Geodetic Positioning of the Aerospace Electronics Research Lab (ERL) Osborne Time Transfer Receiver (TTR) using the GPS NAVSTAR Block I Satellites

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Abstract

Aerospace has routinely processed the Osborne Time Transfer Receiver (TTR) data for the purpose of monitoring the performance of ground and GPS atomic clocks in near real-time with on-line residual displays and characterizing clock stability with Allan Variance calculations. Recently, Aerospace added the ability to estimate the TTR's location by differentially correcting the TTR's location in the WGS84 reference system. We exercised this new feature on a set of TTR clock phase data and obtained sub-meter accurate station location estimates of the TTR at the Aerospace Electronic Research Lab (ERL).

BACKGROUND

The Osborne Time Transfer Receiver's (TTR) primary function is to provide a means to monitor a local laboratory frequency standard by comparing this standard to GPS system time using the GPS Navstar satellites. In order to accomplish this task, it is necessary to set up initially, the local receiver coordinates into the receiver's memory. This initialization is assisted by using the built-in positioning capabilities of the receiver whereby real-time location estimates are obtained during user specified tracks. The TTR is a single channel Clear Acquisition (CA) receiver, thus the real time estimates of location are obtained sequentially in time from each satellite. This operation is in contrast to position receivers designed to receive signals from four or more satellites simultaneously using the Precise (P) Code, which is an order of magnitude more accurate than the CA code. The lack of simultaneity of the measurements further degrades the position estimates from the TTR. Therefore the initially entered station coordinates are only first estimates and subject to error.

These deficiencies are unessential to the receiver's operation as a time transfer device, and consequently the TTR has been most useful and helpful in quickly detecting atomic clock anomalies not only in the laboratory standard, but also, satellite atomic clocks themselves. This latter ability is due to the high precision and stability of the laboratory clock.

The initially entered receiver locations were obtained by observing the TTR's solution on a daily basis for several days. Since no consistent daily solution could be obtained, the locations were finally selected and entered by a trial and error method. The receiver's initial position errors are clearly manifested in the clock phase difference measurements as pronounced systematic diurnal signatures in some the satellites, (Satellites ID PRN 9 and PRN 3) as seen in figure 1. The long non-diurnal systematic effects observed for each satellite seen in this figure is the effect of clock steering (confirmed by independent data from USNO) performed by the GPS control segment. The clock steering is necessary to keep GPS system time to within a microsecond of USNO time.

The program which was used to monitor atomic clock performance and characterize clock stability by computing the Allan Variance was modified with additional code for an iterative least squares procedure. The initial estimate is refined by inputting the time difference measurements from the receiver as data into this least squares program and thus derive a better estimate for the station coordinates. Since the original station entries are referenced to the WGS84 geodetic coordinates as given by the TTR, and the broadcasted ephemerides are also based on the WGS84 system, the resulting least squares corrections will be referenced to the WGS84 coordinates.

ANALYSIS

We note that the Δr, the range difference (i.e.time difference measurement error times the speed of light) is merely the negative sum of the projections of the South, East and height components of the station error into the line of sight from the station to the satellite. A derivation of this is given in the Appendix section.

$$\Delta r = -\cos AZ \cos el(\text{ South error}) - \sin AZ \cos el(\text{ East error}) - \sin el(\text{ Height error}) \quad (1)$$

where (el) and (AZ) are the observe elevation and azimuth angles. Azimuth is defined as measured from North round by East. or:

$$\Delta r = -\cos AZ \cos el(R\Delta\phi) - \sin AZ \cos el(R\cos\phi\Delta\lambda) - \sin el(\Delta h) \quad (2)$$

where Δh, $\Delta\phi$ and $\Delta\lambda$ are the respective height correction above a reference geoid, and corrections in latitude and longitude of the TTR. The system of normal equations necessary for a least squares solution is given by equation (3), where the left hand side contains the data, and the right hand side consists of the station's South, East and height errors to be determined from the data. Each row of the matrix of normal equations $||A||$ is formed from the coefficients of equation (2). A vector of measurements, $|\Delta\vec{r}|$, is formed from the data set, Δr. The solution proceeds in an iterative two step procedure. The data are first fitted to a polynomial up to sixth degree in time to remove the clock phase wander and other systematic effects. The remaining difference, Δr, is used in a standard least squares solution for the station off-sets in the South, East, and height directions, as follows:

$$\begin{pmatrix} \text{(South error)} \\ \text{(East error)} \\ \text{(Height error)} \end{pmatrix} = \left[||A||^T ||A||\right]^{-1} ||A||^T |\Delta\vec{r}| \quad (3)$$

As many as 15 iterations are necessary to converge the final solution. Each iteration includes a reestimation of the polynomial to separate out the clock wander and steering effects from the station location estimation.

DATA DESCRIPTION AND RESULTS

A span of clock phase difference data between 2/8/90 to 4/19/90 from the TTR5 was selected for the receiver coordinate adjustment. This data also contained the elevation and azimuth angles as computed by the TTR5, which are needed in equation (2). The iterative least squares correction for the coordinates is obtain by computing the weighted correction from each NAVSTAR. At the end of each iteration, the sum of the average data residual (standard deviation or s.d.) from each NAVSTAR is computed, and the iteration ceases when this sum reaches a minimum. The result and solution from this procedure are summarized in Table 1.

As seen in this table, the final post-fit data s.d. from each NAVSTAR are substantially reduced from the initial pre-fit data s.d. The columns labelled "corrections" are the final corrections determined from data from each NAVSTAR, and show considerable scatter from NAVSTAR to NAVSTAR (e.g. -11.726 meters of West error for NAVSTAR 9). The individual s.d. for each NAVSTAR for each coordinate component are listed under the column labelled "sig". The final solution for the South, East, height correction components and the associated composite s.d from all the NAVSTARs are listed under the columns labelled "weighted mean" and "composite sigma" respectively. The corrections are 7.366m South, 5.612m West and 16.12m low.

These formal s.d. need to be multiplied by a factor of about 6 to 7 corresponding to the postfit average data residual as described by the data s.d. The covariance implicit in equation (3) assumes a data sigma of one meter, whereas it is more appropriate to use the actual data sigma of about 7 meters. Consequently, the more realistic uncertainties are 0.60 meters, 0.39 meters and 0.36 meters in the respective coordinates. Finally, when the linear corrections are converted to arc second measure, the corrections are 0.2382" South in latitude and 0.2187" West in longitude.

When these corrections are used in equation (2) to compensate the data for station error, a marked improvement is seen when Figure 2 is compared to Figure 1. The most noticeable effect is the reduction in the diurnal phase errors for NAVSTAR PRN 9 and 3 as well as moderate error reduction for the other NAVSTAR PRNs.

For further confirmation of the validity of these results, we performed a "blind test" verification by using the corrective factors derived from data between 2/8/90 to 4/19/90 into a data span not used in the solution. We selected a earlier set between 10/3/89 to 11/12/89. The result of this test is shown in Figures 3 and 4. Figure 3 shows the errors when the station correction is left out. As can be seen, sizable errors for PRNs9 and 3 are there and of the same magnitude as seen in Figure 1, where the station corrections are also left out. In Figure 4, where the corrections are applied, significant reductions in the diurnal errors especially for PRNs 9 and 3 are achieved. As an additional note, the data selected were time differenced measurements between the Aerospace laboratory standard and each of the NAVSTAR clocks. Hence, as can been seen in Figures 3,and 4, the individual satellite clock behavior is exhibited. A summary of each of the clock's frequency labeled as FRQ and aging labeled as DFQ between each satellite and the laboratory is shown in the upper right hand corner of the figure. The units for frequency difference between laboratory standard and satellite clock are in picoseconds/second and the aging factor are in nanoseconds/days^2.

SUMMARY AND CONCLUSION

A few meters of TTR location input error will result in noticeable effects in the data. Significant improvements resulted when station corrections of 0.2382" South in latitude, 0.2187" West in longitude and -16.12 meters in receiver height are applied. The results here show that the TTR with CA coded data provides sufficient precision for the receiver to act as a WGS84 position locator with sub-meter accuracy as well as a time-transfer device.

APPENDIX

For the purpose of differentially correcting the station coordinates, we formulate the rectangular station components for a spherically shaped earth,

$$\begin{aligned} X &= (R+h)\cos\phi\cos\lambda \\ Y &= (R+h)\cos\phi\sin\lambda \\ Z &= (R+h)\sin\phi \end{aligned}$$

where R, h, ϕ and λ are the respective radius, height above a reference geoid, geocentric latitude and longitude of the TTR.
We define a set of unit vectors $\vec{L}$, $\vec{A}$, $\vec{D}$ as:

$$\begin{array}{lll} L_x = \cos\phi\cos\lambda & A_x = -\sin\lambda & D_x = -\cos\lambda\sin\phi \\ L_y = \cos\phi\sin\lambda & A_y = \cos\lambda & D_y = -\sin\lambda\sin\phi \\ L_z = \sin\phi & A_z = 0 & D_z = \cos\phi \end{array} \tag{1}$$

so that errors in the rectangular components are related to errors in the spherical components through the matrix equation as:

$$\begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} = \begin{pmatrix} L_x & A_x & D_x \\ L_y & A_y & D_y \\ L_z & A_z & D_z \end{pmatrix} \begin{pmatrix} \Delta h \\ R\cos\phi\Delta\lambda \\ R\Delta\phi \end{pmatrix} \tag{2}$$

In order to obtain the errors ΔX_h, ΔY_h, and ΔZ_h in a local horizon tangent plane, with the normal to the plane in the local zenith direction, we need to rotate ΔX, ΔY, ΔZ by a rotation matrix:

$$\begin{pmatrix} \Delta X_h \\ \Delta Y_h \\ \Delta Z_h \end{pmatrix} = \begin{pmatrix} -D_x & -D_y & -D_z \\ A_x & A_y & A_z \\ L_x & L_y & L_z \end{pmatrix} \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} \tag{3}$$

Substituting ΔX, ΔY, ΔZ from equation (2) into (3), we obtain:

$$\begin{pmatrix} \Delta X_h \\ \Delta Y_h \\ \Delta Z_h \end{pmatrix} = \begin{pmatrix} -D_x & -D_y & -D_z \\ A_x & A_y & A_z \\ L_x & L_y & L_z \end{pmatrix} \begin{pmatrix} L_x & A_x & D_x \\ L_y & A_y & D_y \\ L_z & A_z & D_z \end{pmatrix} \begin{pmatrix} \Delta h \\ R\cos\phi\Delta\lambda \\ R\Delta\phi \end{pmatrix} \tag{4}$$

which reduces simply to:

$$\begin{pmatrix} \Delta X_h \\ \Delta Y_h \\ \Delta Z_h \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta h \\ R\cos\phi\Delta\lambda \\ R\Delta\phi \end{pmatrix} \tag{5}$$

The topocentric rectangular measurement error vector, $\Delta\vec{\rho}$, with components, Δu, Δv, Δw, is the negative of the station error vector $\Delta\vec{R}$, (assuming no errors for the NAVSTARs' position).

$$\Delta\vec{\rho}(\Delta u, \Delta v, \Delta w) = -\Delta\vec{R}(\Delta X_h, \Delta Y_h, \Delta Z_h) \tag{6}$$

A triad of unit vectors in the topocentric system is defined for the observed elevation (el) and azimuth (AZ) angles as follows:

$$\begin{array}{lll} L_{xh} = -\cos el \cos AZ & A_{xh} = \sin AZ & D_{xh} = \cos AZ \sin el \\ L_{yh} = \cos el \sin AZ & A_{yh} = \cos AZ & D_{yh} = -\sin AZ \sin el \\ L_{zh} = \sin el & A_{zh} = 0 & D_{zh} = \cos el \end{array}$$

so that:

$$\begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} = \begin{pmatrix} L_{xh} & A_{xh} & D_{xh} \\ L_{yh} & A_{yh} & D_{yh} \\ L_{zh} & A_{zh} & D_{zh} \end{pmatrix} \begin{pmatrix} \Delta r \\ r\cos el \Delta AZ \\ r\Delta el \end{pmatrix} \tag{7}$$

The inverse is then:

$$\begin{pmatrix} \Delta r \\ r\cos el \Delta AZ \\ r\Delta el \end{pmatrix} = \begin{pmatrix} L_{xh} & L_{yh} & L_{zh} \\ A_{xh} & A_{yh} & A_{zh} \\ D_{xh} & D_{yh} & D_{zh} \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} \tag{8}$$

Noting the relationship between $\Delta\vec{\rho}$ and $\Delta\vec{R}$ in equation (6), and substituting (5) into (8) we get for first component, the range expression, Δr as:

$$\Delta r = -\cos AZ \cos el(R\Delta\phi) - \sin AZ \cos el(R\cos\phi\Delta\lambda) - \sin el(\Delta h) \tag{9}$$

or:

$$\Delta r = -\cos AZ \cos el(\text{ South error}) - \sin AZ \cos el(\text{ East error}) - \sin el(\text{ Height error}) \tag{10}$$

AEROSPACE ELECTRONIC LAB TTR
LEAST SQUARES STATION ADJUSTMENT
(meters)

ITERATIVE SOLUTIONS FOR EACH NAVSTAR PRN

PRN	stnd deviation pre-fit residual	stnd deviation post-fit residual	final South correction correction	sig	final East correction correction	sig	final Height correction correcton	sig
6	6.809	6.415	-2.776	1.115	- 1.013	0.971	1.667	0.568
9	9.298	7.350	-0.215	0.116	-11.726	0.801	-0.624	0.083
11	7.146	6.557	-6.023	0.430	5.883	0.410	1.002	0.150
12	6.044	5.888	1.054	0.203	-2.926	0.209	-0.768	0.108
13	4.640	3.691	-2.125	0.331	-0.064	0.081	1.285	0.206
3	7.690	6.570	1.857	0.179	0.234	0.081	-0.661	0.102

WEIGHTED SOLUTION

South Correction weighted mean	South Correction composite sig	East Correction weighted mean	East Correction composite sig	Height Correction weighted mean	Height Correction composite sig
-7.366	0.600	-5.612	0.387	-16.122	0.360

AEROSPACE ELECTRONIC LAB TTR
STATION LOCATION

	Latitude			Longitude			Height
Old value:	+33deg	54min	54.5778sec	241deg	37min	13.5410sec	25.00 meters
Correction:			-0.2382sec			-0.2187sec	-16.12 meters
New value:	+33deg	54min	54.3396sec	241deg	37min	13.3223sec	8.78 meters

TABLE 1.
Summary of solutions and estimates of TTR5
station location in WGS84 reference system

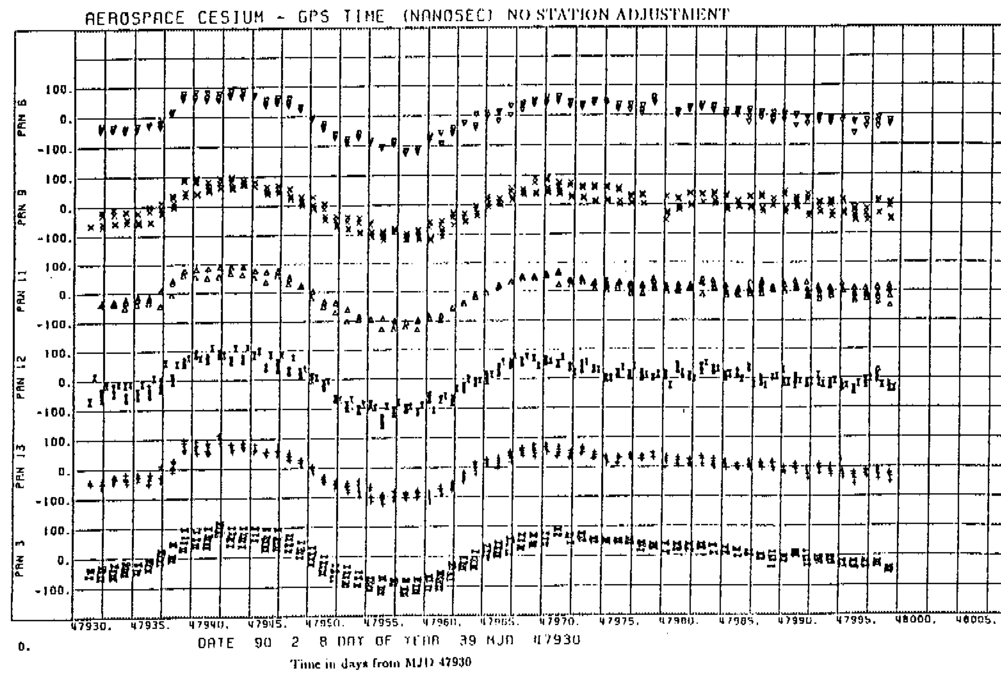

Figure 1. Clock phase residuals between ground cesium and GPS system time via each NAVSTAR PRN. Same systematic effect for satellite is due to GPS time steering. Diurnal "scatter" is due to station location error.

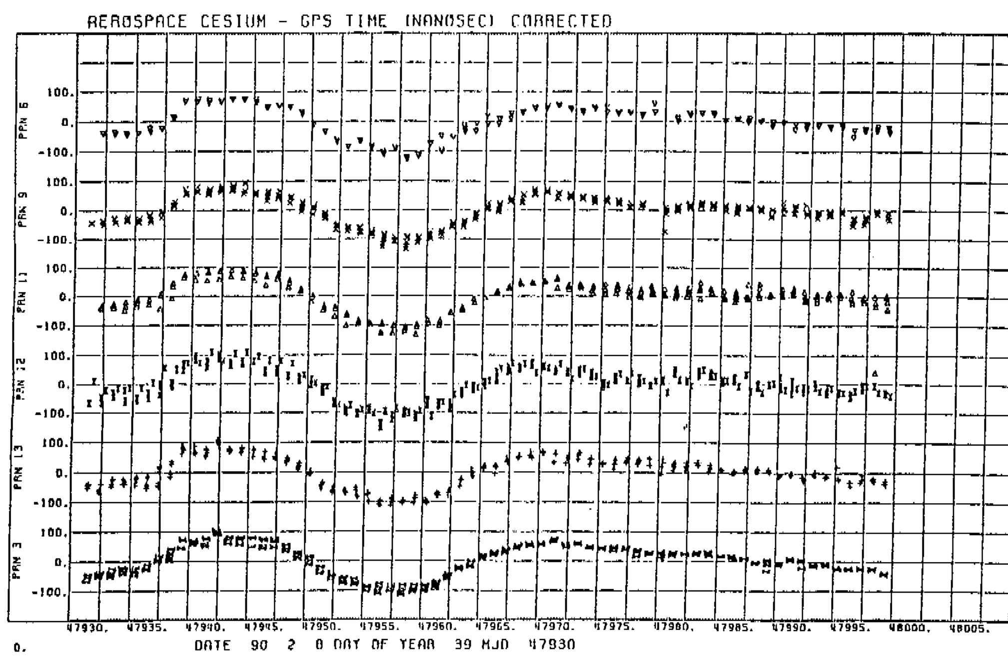

Figure 2. Clock phase residuals between ground cesium and GPS system time via each NAVSTAR PRN. Diurnal "scatter" considerably reduced from Figure 1. Reduction is due to corrected station location.

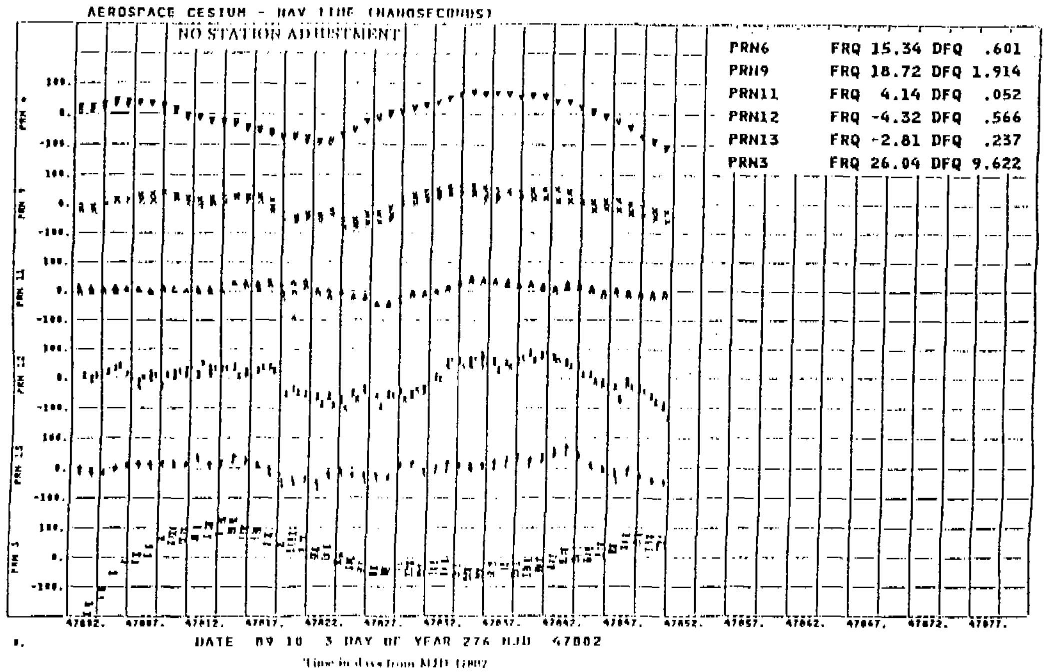

Figure 3. Clock phase residuals between ground cesium and each NAVSTAR PRN. The individual satellite clock variations are apparent. The table on the upper right corner of figure lists the frequency offset (FRQ) in units of picosec/sec. and ageing (DFQ) in units of nanosec/days^2. Diurnal "scatter" is due to station location error.

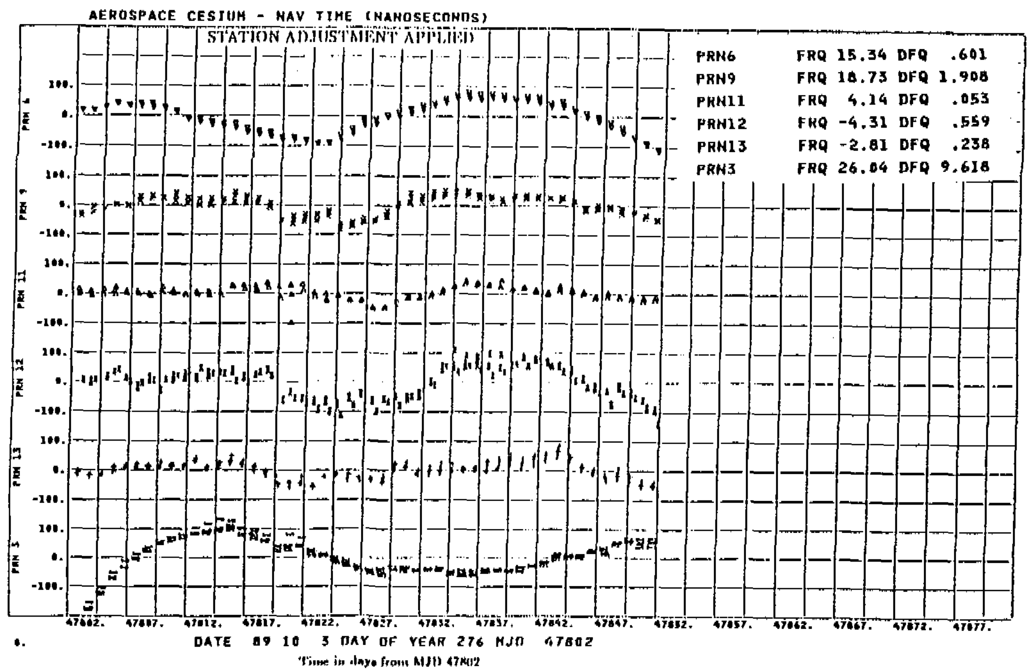

Figure 4. Clock phase residuals between ground cesium and each NAVSTAR PRN. The table on the upper right corner of figure lists the frequency offset (FRQ) in units of picosec/sec. and ageing (DFQ) in units of nanosec/days^2. Diurnal "scatter" considerably reduced by using same station locations values as used in Figure 2. and listed in Table 1.