ON THE LENGTH OF THE DRIFT REGION IN THE RAMSEY CAVITY

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Abstract

The interaction of atoms in a beam with the microwave field in a separated field geometry such as a Ramsey cavity is generally described in terms of the three regions traversed successively by the atoms, namely two interaction regions of length ℓ separated by a "drift", or "free precession", region of length L. For a monokinetic beam of velocity v, the linewidth of the central fringe in the Ramsey resonance pattern is usually expressed as $\Delta\omega = \pi\, v/L$.

A more detailed calculation shows, however, that the linewidth is equal to $\pi\, v/L^*$, where the equivalent drift L^* is larger than L by an amount of the order of ℓ/L. The correction depends on the field distribution in the interaction regions. Its origin lies in the fact that atomic precession is not limited to the field-free regions but also occurs in the interaction regions, where atomic coherence builds up or decreases continuously.

Although the correction to the equivalent length of the drift region is small, it may be relevant to the evaluation of the second-order Doppler effect bias in primary cesium-beam standards to the extent that the atomic velocity is deduced from the lineshape and from the geometrical parameters of the cavity. It is shown that in current and projected standards with atoms of average thermal velocity, use of corrected dimensions may lead to a change of the calculated bias of the order of 10^{-14}, which is significant at the levels of accuracy considered nowadays.

1. INTRODUCTION

The velocity distribution of atoms in atomic beam standards often needs to be known with considerable accuracy because of its relevance to the determination of the second-order Doppler shift $\delta\nu_D = -(v/c)^2/2$. Several methods have been proposed and used to determine the velocity distribution [1-6]. Most of the methods actually yield a transit-time distribution, through an analysis of the Ramsey pattern lineshape [6], the power dependence of the transition probability at resonance [5], or through the response to RF pulses of varying periodicity [2]. In order to deduce the average velocity and the velocity distribution from such transit-time information, one must know precisely what physical length has been travelled during the measured transit-time. It is often assumed that the relevant length is the separation L between the end of the first interaction region and the beginning of the second. The following calculation of the linewidth of the central fringe of the Ramsey pattern takes into account the non-zero length ℓ of the interaction regions, which is usually neglected. The calculated linewidth is indeed affected by the finite interaction length, the amount of the correction being of order ℓ/L and depending somewhat on the RF field profile inside the interaction regions.

2. CALCULATION OF THE EFFECTIVE LENGTH

The purpose of the calculation is to establish a precise relationship between the linewidth of the resonance curve, which is directly accessible to measurement, the length of the Ramsey cavity, which is known by construction, and the atomic velocity which is the parameter to be determined. For the sake of simplicity a monokinetic beam of velocity v will be considered; the results can then be extended to actual velocity distributions.

The two levels of the clock transition (F = 4; m = 0 and F = 3; m = 0 in the case of cesium) are coupled to a near-resonant RF magnetic field. We write the familiar Bloch equations for the three components of the magnetic dipole (fictitious spin 1/2) associated to this two-level system. It is convenient, as usual, to write these equations in a frame rotating with the resonant part of the RF-field, and to neglect the fast oscillating antiresonant part. The equations of motion then read

$$\dot{x} = \alpha y \tag{1a}$$

$$\dot{y} = -\alpha x - \beta(t) z \tag{1b}$$

$$\dot{z} = \beta(t) y \tag{1c}$$

where x and y are the transverse components of the fictitious dipole in phase and in quadrature with the RF-field; z is the longitudinal component of the fictitious dipole and the population inversion of the real two-level system (z = 1 if F = 4, m = 0; z = -1 if F = 3, m = 0).

The detuning α is the difference between RF frequency and atomic frequency: $\alpha = \omega_{RF} - \omega_o$; $\hbar\beta(t) = \mu_B B(t)$ is the coupling energy between atom and RF-field (B(t) is the amplitude of the RF field, directed along Ox in the fictitious dipole space).

Since there is no relaxation in this system, the representative vector is of constant length ($x^2(t) + y^2(t) + z^2(t) = 1$ at all times) and its motion is a rotation about the instantaneous rotation vector $\vec{\Omega}(t) = (\beta(t), 0, -\alpha)$.

We will consider the two standard configurations of Ramsey cavities used in cesium standards, namely the E-bend cavity where the RF field amplitude is constant in each interaction region, and the H-bend cavity where the RF field amplitude has a sine envelope. The corresponding time sequences for B(t) are shown in Fig. 1.

2.1 E-BEND CAVITY, EXACT SOLUTION

Since the amplitude of the RF field is constant, B is time-independent during each of the three parts of the evolution and the equations of motion can be integrated analytically in a standard manner [7]. The quantity of interest is the population inversion z at time $T_P + 2T_R$, as a function of the detuning α.

For small detunings the exact solution can be expanded in power series of the ratio α/β of the detuning to the Rabi frequency at resonance. Optimum RF power is assumed $(\beta T_R = \pi/2)$; for detunings such that $|\alpha T_P| \leq \pi/2$, we have $|\alpha|/\beta \leq \ell/L$.

The usual expression for the lineshape near resonance is the zero order term of the expansion:

$$z\,(T_P + 2T_R) \cong -\cos\,(\alpha T_P) \tag{2}$$

The approximate linewidth $\Delta\omega$ (FWHM) is equal to π/T_P and the atomic velocity is related to the linewidth by

$$v = \frac{L}{T_P} \cong \frac{L\Delta\omega}{\pi} \tag{3}$$

If, however, terms of order 1 in α/β are kept in evaluating z, the main change is a narrowing of the fringe spacing and width:

$$z\,(T_P + 2T_R) = -\cos\left[\alpha\,(T_P + \frac{4}{\pi}\,T_R)\right] \tag{4}$$

The linewidth is now $\Delta\omega = \pi/T^*$, where the effective transit-time T^* is equal to $T_P + \frac{4}{\pi}\,T_R$.

The distance travelled by the atoms during the effective transit-time T^* is what we call the effective length L^* of the Ramsey cavity

$$L^* = L + \frac{4}{\pi}\,\ell \quad , \tag{5}$$

to which the velocity is now related by

$$v = \frac{L^*\Delta\omega}{\pi} \quad . \tag{6}$$

Equation (4) means that the time interval over which the phase of the atomic dipole and the phase of the RF field are allowed to drift apart between the two atom-field interactions extends beyond the field-free interval to include part of the interaction times. The transit-time which can be inferred from the width of the central fringe in the simple case of a monokinetic beam is thus the time T^* to travel L^*, the effective length, instead of the time T_P to travel L in the approximate derivation. Consequently the velocity that can be deduced from this effective time of flight is higher by a factor $1 + \frac{4}{\pi}\frac{\ell}{L}$ (eq. (5)). It may be expected that a similar correction will apply in the case of a real beam when converting the transit-time distribution into a velocity distribution (see also below).

2.2 APPROXIMATE INTEGRATION OF THE BLOCH EQUATIONS

The origin of the additional transit time can best be seen by integrating directly the equations of motion of the fictitious spin 1/2. Although an exact integration is possible in the case of a constant field amplitude, one can settle for an approximate integration where terms of order two in α/β are neglected. This procedure will also allow us to evaluate the effective transit-time and length of an H-bend cavity where the field amplitude is not constant and no analytical solution to eqs. (1) can be found.

We find that the FWHM linewidth, i.e. twice the detuning required for the final inversion to be equal to zero, can be expressed as

$$\Delta\omega = \frac{\pi}{T^*} \quad , \tag{7}$$

$$\text{with } T^* = T_P + 2\int_o^{T_R} \sin\left[\int_o^t \beta(t')\,dt'\right] dt \tag{8}$$

Introducing the explicit field profile $\beta(t)$ (Fig. 1) in the integral, we get

$$\text{E-bend cavity:} \quad T^* = T_P + \frac{4}{\pi}T_R \tag{9}$$

$$\text{H-bend cavity:} \quad T^* = T_P + \sqrt{2}\;\; J_o(\pi/4)\; T_R \tag{10}$$

The E-bend result is identical to the exact result (eq. (5)) and thus validates the approximate integration procedure used to derive eq. (8).

3. DISCUSSION

The implication of the results above for the second order Doppler shift bias depends on the average atomic velocity and the geometry of the Ramsey cavity. In an optically pumped standard with $L = 1$ m, $\ell = 10^{-2}$ m, $v_{rms} = 300$ m/s, the Doppler biases calculated with L and L^* would differ by $1.3 \cdot 10^{-14}$, a value that cannot be neglected anymore.

The linewidth of the central fringe is of course not the only way of measuring transit-times. It seems obvious, however, that the effective transit-time and length described here are the relevant parameters in relating spectral features of the atomic resonator to the atomic velocity or velocity distribution.

The practical case of a velocity distribution introduces a complication in that the optimum power condition assumed in the monokinetic case can no longer be satisfied by all atoms. This has a consequence on the lineshape but it can be shown that the effect on the effective transit time is zero if the transit-time distribution is symmetric. As an illustration of this point we compare the monokinetic results (eqs. (9) and (10)) with results [9] obtained by computing the linewidth of the Ramsey pattern for a real, asymmetric, velocity distribution with a width equal to ~ 10% of average velocity. In the H-bend case, the equations of motion were integrated numerically:

$$\text{E-bend:}\begin{cases} L^* \text{ (monokinetic, eq. 9)} & = L + 1.27\ \ell \\ L^* \text{ (velocity distribution)} & = L + 1.28\ \ell \end{cases}$$

$$\text{H-bend:}\begin{cases} L^* \text{ (monokinetic, eq. 10)} & = L + 1.20\ \ell \\ L^* \text{ (velocity distribution)} & = L + 1.23\ \ell \end{cases}$$

Considering that in the H-bend case ℓ/L was .03, the agreement is within the precision of the monokinetic estimate where terms ~ $(\ell/L)^2$ have been neglected.

CONCLUSION: The concept of effective transit-time and effective length of a Ramsey cavity has been pointed out. The use of these effective parameters in determining atomic velocities may lead to a significant improvement in the evaluation of the second order Doppler bias in atomic beam primary standards.

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References

[1] A.G. Mungall, Metrologia 7, 49 (1971).

[2] H. Hellwig et al, Proc. 27th Annual Symposium on Freq. Control, Fort Monmouth, NJ (1973), p.357.

[3] S. Jarvis, Metrologia 10, 87 (1974).

[4] D.A. Howe et al, Proc. 28th Annual Symposium on Freq. Control (1974), p. 362

[5] T. Heindorff et al, PTB Mitteilungen 94, 318 (1984).

[6] J.-S. Boulanger, Metrologia 23, 37 (1986).

[7] J. Vanier, C. Audoin: The Quantum Physics of Atomic Frequency Standards p. 628 (Hilger, 1989).

[8] N. Ramsey: Molecular Beams (Oxford Univ. Press, 1956).

[9] A. Bauch, private communication.

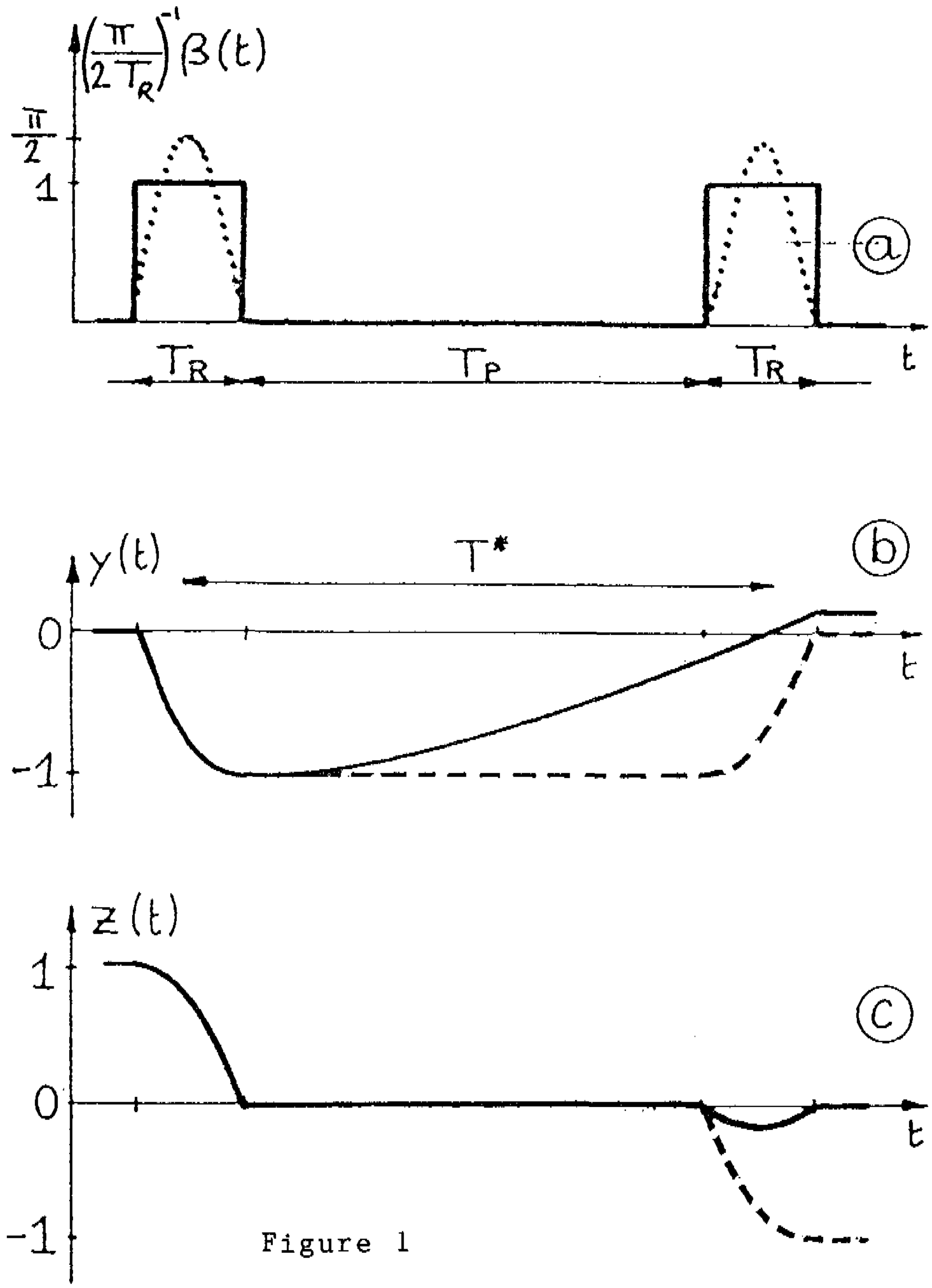

Figure 1

a) Time-dependence of the RF magnetic field B(t) (in Rabi frequency units). Solid line: E-bend cavity; dotted line: H-bend cavity.

b, c) Time-dependence of the atomic dipole y(t) (b) and of the population inversion z(t) (c)
Solid lines: $-\alpha T^* = \frac{\pi}{2}$; dashed lines: $\alpha = 0$

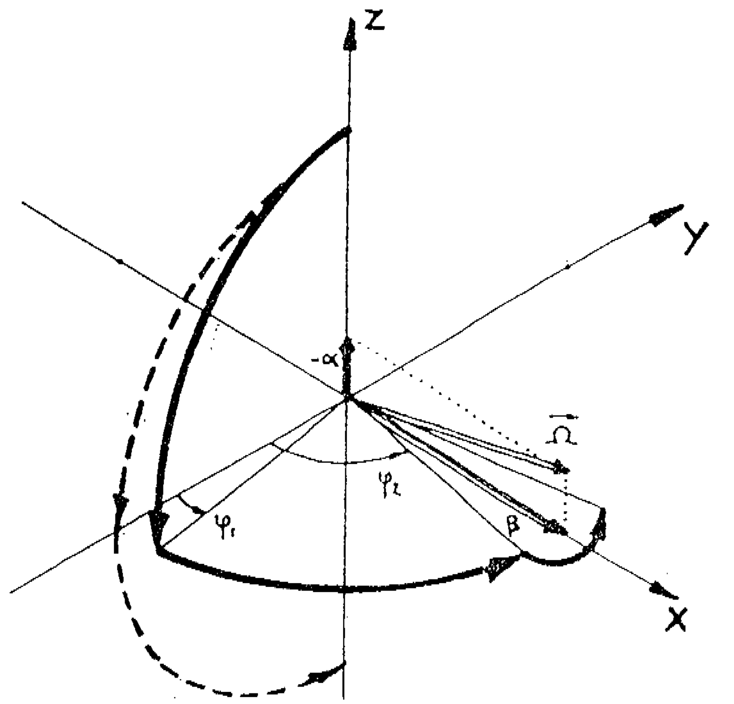

Figure 2: Path of the fictitious spin $\frac{1}{2}$ vector on the Bloch sphere. Solid lines: $-\alpha T^* = \frac{\pi}{2}$; dashed line: $\alpha = 0$.

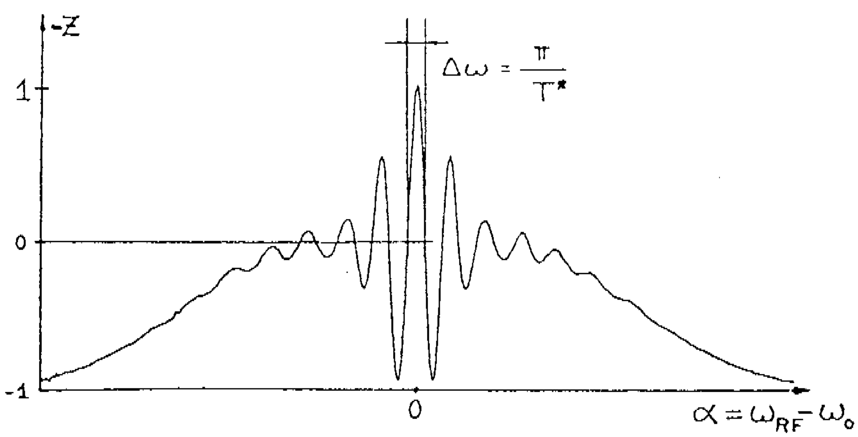

Figure 3: Typical Ramsey fringe pattern and linewidth.