# AN UNBIASED WEIGHTING METHOD FOR DATA IN GPS COMMON VIEW

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## *ABSTRACT*

***The mode parameters for GPS common view time transfer is estimated and an unbiased weighting method (UBWA) for data in GPS common view is discussed in this paper. The one Σ jitter of each time series is determined by the standard deviation of residuals of polynomial regression, which define weights used to compute a weighted average. Averaging the double difference observables during an integral length (30 days), the systematic biases existing between every two time series are corrected in constant monthly relative to a reference time series. The GPS common view data among NRC in Ottawa, NBS in Boulder and USNO in Washington DC, are calculated by using three different methods: UBWA, equal weights average method (EWA), and direct weighting method (DWA). The final weighted averages are compared in terms of Σ (standard deviation of residual), Allan variance (frequency stability) and time discontinuity (caused by increasing or decreasing time series). Results showed that UBWA is superior to the others.***

## ESTIMATE FOR MODE PARAMETERS

GPS single difference observables between stations reflect receiver clock differences between stations directly and apply to high precision time comparison widely, i.e. the so called common view mode. Its model equation can be expressed as;

$$t_{BAj}(\tau) = t_{Aj}(\tau) - t_{Bj}(\tau) =$$

$$1/c\{R_{Bj}(\tau)\} + ((f_B - f_j)/f_j - 9f_A - f_j)/f_j)(\tau - \tau_0) + t_{AB}(ion) + t_{AB}(trop) \qquad (1)$$

where A and B refer two stations, i refers the satellite, $\tau$ is the epoch for measurement. $t_{Aj}(\tau)$ and $t_{Bj}(\tau)$ are the time readings at stations A and B respectively, and the corrections have been made on these readings for the offset of the receiver system time delay. $R_{Aj}$ and $R_{Bj}$ are the distances from satellite j to stations A and B respectively. $f_A$ and $f_B$ are the receiver clock frequencies at stations A and B respectively. $f_j$ is the satellite clock frequency. $t_{AB}(ion)$ and $t_{AB}(trop)$ are the differenced ionosphere error correction and the differenced troposphere error correction, respectively.

A common view track which repeats every sidereal day defines a time series comparing the clocks at the two stations. Each time series represents the time measurement of signal propagation over a specific path from one station to the satellite to another station. Change in path is caused by different satellite or different measurement time, and forms different time series. Some experiments have reported that the different time series between two stations has not only the different time jitter, but also obvious systematic bias. Through analyses of GPS various difference observables, the reason for existing jitters and biases could be understood. For different measurement geometry and measurement epoch, GPS various error sources which include ephemeris, ionospheric and tropospheric models, receiver coordinates as well as frequency offset between the reference clocks have different effect on GPS single difference observable.

Assuming GPS common view time series is a constant linear discrete time system. For a specific common view time series i, $Z_i(k)$ is defined as the optimal estimate of the $i^{th}$ common view measurement $X_i(k)$ the observation equation is:

$$X_i(k) = \Phi_i Z_i(k) + U_i(k) + E_i(k) \tag{2}$$

Where, for one dimension scalar time comparison, the coefficient matrix $\Phi_i = (1, 0)$: i refers to the $i^{th}$ common view measurement $(i = 1, n)$, k refers to the $k^{th}$ day $(k = 1, m)$, $U_i(k)$ is the systematic part of the error which includes the frequency offset and drift between the reference clocks, propagation error, ephemeris error as well as receiver coordinates error. $E_i(k)$ is the random perturbation part of error which includes the random perturbation of atomic clocks as well as the measurement noise caused by propagation, ephemeris and receiver.

The systematic bias $U_i(k)$ between the $i^{th}$ time series and the $j^{th}$ time series is determined by the GPS double difference observable $t_{ABij}(\tau_{ij})$ over an integral data length (about 30 days) and the average value is Bias which is used to perform correction in constant relative to reference time series. The simulation of the deterministic properties of a time series is using polynomial,

$$X_i = a + bT_1 + bT_2^2 + \ldots + bT_n^n \tag{3}$$

App]ying linear regression to measurement data set $T_k$ and $X_i(k)$, the coefficients of the polynomial are determined by least mean-square estimate. Thus $Z_i$, the optimal estimate of the measurement $X_n$, is obtained. It is important to examine the confidence in the result by taking the residual $\Delta(k) = x_i(k) - Z_i(k)$ as the estimate of $E_i(k)$.

First, the sampling time, $\tau$, for common view time series is usually one day. Assuming $E_i$ is Gaussian noise process, it includes mainly the white frequency noise and flicker frequency noise. So, $E_i$ is well modeled by equation (4), Allan variance. The mean squared time error is expressed by equation (5).

$$\sigma_y^2(\tau) = (h_0/2)\tau^{-1} + h_{-1}2\ln(2) \tag{4}$$

$$\epsilon^2(\tau) = h_0\tau + 4h_{-1}\ln(2)\tau^2 \tag{5}$$

When the calibration interval $\tau_c = \tau = 1\ \mathrm{day}$, $\epsilon^2$ is twice as large as $\sigma_y^2$, and it is not divergent.

Secondly, the typical residuals curve for a common view data length of about 135 days is in Figure 1. Let

$$X = [(\Delta(k) - E_m)/\sigma] = 300 \tag{6}$$

$$F(x) = \sqrt{(2\pi)^{-1} e^{-x^2/2}} \tag{7}$$

where, mean value $E_m = 0$ so, the normalized distribution curve $[F(x) - X]$ is in Figure 2. It conforms obviously normal distribution $N(0, \sigma)$.

Hence it is reasonable to have $\sigma$, the standard deviation of residuals as the one sigma jitter of time series. RMS, the standard deviation of GPS double difference gives an estimate of measurement noise of double difference observables. The variance of double difference between the $i^{th}$ and the $j^{th}$ common view time series is $S_{ij}^2$. Applying the N-corner hat technique, the variance estimate $\mathrm{RMS}^2$ of the $i^{th}$ can be completed:

$$\mathrm{RMS}^2 = (n-2)^{-1}\left(\sum_{j=1}^{n} S^2 - B\right) \tag{8}$$

$$B = (2n-4)^{-1}\left(\sum_{(k=1)}^{n}\sum_{j=1}^{n} S_{kj}^2\right)(S_{jj}^2 = 0) \tag{9}$$

Thus, RMS can be considered as the estimate of measurement noise $V_k$.

Bias, Σ and RMS are three major parameters of GPS common view time series. The common view data from MJD47011 to MJD47278 among NRC, NBS and USNO are divided and calculated monthly. Table 1 gives the data status. Bias, Σ and RMS are calculated in each data section. Taking an example, Table 2 lists the values of Bias, Σ and RMS for NRC-USNO link and data sections from 1 to 5.

The results in Table 2 are significant.

1. The Biases of time series over five data sections are quite stable the variable value is only several nanoseconds. By using linear regression, the average correlation coefficients are 0.15 and 0.27 for Σ-Bias and RMS-Bias respectively. This indicates a fact that systematic Bias is independent of random perturbation, hence it is acceptable to correct bias in constant.

2. Usually the Σ value is larger than the RMS value. The Σ value seems to be correlative to the data section, e.g. the Σ values for all twelve time series become large in data section 3, but the RMS value is not correlative to the data section. These conform the above analyses: Σ is the sum of system noise and measurement noise, while RMS is mainly the measurement noise.

3. The values of Σ and RMS are correlative to the time series basically, and the Bias for each time series is quite stable. Thus, the time series could be referred to as independent each other and to weight in one over variance is optimum. Because the correlation coefficient of Σ-Bias is small than that of RMS-Bias, to have Σ as one sigma jitter of time series is preferred.

4. The above results are obvious not only for NRC-USNO link but also for NRC-NBS link.

## UNBIASED WEIGHTING METHOD

Based on the above analysis, an algorithm for processing data of GPS common view time comparison is recommended to be referred to as Unbiased Weighting Method (UBWA). It has the following features.

1. Averaging the double difference observables during an integral length (~30 days), the systematic biases existing between every two time series are corrected in constant monthly relative to a reference time series.

2. The one sigma jitter of each time series is determined by the standard deviation Σ of residuals of polynomial regression, which defines weights in one over Σ square.

3. Using raw measurement data at all, no interpolation or extrapolation for losing data points. For bad points, it can be identified and removed by computer program.

The steps and formulae for UBWA are as follows. The $k^{th}$ day and the $i^{th}$ path common view data between two stations is $X_{i(k)}$, and the $i^{th}$ path common view data form a time series:

$$X_i(k) \qquad\qquad (k = 1, m) \qquad\qquad (10)$$

Where, m is total measurement days, in practice M=30 as a data section. i=1, n, n is the number of common view paths each day between two stations. So total has n time series. Do least square fit for the $i^{th}$ time series and calculate the standard deviation $\Sigma_i$ of residuals, then the optimum weight for the $i^{th}$ series is:

$$W_i = 1/\Sigma_i^2 \qquad\qquad (11)$$

Choose a reliable and continuous time series as reference one, namely the $j^{th}$. The double differences for all other series relative to reference one can be calculated,

$$T_{ij}(k) = X_i(k) - X_j(k) \qquad (i = 1, n-1;\ j \neq i;\ k = 1, m) \tag{12}$$

The average systematic bias $S_i$ between the $i_{th}$ series and $j_{th}$ series is:

$$S_i = \sum_{k=1}^{m} T_{ij}(k)/m \tag{13}$$

The $S_i$ value is used to correct the daily data of the $i^{th}$ time series,

$$\left.\begin{array}{ll} X_{ik} = X_i(k) + S_i & (i \neq j;\ k = 1, m) \\ X_{ik} = X_i(k) & (i = j;\ k = 1, m) \end{array}\right\} \tag{14}$$

Considering the losing data points, the daily weight for the $i^{th}$ time series needs to be calculated,

$$\left.\begin{array}{ll} W_{ik} = W_i & X_i(k) \neq 0 \\ W_{ik} = 0 & X_i(k) = 0 \end{array}\right\} \tag{15}$$

So,

$$AW_{ik} = W_{ik} / \sum_{i=1}^{n} W_{ik} \tag{16}$$

Finally, the weighted average of n time series at the measurement epoch of the $j^{th}$ time series can be obtained,

$$X_k = \sum_{i=1}^{n} AW_{ik} X_{ik} \qquad (k = 1, m) \tag{17}$$

## RESULTS

The common view data among NRC, NBS and USNO are computed by using three different methods from data files extending for a length of about nine months. The time series of final weighting average are compared in three performance characteristics. The three methods are: Unbiased Weighting method (UBWA), Direct Weighting method (DWA) which does not correct the systematic bias, as well as Equal Weights method (EWA). The three characteristics are: the standard deviation $\Sigma$ of residuals of polynomial regression, the frequency stability Allan variances at different sampling time and the time discontinuity caused by increasing or decreasing the number of time series. Results showed that UBWA is prior to the others.

1. Data file is divided into ten time sections showed in Table 1 and computed in three methods monthly. The weighted average series UBWA, DWA and EWA are obtained respectively. The standard deviations $\Sigma$ of residuals of three time series are listed in Table 3. The UBWA results are better than the others, they are about 4.9 ns for NRC-NBS link and about 2.4 ns for NRC-USNO link. The $\Sigma$ value is the indication of time perturbation of time series.

2. The frequency stability Allan variances at sampling times of 1, 2, 3, 7, 12, 16, and 20 sidereal days are calculated for UBWA, DWA and EWA weighted average time series respectively. Results are plotted in Figure 3 which has coordinates log $\log\sigma_y(\tau) - 10\log\tau$ and $\tau$ in seconds. UBWA series has the best frequency

stabilities. The $\sigma_y(\tau)$ is better than 1 part in $10^{13}$ for sampling times of five days and longer, this meets the requirements of high precision time and frequency comparison between timing laboratories.

3. Because of the existence of systematic bias, the time discontinuity could appear at the conjunction point between two consecutive data section when the weights of time series changed largely or the number of time series increased or decreased especially. Due to correction of Bias. this effect should be not obvious for UBWA method. During data sections 1 to 5 from MJD 47144 to 47278, the number of time series was changed intentionally to exam the possible time discontinuity. The computed results are listed in Table 4 and the time discontinuity is the smallest for UBWA method.

4. In UBWA method, it is required to choose a reference time series and to correct systematic biases for other time series relative to the reference one. For large geographic area or multistation GPS common view data, it is not possible to find a common reference time series. But, this can be solved by using reference time series relay method, and time accuracy could be ensured.

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Table 1. Data status

| section | MJD | Interval (days) | No. of time series |
|---|---|---|---|
| 1 | 47248-47278 | 31 | 12 |
| 2 | 47218-47248 | 31 | 12 |
| 3 | 47188-47218 | 31 | 12 |
| 4 | 47158-47188 | 31 | 12 |
| 5 | 47144-47158 | 15 | 12 |
| 6 | 47113-47143 | 31 | 10 |
| 7 | 47083-47113 | 31 | 10 |
| 8 | 47053-47083 | 31 | 10 |
| 9 | 47023-47053 | 31 | 10 |
| 10 | 47011-47023 | 13 | 10 |

Table 2. Parameter estimate for common view time series (NRC-USNO) (ns)

| | Path | 031 | 061 | 091 | 092 | 093 | 111 | 112 | 113 | 121 | 122 | 123 | 131 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bias | 1 | | -4 | 23 | -13 | 14 | -1 | 17 | 23 | 20 | -18 | 20 | 16 |
| | 2 | | -5 | 24 | -13 | -12 | -4 | -16 | 22 | 18 | -16 | 20 | 20 |
| | 3 | | -1 | 28 | -7 | -13 | -4 | -15 | 22 | 24 | -18 | 25 | 16 |
| | 4 | | -2 | 24 | -13 | -14 | -5 | -16 | 23 | 21 | -19 | 22 | 17 |
| | 5 | | 3 | 22 | -13 | -7 | 0 | -14 | 25 | 21 | -16 | 21 | 19 |
| Σ | 1 | 3.6 | 3.6 | 3.4 | 4.1 | 4.6 | 3.4 | 3.8 | 3.4 | 4.6 | 4.5 | 5.1 | 3.7 |
| | 2 | 3.8 | 2.5 | 3.6 | 3.1 | 2.6 | 4.6 | 3.3 | 3.4 | 4.2 | 3.3 | 4.9 | 3.3 |
| | 3 | 8.6 | 8.4 | 12.5 | 7.4 | 8.9 | 8.0 | 9.9 | 8.6 | 9.4 | 9.2 | 10.2 | 6.9 |
| | 4 | 4.8 | 5.4 | 5.2 | 5.2 | 4.9 | 4.5 | 5.3 | 4.0 | 4.8 | 3.4 | 5.3 | 5.6 |
| | 5 | 5.2 | 4.0 | 4.9 | 4.2 | 5.8 | 4.5 | 4.7 | 6.0 | 6.1 | 3.3 | 6.1 | 4.8 |
| RMS | 1 | 2.8 | 3.6 | 3.3 | 3.2 | 4.2 | 4.0 | 3.0 | 1.6 | 4.4 | 3.5 | 4.2 | 3.4 |
| | 2 | 3.1 | 2.1 | 5.2 | 4.5 | 2.8 | 4.4 | 3.0 | 2.7 | 3.6 | 2.7 | 4.7 | 3.0 |
| | 3 | 2.7 | 2.1 | 5.8 | 2.3 | 2.6 | 2.2 | 4.8 | 2.9 | 2.0 | 3.3 | 5.0 | 3.8 |
| | 4 | 3.2 | 3.2 | 4.4 | 2.8 | 2.5 | 1.9 | 4.6 | 2.6 | 5.6 | 2.4 | 4.5 | 3.0 |
| | 5 | 1.6 | 2.6 | 5.8 | 4.9 | 7.8 | 1.5 | 1.7 | 2.1 | 4.8 | 2.2 | 3.0 | 3.4 |

Table 3. Σ values of weighted average time series

| section | NRC--NBS (ns) | | | NRC-USNO (ns) | | |
|---|---|---|---|---|---|---|
| | UBWA | DWA | EWA | UBWA | DWA | EWA |
| 1 | 5.04 | 4.94 | 5.08 | 2.28 | 2.93 | 3.02 |
| 2 | 3.79 | 5.24 | 5.24 | 1.73 | 1.73 | 1.81 |
| 3 | 10.15 | 11.86 | 11.62 | 8.11 | 8.16 | 8.53 |
| 4 | 3.87 | 4.08 | 7.02 | 3.56 | 3.73 | 3.78 |
| 5 | 2.93 | 3.71 | 3.45 | 4.02 | 4.18 | 4.34 |
| 6 | 5.68 | 5.91 | 5.97 | 4.11 | 5.32 | 5.82 |
| 7 | 6.90 | 7.48 | 7.75 | 3.26 | 3.74 | 3.67 |
| 8 | 4.38 | 4.54 | 4.68 | 2.76 | 3.82 | 5.10 |
| 9 | 4.18 | 7.82 | 9.87 | 4.13 | 4.26 | 4.29 |
| 10 | 1.66 | 2.60 | 2.71 | 0.44 | 4.94 | 4.10 |
| Σ | 4.86 | 5.82 | 6.34 | 3.44 | 4.28 | 4.45 |

Table 4. Time Discontinuity (nanoseconds)

| | Data section | 5 -> 4 | 4 -> 3 | 3 -> 2 | 2 -> 1 |
|---|---|---|---|---|---|
| | Number of time series | 12 -> 10 | 10 -> 12 | 12 -> 10 | 10 -> 12 |
| | Conjunction Date | 47158 | 47188 | 47218 | 47248 |
| NRC | UBWA | +1.7 | -2.0 | +1.9 | -2.0 |
| / | DWA | -1.7 | -7.3 | +2.3 | +3.2 |
| NBS | EWA | +4.2 | -4.2 | +4.0 | -3.7 |
| NRC | UBWA | -2.0 | +2.4 | -0.4 | -1.1 |
| / | DWA | 2.2 | -1.2 | +5.4 | -5.2 |
| USNO | EWA | +3.6 | -2.3 | +3.5 | 0.7 |

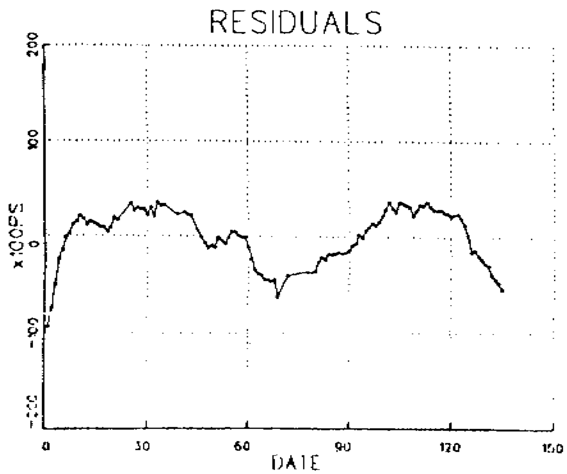


Figure 1. Residuals of time series in GPS common view

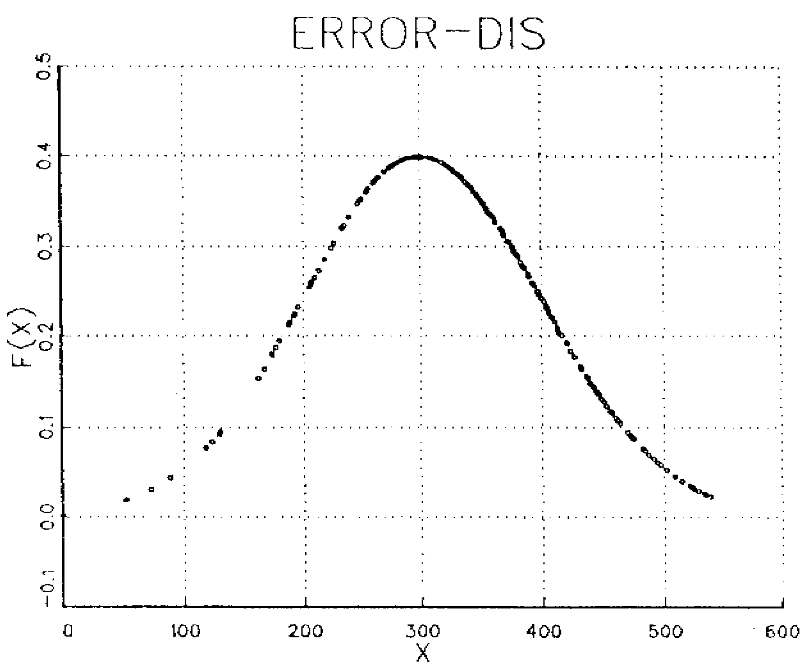


Figure 2. Normal distribution of residuals

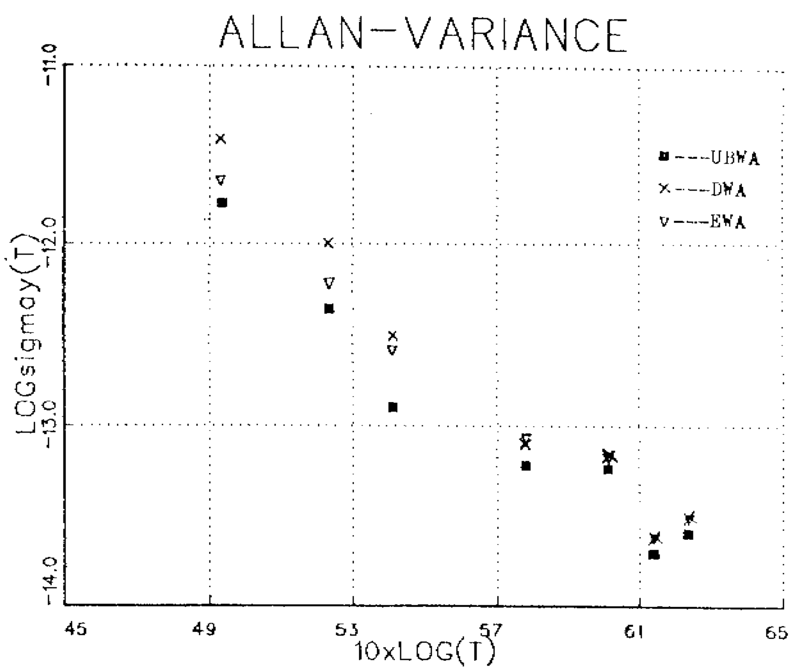


Figure 3. Allan variance results for UBWA, DWA and EWA