THE EFFECTS OF DATA PROCESSING AND ENVIRONMENTAL CONDITIONS ON THE ACCURACY OF THE USNO TIMESCALE

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Abstract

Humidity has significant long–term effects on cesium clock rates that may be controlled environmentally, but not during data processing. Allan variances are minimized at a temperature depending on the clock and clock type. There is no dependence of Allan variance on manufacturing batch. A surprisingly large fraction of clock rate and variance changes may be attributed to human interference or the need for it. Little or no improvement is obtained by altering the unity–or–zero weighting scheme of the current USNO timescale algorithm. An algorithm based on robust ARIMA modelling yields a timescale that may differ markedly and is in most respects inferior to that generated by the current algorithm. The NIST algorithm is comparable in accuracy and stability to the current algorithm, except on the short term, where it is significantly less stable.

INTRODUCTION

USNO is in the process of reevaluating the accuracy and stability of its timescale algorithm relative to other algorithms and determining how best to minimize the effects of changes in environmental conditions on the cesium–beam atomic clocks that generate our timescale. The first phase of this study [1] found that temperature effects on the clock rates (relative frequencies) were negligible when the clock vault temperature was controlled to within ± 1 deg C. No short–term humidity effects were evident, but there appeared to be an annual variation in the clock rates dependent on absolute humidity, confirming results found at PTB [2] and IEN [3] and being in turn confirmed at NBS [4].

DATA

The data consisted of two years of hourly clock differences for each of 44 commercial cesium frequency standards located in six well–separated vaults at USNO, as well as temperature and relative humidity measurements for each vault. The absolute humidity was computed from the temperature, relative humidity, and Figure 15 in [5]. The clocks were restricted to those that were weighted, i.e. that contributed to the USNO timescale, in order to be certain that we were dealing with well–known and well–behaved clocks.

Frequency instability in the time domain is defined by the Allan, or two–sample, variance [6]. Allan variances were computed for each of the clocks at six different sampling times, τ: 1 hour, 5 hours, 1 day, 5 days, 25 days, and 50 days. The mean values of their square roots are given in Table 1, the associated errors having been calculated from the 30–day (for the first four τ's), 180-day (for τ = 25 days), or 360-day (for τ = 50 days) binned values. In some cases, the data were not extensive enough to compute all the values or their errors. Next, the value and time of minimum Allan variance were estimated by second–order interpolation to the lowest three values.

ENVIRONMENTAL EFFECTS

Linear regression of the rates against absolute humidity (both averaged over 10 days) for 5 clocks in the Building 1 vault yielded the coefficents in Table 2. The coefficients agree well with results obtained elsewhere [3,4,5], but are not well determined due to imprecise temperature control. Hence, correction for humidity effects during data processing is not feasible. Only one of our vaults is presently humidity controlled, but two new vaults now under construction will be.

In order to gauge the effects of ambient conditions on frequency stability, the vault temperatures, relative and absolute humidities, and their time gradients were investigated for correlations with the square root of the Allan variance for τ = 1 hour, 5 hours, 1 day, and 5 days. The only significant correlations were with temperature; the mean absolute values of the linear correlation coefficients ranged from 0.21±0.03 to 0.37±0.04, depending on τ, corresponding to confidence levels of from 86% to 99%. The coefficients varied from negative to positive, depending on the clock, suggesting that each clock has a peculiar temperature at which its variance is minimized. No dependence on age or manufacturing batch were noted.

The mean absolute values of the temperature coefficients are given in Table 3. J45–option (high–performance/high–stability beam tube) clocks had temperature coefficients that were significantly more positive than those of the 004–option (high–performance tube) clocks for most of the τ's, indicating that the former either are more temperature–sensitive, have a lower variance–minimizing temperature, or both.

MAINTENANCE ACTIVITY EFFECTS

In a search for possible effects of human activity (e.g. routine maintenance) on clock rates, maintenance logs were compared with the times of significant nontransient rate changes (arbitrarily defined as ¿ 2 ns/day). After elimination of rate changes caused by nonroutine events (e.g. gross movement or degaussing of clock; large temperature excursions; etc.), a background level for the frequency of these changes was established. Then, excess occurrences of rate changes were looked for around the times of maintenance checks. It was found that a typical clock underwent 2.0 unexplained (and presumably spontaneous) rate changes per year and 0.9 rate changes per year attributable to maintenance checks, even if no adjustments were made. If controls had to be adjusted, there was an 13% chance that a rate change would result. Of the adjustments that caused rate changes, 82% were of the second–harmonic control.

A similar investigation was made of the square root σ of the Allan variance for τ = 1 hour, 5 hours, 1 day, and 5 days. Defining a significant change as twice the standard error of σ or more, eliminating changes caused by nonroutine events, and averaging over all clocks and τ's, it was found that 55% more changes in σ occurred when an adjustment had to be made; virtually all of these adjustments were of the second–harmonic control.

Inasmuch as the data were binned in intervals of 5 days (for rates) or 30 days (for σ's), the time resolution was inadequate to determine what was cause and what was effect, i.e. whether it was the condition that necessitated the adjustment, or the adjustment itself, that altered the rate or σ.

We also looked for any dependence of Allan variance on clock serial number (hence, manufacturing batch) or, roughly, age. Figure 1 shows no such dependence, but one must keep in mind that any clock showing significant aging would produce degraded data that would have been rejected from our data set by the deweighting of the clock to zero.

CLOCK WEIGHTING

The USNO uses a linear timescale algorithm (the "old" one described by Percival [7]), which is generated hourly and reprocessed about twice a week, at which time clock weights and rates may be revised, based on an examination of least–squares solutions for the rates generally solved for every 5 days. The mean ("paper") timescale is generated from the average of each of the individual clock rates (after removal of a nominal rate relative to the rest of the ensemble), wherein clocks are either weighted unity or zero. Clocks are deweighted in the preliminary timescale if their rate change exceeds 3 parts in 10^{13}. During reprocessing, they are deweighted if a change in rate exceeds about 6 ns/day, its rms frequency error of unit weight exceeds about 5 ns, or if the rate appears to be drifting. Consequently, the algorithm is iterative and "unweighted" and the data are filtered.

In order to provide a physical realization of this time, called UTC (USNO), a cesium clock, Master Clock (MC) #1, is steered toward the mean timescale, as is one of our active hydrogen masers, MC #2. GPS and all other USNO–derived timescales are traceable to MC #2. The mean timescale itself may be steered by steering the MC's (to which it and all clock time differences are referred), as is currently being done, toward TAI. The steered and unsteered mean timescales relative to TAI are shown in Figure 2 for the years 1986–87.

In order to investigate the effect of altering the weights, seven different mean timescales were computed using weights equal to the inverse Allan variances for the sampling times in Table 1, as well as the inverse minimum Allan variance. Attention was paid to significant changes in each clock's variance with time, since, as noted in the previous section, these did occasionally occur.

The short–term errors of the unweighted and weighted timescales were evaluated by comparing them to our MC #2 maser, which is more stable than cesium clocks at sampling times shorter than a few days. Table 4 lists the mean square roots of the Allan variance of the differences between the timescale and the maser for the six sampling times. Weighting with 5-day variances yields better results than not weighting at all, but to such a small extent that one questions whether it justifies even the relatively little extra labor involved.

The long–term systematic errors of the weighted timescales were studied by solving by least squares for the slope of their drifts relative to the unweighted timescale. According to Table 5, none of the weighted timescales drift significantly from the unweighted timescale.

The long–term nonuniformities of the unweighted and weighted time– scales were analyzed by averaging daily time differences over 10 days, referring the averages to TAI (available only every 10 days), removing the steering, fitting a parabola or a line (whichever fit best) to the 1986 and 1987 data by least squares, and looking at the standard error of the estimate (the scatter after removal of the parabolic or linear trend). Allan variances would have underestimated the errors due to the serial correlation of the residuals. Since the standard errors of the standard errors themselves are about ± 8 ns and ± 5 ns for 1986 and 1987 respectively, it is evident from Table 5 that weighting does not significantly decrease the long–term nonuniformities.

Therefore, little or nothing is to be gained by weighting our clocks any differently, at least with our current algorithm. This is probably due to the homogeneity of the clock types in our ensemble. It also implies that the Allan variance is not the best measure of the contribution of a clock to the stability of a timescale. Previous discussions of this matter [8, 9] have assumed that the outputs of clocks are statistically independent. However, it has been shown [1] that USNO clocks are significantly intercorrelated and even that study would not have been able to detect correlations apparent only with reference to clocks of other types and in other locations. These correlations cause one to underestimate the errors when using an internal estimate like the Allan variance. Apparently, they do so to such an extent that the Allan variance overestimates a clock's contribution when the "optimum–weighting method' [9] is employed.

ROBUST ARIMA ALGORITHM

Percival [7] proposed an algorithm based on ARIMA forecasting. An- ARIMA (Autoregressive Integrated Moving Average) model is described by:

$$z_t - \sum_{j=1}^{p} \phi_j z_{t-j} = \theta_0 + a_t - \sum_{j=1}^{q} \theta_j a_{t-j} \tag{1}$$

where z_t is the d^{th} finite time difference; d is the number of times the elements of the time series have to be differenced to render the series stationary; $\phi_1, \ldots, \phi_p$ are autoregressive parameters; θ_0 is a correction for a systematic d^{th}–order drift; $\theta_1, \ldots, \theta_q$ are moving–average parameters; and $a_t, \ldots, a_{t-q}$ are uncorrelated random variables with zero mean and constant variance. Such a model allows one to make use of short–term trends in the data to predict a real–time timescale and, hence, quickly to sense any significant rate change and downweight such a clock until its rate is stable again.

Downweighting was accomplished by means of a robust filter, namely Hampel's psi function (Figure 3), as also proposed by Percival [10]. The weight of a clock was given by:

$$\frac{\sigma_t \psi(e_t/\sigma_t)}{e_t} \tag{2}$$

where σ_t is the square root of the Allan variance at a sampling time t (the time interval of rate and weight revision) and e_t is the difference between the observed and the ARIMA–predicted frequency.

Each clock's data were divided into segments of constant rate and variance and at least 50 t in length; an initial model was assumed for the clock; and the model was iteratively improved until the rms error of the fit to the data was minimized. The results of the modelling were as follows: (1) d = 2; (2) t = 1 day was the smallest time step for which the minimal number of parameters needed to be solved for; (3) p = 0; (4) θ_0 could not be determined because the coefficients of each model changed significantly too frequently, typically every few months at one or more of the times of rate change; (5) 96% of the time, q = 1, and the rest of the time, q = 2; and (6) θ_1 and θ_2 averaged 0.721±0.015 and 0.819±0.019 respectively.

A timescale was computed hourly with the robust ARIMA algorithm by enforcing equality with the unweighted USNO timescale for the first 10 days (during which the ARIMA prediction errors, required by the weighting procedure, initialized themselves) and then letting the ARIMA algorithm take over for the remainder of the year. The results for 1986 and 1987 are shown in Figure 4. Little difference was found between using weights based on the variance of the fit rather than the Allan variance.

Of the weighted clocks used by the USNO algorithm, those given full weight by the ARIMA algorithm averaged 93.4%, those given partial weight averaged 5.5%, and those given no weight averaged

1.1%. For a normal distribution of errors, the sum of the last two would be 4.6%. In practice, the ARIMA algorithm could have made partial use of some of the clocks given zero weight by the USNO algorithm.

Evaluating the short–term errors, again through comparison with our maser, we obtained the mean square roots of the Allan variance given in Table 6. The ARIMA timescale is significantly less stable than the USNO timescale for every sampling time except 1 day (probably because of the 1–day weighting). Also, its time of minimum variance occurs at a shorter sampling time.

The stability of the robust ARIMA timescale could probably be improved, at least over times longer than a day, by increasing t from 1 day to, say, 5 days (nearer the average time of minimum Allan variance for our clocks). As it stands, the ARIMA algorithm corrects the rate daily, which may be decreasing the ensemble's stability by following the cesium clocks too closely, i.e. tracking some of their noise. Unfortunately, increasing t decreases the number of data points by the same factor, which makes it more difficult to derive the ARIMA coefficients with sufficient accuracy. ARIMA modelling requires a minimum of about 50 data points, but our clocks went only an average of 126 days between significant rate changes (at which time the ARIMA coefficients usually changed significantly). The inaccuracy of the ARIMA parameters could also have contributed to the timescale's instability.

As is evident from Figure 4 and Table 7, the ARIMA algorithm on the long term can drift markedly relative to the USNO timescale in either direction, depending on the prevailing ARIMA parameters. The ARIMA timescale was significantly less stable than the USNO timescale during 1986, but was about as stable during 1987. Since there were fewer clocks and more rate changes in 1987 than in 1986, the only apparent reason for this difference in stability between years is a greater susceptibility of the ARIMA algorithm to nonuniformities in the data; that these nonuniformities were larger in 1986 than in 1987 is clear from the corresponding errors for the USNO algorithm.

Consequently, ARIMA modelling yields a timescale that is no better than, and is in most respects significantly inferior to, that generated by the USNO timescale, for reasons connected with the practical determination of the ARIMA parameters and their apparent sensitivity to noise and nonuniform data.

NIST ALGORITHM

Another important algorithm is that used by NIST (formerly NBS) to generate their AT1 timescale. The algorithm is an asymptotic form of a Kalman filter designed for a steady-state ensemble manifesting the types of noise typical of cesium clocks [11, 12]. Either linear or nonlinear, depending on the clock, it utilizes exponential filters in the rate correction and weight revision. As used here and at NIST, it utilizes daily measurments and weights based mainly on the clock stability at a sampling time of 1 day. We did not use the nonlinear option because any clocks with drifting rates had already been eliminated from our data.

Using initial weights based on 1-day Allan variances, a timescale was generated with the NIST algorithm. Evaluating the short-term errors as before, we obtained the results in Table 6. Both the USNO and the ARIMA timescales are more stable on the short term than the NIST time–scale. The reason for this is unclear. The ratio of the highest clock weight to the lowest clock weight averaged 21.1±1.1 – this for clocks all given equal weight by the USNO algorithm and whose time–scale computed therefrom has been shown above not to be improved by weighting. Experiments with setting an upper bound on the weights and with increasing the time constant of the rate–correction filter failed to improve the NIST timescale's short–term stability.

As seen in Figure 5 and Table 7, on the long term the NIST time–scale agrees well with that of the USNO algorithm as far as both drift and nonuniformities are concerned, as may be expected because of their basic linearity.

Thus, use of the NIST algorithm would not improve the long–term accuracy or stability of the USNO timescale and would apparently degrade its short–term stability.

CONCLUSIONS

Humidity should, and increasingly will be, controlled in our clock vaults. A significant fraction of clock rate and variance changes might be eliminated by reducing the extent of human interaction or the need for it. No improvement in either accuracy, short-term stability, or long-term stability would be achieved by weighting the individual clocks differently from unity or zero with the current USNO algorithm or by employing either the NIST algorithm or one based on ARIMA modelling.

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Table 1. Mean square roots σ of the Allan variance for 44 USNO cesium clocks in units of parts in 10^{13} at 6 different sampling times τ, with standard errors in units of parts in 10^{16} given in parentheses. The clock type is given by a key at the bottom. The J45–option clocks do have significantly lower variances, as advertised. There is no dependence of variance on vault or location therein.

Clock Ser. No.	Sampling Times 1^h	5^h	1^d	5^d	25^d	50^d	Type	Vault in Bldg. No.
0116	2.791(48)	0.679(08)	0.284(22)	0.310(49)	0.39(186)	0.723	b	82
0133	3.039(65)	.021(17)	0.508(43)	0.367(49)	0.305(27)	0.571	d	16
0571	3.45(100)	1.171(46)	0.543(41)	0.360(48)	0.274		b	52
0571	3.662(66)	1.169(14)	0.491(13)	0.294(08)			b	82
0653	3.367(56)	1.250(22)	0.546(19)	0.402(29)	0.373(38)	0.465	b	1
0656	4.416	1.666	0.844	0.346			b	16
0752	6.205	2.528	1.365	0.807			b	3
0778	3.860(93)	1.435(48)	0.630(29)	0.400(25)	0.395(59)	0.753	a	16
0787	2.691(60)	0.826(40)	0.391(45)	0.47(101)	0.776		b	52
0837	3.020(69)	0.985(29)	0.446(19)	0.300(19)	0.312(92)	0.48(204)	b	16
0862	2.766(43)	0.879(13)	0.366(09)	0.269(20)	0.334(40)	0.619(93)	b	16
0873	7.178	2.835	1.264	1.157			b	52
0875	2.718(27)	0.807(12)	0.337(10)	0.292(20)	0.300(83)	0.49(151)	b	16
1025	2.562(46)	0.744(18)	0.313(33)	0.354(64)			b	1
1028	2.794(94)	0.850(08)	0.383(25)	0.285(56)			b	52
1094	2.803(40)	0.711(25)	0.279(10)	0.241(34)			b	78
1104	2.872(37)	0.802(20)	0.328(16)	0.268(50)	0.290		b	1
1117	.636(48)	1.806(30)	0.858(28)	0.441(20)	20.318(89)	0.51(308)	b	1
1300	3.680(38)	1.321(20)	0.615(21)	0.398(27)	0.32(140)	0.36(224)	b	1
1301	2.767(74)	0.815(08)	0.347(09)	0.256(44)	0.36(199)	0.675	b	16
1301	2.996(36)	0.932(08)	0.353(17)	0.229(02)			b	82
1305	3.672(61)	1.340(42)	0.599(20)	0.399(63)	0.413		b	52
1343	3.04(125)	0.859(17)	0.371(29)	0.318(36)	0.431		b	16
1423	3.954(28)	1.484(18)	0.660(12)	0.431(30)	0.33(121)	0.677	b	16
1449	2.494(71)	0.763(26)	0.252(19)	0.193(18)			b	1
1452	3.659(56)	1.364(24)	0.662(15)	0.530(56)	0.43(202)	0.615	b	1
1586	2.837(62)	0.848(26)	0.367(27)	0.322(38)	0.40(164)	0.403	b	16
1586	2.945(47)	0.877(51)	0.415(59)	0.288(37)			b	82

Type Key:
- a: Hewlett–Packard model 5061A
- b: Hewlett–Packard model 5061A option 004
- c: Hewlett–Packard model 5061A option J45
- d: Frequency and Time Systems model 4050 with high-performance tube

Table 1. Continued

Clock Ser. No.	Sampling Times 1^h	5^h	1^d	5^d	25^d	50^d	Type	Vault in Bldg. No.
1605	3.645(44)	1.336(11)	0.612(14)	0.387(20)	0.374(68)	0.63(225)	b	1
1809	2.992(24)	0.953(13)	0.430(22)	0.328(31)	0.346(57)	0.68(113)	b	16
1846	2.936(33)	0.957(15)	0.389(14)	0.266(17)	0.301(74)	0.51(274)	b	52
1986	3.99(110)	1.52(103)	0.79(126)	0.703			b	1
2098	2.817(50)	0.819(10)	0.349(12)	0.297(23)	0.345(94)	0.409	b	16
2098	2.862(41)	0.915(20)	0.415(59)	0.315(63)	0.323		b	82
2100	2.621(42)	0.724(09)	0.288(11)	0.274(31)	0.312(44)	0.41(150)	b	1
2157	2.691(34)	0.753(13)	0.323(17)	0.262(29)	0.191		b	1
2277	2.764(43)	0.693(11)	0.296(10)	0.310(23)	0.37(123)	0.643	b	1
2285	2.672	0.860	0.374	0.446			b	1
2314	2.64(294)	0.703(12)	0.253(01)	0.179(07)			b	1
2315	2.600(60)	0.691(10)	0.361(54)	0.369(50)	0.382		b	1
2481	2.809(32)	0.838(18)	0.340(13)	0.280(28)	0.418(71)	0.54(198)	c	16
2482	3.08(161)	0.86(102)	0.355(67)	0.273(72)	0.250(70)	0.226	c	52
2483	2.667(30)	0.751(13)	0.359(19)	0.320(39)	0.349(90)	0.53(180)	c	52
2484	2.453(48)	0.693(07)	0.271(07)	0.198(18)	0.408(90)	0.390	c	1
2485	2.71(103)	0.787(57)	0.53(125)	0.98(354)			c	52
2485	2.811(61)	0.743(17)	0.296(17)	0.285(32)	0.377(82)	0.488	c	78
2487	2.630(52)	0.765(13)	0.322(14)	0.268(27)	0.193(07)	0.108	c	3
2488	2.86(125)	0.824(53)	0.366(21)	0.405(60)	0.510		c	52
2493	2.555	0.716	0.327				c	1

Type Key:
- a: Hewlett–Packard model 5061A
- b: Hewlett–Packard model 5061A option 004
- c: Hewlett–Packard model 5061A option J45
- d: Frequency and Time Systems model 4050 with high–performance tube

Table 2. Mean coefficients relating the frequency and the absolute humidity for 5 cesium clocks in the USNO Building 1 vault.

Clock Serial No.	Absolute Humidity Coefficient
1117	$-0.95 \pm 0.72 \times 10^{-14} g\ m^{-3}$
1300	$-1.01 \pm 0.62 \times 10^{-14} g\ m^{-3}$
2100	$+1.64 \pm 0.60 \times 10^{-14} g\ m^{-3}$
2277	$-1.71 \pm 0.56 \times 10^{-14} g\ m^{-3}$
2484	$-0.51 \pm 0.43 \times 10^{-14} g\ m^{-3}$

Table 3. Mean absolute values for the coefficients relating the square root of the Allan variance and temperature in units of parts in 10^{15} per $°C$ for two types of Hewlett–Packard cesium clocks at USNO.

Clock Type	Sampling Time 1^h	5^h	1^d	5^d
HP option 004	7.2 ± 1.6	4.0 ± 0.9	3.5 ± 0.7	2.8 ± 0.5
HP option J45	11.3 ± 3.0	6.3 ± 1.7	4.1 ± 1.2	0.4 ± 1.3

Table 4. The mean square roots of the Allan variances of the maser relative to the USNO unweighted mean timescale and seven weighted mean timescales in units of parts in 10^{13} for 6 sampling times, with standard errors in units of parts in 10^{16}

Mean Computed Using:	Sampling Time 1^h	5^h	1^d	5^d	25^d	50^d
no weights	2.044 ± 27	0.489 ± 08	0.252 ± 27	0.180 ± 11	0.086 ± 22	0.063 ± 01
$1/\sigma^2(\tau = 1^h)$	2.080 ± 26	0.486 ± 09	0.248 ± 28	0.172 ± 11	0.097 ± 21	0.067 ± 03
$1/\sigma^2(\tau = 5^h)$	2.061 ± 25	0.474 ± 08	0.242 ± 27	0.172 ± 11	0.089 ± 23	0.061 ± 10
$1/\sigma^2(\tau = 1^d)$	2.066 ± 31	0.476 ± 08	0.242 ± 27	0.169 ± 10	0.091 ± 24	0.063 ± 01
$1/\sigma^2(\tau = 5^d)$	2.014 ± 27	$0.473 \pm 07^*$	0.243 ± 27	0.172 ± 10	0.090 ± 21	$0.056 \pm 04^*$
$1/\sigma^2(\tau = 25^d)$	2.056 ± 27	0.489 ± 08	0.253 ± 26	0.177 ± 10	0.086 ± 20	0.061 ± 06
$1/\sigma^2(\tau = 50^d)$	2.098 ± 25	0.518 ± 08	0.271 ± 24	0.192 ± 09	0.108 ± 15	0.116 ± 25
$1/\sigma^2(\tau = \tau_{min})$	2.059 ± 21	0.486 ± 08	0.252 ± 26	0.177 ± 10	0.089 ± 18	0.071 ± 09

* The only values significantly smaller than for the unweighted mean.

Table 5. The drifts and nonuniformities of the unweighted and weighted USNO timescales. The standard errors of the estimate are for a parabolic (1986) or linear (1987) fit.

Mean Computed Using:	Drift Relative to Unweighted Mean (ns/day)	Standard Error of Estimate of TAI–Unsteered USNO Mean (ns) 1986	1987
no weights		±65.3	± 39.9
$1/\sigma^2(\tau = 1^h)$	0.011 ± 0.006	±68.6	±37.9
$1/\sigma^2(\tau = 5^h)$	0.039 ± 0.007	±69.0	±38.8
$1/\sigma^2(\tau = 1^d)$	0.011 ± 0.011	±70.2	±36.5
$1/\sigma^2(\tau = 5^d)$	-0.041 ± 0.010	±68.2	±38.1
$1/\sigma^2(\tau = 25^d)$	0.018 ± 0.012	±69.7	±37.8
$1/\sigma^2(\tau = 50^d)$	0.004 ± 0.021	±72.5	±38.1
$1/\sigma^2(\tau = \tau_{min})$	0.051 ± 0.009	±67.6	±41.1

Table 6. The mean square roots of the Allan variances of the maser relative to the timescale generated by three algorithms (using USNO cesium data) in units of parts in 10^{13} for 6 sampling times, with standard errors in units of parts in 10^{16}

Algorithm	Sampling Time 1^h	5^h	1^d	5^d	25^d	50^d
Unweighted USNO	2.044 ± 27	0.489 ± 08	0.252 ± 27	0.180 ± 11	0.086 ± 22	0.063 ± 01
Robust ARIMA	2.260 ± 23	0.570 ± 11	0.265 ± 29	0.215 ± 15	0.257 ± 64	0.417 ± 63
NIST			0.469 ± 39	0.348 ± 34	0.095 ± 16	0.075 ± 21

Table 7. The drifts and nonuniformities of timescales generated by three algorithms using USNO data for the years 1986 and 1987

Algorithm	Drift Relative to Unweighted USNO Mean (ns/day)		Standard Error of Estimate of TAI–Unsteered Mean (ns)	
	1986	1987	1986	1987
Unweighted USNO			±65.3*	±39.9
Robust ARIMA	-3.40 ± 0.18	3.95 ± 0.07	158.7*	44.7*
NIST	0.0113 ± 0.0096		70.3	40.6

*Quadratic fits; the other fits are linear.

FIG. 1. SQUARE ROOT OF ALLAN VARIANCE VS. SERIAL NUMBER (TAU OF MIN. VARIANCE)

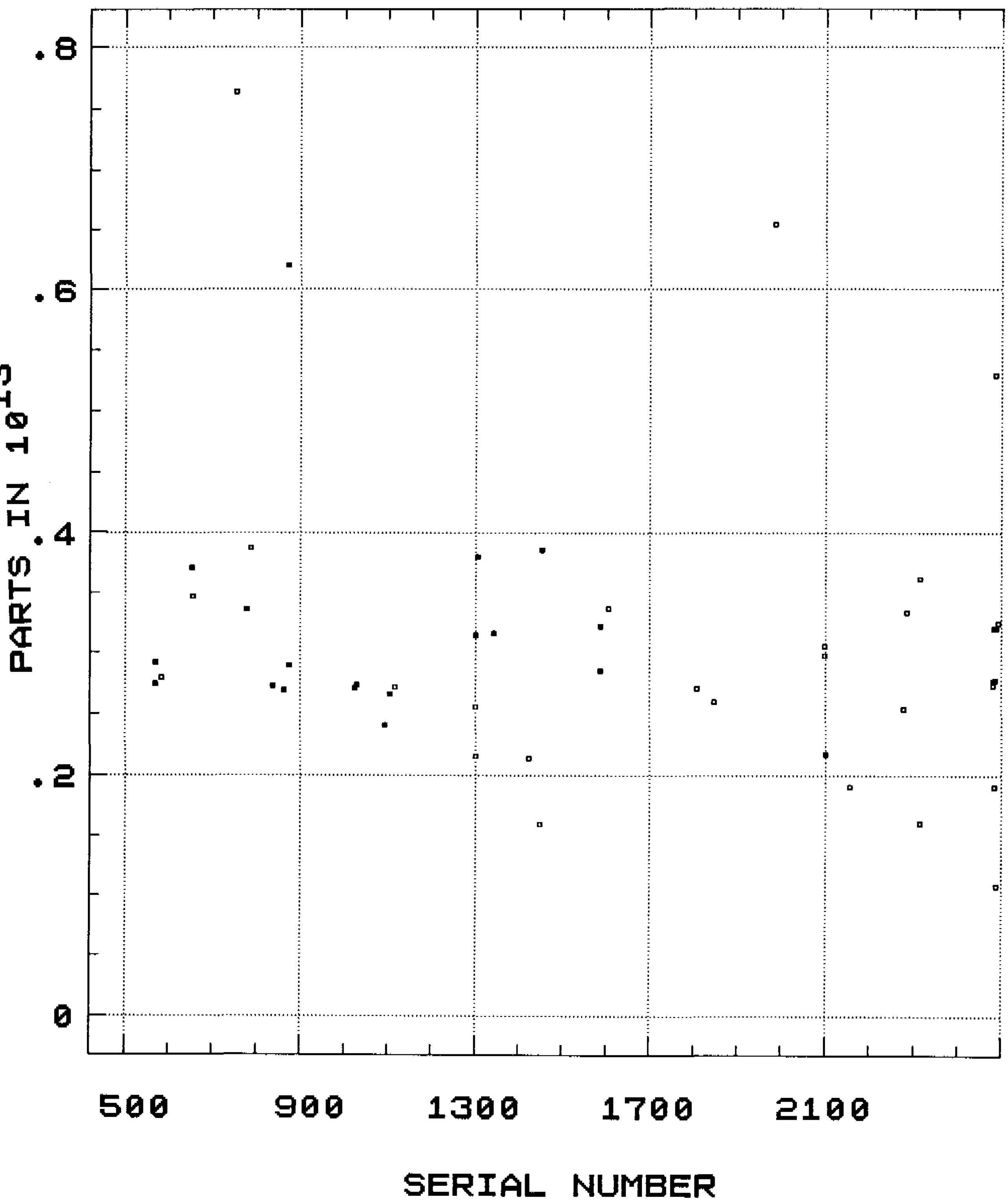

FIG. 2. TAI - USNO MEAN

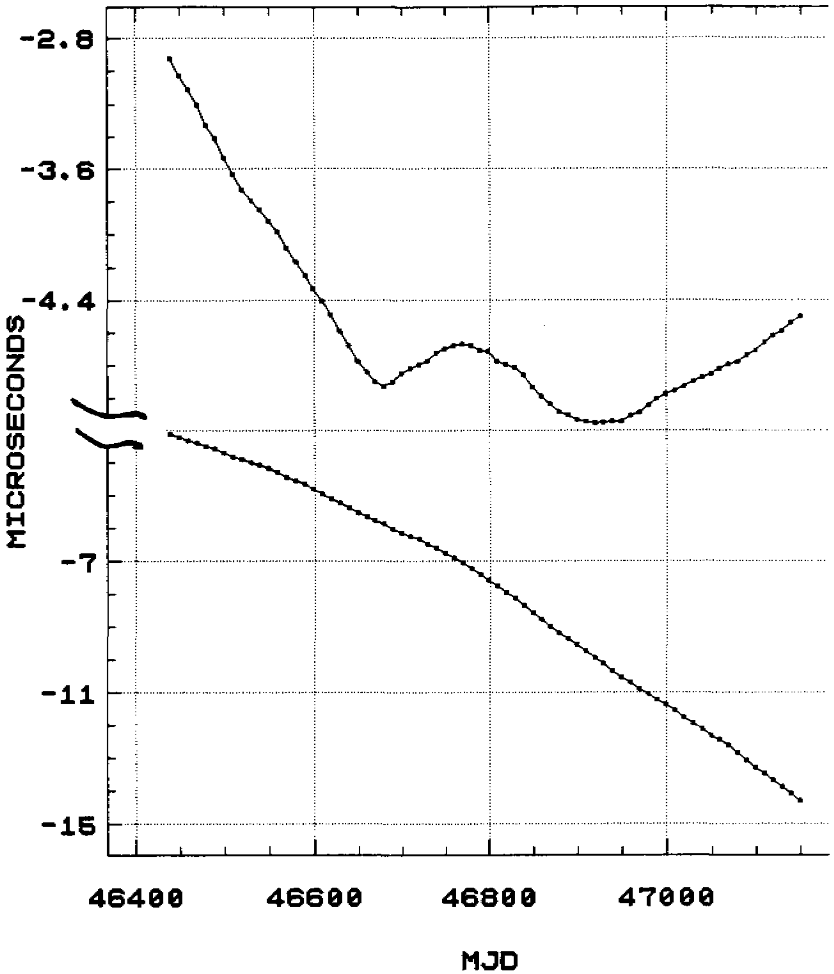

FIG. 3. HAMPEL'S PSI FUNCTION

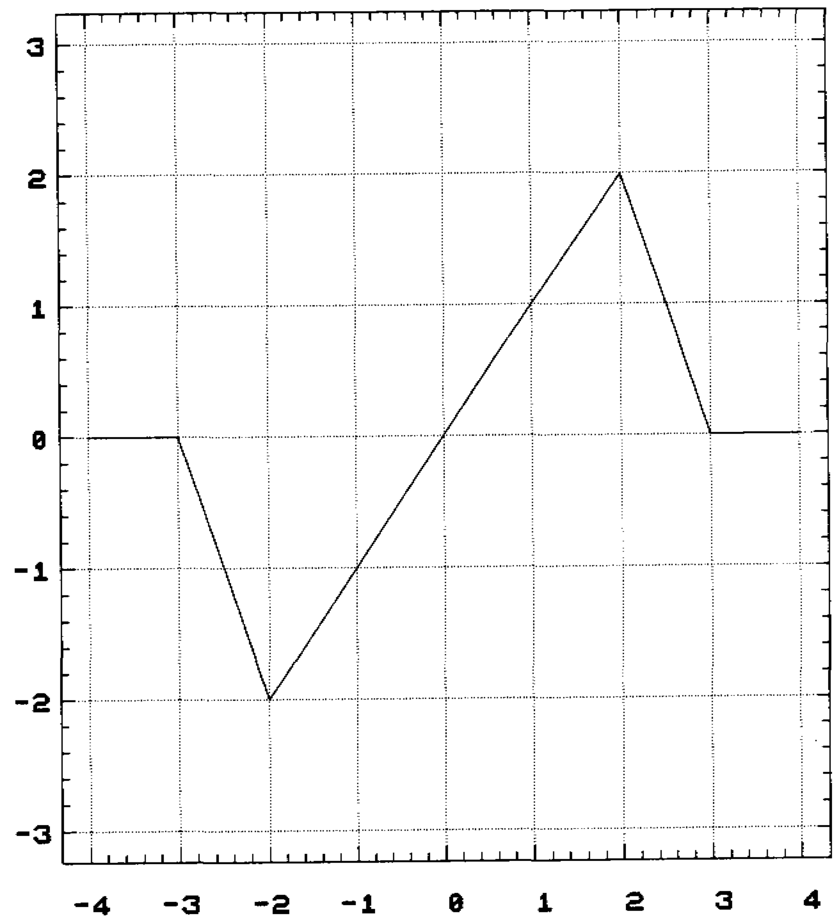

FIG. 4. USNO MEAN - ROBUST ARIMA

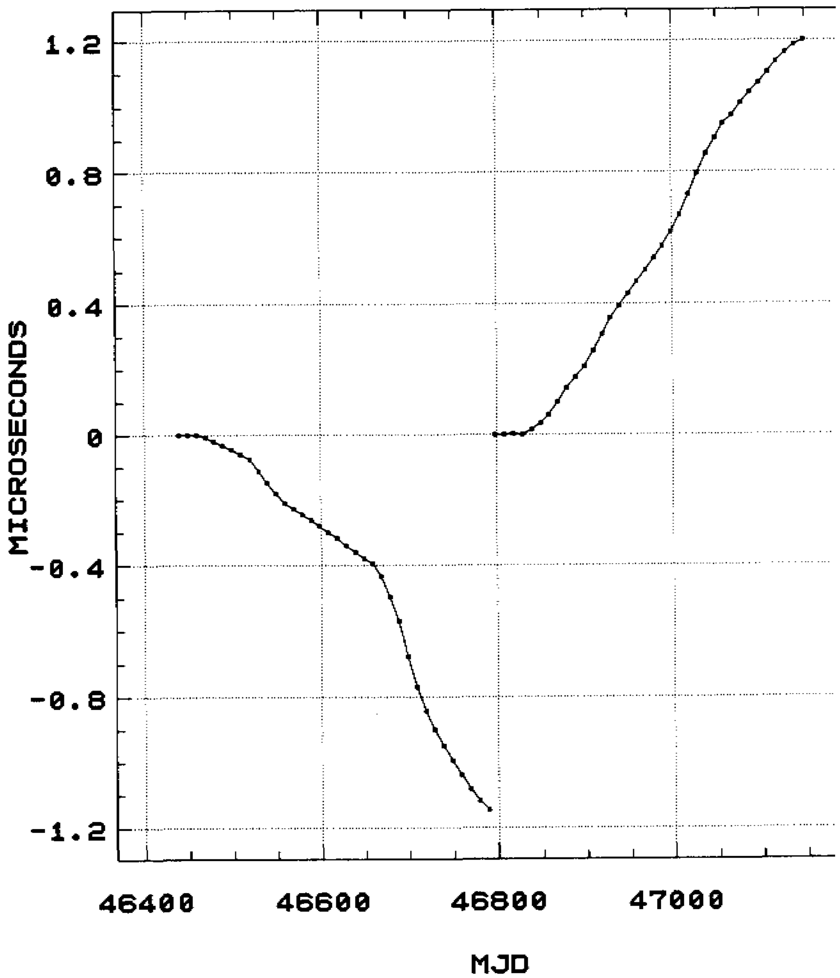

FIG. 5. USNO MEAN - NIST MEAN