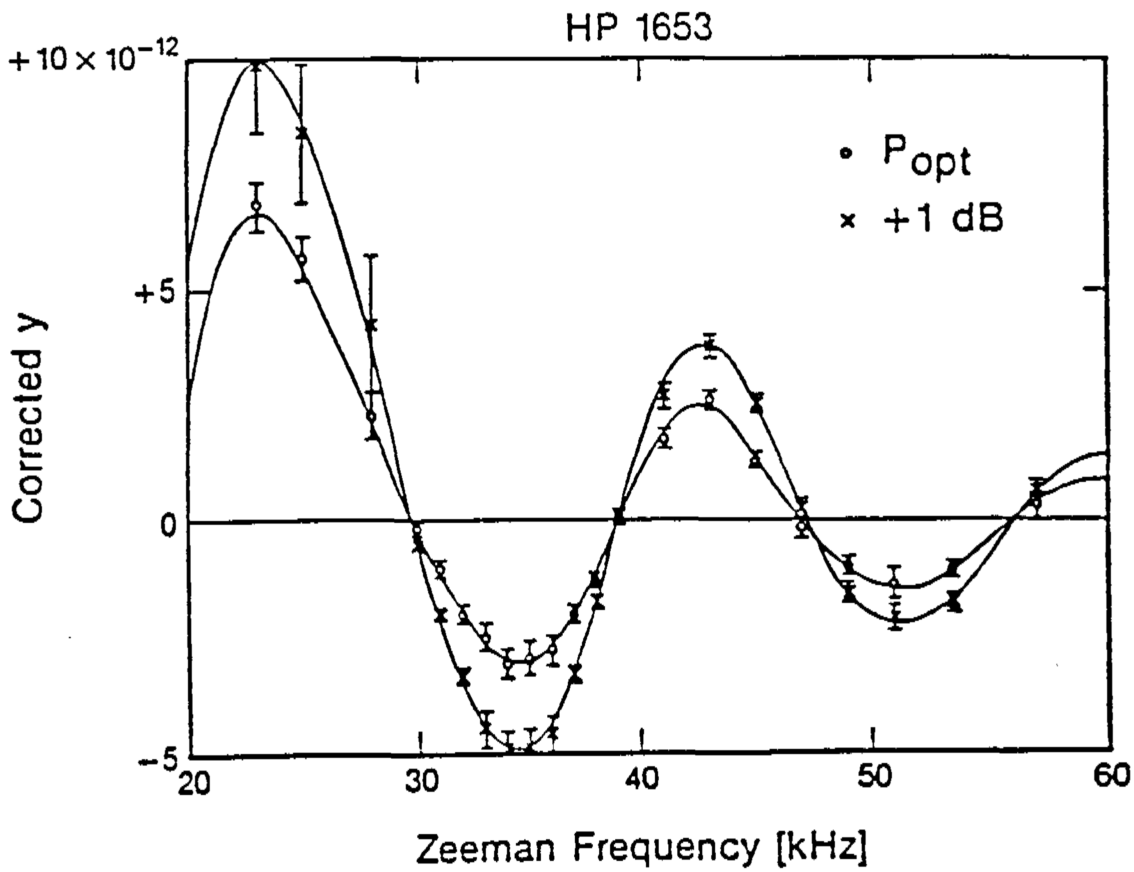


Figure 3. Example of measured Rabi pulling relative frequency offset versus a measure of the static magnetic induction (from ref. 5).

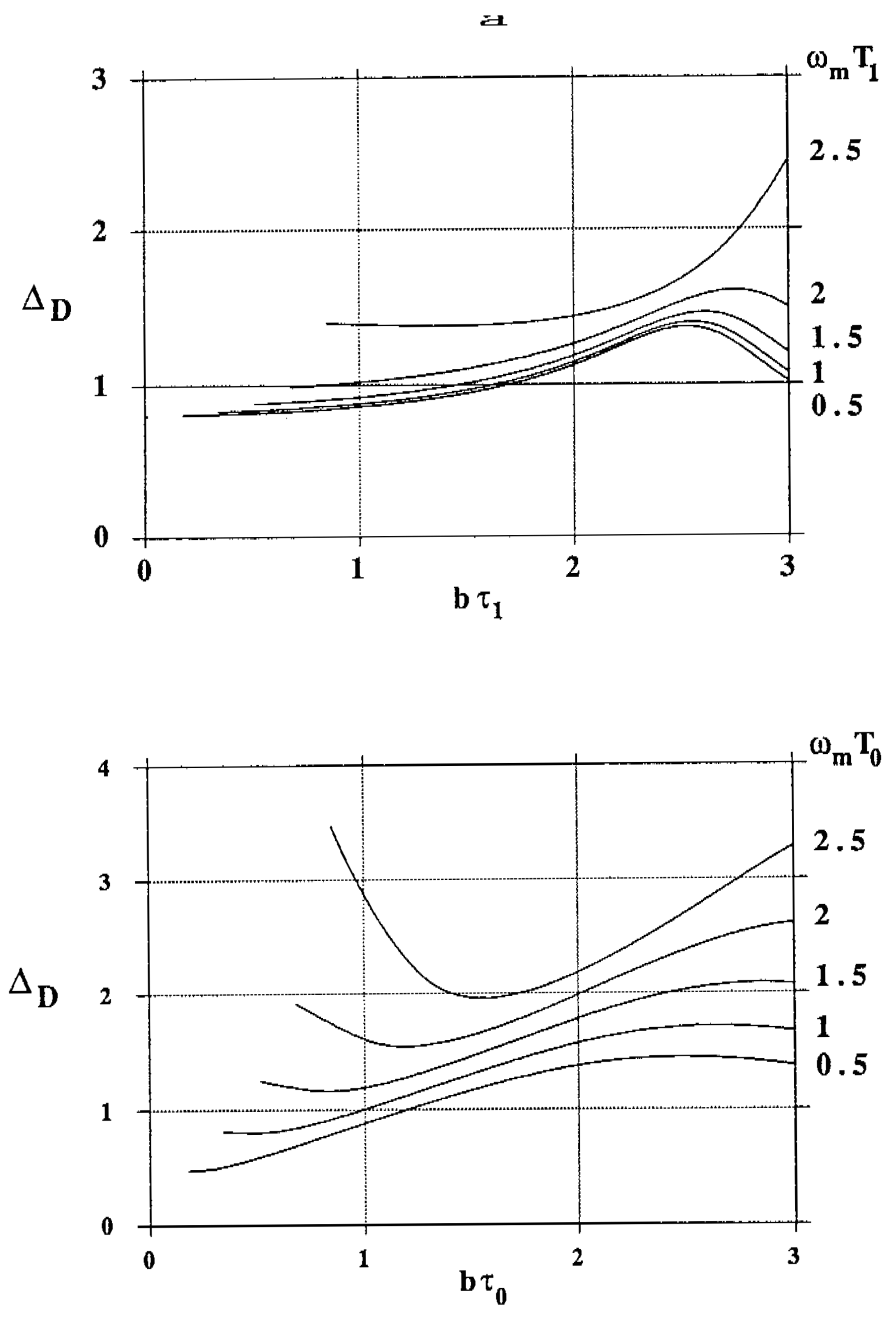


Figure 4. Second order Doppler effect, assuming slow square wave frequency modulation. Variation of $\Delta_D$ versus $b\tau_1$ or $b\tau_0$ for different values of $\omega_m T_1$ and $\omega_m T_0$.

a) Tube with magnet state selector

b) Tube using a single laser for state selection and detection.

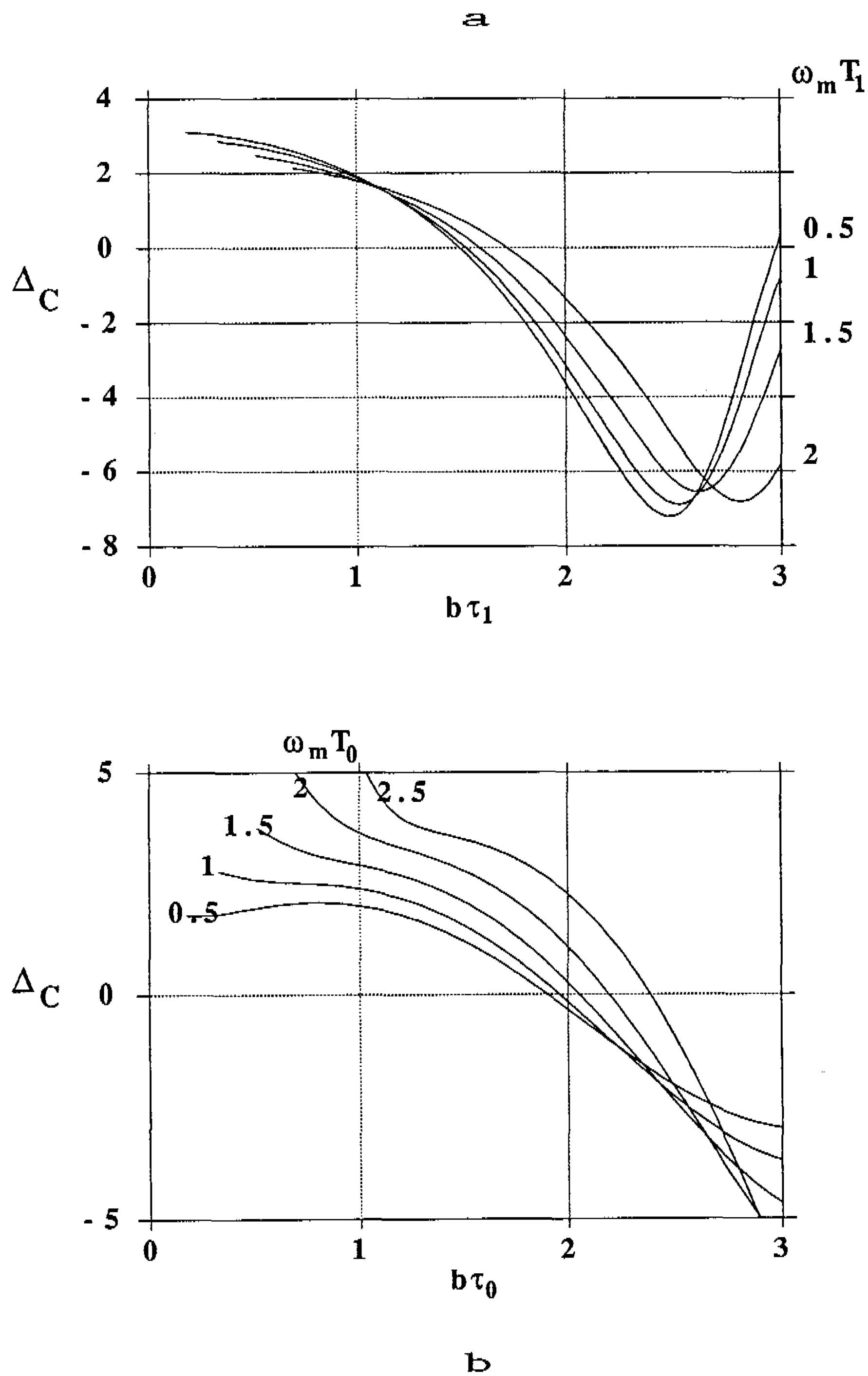


Figure 5. Cavity pulling, assuming slow square wave frequency modulation. Variation of $\Delta_c$ versus $b\tau_1$ and $b\tau_0$.

a) Tube with magnet state selector

b) Tube with a single laser for state selection and detection.

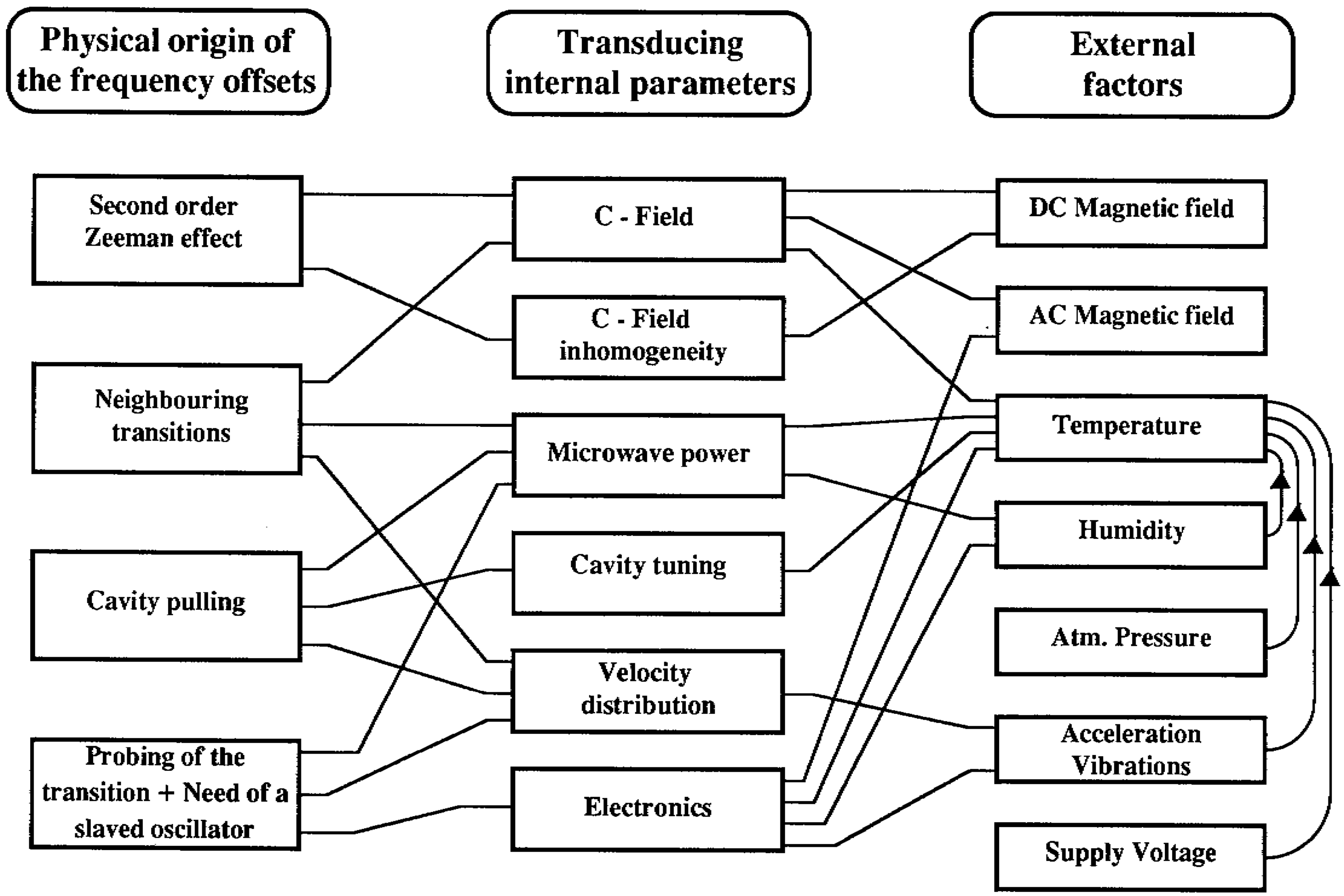


Figure 6. Pathways between the external stimuli and some of the physical origins of the frequency offsets, via internal characteristics of the device.