AUTOMATIC CALIBRATION OF SYSTEMATIC ERRORS IN FAST PULSE MEASUREMENTS

David C. Chu

Hewlett Packard Company
5301 Stevens Creek Blvd
Santa Clara, California
U. S. A. 95051

ABSTRACT

In calibrating a precision time-interval measuring instrument such as a counter, one needs two signals separated by a known time interval at the trigger level. The waveforms of these sources should imitate as much as possible the signals to be measured in amplitude, slope, transition time etc. This paper describes a calibrator which splits a given input waveform linearly and passively into two signals which are either identical or mirrored about a given trigger level. These are measured by the counter either directly or swapped via rf relays. Through an algorithm of measurements (via HPIB) and computation, counter systematic errors can be extracted for any particular waveform type. Accuracy of the calibrator itself is better than 10 ps, traceable to the NBS, and for the counter, better than 100 ps after calibration.

INTRODUCTION

A time interval counter triggers at a particular voltage of the start- and stop- channel waveforms. A variety of techniques are used to measure the outputs of these input amplifier/triggers. The errors introduced can be classified into two main categories: random, ie those not repeatable from one measurement to the next; and systematic, those which are constant for a particular type of measurements, but may change from one type to another. The following is a summary of these errors:

TIME INTERVAL MEASUREMENT ERRORS

RANDOM (Resolution)
- Quantization
- Trigger

SYSTEMATIC (Accuracy)
- Timebase
- Differential Linearity
- Skew

An example of random error is the plus-or-minus one count quantization. Quantization error is minimized by using a fast time base clock and by interpolation [1],[2]. Another example is trigger error, which is the effect of noise on the input signal, amplifier, trigger level, or the signal. It is most pronounced when the signal is small or slow and can therefore be minimized by having signals of sufficient amplitude and slew-rate. Both errors can be further reduced by averaging [3] over many measurements. Other techniques [4] exists to improve resolution using CW mixing, but these are not readily applicable to measurement of fast random pulses. When these as well as other techniques are taken, it is not uncommon for resolution to be reduced to 1 or 2 ps. Accuracy, unmasked by reading jitter, can now be dealt with.

One aspect of accuracy is that of the time base clock. Time base accuracy is important only in measuring longer time intervals. It is hard to imagine a crystal oscillator bad enough not to give 100 ps accuracy in measuring 100 ns time interval. On the other hand, having an extremely accurate timebase oscillator will not generate benefit that can be realized if the time intervals are short.

Another source of error is differential linearity, which is the change of measurement result per change of input as a function of input. It is caused mainly by cross-talk and line mismatches and tends to be data-dependent. It is minimized when active edges are isolated.

The last item is skew or offset. It has to do with the ability to define zero time accurately. The measurement and removal of skew is the topic of this paper. Skew is by far the largest error, typically an order of magnitude larger than differential linearity and two orders of magnitude larger than resolution. Furthermore, it is not a single number one can attach to a counter, but rather it varies with each measurement situation, such as slope direction, amplitude, slew rate, trigger level...etc. It may involve cable length, probe delay as well as electrical path length differences inside the counter.

To identify and remove skew, that is, to accurately zero out the counter under different measurement conditions, we make use of an instrument J06-5999A Time Interval Calibrator we developed at Hewlett Packard. The instrument precisely splits a given input signal linearly and passively, with and without polarity inversion. The two signals generated are then delivered to the counter for measurement . Linearity and passivity ensures equal turn-on and turn-off delays, and that delays change negligibly with signal amplitude, time and (within a band) frequency. Furthermore, the signals are AC-coupled, and therefore timing is automatically correlated at ground, i.e. inverted signals are symmetrical about ground. A biasing scheme allows the same time-correlation at ground to take place at any other trigger level. Swapping and averaging remove the effect of skew within the calibrator itself.

Using this calibrator, the user chooses his own calibration waveform to simulate as closely as desired the actual signals to be measured. Through an algorithm to be described, the counter systematic errors can be extracted from the result of the calibration process. The process can be automatically executed by a program running the calibrator, counter, and an optional active probe via HPIB.

One such calibrator was calibrated at the Fast Pulse Metrology Department of NBS [5]. Through comparison with this unit, the accuracy of any other calibrator is traceable to the NBS data.

COUNTER MODEL

In Figure 1, we show a model of a time interval counter as a perfect counter preceded by two input amplifiers whose propagation delays are unknown, but are assumed to to constant for a particular input waveform. The counter is "perfect" in the sense that errors from resolution, linearity and other sources are negligible compared with counter offset systematic error. We further assume that the offset errors are different for + and - slopes, and that the delay of A is different from that of B. If a given waveform, say, a square wave of +/- 1 volt is fed to both channels, then all 4 edges in one period are delayed by different amounts A+, A-, B+, and B- at 0 volt, the trigger level. The symbol A+ stands for the delay at the A channel for a positive slope at 0 volt, etc. The perfect counter then measures the delayed version of the input. Depending on the slopes selected, and assuming the positive half-cycle of the input is H and the negative half L, the following results will be obtained:

RESULT	SLOPE SELECTION
B+ - A+	for A-pos and B-pos
B- - A-	for A-neg and B-neg
B- - A+ + H	for A-pos and B-neg
B+ - A- + L	for A-neg and B-pos

that if the signals to the counter were truly simultaneous, the first two measurements would give the calibration constants for the same-slope cases. The last two measurements would give the opposite-slope constants only if one knew what H and L were. Since usually we have no assurance of simultaneity nor any

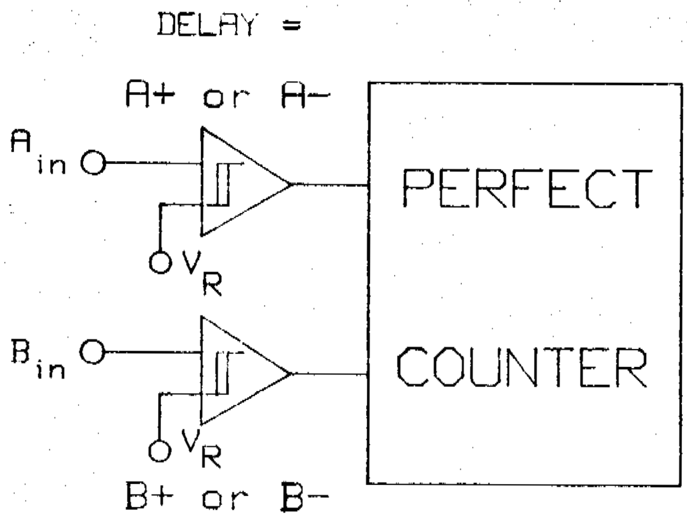

Figure 1. Model of a time interval counter: a perfect counter preceded by amp/triggers of unknown propagation delays.

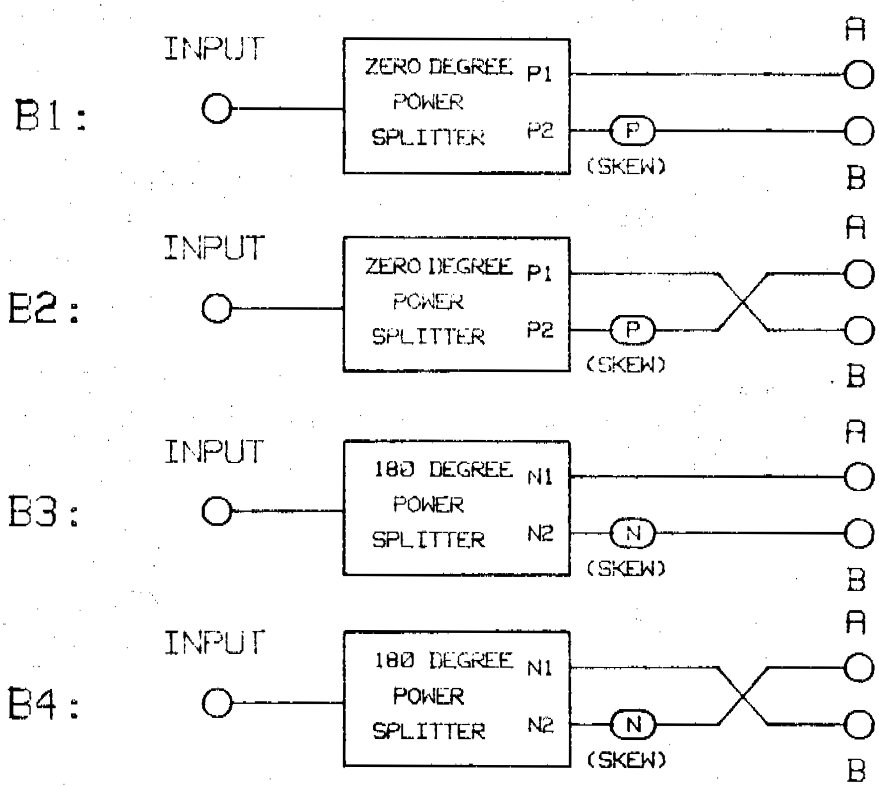

Figure 2

Model of the Hewlett Packard J06-59992A Time Interval Calibrator. The four states B1 to B4 are switched from one to another by rf-relays.

accurate knowledge of H and L, these four measurements are not sufficient to produce the required calibration constants.

CALIBRATOR MODEL

Figure 2 shows a model of the time interval calibrator. In the first two states, B1 and B2, a zero-degree power splitter is used. An input signal is split into two identical waveforms and routed to the outputs A and B either directly or swapped by the action of an rf-relay. We allow for a skew P between the waveforms. The line-lengths from the cross-switch relay to the outputs are matched to better than 10 ps. States B3 and B4 are similar but use a 180-degree power splitter instead. The output waveforms at N1 and N2 are mirror images of each other about, for the moment, ground. We also allow for a possible skew N between outputs N1 and N2. The power splitters are AC-coupled and is matched to 50 ohms at all ports. There is a nominal loss of 3 dB from input to output. Some biasing circuits are omitted for the sake of clarity. Switching from one state to another are done by rf-relays either manually or through HPIB remotely.

Since we allow for skew between power splitter outputs, no effort was made to match line lengths preceding the ross-switch relay. Every effort, however, was made to ensure the lines are matched after the cross-switch relay, typically to well below 10 ps.

TIME INTERVAL CALIBRATION

In time interval calibration, we connect calibrator output A to counter START input, and output B to STOP input. We intend to obtain 4 calibration constants corresponding to the 4 combinations of slopes. We assume that the user has chosen a signal so that the signals from the calibrator outputs resemble the signals he will be measuring. The connecting cables, which can be probes also, have skews which are lumped with the counter skew and removed with the calibration constants. We perform 8 time-interval measurements tabulated below and label the results T1 through T8.

#	Calibrator State	A slope	B slope	Result
1)	B1	+	+	T1
2)	B1	-	-	T2
3)	B2	-	-	T3
4)	B2	+	+	T4
5)	B3	+	-	T5
6)	B3	-	+	T6
7)	B4	-	+	T7
8)	B4	+	-	T8

Tracing signal paths from beginning to end in Figure 3 and noting path differences, we expect the measurement results to be as follows:

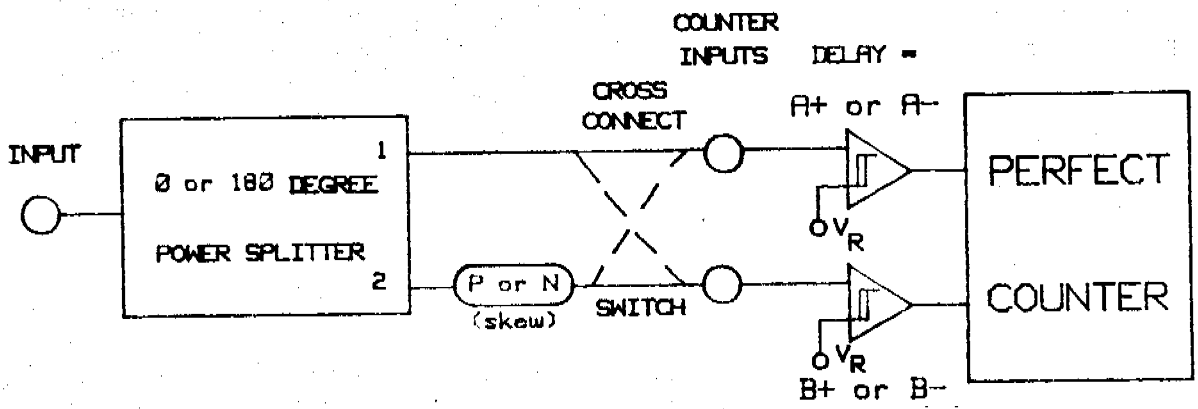

Figure 3 Instrument Connection for time interval calibration.

$$T1 = B+ - A+ + P \qquad (1)$$
$$T2 = B- - A- + P \qquad (2)$$
$$T3 = B- - A- - P \qquad (3)$$
$$T4 = B+ - A+ - P \qquad (4)$$
$$T5 = B- - A+ + N \qquad (5)$$
$$T6 = B+ - A- + N \qquad (6)$$
$$T7 = B+ - A- - N \qquad (7)$$
$$T8 = B- - A+ - N \qquad (8)$$

The four calibration constants of interest are simply the differences between the B delays and the A delays with the four combinations of slopes. Using the pseudoinverse method [6], the minimum-variance estimates for the calibration constants are readily obtained from the eight equations as:

$$T{++} = B+ - A+ = 0.5*(T1 + T4) \qquad (9)$$
$$T{--} = B- - A- = 0.5*(T2 + T3) \qquad (10)$$
$$T{+-} = B- - A+ = 0.5*(T5 + T8) \qquad (11)$$
$$T{-+} = B+ - A- = 0.5*(T6 + T7) \qquad (12)$$

We make use of the extra degrees of freedom in the system of equations to check for consistency and experimental errors. For that purpose, let us make the P's in equations (1) and (2) P+, and those in (3) and (4) P-. This implies that the calibrator does not have to have the same skew for positive and negative slopes. We can check this by solving for P+ and P- and see if they are the same. With some manipulation, we get

$$P+ - P- = (T1 - T2 + T3 - T4)/2 \qquad (13)$$

Similarly, by making the N's in (5) and (6) to be N+ and those in (7) and (8) to be N-, we solve for the difference

$$N+ - N- = (T5 - T6 + T7 - T8)/2 \qquad (14)$$

The consistency parameters obtained by (13) and (14) give us an indication of the typical experimental errors. In practice they turn out to be quite small.

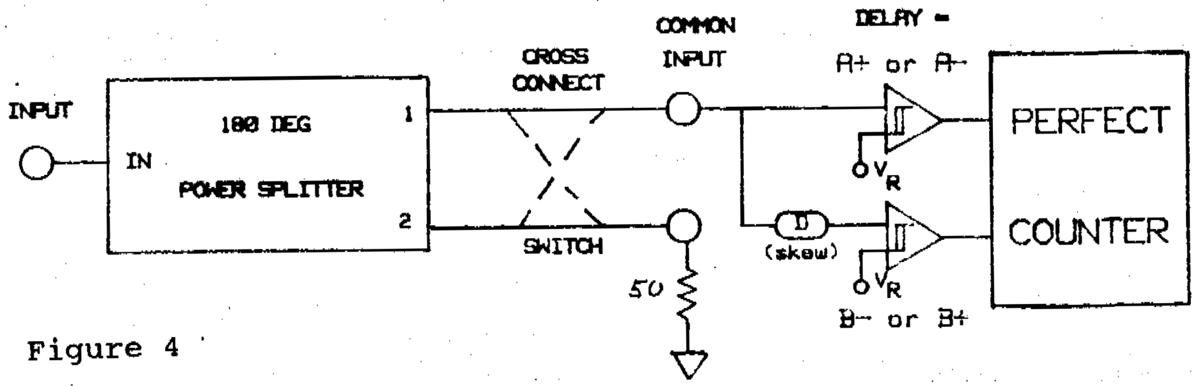

Figure 4

Instrument connection for width, rise/fall time calibration.

WIDTH CALIBRATION

Width is a single-channel measurement where the counter splits the signal internally to make an opposite-slope time interval measurement. The effect of the internal splitter D must be taken into account. Figure 4 shows a model of the counter with a common input, connected to the calibrator. The unused terminal provides termination and is only incidental to the calibration process. The input waveform is assumed to be an approximate square-wave with the positive half-wave H and the negative L. Four width measurements are made, two with calibrator in B3 and two in B4, labeled W1 through W4. Figure 5a shows the three waveforms, input to counter, inputs to perfect counter ch A and ch B, with calibrator at B3. Figure 5b shows the same waveforms with the calibrator at B4. We want to point out that the input-to-counter waveforms in 4a and 4b are mirrored images of each other. The four measurements are tabulated below:

#	Calibrator State	A slope	B slope	Result
1	B3	+	-	W1
2	B3	-	+	W2
3	B4	-	+	W3
4	B4	+	-	W4

Through signal tracing, or studying Figures 5a and 5b, we can make up the composition of W1-W4, as

$$W1 = B- - A+ + D + H \tag{15}$$
$$W2 = B+ - A- + D + L \tag{16}$$
$$W3 = B+ - A- + D + H \tag{17}$$
$$W4 = B- - A+ + D + L \tag{18}$$

Lastly a period average measurement of the signal is made with a 100 ms gate. We assume negligible error in such a measurement, and the period obtained, Per, is the sum of H and L, i.e.

$$Per = H + L \tag{19}$$

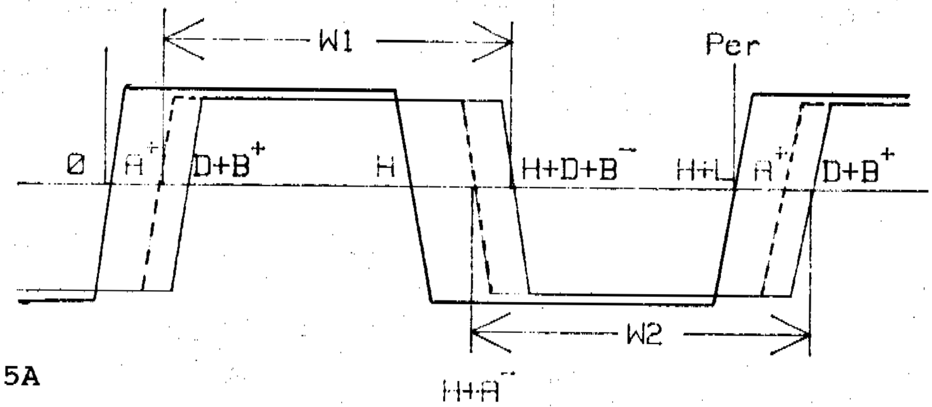

Figure 5A

Waveform for during width calibration using calibrator state B3.

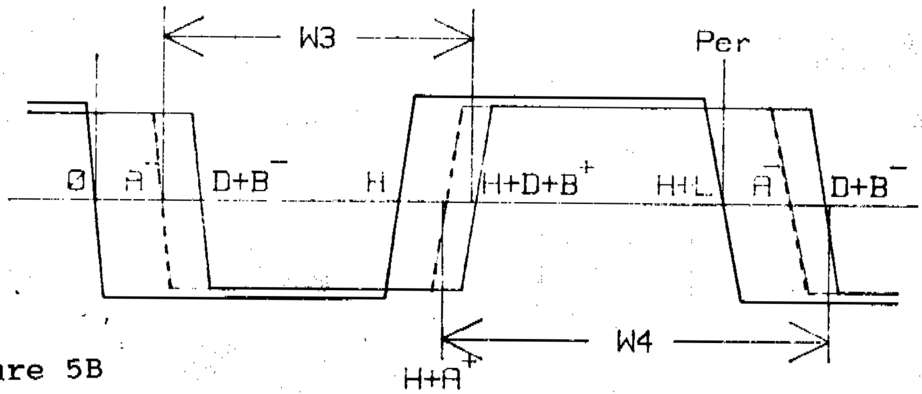

Figure 5B

Waveform during width calibration using calibrator state B4. Note input waveform is the mirror image of that in Fig. 5A

There are two width calibration constants , W+- for positive pulse and W-+ for negative pulse, W+- being (B- - A+ + D) and W-+ being (B+ - A- +D). Substituting (19) as a constraint for (15) through (18) and again using the pseudoinverse method, the minimum variance estimates for W+- and W-+ can be computed from W1 thru W4 and Per as,

$$W{+-} = B- - A+ + D = (W1 + W4 - Per)/2 \qquad (20)$$
$$W{-+} = B+ - A- + D = (W2 + W3 - Per)/2 \qquad (21)$$

Excess degrees of freedom are also found here. We can get W+-(a) and W+-(b) to bracket the W+- in (20), and W-+(a) and W-+(b) in (21) for W-+:

$$W{+-}(a) = W1 - Per/2 - (W1 - W2 + W3 - W4)/4 \qquad (22)$$
$$W{+-}(b) = W4 - Per/2 + (W1 - W2 + W3 - W4)/4 \qquad (22)$$
$$W{-+}(a) = W2 - Per/2 + (W1 - W2 + W3 - W4)/4 \qquad (23)$$
$$W{-+}(b) = W3 - Per/2 - (W1 - W2 + W3 - W4)/4 \qquad (24)$$

It is essential that the input signal be stable during width calibration as H and L are assumed to be constant throughout. Also H and L should be approximately equal so that calibration is performed near the midpulse point for both edges, minimizing secondary effects.

RISE AND FALL SKEW CALIBRATION

At the time W1-W4 were taken, 2 same-slope measurements could be taken. We name the results R for ++ and F for --. These calibration constants R and F are useful in nulling out skews at mid-pulse prior to making rise- and fall-time measurements at say 20 to 80%.

ARBITRARY TRIGGER LEVEL

Up to this point, all calibration measurements have been done with trigger level at 0.00 volt. This is not desirable for users who would like to calibrate at the threshold of the logic family he will be testing, e.g. VBB for ECL. It is possible to do so with the addition of bias current sources and blocking capacitors to the calibrator outputs. Figure 6 shows the hardware needed. Current sources are controlled by the op-amps which sense the dc voltages of the transmission lines and compare them to the reference voltage input which should be the same as the trigger level of the counter. The blocking capacitors keep the dc currents from flowing into the calibrator. With this scheme, the entire waveforms are raised to precisely (within 2 mV), equal to the trigger level.

Trigger level errors will indeed lead to errors in calibration, but the effect will be minimized if fast pulses are used. On the other hand, hysteresis effect of the counter input circuitry is cancelled out, at least for the first order, if the middle of the hysteresis band is taken to be the trigger level. This is true for both same- and opposite-slope measurements. A detailed discussion of these issues is omitted for clarity.

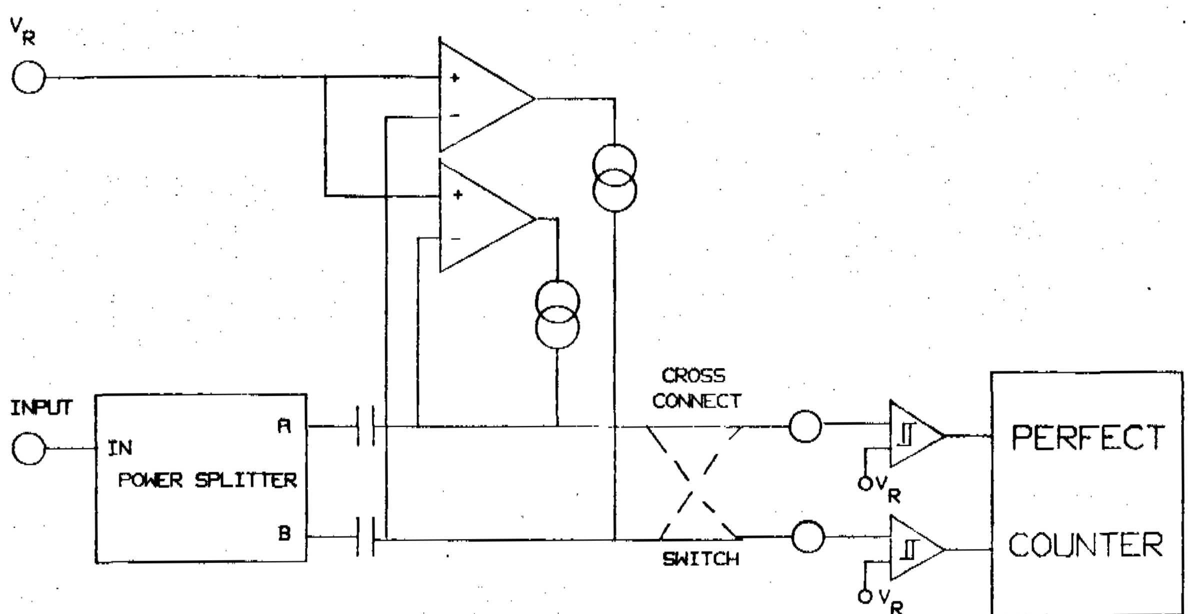

Figure 6

Time Interval Calibrator shown with biasing circuitry to offset biasing signals to match the trigger level used.

EXPERIMENTAL VERIFICATION

During the course of development, we at one time calibrated 50 counters as they were being manufactured. All the counters were made to measure a time interval and a width from the same source, with and without this calibration. The results are shown in Figure 7. Although the measurements were made over several weeks and by different technicians, the calibrated spread is well under 100 ps.

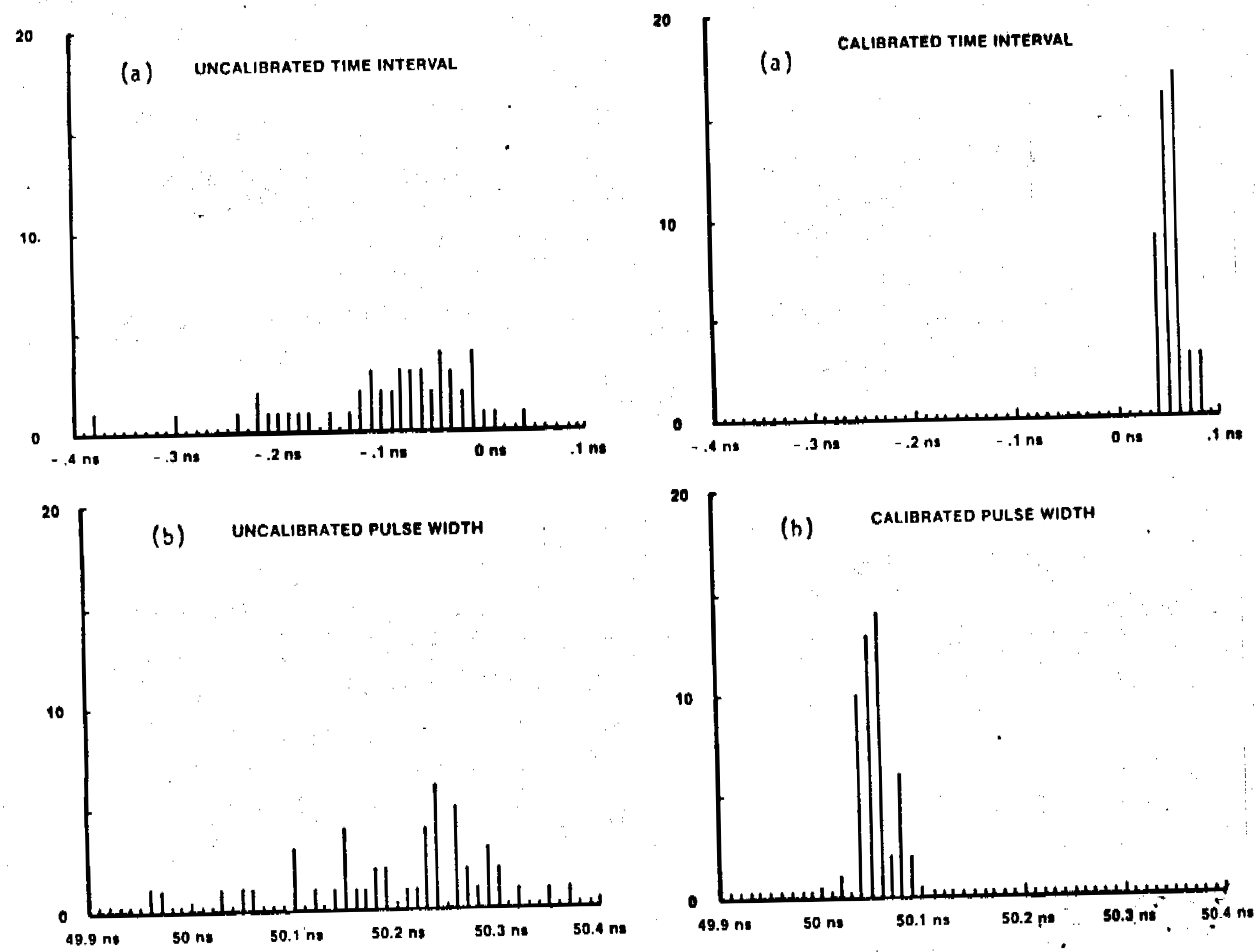

Figure 7

Experimental result of calibrating 40 units of 5370B counter for (a) time interval (opposite-slopes), and (b) width.

NBS TRACEABILITY

An early version of J06-59992A calibrator was sent to the Fast Pulse Metrology Department of the National Bureau of Standards for calibration. Eight time interval measurements were made on the unit at 100 MHz with approximate square waves 1 volt peak to peak centered around ground, or 0.00 volt. The resulting time intervals from Output A to Output B, as reported by NBS, are the following:

T11 =	+ to +	transitions	@ calibrator state B1 =	12	ps
T21 =	+ to +	transitions	@ calibrator state B2 =	-34	
T31 =	- to +	transitions	@ calibrator state B3 =	61	
T41 =	+ to -	transitions	@ calibrator state B4 =	-89	
T12 =	- to -	transitions	@ calibrator state B1 =	13	
T22 =	- to -	transitions	@ calibrator state B2 =	-33	
T32 =	+ to -	transitions	@ calibrator state B3 =	77	
T42 =	- to +	transitions	@ calibrator state B4 =	-100	

The accuracy of these measurements is 9 ps. However, there is an offset of 11.6 ps from a power-splitter used. It is not clear whether the above numbers have taken into account this 11.6 ps. This uncertainty is immaterial, however, because a systematic offset to these measurements can be measured.

Any unit of J06-5999A can be traced to the NBS calibration as follows: a pulse generator at 100 MHz, 1.5V p-p, and ~50% duty cycle is fed to the NBS unit. The same eight measurements are made with an HP5370B counter. Then the test unit is substituted for the NBS unit, and again the same eight measurements are made. The difference is added to the data above to give the traced value for the test unit.

Tij(traced) = Tij(test unit) - Tij(NBS unit) + Tij(NBS data)

The process is facilitated by the use of a BASIC program orchestrating the instruments under HPIB. Since only differences are used, systematic errors in the measurements are irrelevent.

Assuming the J06- Calibrator skews are:

P	:	P power-splitter skew
N	:	N power-splitter skew
Ofs	:	B channel to A channel post-cross switch skew

then these parameter are related to Tij(traced) by:

$$
\begin{bmatrix} 1 & & 1 \\ 1 & & 1 \\ -1 & & 1 \\ -1 & & 1 \\ & 1 & 1 \\ & 1 & 1 \\ & -1 & 1 \\ & -1 & 1 \end{bmatrix}
\begin{bmatrix} P \\ N \\ Ofs \end{bmatrix}
=
\begin{bmatrix} T11 \\ T12 \\ T21 \\ T22 \\ T31 \\ T32 \\ T41 \\ T42 \end{bmatrix}
$$

This can be expressed in matrix notation as,

$$A * X = T$$

Using the again the pseudoinverse method from statistics, the minimum variance estimate of X is, (A' is the transpose of A),

$$X\ (\text{estimate}) = (A'A)^{-1} * A' * T$$

or

$$\begin{bmatrix} P \\ N \\ Ofs \end{bmatrix} = \begin{bmatrix} 1/4 & 1/4 & -1/4 & -1/4 & & & & \\ & & & & 1/4 & 1/4 & -1/4 & -1/4 \\ 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 \end{bmatrix} * \begin{bmatrix} T11 \\ T12 \\ T21 \\ T22 \\ T31 \\ T32 \\ T41 \\ T42 \end{bmatrix}$$

It is seen that the best estimate of "Ofs" is simply the average value of all the eight T's. Applying it to the NBS data, the estimate of the offset is -11.6 ps. This offset value depends directly on the absolute accuracy of the NBS measurements.

In the absence of any offset and other measurement errors the following relationships are considered ideal,

$$\begin{aligned} T11 &= -T21 \\ T12 &= -T22 \\ T31 &= -T41 \\ T32 &= -T42 \\ T31 &= T32 \\ T41 &= T42 \end{aligned}$$

OFFSET MEASUREMENTS

Offset of the calibrator can be measured independent of the absolute accuracy of the measuring instruments. The procedure is outlined below: Eight measurements are needed, four of which are the same as T11, T12, T31 and T32 above. The other four, T'21, T'22, T'41, T'42 are similar to T21, T22, T41, and T42 except that the cables from the test unit to the counter are swapped at the calibrator output ends. Pre-crossswitch skews and counter skews are switched twice, and are therefore not effectively unswitched. any offset between the cross-switch and output, however, is not switched and appears as a difference. This process is illustrated in Figures 8 and 9.

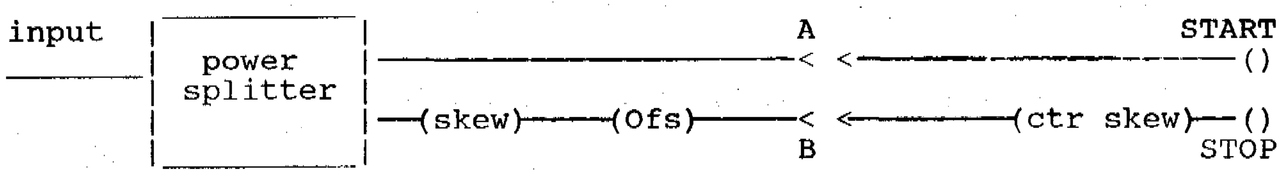

Figure 8. Condition for measuring T11, T12, T31, and T32

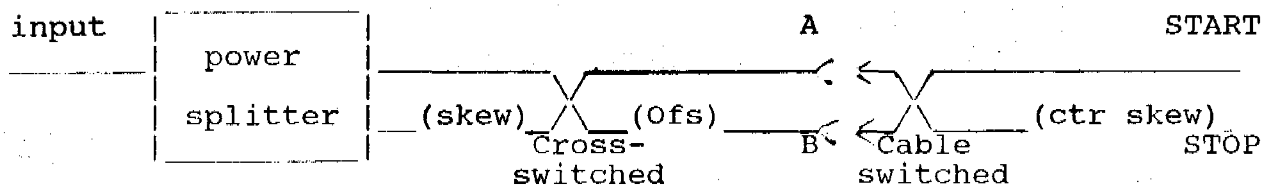

Figure 9. Condition for measuring T'21, T'22, T'41 and T'42

The difference, say between T11 and T'21, is twice "Ofs" as other skews are effectively unswitched. Similar relationships exist between T12 and T'22, T31 and T'41, T32 and T'42. The unswitched skews can be lumped together as "Pskw" or "Nskw" and the mathematical relationships between Ofs, skw's and T's are expressible as:

$$\begin{bmatrix} 1 & & 1 \\ 1 & & 1 \\ & 1 & 1 \\ & 1 & 1 \\ 1 & & -1 \\ 1 & & -1 \\ & 1 & -1 \\ & 1 & -1 \end{bmatrix} \begin{bmatrix} \mathrm{Pskw} \\ \mathrm{Nskw} \\ \mathrm{Ofs} \end{bmatrix} = \begin{bmatrix} T11 \\ T12 \\ T31 \\ T32 \\ T'21 \\ T'22 \\ T'41 \\ T'42 \end{bmatrix}$$

This is in the form of B * X = D in matrix notation. The min-squared error estimate of Ofs, using the pseudo-inverse method, X(estimate) is given by,

$$X \text{ estimate} = (B'B)^{-1} * B' * D$$

or,

$$\begin{bmatrix} \mathrm{Pskw} \\ \\ \mathrm{Nskw} \\ \\ \mathrm{Ofs} \end{bmatrix} = \begin{bmatrix} 1/4 & 1/4 & & & 1/4 & 1/4 & & \\ & & & & & & & \\ & & 1/4 & 1/4 & & & 1/4 & 1/4 \\ & & & & & & & \\ 1/8 & 1/8 & 1/8 & 1/8 & -1/8 & -1/8 & -1/8 & -1/8 \end{bmatrix} * \begin{bmatrix} T11 \\ T12 \\ T31 \\ T32 \\ T'21 \\ T'22 \\ T'41 \\ T'42 \end{bmatrix}$$

The "Skws" are counter dependent and are therefore arbitrary. On the other hand, "Ofs" is computed independent of both counter and splitter skews. The best estimate of Ofs is simply,

$$\mathrm{Ofs}_{\text{estimate}} = (T11 + T12 + T31 + T32 - T'21 - T'22 - T'41 - T'42)/8$$

BEST HP ESTIMATE BASED ON NBS DATA

Using this value of Ofs which is independent of instrument systematic errors we can trace the test unit performance to NBS data up to a constant. The best hp estimate Tij for the eight time intervals is given by:

Tij(best) =	Tij(test unit) -	Tij(NBS) +	Tij(NBS) +	11.6 +	Ofs
hp estimate	measured	measured	data	ps	estimate

Thus a J06-59992A Time Interval Calibrator can be traced to the NBS calibration unit in two ways. It can be traced absolutely; accepting the NBS data as absolutely accurate. It can also be traced to the NBS data up to a constant, and the constant can be measured.

CONCLUSION

In summary, we have developed a new way of calibrating out systematic timing error in time interval measurement. It is good for same-slope as well as opposite-slope, for timing between channels as well as within the same channel, It can be performed in conditions resembling the actual measurements, be it square wave, sine wave or triangular wave, with or without bias. It is linear and passive, and therefore lends itself to be certifiable by NBS either directly or traced through Hewlett-Packard. It is automated and can be done by relatively unskilled person. The method and apparatus can be applied to any time interval measuring instrument, be it counter, oscilloscope, or VLSI tester.

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