NEW ION TRAP FOR FREQUENCY STANDARD APPLICATIONS*

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Abstract

We have designed and built a novel linear ion trap which permits storage of a large number of ions with reduced susceptibility to the second order Doppler effect caused by the RF confining fields. This new trap should store about 20 times the number of ions as a conventional RF trap with no corresponding increase in second order Doppler shift from the confining field. Other comparisons to standard RF ion traps will be made.

Introduction

We have designed and constructed a hybrid RF/DC linear ion trap for use as a frequency standard. This new trap has about 20 times the storage volume as a conventional RF trap with hyperbolic electrodes with no corresponding increase in second order Doppler shift from the micro-motion induced by the RF trapping fields. Alternatively, if loaded with $2 \cdot 10^6$ ions, the Doppler shift from the trapping fields is reduced by a factor of 10 below comparably loaded hyperbolic traps.

Second Order Doppler Shift for Ions in an RF Trap

Figure 1 shows a conventional RF ion trap along with the applied voltages. The trapping forces are generated by the driven motion of the ions (at frequency Ω) in the inhomogeneous electric field created by the trap electrodes[1]. Ions are trapped around the node point of the oscillating electric field at the center of the trap for certain trap voltages, ionic masses, etc. The motion in each of three directions for a single ion in an RF trap is characterized by two frequencies , the fast driving frequency Ω and a slower secular frequency ω. An exact solution to the equations of motion shows that frequencies $k \cdot \Omega \pm \omega$, k = 2, 3, ... are also present. However, in the limit $\omega/\Omega \ll 1$ the ω and $\Omega \pm \omega$ frequencies dominate and the kinetic energy of a particle, averaged over one cycle of Ω, separates into the kinetic energy of the secular motion and the kinetic energy of the driven motion. The average kinetic energy is transferred from the secular to the driven motion and back while the sum remains constant just as a harmonic oscillator transfers energy from kinetic to potential and back. The second order Doppler shift for a small and/or hot ion cloud, where interactions between ions are negligible is:

*This work represents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract sponsored by the National Aeronautics and Space Administration.

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\begin{aligned}
\frac{\Delta f}{f} = -\frac{1}{2}\frac{< v^2 >}{c^2} &= \frac{< \text{total K.E.} >}{mc^2} \\
&= \frac{< \text{secular K.E.} + \text{driven K.E.} >}{mc^2} \\
&= 2\frac{< \text{secular K.E.} >}{mc^2} \\
&= \frac{3k_BT}{mc^2}.
\end{aligned}
$$

where $<>$ indicates a time average over one cycle of Ω. We have also averaged over one cycle of ω to equate the secular and driven K.E. This is the same as a simple harmonic oscillator where the average K.E. is equal to the average potential.

We now consider the case where many ions are contained in a trap and interactions between ions dominate. In this cold cloud model[2] of the trapped ions displacements of individual ions from the trap center is primarily due to electrostatic repulsion between the ions and random thermal motion associated with temperature can be assumed to be small compared to driven motion due to the trap fields. Such clouds will have a constant ion density out to the edge of the plasma where the density falls off in a distance characterized by the Debye length[3]:

$$\lambda_D = \sqrt{\frac{k_BT\epsilon_o}{n_oq^2}}. \tag{1}$$

This cold cloud model should be useful provided the ion cloud size is large compared to the Debye length. For room temperature Hg ions held in a trap with 50 kHz secular frequency the Debye length is about 1/5 mm.

The trap shown in Figure 1 is described by a pseudo-potential energy[4]:

$$\phi = \frac{1}{2}\left(m{\omega_\rho}^2\rho^2 + m{\omega_z}^2z^2\right), \tag{2}$$

where:

$${\omega_\rho}^2 = \frac{2q^2{V_o}^2}{m^2\Omega^2\epsilon^4} + \frac{2qU_o}{m\epsilon^2}, \tag{3}$$

and:

$${\omega_z}^2 = \frac{8q^2{V_o}^2}{m^2\Omega^2\epsilon^4} - \frac{4qU_o}{m\epsilon^2}, \tag{4}$$

and ϵ describes the trap size.

We assume that the DC and RF voltages are adjusted to make the trapping forces spherical so that the ion cloud is a sphere containing N ions out to radius R_{sph}.

This pseudo-potential depicts trap forces as arising from a uniform "background" charge density computed from Poisson's equation with the above pseudo-potential:

$$n_o = \frac{3\epsilon_o m\omega^2}{q^2}. \tag{5}$$

Trapped positive ions neutralize the negative background of charge, matching its density out to a radius where the supply of ions is used up.

The oscillating electric field which generates the trapping force grows linearly with distance from the trap center. The corresponding amplitude of any ion's driven oscillation is proportional to the strength of the driving field, i.e., also increasing linearly with the distance from the trap center. The average square velocity of the driven motion for an ion at position (ρ,z) is:

$$< v^2 >= \frac{1}{2}\omega^2 \left(\rho^2 + 4z^2\right). \tag{6}$$

For a given trapping strength, reflected in force constant ω^2, the density is fixed by eq (5) while the radius of the spherical cloud is determined once the ion number N has been specified. The second order Doppler shift due to the micro-motion is the spatial average of $-(1/2)(<v^2>/c^2)$ over the spherical ion cloud. Using eq (6) for the spatial variation of the micromotion:

$$\begin{aligned} \left(\frac{\Delta f}{f}\right)_{sph} &= -\frac{1}{2}\frac{\overline{< v^2 >}}{c^2} && (7)\\ &= -\frac{3}{10}\frac{\omega^2 {R_{sph}}^2}{c^2} && (8)\\ &= -\frac{3}{10}\frac{1}{c^2}\left(\frac{N\omega q^2}{4\pi\epsilon_o m}\right)^{2/3}. && (9) \end{aligned}$$

For typical operating conditions[2], $N = 2 \cdot 10^6$ and $\omega = (2\pi) \cdot 50\text{kHz}$, $\Delta f/f = 2 \cdot 10^{-12}$. This corresponds to a second order Doppler shift that is about 10 times higher than the shift for free ^{199}Hg ions at room temperature, $\Delta f/f = 3k_BT/2mc^2 = 2 \cdot 10^{-13}$.

For increased signal to noise in the measured atomic resonance used in frequency standard applications, it is desirable to have as many trapped ions as possible. However, as we have just seen larger ion clouds have larger second order Doppler shifts.This frequency offset must be stabilized a high degree in order to prevent degradation of long term performance.

To reduce this susceptibility to 2nd order Doppler shift we have designed and constructed a hybrid RF/DC ion trap which replaces the single field node of the hyperbolic trap with a line of nodes. The RF electrode structure producing this line of nodes of the RF field is shown in Figure 2. The ions are trapped in the radial direction by the same RF trapping forces used in a conventional RF trap and we assume a secular motion in that direction of frequency ω. To prevent ions from escaping along the axis of the trap DC biased "endcap" needle electrodes are mounted on each end. Unlike conventional RF or Paul traps this linear trap will hold positive or negative ions but not both simultaneously.

Near the central axis of the trap we assume a quadrupolar RF electric field:

$$\phi = \frac{V_o\left(x^2 - y^2\right)\cos(\Omega t)}{2R^2}, \tag{10}$$

which gives the corresponding pseudopotential energy:

$$\begin{aligned} \Phi &= \frac{q^2 {V_o}^2}{4R^4 m\Omega^2}\left(x^2 + y^2\right) && (11)\\ &\equiv m\frac{\omega^2}{2}\rho^2. && (12) \end{aligned}$$

Poisson's equation leads to a limiting charge density,

$$n_o = \frac{2\epsilon_o m\omega^2}{q^2}, \tag{13}$$

for ions held in this linear trap, where:

$$\omega^2 = \frac{q^2 V_o{}^2}{2m^2\Omega^2 R^4}. \tag{14}$$

The motion induced by the RF trapping field is purely transverse and is given by:

$$< v^2 >= \omega^2\rho^2. \tag{15}$$

As before we average this quantity over the ion cloud to find the second order Doppler shift:

$$\left(\frac{\Delta f}{f}\right)_{lin} = -\frac{1}{2}\frac{\overline{< v^2 >}}{c^2} = -\frac{\omega^2 R_c{}^2}{4c^2}. \tag{16}$$

We have assumed for simplicity a cylindrical ion cloud of radius R_c and length L. Equation (16) can be written in terms of total ion number, N, and trap length, L,

$$\left(\frac{\Delta f}{f}\right)_{lin} = \left(\frac{q^2}{8\pi\epsilon_o mc^2}\right)\frac{N}{L}. \tag{17}$$

We can compare the second order Doppler shift for the two traps assuming both hold the same number of ions and that ions bound in both traps have the same secular frequency, ω. The cloud radii in the two traps then satisfy $(1/2)R_c{}^2 L = R_{sph}{}^3$ giving:

$$\left(\frac{\Delta f}{f}\right)_{lin} = \frac{5}{3}\frac{R_{sph}}{L}\left(\frac{\Delta f}{f}\right)_{sph}. \tag{18}$$

As more ions are added to the linear trap their average second order Doppler shift will increase. It will equal that of the spherical ion cloud in the hyperbolic trap when:

$$N_{lin} = \frac{3}{5}\frac{L}{R_{sph}}N_{sph}. \tag{19}$$

It should be repeated that these expressions are valid in the cold cloud limit, where the radius of each of the clouds is large compared to the Debye length given in eq(1).

A linear trap can thus store $(3/5)(L/R_{sph})$ times the ion number as a conventional RF trap with no increase in average second order Doppler shift. For the trap we have built , L = 75 mm. Taking $R_{sph} = 2.5$ mm for $2 \cdot 10^6$ $^{199}Hg^+$ ions[2] we see that our linear trap should hold about 18 times the number of ions as that of Reference 2.

Construction of a Linear Ion Trap

We have built a linear trap consisting of 4 molybdenum rods equally spaced on an approximately 1 cm radius. OFHC copper pins with dc bias are located at each end to confine ions in the axial direction and are about 75 mm apart.

The input optical system which performs state selection and also determines which hyperfine state the ions are in has been modified from the previous system[4]. The present system illuminates about 1/3 of the 75 mm long cylindrical ion cloud. An ion's room temperature thermal motion along the axis of the trap will give an average round trip time of 1.4 msec, a value which is much smaller than any optical pumping,interrogation or microwave resonance time. Thus, all the ions will be illuminated, but with a lower average intensity.

In order to operate within the Lambe-Dicke regime the 40.5 Ghz microwave resonance radiation will be propagated perpendicular to the line of ions. The ions should then all feel phase variations of this radiation which is less than π so that the 1st order Doppler absorption in sidebands induced by an ions motion will not degrade the 40.5 Ghz fundamental.

The optical axis of the fluorescence collection system is perpendicular to the axis of the input optical system as in the previous system. There is one difference, however. In the hyperbolic trap the collection has in its field of view the ion cloud and the semitransparent mesh of both endcap trap electrodes. This mesh can scatter stray light into the collection system which will degrade the signal to noise ratio in the clock resonance. This linear trap has no trap electrodes, mesh or otherwise, in its field of view and, consequently, should have less detected stray light.

In summary, we have designed an constructed an ion trap which confines ions with a combination of RF and DC electric fields. This trap has 15 to 20 times the ion storage volume as conventional RF traps with no increase in second order Doppler shift from the trapping fields.

Acknowledgements

We wish to thank Dave Seidel for assisting in the design of the linear trap described here.

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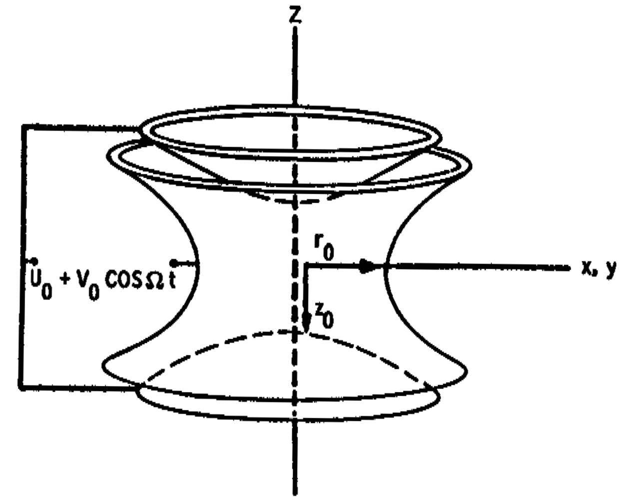

Figure 1. A conventional hyperbolic RF ion trap. A node of the RF and DC fields is produced at the origin of the coordinate system shown.

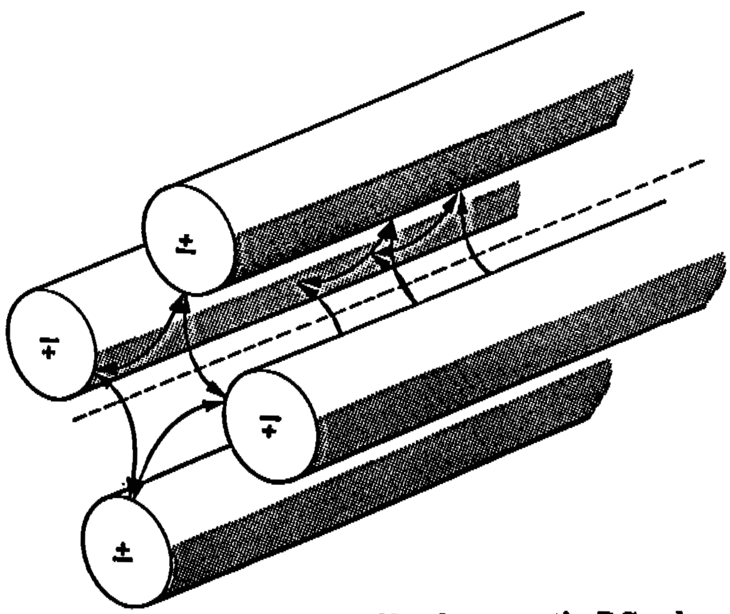

Figure 2. The RF electrodes for a linear ion trap. Not shown are the DC endcap needle electrodes used to prevent the ions from escaping along the longitudinal axis. Ions will be trapped around the line of nodes of the RF field with reduced susceptibility to second order Doppler frequency shift.

QUESTIONS AND ANSWERS

DAVID ALLAN, NIST: Can you remind us, from fundamental physical principles, what you hope to get in stability and accuracy once you have achieved your goals?

DR. PRESTAGE: This will probably start out at one second below ten to the minus twelve. One of the nice things about this concept is that we can scale this up to have many more ions. We are down many orders of magnitude compared to a hydrogen maser in terms of number of atoms or ions. One of the approaches to improve the mercury device is to get better signal–to–noise. It is sort of feeble now, ten to the sixth ions is not a lot. If we can get a factor of a hundred over that it will show up in the short term performance. We then would also have very good accuracy, the best of cesium, say, and the best of hydrogen masers. That is the goal.

DR. ED MATTISON, SAO: What does the cylindrical geometry do to your detection and optics scheme? Does it make it considerably more complicated?

DR. PRESTAGE: We haven't done measurements yet, but it actually makes it simpler. There are fewer scatterers in the field of view than there are in the present trap.

DR. MATTISON: Do you have to change your mirror and detector geometry to capture the fluorescence from that cylindrical arrangement?

DR. PRESTAGE: Not really.