ALTERNATIVE TIMING NETWORKS WITH GPS

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Abstract

An approach to determining accurate time from GPS with an independent network of receiving stations has been investigated. The methods of using the Global Positioning System (GPS) for transferring time in previous work has been by the "common view" and "melting pot" methods. Both of these techniques have used simplified single frequency receivers operating on the clear/acquisition (C/A) GPS codes and assumes that the satellite transmissions are quality observables producing "GPS time", accurately traceable to UTC(USNO). In the case of "common view", the position of the satellite is assumed to be accurately known from the satellite transmissions. Then the time delays due to position at the two common view sites may be accurately measured for time comparisons. In the "melting pot" method, an individual site measures "GPS Time" as determined from observing all GPS satellites in view resulting in an accurate over–solution of the GPS system time. The satellite broadcasts then provide the UTC – GPS time correction. The investigation into an independent network was performed on the basis of using the simplified C/A receiving equipment to produce accurate timing information regardless of the GPS broadcast information accuracy. The technique can be used to improve the inherent capabilities of these single frequency receivers or maintain accuracy with degraded GPS signals. The similarities with geodetic positioning using GPS will be described. A proof–of–concept experiment will be discussed and data presented to verify the technique.

INTRODUCTION

Two closely related techniques have been developed over the past twenty years to obtain time and/or position information from range or range–like data between ground stations and satellite positions, with no need to model the satellite orbits dynamically. The more recent is the common view technique. International synchronization of clocks in widely separated timing laboratories has been achieved using GPS space vehicles (SVs) in common view of the laboratories. Two or more stations receive almost simultaneously the same pulse from one SV. They record the times of arrival against their clocks, and correct for the differences in propagation times from the SV to the two different stations by knowing the SV position. The behavior of the SV clock is immaterial, since the data is differenced and the SV clock contribution, being the same at both sites, is eliminated. The radial distance of the SV from the Earth's center is a common factor, and only a small component of the SV along–track and cross–track error corrupts the determination of the time offset between the two ground clocks. Common view was developed by the Time and Frequency Division of the National Bureau of Standards (NBS), now the National Institute of Standards and Technology (NIST)[1,2]. Since 1986, the responsibility of scheduling and coordinating common view observations between international timing observatories has been assumed by the Bureau International des Poids et Mesures (BIPM) in Paris[3].

The other, older technique for using ranging data for positioning is multilateration. A number of ground stations (M) simultaneously range to N SVs, that is, either to several SVs or to several points on the orbit of one SV. The ranges determine the coordinates of all the points, on the ground and in the sky, relative to one another. A relative reference frame is then established with the coordinates of three stations. Station 1 will be the origin of the relative coordinates and the reference planes established by the positions of the other stations used to range to six SV's. In this coordinate system the number of observations (24) will equal the number of unknowns (also 24, namely the 30 spatial coordinates of the 10 points, minus the 6 coordinates which are fixed). It has been shown in the extensive literature on multilateration[4–9], that good solutions can be obtained for all the relative spatial coordinates, if certain singular geometric configurations of the points are avoided, e.g., in this case, if the ground stations do not lie all on one plane. Common view is a variant of multilateration, in which the spatial coordinates have been held fixed, and time coordinates are added and solved for. The theory of multilateration developed for geodesy can therefore be applied to the practical problems of time transfer.

DYNAMIC *VS* GEOMETRIC METHODS OF MEASUREMENT

The two basic methods of using artificial satellites for geodesy are the dynamic and the geometric. Dynamic modeling is often the more practical. A model of all the forces known to be acting on the SVs is used to develop a theory or a numerical model of their motion, and the parameters of the model are fitted to the range data. For GPS, this is done routinely by the National Geodetic Survey, the Jet Propulsion Laboratory, and the Center for Space Research of the University of Texas[10–12]. Dynamic modeling is the method of choice when, (i) the force model is well known, or (ii) anomalous forces acting on the SVs are themselves of primary interest, and (iii) data are non–simultaneous and from widely separated locations. Conversely, geometric methods are better when (i) the force model is incomplete, (ii) improving the force model is not the task at hand, and (iii) when observations can be made simultaneously from stations of suitable number and distribution. Multilateration is the technique of measuring ranges to targets in unknown positions from several locations simultaneously. It treats every SV point location as independent of every other, uses no dynamic modeling, and therefore requires no force model or software for trajectory integration. Computer programs for multilateration tend to be very simple. Of course, hybrid dynamic/geometric systems are also used, e.g., short–arc solutions[9].

Common view has two special advantages for international timekeeping. First, the necessary software can be kept very simple. Second, one need not handle precise ephemerides of GPS satellites which may not be readily available. A simple modification of the common view technique retains these advantages, even when the broadcast ephemeris is not sufficiently accurate to be useful. In geodesy, there are at most three unknowns per point, the three space coordinates. Using GPS, we have clock offsets to determine, so the number of unknowns increases from three to four. Nevertheless, using enough stations, solutions can always be found for all the unknowns. The basic reason could be called "the fundamental theorem of multilateration". Given any values of A (station unknowns), B (SV point unknowns), and C (unknowns defined as reference knowns), there always exists a pair of numbers (M, N), (M stations, N SV points) such that,

$$MN \geq AM + BN - C$$

so that the number of measurements exceeds the number of unknowns. But will the normal equation matrix be well conditioned? Every application of multilateration gives rise to a particular mathematical case, which must be specially examined.

FIVE CASES OF MULTILATERATION/COMMON VIEW

Designing a common view time transfer experiment addresses three questions:

1. Do we solve for time offsets only, or also correct SV and/or station coordinates?
2. If we solve for spatial coordinates, then do we correct SV coordinates only, or station locations as well?
3. Do we solve for three dimensions, or only for two?

For, if all stations are in a common plane and an SV passes nearly through that plane, good solutions can be obtained by estimating only two spatial coordinates of each point[13–14].

Logically, these three questions give rise to five possible cases. For each case, there is a minimum number of stations and SVs which will give a solution.

CASE A: Stations in known locations measure pseudo–ranges to SVs in known locations and estimate time offsets only.

This is the classical common view case. Take station 1 as the reference station, whose time is known by definition. The remaining M-1 stations contribute M-1 unknowns, their clock offsets at the moment of measurement. Each SV contributes one unknown, its clock offset from station 1. Two stations ranging to one SV are sufficient.

CASE B: Stations in known locations measure pseudo–ranges to SVs in poorly known positions. Corrections to SV coordinates (X, Y, and Z) must be estimated as well as all time offsets relative to station 1. Five stations ranging to four SV points suffice. There are 20 unknowns (4 station clocks relative to station 1, 4 SV clocks, and 12 SV coordinates) and 20 measurements.

CASE C: Stations in poorly known locations measure pseudo–ranges to SVs in poorly known positions. Corrections to the coordinates (x, y, and z) of all stations and SV points must be determined in the reference frame of the first three stations, as well as all clock offsets relative to Station 1. Five stations ranging to thirteen SV points suffice. There are 65 unknowns (9 station coordinates, 4 station clocks relative to station 1, 13 SV clocks, and 39 SV coordinates of position) and 65 measurements. Six stations ranging to nine SV points also suffice.

CASE D: Nearly co-planar stations, that is, stations nearly on a great circle on the surface of the Earth, in known locations measure pseudo–ranges to an SV passing through the plane and correct the along–track and radial components of SV positions. Four stations ranging to three SV points suffice. There are 12 unknowns (3 station clocks relative to station 1, 3 SV clocks and 6 SV coordinates) and 12 measurements.

CASE E: Nearly co–planar stations in poorly known locations measure pseudo–range to an SV passing through the plane, and estimate time offsets and all relevant coordinates. Four stations ranging to eight SV points suffice. There are 32 unknowns (5 station coordinates, 3 station clocks relative to station 1, 8 SV clocks, and 16 SV coordinates of position) and 32 measurements. Five stations ranging to six SV points also suffice.

The cases above are based on a minimum number of measurements. Taking additional points from the SVs as individual measurements increases the data from a clock comparison point of view but also increases the number of unknowns. The minimum number of measurements for each case to provide a solution is given by the geometric arrangement of the stations in accordance with the formulas in Table 1.

Table 1. Case Definitions
(M = stations, N = SV points)

Case A	M + N - 1
B	M + 4N - 1
C	4M + 4N - 7
D	M + 3N - 1
E	3M + 3N - 4

Numerical simulations seem to show that the accuracy with which the ground clock time offsets are determined increases roughly with the square root of N, even though the accuracy of the SV coordinate corrections may remain small. The reason is probably that those time offsets are common to the whole solution, and so the accuracy with which they are determined increases steadily with N, whereas each SV point is independent of every other in multilateration, and increasing N cannot increase the accuracy of any particular point. To increase accuracy of the SV points they would need to be connected dynamically, which would introduce the complexity we are attempting to avoid, determination of highly precise GPS orbits. To accomplish synchronization of ground clocks, the accuracy of simple correction of independent clock SV points appears adequate.

However, it is not enough to count unknown quantities. The configuration of the ground stations is critical to this geometric approach to avoid what Killian and Meissl called "gefachrlichen Oerter" [5], "dangerous regions" or "critical configurations"[6]. Geometrical arrangements of stations and/or SV points for which the solution matrix is singular and no unique solution for coordinates can be obtained.

Using GPS, we measure pseudo–range which includes a time offset error (t). If t[r] is the offset of the station i clock with respect to some reference standard, T is the offset of SV number j with respect to the same standard and c is the speed of light, then we measure t where

$$t(ij) = t[r](i) + T(j) - R(ij)/c$$

The general form of the differential equation for the corrections to all these quantities is therefore,

$$dt(ij) = dt[r](i) + dT(j) - (1/cR(ij))[(X(j) - x(i))(dX(j) - dx(i)) \ldots$$

In our Case C, in which one solves for corrections to clocks and to both station and SV coordinates, the above is the observational equation. In the other cases, one or more of the terms in the unknowns dx, dy, dz, and/or dX, dY, dZ, are omitted. To anchor the coordinate frame, however, one must impose reference conditions. Set dt[r](1) = 0, and dx(1), dy(1), dz(1), dy(2), dz(2), and dz(3) = 0. This establishes the station 1 clock as the reference clock, and fixes the coordinate system.

In Case A, all the space coordinate terms are omitted, and the solution is always straightforward. In Case B the normal equations will contain terms linear in time and linear in space coordinates. Generally speaking, the case will be singular only if all stations lie in a straight line, a circumstance easy to avoid. However, our Case C falls under all the restrictions discussed by George Blaha[6]. In Case C, the normal matrix contains parallel columns of quadratic terms in the space coordinates. No solution can be obtained for both station and SV coordinates if all stations lie on a second-degree plane curve (e.g., an ellipse), or if all stations and SV points lie on a second-degree surface (e.g., an ellipsoid). That implies that the five stations cannot lie all in one plane, because fewer than six points on a plane can always be fitted by a second-degree plane curve. Therefore, Case C requires either that the five stations be spread over an area large compared to the Earth's radius, or that we use at least six stations. Likewise, the stations must not lie on two intersecting straight lines which would be the asymptotes of a family of hyperbolas. Case E (the co-planar case, adjusting stations and SV points) is subject to a similar but less troublesome restriction. All points, ground and sky, must not lie on a second-degree curve, but they seldom will. Case D (adjusting stations only) presents no singularities apt to arise in actual practice.

SIMULATING REALISTIC TIME TRANSFER EXPERIMENTS

In practice, GPS pseudo-range measurements cannot be made in an instant of time. Each measurement is an integration over a short time interval. Then will it be necessary to estimate corrections to SV velocity as well as range? Over short observing times such as those now used in common view comparisons (90 to 790 seconds), the SV positional correction could act as a mean correction to the X, Y, and Z coordinates of the SV over that time interval. Estimation of the time comparison errors involved with varying error contributions, such as velocity, will need further simulation.

The simulations discussed here are preliminary results of investigating the technique. They will ultimately be used to design an experiment to compare actual results with other time transfer techniques. The receiver locations were restricted to the continental United States and, for the most part, represent receivers at precisely known locations. Five stations were assumed to be located at Seattle, Washington (1), Los Angeles, California (2), the Naval Observatory (3), Colorado Springs (4), and Richmond, Florida (5).

For the simulation, the Block I GPS constellation on day 120 (30 April) 1989 was chosen for the SV points, since the precise trajectory data was available for NAVSTARs 3, 6, 8, 9, 10, and 11. The data selection was restricted to the times when four or more satellites were in simultaneous view at all receiver sites. Figures 1 and 2 depict the SV points at 02:45 and 05:15. These are the times of the best and worst solution results from the cases examined. The dashed lines represent the limits of coverage using a 10 degree elevation mask angle for the stations shown. The solid lines are the satellite ground tracks and the symbols represent the SVs at the specified times.

DATA GENERATION

Two methods of data processing were used. Method one made independent time transfer calculations for Case A and Case B every fifteen minutes. Method two made a single time transfer calculation based on eleven time samples of data taken every fifteen minutes. Earlier simulation results using method one are discussed in Reference 15. The uncertainties associated with the different measurements were generated with random noise. The statistics on each were set independently and the range of values is shown in Table 2. The selection of a one–meter sigma for the range measurement is based on the typical values observed in the operation of timing receivers. The values for SV point uncertainties ranged from 1.0 meters for precision SV points to 100,000 meters, five times the value of five–week–old almanac data. Station position may be known to 0.1 meters but 100 meters is used as a reasonably high limit. These noise uncertainties were considered to be spherical values. The time offsets and clock errors were not critical to this simulation, therefore the uncertainties were fixed at the equivalent of 10,000 meters. For simplicity in the simulation the calculations were performed in meters, and at least 10 trials were run to generate more information on performance. New values for the random terms were used for each trial. Method two was necessary for Case C to give a sufficient number of SV points, and the data was used to compute solutions for Case A and B for comparison.

Table 2. Measurement Uncertainties
1 sigma Values (meters)

A(SV,S)	Both considered known
B(SV)	1.5, 10, 10^3, 10^4, 10^5

Table 3 is a comparison of Cases A, B and C using Method Two. Case A calculations were done for two receiver position uncertainties and two SV point uncertainties. Case B calculations were on done for two receiver position uncertainties because Case B results are independent of SV point uncertainties. Case C calculations are independent of both uncertainties.

The multilateration method can improve the results of time transfer. In the case of a minimum amount of data, 4 satellites for Case B or 13 SV points for Case C, improvement is found only when there is a large uncertainty on the positions. With a larger data base, greater than 60 sky points, improvements were obtained even when position errors were in the order of a 100 meters for either the SV point or receiver.

With Method Two there is a significant increase in the number of unknowns. In Method One the number of unknowns for Case B with 5 receivers and 5 SV points was 24, but with Method Two and 62 SV points the number of unknowns increase to 252 for Case B, and 261 for Case C. For the same number of SV points and five receivers Case C requires only nine more unknowns.

FUTURE WORK

An experiment will be designed using the stations simulated in this study. A new refined simulation that is optimized and accounts for propagation and other effects will be used to guide the experiment. Work on the data generation algorithm will include ionosphere, troposphere, receiver clock frequency offsets and aging, position bias and velocity terms. In the present approach, no solution constraints

TABLE 3. Method Two Simulation Results
(Uncertainties in Meters)

CASE	A	A	A	B	B	C
Station Position	1	1	100	1	100	1000
SV Position	10	100	100	10000	10000	10000
Unknowns	66	66	66	252	252	261
Station Pairs						
1-2	0.35	0.46	31.70	2.37	195.66	9.14
1-3	0.32	0.68	26.48	2.20	217.77	11.49
1-4	0.24	0.40	24.14	1.69	123.06	10.59
1-5	0.39	0.78	33.24	3.11	262.39	13.80
2-3	0.40	0.60	40.43	2.69	292.74	12.63
2-4	0.31	0.33	28.69	1.14	116.59	3.67
2-5	0.39	0.57	39.86	2.01	215.12	10.76
3-4	0.41	0.53	41.88	1.64	182.12	10.86
3-5	0.20	0.31	20.00	1.72	158.80	5.92
4-5	0.44	0.58	44.49	1.62	154.49	8.44

Ranging Noise	1 meter
Station Clock Uncertainty	10000 meters
SV Clock Uncertainty	10000 meters
Start/Stop Time of Day	1:45 / 5:15 GMT
Stations = 5	SV Positions = 62

have been placed on the uncertainty in position of the satellites or receivers. Statistical knowledge of these uncertainties should be included in the processing. Additional terms for the receiver clock frequency offset and aging may be appropriate if satellite motion is significant.

An international experiment will also be examined. The simplest is one which requires the fewest stations and where the stations lie nearly in the orbital plane of a GPS satellite (Cases D and E).

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Figure 1, GPS Coverage, Time = 02:45

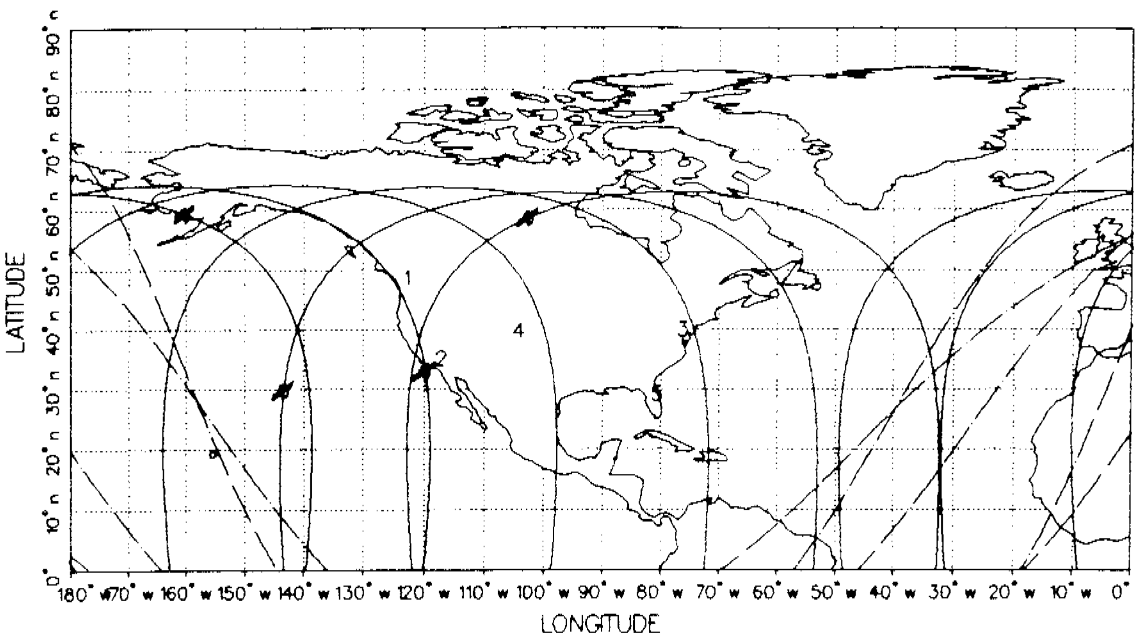

Figure 2, GPS Coverage, Time = 05:15