

# THE CLASSICAL MICROWAVE FREQUENCY STANDARDS

Giovanni Busca, Pierre Thomann, Laurent-Guy Bernier,  
Philippe Willemin and Hartmut Schweda  
Observatoire de Neuchâtel  
Rue de l'Observatoire 58  
CH-2000 NEUCHATEL/Switzerland

## 1. Introduction

In this paper we present some key problems encountered in the classical microwave frequency standards which are still not solved today. The point of view expressed here benefits from the experience gained both in the industry and in the research lab, on the following classical microwave frequency standards: active and passive H, conventional and laser pumped Cs beam tube, small conventional and laser pumped Rubidium.

The accent is put on the Rubidium standard, the other topics being covered in the following papers. Table 1 presents a simple comparison among the microwave standards.

## 2. Conventional Rubidium Standards

Models for predicting signal, noise and linewidth are available [1]. However only crude calculations exist concerning the output frequency [2]. The basis of this calculation is a linearization of the equation of the isotopic filter induced optical displacement  $\Delta\nu_{op}$  given in REF [3] and a linearization of the equation for light absorption.

The light-shift  $\Delta\nu_{LS}$  experienced by the first layer of atoms in the absorption cell can be expressed with small changes from REF [2], as

$$\Delta\nu_{LS} = I_o [1 - K_o(\theta - \theta_o)] K_1(\theta - \theta_o) \quad (1)$$

where  $\theta$  is the filter temperature

$I_0$  is the light intensity impinging on the filter

$I_0[1-K_0(\theta-\theta_0)]$  the light intensity at the output of the filter and entering the Rubidium cell,

$K_1$  a light shift parameter depending upon the spectral shape of the pumping line and

$\theta_0$  the filter temperature for which the light shift goes to zero.

With the same meaning of the parameters, considering a mixture of isotopes in the lamp and, for the integrated filter approach, a mixture of isotopes in the cell, we have the general equation giving the frequency vs light intensity and filter cell temperature:

$$\nu = \nu_0 - I_0^- \left[ 1 - K_0^- (\theta - \theta_0^-) \right] K_1^- (\theta - \theta_0^-) + I_0^+ \left[ 1 - K_0^+ (\theta - \theta_0^+) \right] K_1^+ (\theta + \theta_0^+) + \delta_m (\theta - \theta_0^-) + \gamma_m (\theta - \theta_0^-)^2 \quad (2)$$

where:

$\nu_0$  is the unperturbed frequency (independent from the temperature and light intensity)

the index - refers to Rb<sup>87</sup> isotope

the index + refers to Rb<sup>85</sup> isotope

$\theta$  is the filter-cell temperature

$\delta_m$  and  $\gamma_m$  are respectively the linear and quadratic temperature dependence given by the buffer gas mixture [2].

Eq. (2) explicits the fact that the Rb<sup>87</sup> gives, for normally used cell temperatures  $> 65^\circ\text{C}$ , a negative light shift; the Rb<sup>85</sup> gives a positive light shift which never goes to zero.

The parabolic behaviour given in eq. (2) has a minimum for a temperature  $\theta = \theta_{\min}$  given by

$$\theta_{\min} - \theta_0^- = \frac{1 + \frac{K_0^+(\theta_0^+ + \theta_0^-)}{p_i^{-1}} - \frac{\delta_m}{I_0[\alpha_i K_1^- - (1-\alpha_i)K_1^+]}}{\frac{2K_0^-}{p_i^{-1}} \left( p_i - \frac{K_0^+}{K_0^-} \right) + \frac{2\gamma_m}{I_0[\alpha_i K_1^- - (1-\alpha_i)K_1^+]}} \quad (3)$$

where

$$I_0 = I_0^- + I_0^+$$

$$\alpha_i = I_0^- / I_0$$

(4)

$$(1-\alpha_i) = I_0^+ / I_0$$

$$p_i = \alpha_i K_1^- / (1-\alpha_i) K_1^+$$

$p_i$  represents the ratio (absolute value) of the negative to the positive light shift, the index  $i$  meaning that the ratio is taken at the input of the filter-cell.

Eq. (3) shows that  $\theta_{\min}$  depends upon the total light intensity  $I_0$  emitted by the lamp if  $\delta_m$  and  $\gamma_m$  are different from zero.

The eq. (2) can be expressed in terms of a measurable output parameter, i.e. light intensity  $I$  transmitted by the cell.

$$\nu = \nu_0 - \alpha_0 I K_1^- (\theta - \theta_0^-) + (1-\alpha_0) I K_1^+ (\theta + \theta_0^+) + \delta_m (\theta - \theta_0^-) + \gamma_m (\theta - \theta_0^-)^2 \quad (5)$$

where

$$I_o^- \left[ 1 - K_o^- (\theta - \theta_o^-) \right] = I^-$$

$$I_o^+ \left[ 1 - K_o^+ (\theta - \theta_o^-) \right] = I^+ \quad (6)$$

$$I = I^- + I^+$$

$$I^- = \alpha_o I$$

$$I^+ = (1 - \alpha_o) I$$

The meaning of index o is that we are referring to output parameters.

Eq. (5) shows that the frequency is a linear function of the total transmitted light in agreement with experiments [2].

From eq. (5) by differentiating with respect to the transmitted light intensity I one obtains that  $\partial\nu/\partial I = 0$  for  $\theta = \theta_{LS=0}$  given by:

$$\theta_{LS=0} - \theta_o^- = \frac{\theta_o^+ + \theta_o^-}{p_o - 1} \quad (7)$$

$$\text{where } p_o = \frac{\alpha_o K_1^-}{(1 - \alpha_o) K_1^+} \quad (8)$$

$p_0$  has the same meaning as  $p_i$  but referred now to the transmitted total light intensity.

For obtaining good standard performances

$$\theta_{\min} = \theta_{LS=0}$$

this relation can be realised by choosing the isotopic ratio in the lamp and in the cell and in addition the buffer gas mixture.

For one commercial Rubidium this occurs at a filter-cell temperature of 75°C [2].

The model shows that the main limitation of the conventional Rubidium is that temperature coefficient and light shift compensation occurs only for a well defined light intensity and well defined cell-filter temperature.

All the parameters occurring in the previous equations are important for a Rubidium drift model.

### 3. Laser Pumped Rubidium

From experiments in our laboratory, and using a laser power corresponding to the "saturated" light shift [4] the following data are obtained:

- a) The light shift vs the laser frequency is  $5 \times 10^{-11}$ /MHz or a Rubidium stability of  $10^{-11} \tau^{-1/2}$  requires a laser stability of  $5 \times 10^{-10} \tau^{-1/2}$ .
- b) The light-shift vs the laser power is:  $\sim 3 \times 10^{-12}$ /%.
- c) The laser locked to the Rubidium cell has a cell temperature induced frequency shift of 9 MHz/°C [4].

- d) From c) and a) the laser servo produce an additional cell temperature coefficient of  $\sim 4.5 \times 10^{-10} / ^\circ\text{C}$ .
- e) In the relevant Fourier frequency range (100 Hz - 1 kHz), the laser intensity noise is  $> 10^{-6} / \sqrt{\text{Hz}}$  while shot noise is  $< 10^{-7} / \sqrt{\text{Hz}}$ .
- f) Aging of the laser parameters in single mode operation, locked to the Rb cell, is commonly experienced.

In view of points a) to f) it appears not a simple work to reach the predicted performance [5]  $\sigma_y(\tau) = 3 \times 10^{-14} \tau^{-1/2}$ .

However when all the previous problem will be solved we will certainly see the realization of the promise of the laser pumping: a very small, low power and high performance Rubidium standard.

#### 4. Cs standards

As a comment to the Table 1 we like to remark that the Cs standard has the lowest intrinsic temperature coefficient, no significant drift and no significant drift mechanism compared to the other standards. In addition, recent advances in a microprocessor controlled electronics [6] have still produced remarkable improvements of the environmental characteristics. One problem to be solved is the aging of the signal due to the degradation of the electron multiplier. This problem exists for high gain ( $\sim 10^5$ ) and low noise figure ( $\sim 1$  dB) multipliers. For low gain and high noise figure multipliers the problems seem solved [7]. However in this case much higher atomic flux is required for obtaining the same short term stability, i.e. the tube lifetime is shortened.

The small optically pumped Cs is appealing. The demonstrated short term stability is  $< 10^{-11} \tau^{-1/2}$  [8]. The potential is: small size and light weight. The problems to be solved: the laser frequency noise and laser aging.

## 5. H-Maser

Relatively to the H-Maser we believe that the problem of the frequency drift is still open. In this context, the cavity pulling has drawn a lot of attention. Today, to our knowledge, 4 different methods of automatic cavity tuning are used for maser in the field: The SE tuning [9], the magnetic relaxation tuning [10], the cavity frequency switching [11] and the auxiliary mode stabilization [12]. Many more have been proposed and tested [2]. As a conclusion we consider that the long term maser frequency drift associated with the cavity pulling is a well known subject due to the high level of precision obtainable in principle by these methods. On the contrary wall shift drift is still very poorly known [13].

From our experience it appears illusory to derive conclusions on wall shift drift vs time a) for masers which are not in continuous operation, b) masers which show a signal amplitude decay which is normally correlated with the line Q decay, c) masers which suffer from magnetic relaxation requiring neck coil current trimming, in order to get the full power operation at a low C field of  $< 1$  mG.

Maser amplitude, line Q and magnetic relaxation should be monitored in addition to the cavity frequency for correct interpretation of the frequency drift data.

As a conclusion wall shift and the associated wall relaxation is still the main problem to be solved.

### Acknowledgments

We thank the Swiss Commission CERS, the European Space Agency and the Swiss Federal Office of Metrology for supporting our work on the atomic frequency standards. We acknowledge also the cooperation of Oscilloquartz S.A. for discussing data on Cs beam tubes.

TABLE 1. CLASSICAL  $\mu$ -WAVE STANDARDS: WHERE DO WE STAND?

	Rubidium Lamp      Laser		Cesium (small) magnetic    optical		H active
Short-term stability	$\sim 10^{-11} \tau^{-1/2}$	?	$10^{-11} \tau^{-1/2}$	$3 \cdot 10^{-12} \tau^{-1/2}$	$10^{-15}$ (1000 s)
S/N	85 dB	?	1000	20000	--
Q	$10^7$	$10^7$	$2 \cdot 10^7$	$1 \cdot 10^7$	$> 2 \cdot 10^9$
Temp. coeff.	$10^{-12}/K$ th. gain: 100		$< 10^{-13}/K$		$< 3 \cdot 10^{-14}/K$ th. gain: $10^2 - 10^4$
Light-shift coeff.	$5 \cdot 10^{-11}/\%$	$10^{-12}/\%$	--	--	--
Magnetic coeff.		$2 \cdot 10^{-11}/G$		$< 10^{-12}/G$	$< 10^{-13}/G$
Long-term stability	$10^{-11}/\text{month}$	?	$3 \cdot 10^{-12}/\text{Life}$		
Life time	$> 5$ years	?	$3 \div 10$ years	?	?



## References

- [1] J.C. Camparo: Proc. of the 4th Symposium on Frequency Standards and Metrology, Ancona 1988, p. 62.
- [2] J. Vanier and C. Audoin: The Quantum Physics of Atomic Frequency Standards (A. Hilger, 1989).
- [3] G. Busca, M. Têtu, J. Vanier: Can. J. Phys. 51, (1973), 1379.
- [4] J.T. Liu et al.: Proceedings of the 4th European Frequency and Time Forum, Neuchâtel, 13-15 March 1990, p. 157.
- [5] J.C. Camparo, R.P. Frueholz: J. Appl. Physics 59, (1986), 3313.
- [6] J. Rabian, P. Rochat: Proceedings of the 2nd European Frequency and Time Forum, Neuchâtel, 16-18 March 1988, p. 461.
- [7] R.P. Frueholz et al.: Proceedings of the 18th Annual Precise Time and Time Interval Applications and Planning Meeting, (1986), p. 11.
- [8] V. Candelier et al.: Proceedings of the 2nd European Frequency and Time Forum, Neuchâtel, 16-18 March 1988, p. 483.
- [9] L.J. Rueger, J. Instn. Electronics and Telecom. Engrs 27 (1981) 493.
- [10] N. Demidov, A. Uljanov: Design and Industrial Production of Frequency Standards in USSR, these Proceedings, paper no. 13.
- [11] H.E. Peters et al.: Hydrogen Masers with Cavity Frequency Switching Servos, these Proceedings, paper no. 22.
- [12] G. Busca and L. Johnson: Proceedings of the 1st European Frequency and Time Forum, Besançon, 18-20 March 1987, p. 339.
- [13] E.M. Mattison and R.F.C. Vessot: Proceedings of the 41th Annual Frequency Control Symposium, 1987, p. 95.

## QUESTIONS AND ANSWERS

**Dave Wineland, National Institute of Standards and Technology:** Concerning the light shift with the lasers on the rubidium standard; what about chopping the laser light to get rid of the light shift?

**Mr. Thomann:** Yes, of course that is a method that has already been proposed with normal lamps. It could more easily be done with lasers because it is very easy to chop lasers, at least in principle. I don't know of anybody that has tried that, but it is certainly worth trying. One problem is that we have to keep the laser frequency locked at the same time as we chop the intensity. This could be a problem. Of course we don't want to use involved choppers in commercial rubidium standards which are traditionally the cheapest available, but there is a choice: we could make a laboratory standard with elaborate techniques, but for a commercial device one should keep with very simple techniques.