STEADY STATE OSCILLATOR ANALYSIS IN THE IMMITTANCE DOMAIN

Benjamin Parzen
Consulting Engineer
San Diego, CA 92103

Abstract

The theory of oscillator analysis in the immittance domain is presented. This theory enables the computer simulation of the steady state oscillator. The simulation makes practical the calculation of the oscillator total steady state performance, including noise. Some oscillator applications of the PC program, BPT, created for the simulation, are listed.

1 INTRODUCTION

During the past 11 years, the writer has been developing theory for oscillator analysis in the immittance domain. "IMMITTANCE DOMAIN" means that the basic relationships are expressed in terms of immittance, $Z = R + jX$ or $Y = G + jB$.

Effective and facile application of the theory to real problems also necessitated the creation of a PC computer circuit analysis program. The results of this effort are:

- A remarkably simple oscillator theory which fully describes the oscillator operation and which is readily translatable into the real world.
- A very user friendly multipurpose circuit analysis program, BPT.

The theory and program are universal in that they apply to all oscillators, past, present, and future. The theory and program are symbiotic, in that:

- To apply the program to oscillators, the theory must be used.
- The program is very helpful and almost indispensable in applying the theory to real oscillator problems including that of improving the theory.

It should be noted that, as BPT does not incorporate any basic information peculiar to oscillators, it follows that the theory may be used, in conjunction with other circuit analysis programs, to analyze oscillators but with much greater, and perhaps prohibitive, difficulty.

The theory is based upon the combination of 2 compatible oscillator models:

- The negative resistance model (2) as the primary model.
- The noise source, amplifier, and filter, model (3) in which positive feedback produces the necessary extremely high effective amplifier gain, as the secondary model.

2 THE NOISELESS OSCILLATOR [2]

2.1 Fundamental relationships

At steady state, in any mesh, of a hypothetically noiseless oscillator,

$$Z_t = \sum Z = 0 \tag{1}$$

From which,

$$\sum R = 0 \tag{2a}$$

$$\sum X = 0 \tag{2b}$$

Defining,

$$RN = \sum R_{negative} \qquad RT = \sum R_{positive}. \tag{3}$$

Then,

$$RT = -RN. \tag{4}$$

2.2 Non–linearity considerations

If the linear and non–linear elements are grouped together, then, from Eqs 2:

$$\sum Z_{lin} + Z_{nlin} = 0 \tag{5}$$

From which,

$$\sum R_{nlin} = -\sum R_{lin} \tag{5a}$$

Similarly,

$$\sum X_{nlin} = -\sum X_{lin} \tag{5b}$$

where the non–linear values are the effective values.

Eqs 5 are very important since they state that it is only necessary to study the linear R and X elements to compute the totals of the non–linear R and X elements. This fact drastically reduces the labor involved in the analysis and, very often, it eliminates the need for considering the detailed behavior of non–linear elements.

2.3 Noiseless oscillator model

Fig 1 is the simplified diagram of the oscillator resonator mesh. (The mesh containing the resonator is usually chosen for analysis as it has the most information.)

As shown in Fig. 1, that part of the oscillator containing the resonator circuitry is called the osci, symbol os. The remainder is called the llator, symbol LL.

From Eq 1,

$$Z_{os} = -Z_{LL}. \tag{6}$$

Also,

$$\Delta f = -\Delta X_{LL}/(\delta X_{os}/\delta f). \tag{7}$$

Eq 7 enables the easy determination of oscillator frequency shifts caused by llator changes.

3 THE REAL OSCILLATOR

3.1 Introduction

The real oscillator always includes one or more llator noise sources in addition to the resonator frequency noise. The calculation of the contribution of the resonator noise to the oscillator noise is very simple (See Section 3.3) However, the calculation of the contributions of the llator noises is extremely difficult and constitutes a major topic of this paper.

3.2 Frequency relationships

The total frequency, F is described by,

$$F = f_0 + f \tag{8}$$

where f_0 is the carrier frequency and f is the offset or Fourier frequency.

3.3 Contribution of the resonator frequency noise to the oscillator noise.

Let the resonator frequency noise be described as:

$$[S_f(f)]_{os}. \tag{9}$$

Then the resonator frequency noise contribution to the oscillator frequency noise at all oscillator locations is identical to that in Eq 9.

The resonator frequency noise contribution to the oscillator phase noise at all oscillator locations is:

$$[S_f(f)]_{os}/f^2. \tag{10}$$

The total phase noise at any oscillator location is given by the sum of that of Eq 10 and the sum of the contributions of the llator noise sources to the phase noise at that location.

3.4 Contributions of the llator noise sources to the oscillator noise at location m.

Let the noise of noise source n be given by power spectrum,

$$PS_n(f). \tag{11}$$

Then the contribution of this source to the oscillator noise power spectrum at location m is:

$$PS_{mn}(f) \tag{12}$$

and

$$PS_{mn}(f) = PS_n(f) * TF_{mn}(f) \tag{12a}$$

where $TF_{mn}(f)$ is defined as the transfer function of noise source n to the oscillator noise at location m .

The total oscillator noise, PS_t at location m, obviously is

$$PS_t(f) = \sum_1^n PS_{mn}(f). \tag{13}$$

3.5 Real oscillator, special case, N configuration.

Fig 2 is the complete diagram for the N configuration (N meaning noise) of a special case real oscillator. The fact that makes this oscillator a special case is that the noise source, V_n , represents the total equivalent noise contributions of all the noise sources in the llator.

All real oscillators always contain a non–physical resistance, dR. The value of dR is determined by the oscillator limiting, or ALC, system which sets the level of the oscillator output.

Fig 2 includes the input, output, and common networks and the multiple potential output points, available in every real oscillator. The noise in the output depends upon the output point location.

Fig 2 also shows RV and XV which do not exist in the real oscillator. Their function is described in Sect 3.7.

The oscillator, at steady state, at $f = 0$, (the carrier frequency) satisfies Eqs 14 to 16, similar to, and derived from, Eqs 1 to 4.

$$Z_t = \sum Z = dR. \tag{14}$$

From which,

$$\sum X = 0, \qquad \sum R = dR \tag{15}$$

and

$$RT = -RN + dR. \tag{16}$$

Also, it has been proven (5) that

$$\left|\frac{RN}{dR}\right|^2 = \left|\frac{V_s(0)}{V_n(0)}\right|^2. \tag{17}$$

This equation is used in calculating the oscillator operating Q.

Let

$$|RN/dR|^2 = (A_r)^2. \tag{18}$$

3.6 Real Oscillator Operation

a. The oscillator is constructed and power is applied.

b. The operator sets the output level by adjusting the limiting circuitry and thus setting dR.

c. The oscillator determines f_0, V_n, RT, dR and all other operating conditions.

d. The oscillator determines the phase noise as a function of f.

3.7 The real oscillator, special case, Z configuration

Fig 3 is the complete diagram for the Z configuration (Z meaning impedance) of the special case real oscillator.

The purpose of this configuration is to enable the precise setup of the necessary and sufficient oscillatory condition of Eq 14. Once this is done, we are certain that this configuration when converted to the N configuration represents a true oscillator at the desired frequency.

As will be noted, the Z configuration differs from the N configuration in the following:

a. The value, V_n, of the noise is 0.

b. The circuit has been broken and a 1 A current source inserted. The value of the voltage, V_z, across this source is the value of the impedance Z_t of Eq 14. When Z_t has been precisely adjusted to dR at the desired frequency, f_0, then this Z configuration is ready for conversion to a N configuration. The adjustment procedure is called "zeroing".

RV and XV are very small value trimmable resistor and reactor, respectively, provided to facilitate zeroing. Their values are identical in the Z and N configurations.

The zeroing procedure has been automatized in the computer simulated laboratory and thus makes practical the computer oscillator simulation.

3.8 Computer simulated oscillator operation

In the program oscillator, Z config. (See Fig 3)

a. The oscillator is constructed by entering the circuit into the computer.

b. The operator sets f_0, $dR = 0$, and $V_n = 0$.

c. The program sets the effective gain of the active circuitry, so that Eq 14 is almost satisfied.

d. The program adjusts RV and XV so that the mesh impedance, $Z_t = V_z$ in Fig. 3, satisfies Eq 14 to a very high degree of precision. The Z configuration is then converted into the N configuration.

In the program oscillator, N config. (see Fig 2)

e. The operator sets V_n to V_n real.

f. The operator sets dR to the value where the desired oscillator output is obtained as measured by the AC current at any 1 point or ac voltage between any 2 points.

g. The program determines the voltages and currents at all other points.

h. The program determines the phase noise at and between all points.

i. The program determines $Z_t(f)$, (see Fig 3) when dR is at the value of Step f, and Vn is set to 0.

3.9 Computation of the transfer functions, $TF_{mn}(f)$ in the real oscillator

This is best done by using the program. The procedure is the following

a. Start with the oscillator configured as in Sect 3.8, Step d. Include all the noise sources, which are assumed to be uncorrelated.

b. Create the equivalent N configuration, with the correct value of dR.

c. To compute $TF_{mn}(f)$, set the magnitudes of all noise sources to 0, excepting the source at location n.

d. Make the source, n, a unit white noise source.

e. Determine the noise response at location m, $PS_{mn}(f)$.

f. Then from Eq 12,

$$TF_{mn}(f) = PS_{mn}(f). \tag{19}$$

g . Repeat steps c to f for all noise sources.

3.10 Oscillator noise, in all real oscillators, due to V_n (See Fig 2)

3.10.1

The resonator current, I_x, phase noise, due to V_n, is given by:

$$\mathcal{L}_{I_x}(f) = \mathcal{L}_{V_n}(f) * \left|\frac{RT}{Z_t(f)}\right|^2. \tag{20}$$

$Z_t(f)$ is obtained with the Z configuration and

$$\mathcal{L}_{V_n}(f) = PS_{V_n}(f)/(V_s)^2. \tag{21}$$

3.10.2 Noise at other locations

The phase noise in V_s is almost the same as that in I_x, except at the higher values of f.

Now that the phase noise in I_x is known, the determination of the phase noise, at all other locations, is straightforward but, depending upon the circuit complexity, can be very difficult and tedious and is best done with the program.

However, those desiring to perform the calculations should keep in mind the following rules when combining noises at any location:

a. Noise voltages and currents, due to the same noise source, should be combined as phasors.

b. Noise powers, due to different uncorrelated noise sources, should be combined as scalars.

3.11

The general relationship between $PS(f)$ and the phase noise, $\mathcal{L}(f)$ at the same location, is

$$\mathcal{L}(f) = PS(f)/\text{CarrierPower} \tag{22}$$
$$= PS(f)/PS(0) \tag{23}$$

for all values of f, when $PS(0)$ does not approach infinity.

Eq 22 is also useful for the case where $V_n(f)$ has a flicker or other noise component which theoretically approaches infinity as f approaches 0 and thus theoretically also makes $PS(f)$ approach infinity as f approaches 0. In this case, the oscillator noise power spectrum, at f when $|X_t(f)| \gg dR$, is independent of dR which is set by the oscillator limiting system at the desired carrier. Therefore, at these values of f,

$$\mathcal{L}(f) = PS(f)/PC \tag{24}$$

where PC is the arbitrary desired carrier reference power.

4 PROGRAM APPLICATIONS IN OSCILLATORS

The following applications to oscillators are cited to demonstrate the utility and power of program BPT when used in conjunction with the theory.

4.1 Automatic zeroing of the Z oscillator configuration (coarse, active device gain setting and fine, impedance trimming) for the derivation of the Zt(f) relationship.

4.2 Automatic calculation of the oscillator operating frequency. Both 4.1 and 4.2 functions are high speed and high resolution executions of their respective functions.

4.3 Calculation of the DC operating point of oscillators.

4.4 Includes a procedure for determining the AC operating point of a self-limiting oscillator.

4.5 Includes a procedure for setting the AC and DC operating points in ALC type oscillators.

4.6 The program is a linear one but is designed to be capable of being interfaced with non linear operating conditions. Items 4.3 to 4.5 are examples.

4.7 Investigation of the Zt(f) or Zt(?) function of an oscillator or circuit such as a llator.

The following 5 examples illustrate the very useful information obtained from llator studies.

4.7.1 The effect of component changes upon frequency.

4.7.2 Overtone and mode selector gain margins and the effect of the resonator overtone and mode selector circuitry upon the oscillator stability.

4.7.3 Starting gain margin (loop gain, ALo)

4.7.4 Effect of power dissipating components upon the loop gain and the operating Q.

4.7.5 Effect of component tolerance and environment upon the above 4 items.

4.8 Determination of bypass and coupling capacitor adequacy.

4.9 The performance of subcircuits such as tuning networks.

4.10 The determination of circuit isolation properties.

4.11 Calculation of operating Q, from Eq 17.

4.12 Studies of impedance properties of resonators and other devices requiring high resolution.

4.13 Investigation of the effect of component and subcircuit noise upon oscillator noise.

4.14 Determination of the circuit configuration for optimum noise performance.

4.15 Calculating and plotting oscillator phase noise. It is not necessary to assume symmetrical noise sidebands.

4.16 Calculating and plotting the resonator current phase noise for a llator white noise source from the Zt(f) relationship.

4.17 In setting up an oscillator, the oscillator Z configuration is first created and then zeroed. (See Item 4.1). A facility is provided to automatically convert the zeroed Z configuration into an N configuration and vice versa.

5 THE PROGRAM AS A RESEARCH TOOL

The following suggested applications illustrate the power and usefulness of the program in research activities:

5.1 The formulation, checking, and confirmation of new theory. and new oscillator designs.

5.2 Determination of component aging from experimentally derived llator aging.

5.3 Determination of component temperature characteristics from experimentally derived llator temperature performance.

5.4 Determination of llator noise by experimentally deriving the oscillator phase noise and then program calculating the Vn(f) function required to produce that oscillator noise.

6 CONCLUSIONS

Extremely simple and powerful oscillator theory has been presented. This theory has been used in the creation of a computer program for the universal analysis of oscillator steady state performance.

The importance of Zt(f), RT, and dR and the tremendous power of analysis in the immittance domain complemented by computer aided analysis have been demonstrated. An important additional advantage is that this analysis method provides considerably greater understanding of the operation of the real oscillator.

7 REFERENCES

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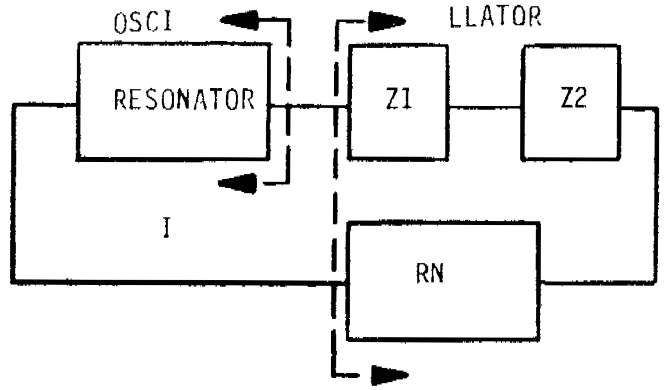

Figure 1: Noiseless Oscillator Model

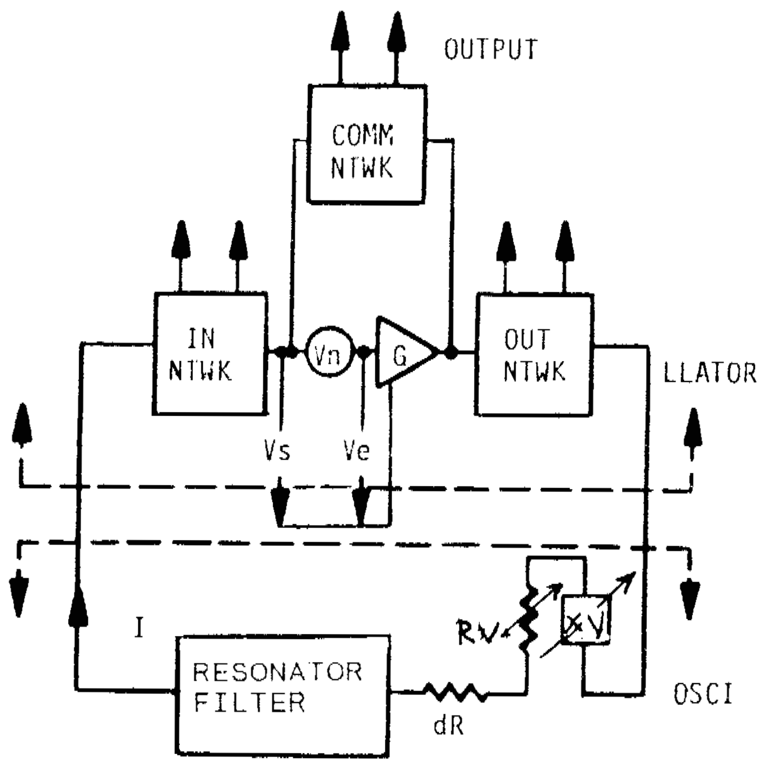

Figure 2: Real Oscillator, Special Case, N Configuration

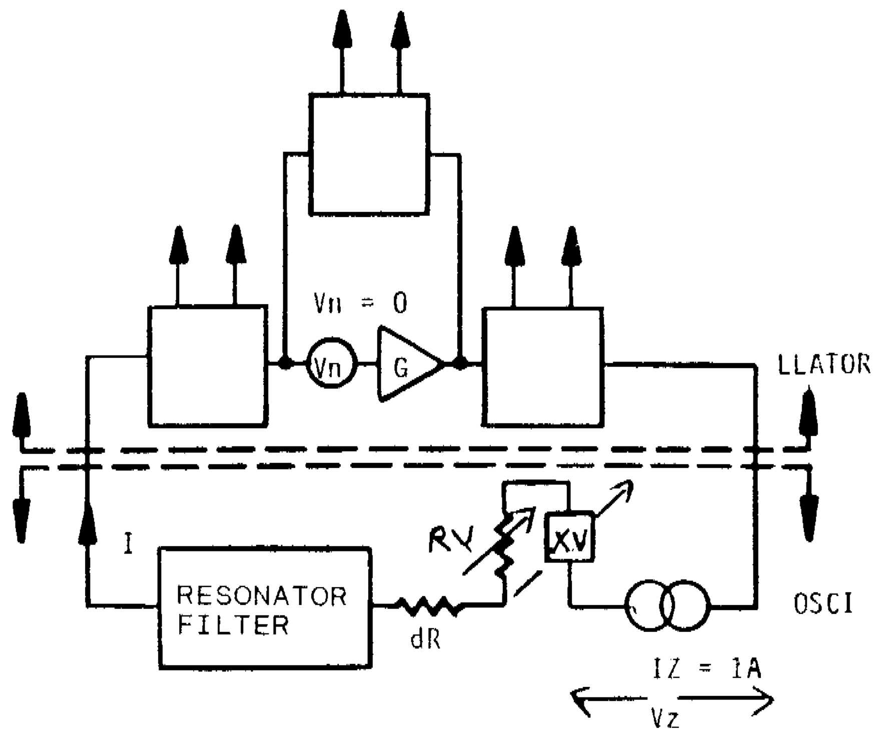

Figure 3: Real Oscillator, Special Case, Z Configuration