ON-ORBIT FREQUENCY STABILITY ANALYSIS OF THE GPS NAVSTAR'S 3 AND 4 RUBIDIUM CLOCKS AND NAVSTAR'S 5 AND 6 CESIUM CLOCKS

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This paper describes the on-orbit frequency stability performance analysis of the GPS NAVSTARs 3 and 4 rubidium clocks, and the NAVSTARs 5 and 6 cesium clocks. Time-domain measurements, taken from the four GPS monitor sites, have been analyzed to estimate the short- and long-term frequency stability performance of the NAVSTAR clocks. The data analyzed includes measurements from 1981, 1982, and the first 100 days of 1983. Short- and long-term results are presented for data collected during 1982. The Allan variance was used as the measure of frequency stability performance in the time domain.

The time-domain noise analysis results indicate a white noise FM process is present, in both rubidium and cesium clocks, for sample times of 900- and 1800-seconds. The projected value of this white noise FM process to a 1-day sample time agrees closely with the 1-day sample time stability results, for both rubidium and cesium clocks, indicating an underlying white noise FM process for sample times ranging from 900 seconds to 1 day.

A random walk FM process was measured for the NAVSTARs 3 and 4 rubidium clocks for sample times of 1 to 10 days. A flicker noise FM process was measured for the NAVSTARs 5 and 6 cesium clocks, for sample times of 1 to 10 days.

The NAVSTARs 3 and 4 rubidium clock long-term frequency stability values are in good agreement with the expected performance. The NAVSTAR-3 short-term stability results indicate an anomally which has peak effect at a 1.25-hour sample time.

The NAVSTARs 5 and 6 cesium clock long-term frequency stability values are in good agreement with the expected performance. For sample times of 2- to 10-days the cesium clocks have better frequency stability results than the rubidium clocks.

INTRODUCTION

The NAVSTAR Global Positioning System (GPS) is a Department of Defense (DOD) space-based satellite system. When operational in the late 1980's, 18 - 24 satellites in six orbital planes will provide accurate navigation and precise time information to users anywhere in the world. Examples of GPS use are weapons delivery, point-to-point navigation, search/rescue operations and passive rendezvous. It can provide navigational updates to platforms with other navigation systems.

One role of the Naval Research Laboratory (NRL) in GPS is to provide space-qualified atomic clocks for use in the NAVSTAR spacecraft. The responsibility of NRL includes pre-flight and post-flight frequency stability analyses (1,2) to insure that on-orbit accuracy and stability requirements are met.

This presentation describes the on-orbit frequency stability performance analysis of the GPS NAVSTARs 3 and 4 rubidium clocks, and the NAVSTARs 5 and 6 cesium clocks. Time-domain measurements, taken from the four GPS Monitor Sites (MS), have been analyzed to estimate the short- and long-term frequency stability performance of the NAVSTAR clocks. The results include long-term (1- to 10-day sample times) results for data collected during 1981, 1982, and the first 100 days of 1983. Short- and long-term results are presented for data collected during 1982.

The first part of the presentation briefly describes the NAVSTAR GPS system, with emphasis on the clock measurements. Equations are then presented which permit the separation of the orbital signal, and other smaller effects, from the clock offset between a NAVSTAR clock and a GPS monitor site clock. A time-domain analysis of the NAVSTARs 3, 4, 5, and 6 clocks is then presented. The Allan variance is used as the measure of frequency stability in the time domain. The results presented include a time-domain noise analysis, whose purpose is to identify the random periodic noise processes that are present in the NAVSTAR cesium and rubidium clocks. Readers who are familar with the mathematical theory of clock analysis may choose to proceed directly to the section on NAVSTAR-3 On-orbit Results.

GPS System Description

The NAVSTAR GPS system is comprised of three major segments:

(1) Control Segment. The current GPS Control Segment consists of a master control station (MCS), located at Vandenberg, CA and four monitor sites. One monitor site is located adjacent to the master control station at Vandenberg; the remaining three remote monitor sites are located at Hawaii, Alaska, and Guam. These four stations track the GPS space vehicles (SV). Data from these sites are transmitted to the MCS and processed to determine SV orbits and clock offsets. A separate Satellite Control Facility (SCF) is used to transmit commands and navigational information to the GPS spacecraft.

(2) Space Segment. During the time period covered in this report, the GPS Space Segment constellation consisted of five NAVSTAR SVs; NAVSTAR 8 was launched on July 14, 1983. The launch dates and clock information are detailed by the following table.

TABLE 1

GPS SV Launch Date	Frequency Standard Currently in use
NAV 1 2/22/78	Quartz
NAV 3 10/07/78	Rubidium
NAV 4 12/11/78	Rubidium
NAV 5 2/09/80	Cesium
NAV 6 4/26/80	Cesium
NAV 8 7/14/83	Rubidium

Each GPS space vehicle continuously broadcasts spread spectrum signals in L-band. The center frequency values are at 1227.6- and 1575.42-MHz, which are designated as L_1 and L_2, respectively. The signal waveform is a composite of two pseudo-random noise (PN) phase-shift-key (PSK) signals transmitted in phase quadrature. These two signals are referred to as the P-signal and the C/A signal.

The P-signal provides the capability for precise navigation, is resistant to ECM and multipath, and could be denied to unauthorized users by means of transmission security (TRANSEC) devices.

The C/A signal provides a ranging signal for users whose navigation requirements are less precise. In addition, this signal serves as an acquisition aid for authorized users to gain access to the P-signal. The C/A designation indicates the "clear" and "acquisition" functions of this waveform.

Orthogonal binary coded sequences, transmitted from each GPS satellite, provide a capability for identifying each individual satellite. This technique is known as Code Division Multiple Access (CDMA). By means of a correlation detector, the apparent time difference between transmission of the signal and arrival of the signal as determined by the user's receiver clock is measured. This apparent time difference is composed of two parts: the signal propagation delay from the satellite transmitter to the user and the unknown offset of the user clock. Each GPS spacecraft transmits a navigation message which is modulated onto the signal, and may be decoded and used in the calculation of the user's position, velocity, and clock offset.

(3) User Segment. The GPS User Segment is comprised primarily of users from DOD and the NATO community. Selective civilian use of GPS is being considered, with appropriate restrictions to limit the accuracy.

A high accuracy GPS navigational solution is obtained from four simultaneous measurements of apparent time difference. These apparent time difference measurements are called pseudo-range measurements because the signal must travel from the GPS spacecraft to the user's receiver, a distance on the order of 20,000 to 25,000 km. Hence this delay is present in addition to the actual clock difference. The time differences are taken between the user receiver clock and each of the NAVSTAR spacecraft clocks. Using a computer-controlled receiver, the GPS user tunes and locks the GPS receiver to signals broadcast from the NAVSTAR SVs, and then makes four simultaneous pseudo-range measurements. The four NAVSTAR SV positions are calculated from the GPS navigation message, which is modulated onto each GPS signal. These four pseudo-range measurements are then used to calculate a navigational solution (3,4) for the user's latitude, longitude, height, and clock offset. GPS provides a near-instantaneous navigation capability for users on a world wide basis.

A GPS navigational solution for the user's velocity and clock-rate may be computed through the use of four additional simultaneous measurements of apparent frequency difference. These apparent frequency difference measurements are called pseudo-range rate measurements, because the relative motion between the GPS spacecraft is present in addition to the clock-rate difference. The basic GPS navigation solution for user position and clock offset is independent of the user's velocity and clock rate; however, the user's position is required for the velocity solution. Alternately, the solution for velocity and clock-rate may be estimated from two or more successive GPS position and clock offset solutions.

GPS On-Orbit Clock Analysis

The GPS instantaneous navigation capability is possible because each NAVSTAR clock is synchronized to a common GPS time. The clock offset, orbital elements, and spacecraft health parameters of all spacecraft in the GPS constellation are periodically determined at the GPS master control station. These clock offsets, orbital elements, and spacecraft health parameters are then uploaded to each NAVSTAR SV and inserted into the GPS navigation message. Each NAVSTAR clock must then keep time, to within GPS specifications, until the next clock update. The time stability of a clock is related to its frequency stability; therefore, a fundamental measure of GPS clock performance is the frequency stability of the clocks. The Allan variance is the statistical measure of frequency stability that is used for reporting clock performance.

The procedure that has been devised at NRL (5, 6) for determining GPS clock performance is presented in Figure 1. The goal of this technique is to separate the clock offset, from the orbital and other smaller effects that are present in the GPS signal. This procedure utilizes a highly redundant set of pseudo-range, and pseudo-range rate measurements, that are collected from all four GPS monitor sites during two-week intervals. This redundant set of measurements allows the determination of smoothed orbit states and bias parameters that are independent of the GPS Master Control Station realtime Kalman estimation procedure. A description of this technique follows, with emphasis on the variables related to clock performance analysis.

Measurements of pseudo-range (PR) and integrated pseudo-range rate are taken between the NAVSTAR SV clock and the MS clock using a spread spectrum receiver. The MS receivers are capable of making measurements from four GPS SVs, simultaneously, whenever four or more SVs are above the MS horizon. The measurements are taken once every six seconds and then aggregated and smoothed once per 15 minutes. Figure 2 presents a plot of a typical pseudo-range signature obtained from a single NAVSTAR satellite pass over a monitor station. Each measurement is corrected for equipment delay, ionospheric delay, tropospheric delay, earth rotation, and relativistic effects. The data are then edited and smoothed after subtracting the predicted SV ephemeris and clock offset, which removes most of the signal. Following the smoothing procedure, the predicted values are added to the smoothed values to produce the smoothed measurements. The apparent clock offset is evaluated near the midpoint of the 15-minute data span, using a cubic polynomial model and both the pseudo-range and the pseudo-range-rate measurements.

The pseudo-range measurements are resolved to 1/64 of a P-code chip, which corresponds to 1.5 ns of time, or 46 cm in range. Nominal values of pseudo-range noise levels are σ_{PR} = 1.3 m for the L_1 measurements, and σ_{PR} = 2.0 m for the L_2 measurements. The L_1 and L_2 measurements are combined to correct for ionospheric refraction, which results in an increase to σ_{PR} = 4.53 m for the corrected pseudo-range measurements. The accumulated delta pseudo-range measurement noise levels are 0.31 cm for L_1 and 0.56 cm for L_2. These measurements are also combined to correct for ionospheric refraction. The smoothing procedure uses the pseudo-range rate measurements to aid in the pseudo-range smoothing of each 15-minute segment of data. This process, as outlined in reference 7, results in a smoothed pseudo-range measurement noise level of 18.5 cm.

The equation that relates the pseudo-range measurement to the clock difference between the NAVSTAR SV clock and the MS clock is:

$$PR = R + c\,(t_{MS} - t_{SV}) + c\,t_A + \varepsilon \qquad \text{Eq (1)}$$

where

PR = the measured pseudo-range

R = the slant range (also known as the geometric range) from the SV (at the time of transmission) to the MS (at the time of reception)

c = the speed of light

t_{MS} = the MS clock time

t_{SV} = the SV clock time

t_A = ionospheric, tropospheric, and relativistic delay, with corrections for antenna and equipment delays

ε = the measurement error

The clock difference, $(t_{SV} - t_{MS})$, is obtained by dividing by c, the speed of light, and rearranging Eq (1) into

$$(t_{SV} - t_{MS}) = R/c + t_A + \varepsilon/c - PR/c \qquad \text{Eq (2)}$$

In Eq (1), the pseudo-range is a measure of distance, typically expressed in kilometers (km). In Eq (2) the unit of measure is time, typically expressed in milliseconds (ms).

The clock difference $(t_{SV} - t_{MS})$ may be defined as a new variable $x(t_k)$.

$$x(t_k) = (t_{SV} - t_{MS}) \qquad \text{Eq (3)}$$

The subscript k is used to denote the time of measurement as determined by the monitor site clock. This definition of $x(t_k)$ is made so that the clock difference notation will agree with referenced literature; the clock difference is also denoted by the variable Δt_k.

$$\Delta t_k = (t_{SV} - t_{MS}) \qquad \text{Eq (4)}$$

The variables $\{\, x(t_k),\ \Delta t_k \,\}$ are equivalent; the choice of variable will be one of convenience.

All of the smoothed pseudo-range measurements from the four GPS monitor sites are collected at the GPS Master Control Station. These measurements are processed to produce a realtime estimate of each of the NAVSTAR clock and ephemeris states. These smoothed measurements are further processed in post-flight analysis to produce a smoothed estimate of the NAVSTAR ephemerides.

The realtime estimates of the NAVSTAR SV clock and ephemeris states are made using a Kalman (8, 9) estimator which has been adapted for GPS use, as described in reference 10. The success of the estimation technique is critically dependent on the stability of the NAVSTAR SV and MS clocks. For example, Figure 2 presents the time delay that occurs as the NAVSTAR signal travels from the spacecraft to a GPS monitor site. Reference to Figure 2 indicates a change in apparent time differences of 15 milliseconds or 15,000,000 nanoseconds during the first 3 hours of this NAVSTAR pass. Current GPS specifications call for a maximum clock uncertainty of less than 5 nanoseconds during the pass. If the NAVSTAR SV clock does not meet this specification, then the Kalman estimator has difficulty in separating the orbit part of the GPS signal from the clock noise. Reference to Eq (2) shows that the monitor site clock has the same weight in the measurement as the NAVSTAR clock; therefore, it is highly desirable to have a MS clock of equal, or better time stability at each GPS monitor site. Figure (3) presents theoretical frequency models for the GPS cesium and rubidium clocks. In figure (3), the on-orbit clocks are preceded by "SV", the monitor site clocks are all cesium and are designated by "MS cesium".

It will be assumed that the reference GPS MS clocks are significantly more stable than the on-orbit GPS SV clocks. This assumption will be tested, and verified, using a clock ensemble composed of the GPS MS clocks.

Smoothed estimates for the NAVSTAR orbits are routinely made by the Naval Surface Weapons Center (NSWC), using an orbit estimation program (11). The model includes dynamics of the satellite motion, solar radiation pressure, pole wander, earth tides and orbit adjust maneuvers. The smoothed orbits are made once per week, using all available observations for a two-week span from each of the four GPS monitor sites. The pseudo-range measurements are differenced to compute delta-pseudo-range values which are used as the measured quantity in the NSWC program. The model incorporates a segmented bias parameter solution, with analysis of the resulting residual patterns of the smoothed orbit estimation.

The major advantage of the smoothed orbit estimate, over the Kalman realtime estimate, is the production of a smoothed orbit which is almost completely determined by the data, without restrictive assumptions on the uncertainty in clock and orbit states.

Time-Domain Clock Analysis

GPS operation requires that the on-orbit NAVSTAR SV clocks keep the current GPS time. Because the clocks are periodically updated, interest is in evaluating clock performance as a function of the sample time, τ, which is the difference between two successive values of running time.

Given two clock measurements, $x(t_k)$ and $x(t_j)$, which were made at running times t_k and t_j (by the GPS monitor site clock), the sample time τ is given by Eq (5).

$$\tau = (t_k - t_j) \qquad \text{Eq (5)}$$

The sample time will be varied from 900 seconds to 10 days (in this presentation) to evaluate clock performance.

One clock model used to describe the NAVSTAR clock as a function of time is a quadratic equation of the form

$$x(t) = x_o(t_o) + y_o(t_o)(t-t_o) + \frac{\dot{y}_o(t_o)}{2}(t-t_o)^2 + \varepsilon(t) \qquad \text{Eq (6)}$$

In Eq (6) $x_o(t_o)$ is the initial clock offset, $y_o(t_o)$ is the clock rate (also known as the fractional frequency offset), $\dot{y}_o(t_o)$ is the drift in the fractional frequency (also known as the aging rate), and $\varepsilon(t)$ is the error term.

By choosing $\tau = (t-t_o)$ and omitting the error term, Eq (6) can be written as

$$x(t) = x_o(t_o) + y_o(t_o)\tau + \frac{\dot{y}_o(t_o)\,\tau^2}{2} \qquad \text{Eq (7)}$$

By holding the value of τ fixed, and evaluating Eq (7) for many data samples of t and t_o, the statistical error in the clock coefficients and the error term can be estimated.

The measure of clock performance used in the analysis of this paper is the Allan variance (12), which is defined by

$$\sigma_y^2(\tau) = \left\langle \frac{(\bar{y}_{k+1} - \bar{y}_k)^2}{2} \right\rangle \qquad \text{Eq (8)}$$

where y_k denotes the average fractional frequency, τ denotes the sample time, and the brackets < > denote the infinite time average. Two other parameters are involved in the Allan variance analysis. The first is the repetition interval T, which is equal to the sample time in Eq (8). The other parameter is f_h, the system noise bandwidth, which does not explicitly appear in the Allan variance equation. The system noise bandwidth is receiver dependent, and depends upon user dynamics. Information on the GPS monitor site receivers may be found in reference 13.

The fractional frequency, denoted by the variable y, is given by

$$y = \frac{(\nu - \nu_o)}{\nu_o} \qquad \text{Eq (9)}$$

where ν denotes the instantaneous frequency, and ν_o is the reference, or nominal frequency. The average fractional frequency, denoted by $\bar{y}_k$, is given by

$$\overline{y}_k = \frac{1}{\tau} \int_{t_k}^{t_k + \tau} y(t)dt \qquad \text{Eq (10)}$$

Reference to Eq (10) shows that $\overline{y}_k$ depends on t_k and τ, as well as y(t); so $\overline{y}_k$ could be written as $\overline{y}(t_k, \tau)$ to show this dependence on t_k and τ. The values for $\overline{y}_k$ used in this report are obtained from values of clock offset, $x(t_k)$, computed according to Eq (3). The average frequency $\overline{y}_k$ may be evaluated in terms of x(t), as given by

$$\overline{y}_k = \frac{1}{\tau} \left[x(t_k + \tau) - x(t_k) \right] \qquad \text{Eq (11)}$$

The infinite time average required in Eq (8) for the Allan variance is, of course, unobtainable in the real world. Therefore, a finite approximation of the Allan variance, given by Eq (12), is used.

$$\sigma_y^2 (2,\tau,M) = \frac{1}{(M-1)} \sum_{k=1}^{M-1} \frac{(\overline{y}_{k+1} - \overline{y}_k)^2}{2} \qquad \text{Eq (12)}$$

The arguments of the finite approximation, $\sigma_y^2(2,\tau,M)$, are 2, τ, M, respectively. The number 2 specifies that pairs of fractional frequencies are used, τ denotes the sample time, and (M-1) denotes the number of frequency pairs.

The difference between $\sigma_y^2(\tau)$ and $\sigma_y^2(2,\tau,M)$ is that $\sigma_y^2(\tau)$ is the desired quantity defined by an infinite series; $\sigma_y^2(2,\tau,M)$ is a partial series obtained from a finite number of data points. The use of a finite number of data points does not introduce any bias in the estimate of $\sigma_y^2(\tau)$, as shown by reference 14. The ratio of the variables $\sigma_y^2(2,\tau,M)$ and $\sigma_y^2(\tau)$ will be used in establishing confidence limits for the finite estimate of $\sigma_y^2(\tau)$.

The convergence of this finite-sample average, $\sigma_y^2(2,\tau,M)$, towards a theoretical limit has been investigated by researchers (13). The confidence of this quantity as a measure of $\sigma_y^2(\tau)$ has also been investigated (14, 15). These theoretical results indicate that a high-confidence estimate of $\sigma_y^2(\tau)$ may be obtained through the use of large data bases, which result in a large number of frequency pairs. In practice it is desirable to have a data base length which is at least a factor of ten larger than the sample time.

The square root of the Allan variance is called the Allan deviation. The Allan deviation is defined by Eq (13).

$$\sigma_y(\tau) = [\sigma_y^2(\tau)]^{1/2} \qquad \text{Eq (13)}$$

Frequency Stability Set Selection Criteria

The GPS measurements may be aggregated into sets for the short- and long-term frequency stability analysis. Figure 4 presents a flow diagram of this procedure. The smoothed pseudo-range measurements are combined with the reference ephemeris to produce smoothed clock offsets. Each smoothed measurement is obtained from up to 150 six-second pseudo-range and 149 delta pseudo-range measurements. This procedure includes corrections for ionospheric, tropospheric, and equipment delays, and a correction for relativity effects. The set selection criteria are then applied to construct subsets of clock offset values, $\{ x(t_k) \}$, which are then used to produce the $\sigma_y^2(\tau)$ Allan variance values.

For the short-term frequency stability analysis, the Allan variance is computed from the set of smoothed clock offsets, using one or more satellite passes. Reference to Figure 4 indicates that sets of 5, 10, or more days have been used to compute one value of $\sigma_y^2(\tau)$.

Figure 5 presents an example of a five day set of NAVSTAR-3 observations as recorded at the Vandenberg MS. The plot presents the elevation angle of NAVSTAR-3 as a function of time for five days beginning at day 180. The elevation angle is computed every 15 minutes, and plotted as a "dot" on Figure 5. Inspection of this plot indicates that a partial pass may occur at the beginning, or ending, of the five-day set.

Note the repeating pass signature in Figure 5 which is characteristic of all GPS orbits. This repeating signature is a result of the 12-sidereal-hr GPS orbits which produce repeating ground tracks. Therefore the number of points-per-pass remains constant. For example, in Figure 5 a total of 28 values of smoothed data are available from this NAVSTAR-3 pass over the Vandenberg MS. A five-day set would contain approximately 140 data segments which could be used to obtain smoothed NAVSTAR clock offset values.

The 5-day set has been chosen for use in this paper because of a tradeoff between the confidence in the Allan variance and the length of the set. The primary reason for choosing the shortest set possible is to see changes in the NAVSTAR clocks as a function of time. The reason for choosing longer sets is to increase the total number of samples in each Allan variance calculation.

The number of Allan variance values that are calculated may be maximized, using a procedure based on the one given in reference 15. This procedure involves defining a base sampling time τ_o, which is defined by Eq (14).

$$\tau_o = \mathrm{MIN}\ \{(t_k - t_j):\ \ j \leq k\} \qquad \text{Eq (14)}$$

For the short-term frequency stability results, a nominal base sampling time of 15-minutes (or 900 seconds) will be used. For the long-term frequency stability results a base sampling time of 1-day will be used. Multiples of the base sampling time may be calculated according to

$$\tau = n\tau_o$$

where the variable n takes on integer values 1, 2, ... A maximum value of n = 8 will be used for the short-term frequency stability analysis, and n = 10 for the long-term frequency stability analysis.

The number of clock offsets in a set will be denoted by the variable N. Assuming that all of the clock offsets are equi-spaced at sample time τ, the total (with maximal use of data) number of Allan variance values is given by the expression (N - 2n). This is not the case with the data used in this report due to the nature of the GPS orbits. For both the short- and the long-term processing algorithms each sample time is calculated according to Eq (5), and each fractional frequency according to Eq (11).

A calculation has been performed to produce typical values for the confidence in the Allan variance. The Allan variance value is then converted to the Allan deviation according to Eq (13). Most of the reference literature expresses frequency stabilities in terms of $\sigma_y(\tau)$ rather than $\sigma_y^2(\tau)$.

The confidence limits are calculated according to the method outlined in reference 15, using a white noise FM process, because this is the dominant noise type that was encountered in the short-term frequency stability values. The confidence limits for a 95% confidence level, calculated for a five-day set (Figure 4), are presented in Table 2. These calculations have been made assuming 25 points-per-pass; however, the total number of samples for the five-day set has been modified to account for the pass-to-pass break in the data. The pass-to-pass break in the data effectively reduces the number of samples that may be computed.

Table 2
95% Confidence Limits
for a
5-Day Set and a
White noise FM process

Sample Time (hrs)	Confidence Limits UPPER	Confidence Limits LOWER	Number of $\sigma_y^2(\tau)$ Samples	Degrees of Freedom
0.25	1.189	.816	115	76
.5	1.211	.798	105	63
.75	1.253	.772	95	47
1.00	1.299	.748	85	36
1.25	1.344	.726	75	29
1.5	1.403	.702	65	23
1.75	1.444	.687	55	20
2.00	1.499	.669	45	17

The typical confidence limits may be used to separate random sampling fluctuations from systematic changes in clock behavior, or other changes in clock performance. For example, if a stability of 1×10^{-12} was computed for a 0.25 hour sample time, the upper 95% confidence would be $(1 \times 10^{-12}) \times (1.189) = 1.189 \times 10^{-12}$. A sequence of Allan deviations, computed using successive five-day sets, could then be analyzed using these 95% confidence limits as a guide to separate random sampling from systematic and other effects. Further inspection of Table 2 indicates that for sample times greater than 2 hours, larger sets would be required to produce acceptable confidence limits in $\sigma_y(\tau)$ values.

The Allan variances obtained from successive 5-day sets can be further averaged to obtain one value for the entire data span. For a 1-year data span, a total of 72 five-day sets would be available for the $\sigma_y^2(\tau)$ calculation. Multiplying the degrees of freedom for a typical 5-day set (Table 2) by 72 five-day sets per year results in 3240 Allan variance samples for a 2-hour sample time, and for a 15-minute sample time. The number of Allan variance samples versus sample time is presented in Figure (7) for a 5-day set, and a 1-year data set from the four GPS monitor sites.

For the long-term (1- to 10-day sample times) frequency stability analysis, one pass of a NAVSTAR SV over a monitor site is used as an operational set selection criteria. Figure 6 depicts one pass of NAVSTAR-6 as observed from the Vandenberg MS. Using the set of clock offsets from one pass, a single value of clock offset is computed for the pass. The epoch for the calculation is chosen to be the Time-of-Closest Approach (TCA) of the spacecraft to the monitor site. This value of clock offset is denoted as $x(t_{TCA})$. A least square objective function is used with data editing to identify and remove statistical outliers, and to limit the set points within $\pm$ 1.5 hours of TCA. In addition to this procedure, a pass-to-pass constraint on the sample time τ must be met before an Allan variance can be computed.

For the long-term frequency stability values, two types of noise processes were encountered. These two types of noise processes were: random walk FM for the NAVSTAR rubidium clocks, and flicker noise FM for the NAVSTAR cesium clocks. Tables 3 and 4 present the 95% confidence limits for a 1-year data set assuming no pass-to-pass break in the data.

Table 3
95% Confidence LIMITS
for a
1-year set and a
Random walk FM noise process

Sample Time (days)	Confidence Limits UPPER	Confidence Limits LOWER	Number $\sigma_y^2(\tau)$ Samples	Degrees of Freedom
1	1.07	0.93	363	364
2	1.09	.91	361	180
3	1.15	.89	359	119
4	1.17	.87	357	88
5	1.20	.86	355	70
6	1.22	.84	353	58
7	1.25	.83	351	49
7	1.25	.83	351	49
8	1.27	.82	349	42
9	1.32	.81	347	37
10	1.33	.80	345	33

Table 4
95% Confidence LIMITS
for a
1-year set and a
Flicker noise FM process

Sample Time (days)	Confidence Limits UPPER	LOWER	Number of $\sigma_y^2(\tau)$ Samples	Degrees of Freedom
1	1.07	.92	363	315
2	1.08	.91	361	224
3	1.13	.90	359	148
4	1.15	.89	357	110
5	1.17	.87	355	87
6	1.20	.86	353	72
7	1.22	.85	351	61
8	1.24	.84	349	53
9	1.25	.83	347	47
10	1.27	.83	345	42

NAVSTAR-3 On-orbit Results

Two types of plots will be used to present the NAVSTAR time domain clock data. The first type of plot presents the Allan deviation, averaged over 5-day sets, as a function of running time. The second type of plot presents the Allan deviation as a function of sample time.

Figure (8) presents the NAVSTAR-3 $\sigma_y(\tau)$ versus running time t, for 1982. A total of 64 five day sets were used to produce these results. These calculations were made using the 1-year average as a reference. The 95% confidence limits used were those presented in Figure (8). Inspection of Figure (8) indicates only two outliers that obviously exceed the 95% confidence limits.

For brevity only one $\sigma_y(\tau)$ versus running time t plot will be presented in this paper. Those readers interested in a more complete treatment may reference the forthcoming NRL report #8778.

Figure (9) presents the NAVSTAR-3 frequency stability results as a function of sample time. The independent variable in Figure (9) is the sample time, which varies from 900 seconds to 10 days. The NAVSTAR-3 aging rate, which averaged -3.5 PP10(13)/day for 1982, was removed before calculating the Allan variance. The aging rate was calculated using a least-squares curve fit to a quadratic equation, given by Equation (6), and a data length significantly greater than 10-days, which was the longest sample time analyzed. The theoretical curves for the SV rubidium and MS cesium clocks are plotted in Figure (9) as solid curves. The on-orbit measured values, which are referenced to each of the GPS monitor site cesium clocks, are plotted as discrete points connected by dashed line segments. The short-term sample times values range from 900-seconds to 2-hours, with increments of

900-seconds. The long-term sample times vary from 1 to 10 days, with increments of 1-day.

Figure (10) presents the NAVSTAR-3 frequency stability results referenced to an ensemble of clocks, consisting of the four GPS MS clocks. The presentation of the NAVSTAR-3 plots, referenced to individual GPS monitor site clocks, allows statistical comparison of the reference clocks, in addition to the NAVSTAR-3 clock.

The NAVSTAR-3 short-term frequency results indicate a value of 21.3 PP10(13) for a 900-second sample time. From 900- to 1800-seconds, the NAVSTAR-3 clock followed a white noise FM process. Inspection of Figure (9) indicates an anomally that was first noticed for a 2700-second sample time. This anomally reaches a peak value of 17.1 PP10(13) for a 1.25-hour sample time. Following presentation of the NAVSTARs 4, 5, and 6 results, it will be concluded that this anomally is entirely due to the NAVSTAR-3 clock.

The NAVSTAR-3 long-term frequency stability results indicate a stability of 1.11 PP10(13) for a 1-day sample time. For sample times of 1- to 6-days, the NAVSTAR-3 rubidium clocks follow a random walk FM process, which was expected for the rubidium type clocks. For sample times of 6- to 10-days, there was a small departure from the random walk FM process. Reference to Figure (9) indicates that this is due to the behavior of data received from the Guam MS.

NAVSTAR-4 On-orbit Results

The NAVSTAR-4 short- and long-term frequency stability results are presented in Figures (11) and (12). Figure (11) presents the NAVSTAR-4 results referenced to each individual GPS MS clock. Figure (12) presents the results referenced to the ensemble of the four GPS MS clocks.

The NAVSTAR-4 short-term frequency stability results indicate a value of 21.3 PP10(13) for a 900-second sample time. From 900- to 1800-seconds, the NAVSTAR 4 rubidium clock follows a white noise FM process. From 1800- to 3600-seconds a small change in slope occurs, which indicates a change in the noise process. From 3600- to 7200-seconds, a distinct change in the noise process is evident. The slope of this data indicates that a flicker noise FM process is present. This solution is obtained by solving the power law model of the Allan variance, given by Equation (15), for the coefficients a and the exponent μ. This solution yields a value of μ=-0.17, which is close to the value of μ=0 for a flicker noise FM process.

$$\sigma_y^2(\tau) = a(\tau)^{\mu} \qquad \text{Eq (15)}$$

The NAVSTAR-4 long-term stability results indicate a stability of 1.34 PP10(13) for a 1-day sample time. For sample times of 1- to 6-days, the NAVSTAR-4 rubidium clock follows a random walk FM process. For between 6- and 10-days sample time, a small change in slope occurs, which is due to data received from the Guam GPS MS.

NAVSTAR-5 On-orbit Results

The NAVSTAR-5 short- and long-term frequency stability results are presented in Figures (13) and (14). Figure (13) presents the NAVSTAR-5 results referenced to each individual GPS MS clock. Figure (14) presents the results referenced to the ensemble average of the four GPS MS clocks. No aging rate correction was calculated, or necessary, for the cesium clock.

The NAVSTAR-5 short-term results indicate a value of 23.5 PP10(13) for a 900-second sample time. The NAVSTAR-5 short-term results are slightly higher than the NAVSTAR-4 results, except for the results referenced to the GPS Guam MS. In the ensemble results, presented in Figure (14), the Guam MS data have been deleted from the ensemble average for sample times between 1800- and 7200-seconds. The ensemble average results indicate a flicker noise FM process for sample times of 3600- to 7200-seconds.

The NAVSTAR-5 long-term stability results indicate a stability of 1.58 PP10(13) for a 1-day sample time. From 1- to 5-days sample time, a small slope is noted. From 5- to 10-days, a flicker noise FM process is present, with a value of 1 PP10(13).

NAVSTAR-6 On-orbit Results

The NAVSTAR-6 short- and long-term frequency stability results are presented in Figures (15) and (16). Figure (15) presents the NAVSTAR-6 results referenced to each individual GPS MS clock. Figure (16) presents the results referenced to the ensemble average. No aging rate correction was required.

The NAVSTAR-6 short-term results indicate a value of 21.6 PP10(13) for a 900-second sample time. From 900- to 1800-seconds, the NAVSTAR-6 clock followed a white noise FM process. From 1800- to 3600-seconds, a small change in slope occurs similar to the NAVSTAR 4 and 5 results. Likewise, a flicker noise FM process was present from 3600- to 7200- seconds sample time.

The NAVSTAR-6 long-term results indicate a stability of 1.11 PP10(13) for a 1-day sample time. The stability continues to improve to a stability of 7.7 PP10(14) for a 4-day sample time. Between 4- and 5-days sample time, a small change is noted. For sample times between 5- and 10-days, the NAVSTAR-6 cesium clock follows a flicker noise FM process.

NAVSTARs 3/4/5/6 On-orbit Results

The NAVSTARs 3 and 4 rubidium clock results, referenced to the GPS MS clock ensemble, are presented in Figure (17). The short-term frequency stability results are in close agreement for 900, 1800, and 2700-seconds sample time. The NAVSTAR-3 anomally, at 1.25 hours sample time, clearly departs from the NAVSTAR-4 results. The long-term results are in close agreement, with both rubidium clocks following a random noise FM process.

The NAVSTARs 5 and 6 cesium clock results are presented in Figure (18). The short-term results are in close agreement at 900-seconds sample time. For sample times of 1800- to 2700-seconds, the NAVSTAR-5 results indicate more noise than the NAVSTAR-6 results.

The long-term results indicate the NAVSTAR-5 cesium clock has more noise for sample times of 1- to 4-days. The cesium clocks agree closely from 5- to 10-days sample time.

The NAVSTARs 3,4,5, and 6 results are presented in Figure (19). The short-term results indicate close agreement between NAVSTARs 3, 4, and 6 for a 900-second sample time. The NAVSTARs 3 and 6 clocks are in close agreement for sample times of 900- to 2700-seconds. The NAVSTAR-5 cesium indicates slightly more noise than the NAVSTAR 3 and 6 results. The NAVSTAR-3 anomally is significiantly different from the other clock results.

The long-term results indicate a random noise FM process for the rubidium clocks, and a flicker noise FM process for the cesium clocks. For sample times of 2- to 4-days, the NAVSTAR-6 cesium clock is the best performer, with a stability of 7.7 PP10(14) for a 4-day sample time.

The Allan variance power law model, given by Equation (15), was fitted to the 900- and 1800-second sample time data from all four NAVSTAR clocks. The projection of the short-term results to a 1-day sample time indicates good agreement with the measured Allan deviation for a 1-day sample time. Because of the close agreement, a fit was made to the 900- and 1-day Allan variance stability results. This solution is given by Equation (16).

$$\sigma_y^2(\tau) = (2.31 \times 10^{-20})(\tau)^{-1.25} \qquad \text{Eq (16)}$$

CONCLUSIONS

* The NAVSTARs 3 and 4 rubidium clock (with drift removed) long-term stability values agreed closely. A random walk FM noise process was present for sample times of 1- to 10-days. These measurements are in good agreement with the expected rubidium long-term performance.

* The NAVSTAR-3 rubidium frequency had a significant departure, from expected performance, at a sample time of 1.25 hours. A possible cause is thermal fluctuations with a 2.5-hour period. Performance is nominal for 900- and 1800-seconds sample time.

* For sample times of 1- to 5-days, the NAVSTAR-6 cesium clock exhibited better performance than the NAVSTAR-5 clock. The NAVSTARs 5 and 6 cesium clock long-term stability values agreed closely for sample times of 6- to 10-days. A flicker noise FM process was present, in both cesium clocks, for sample times of 1 to 10 days.

* For sample times of 2- to 10-days, the NAVSTARs 5 and 6 cesium clocks have better frequency stability results than the NAVSTARs 3 and 4 rubidium clocks.

* White noise FM was measured, for both rubidium and cesium clocks, for short-term sample times of 900- and 1800-seconds. For sample times of 2700 seconds to 2-hours, a gradual transition to an apparent, as yet

unexplained, flicker noise FM process was observed.

* The 1-day sample time frequency stability measurements, for both cesium and rubidium clocks, were in close agreement with an average value of 1.3 PP10(13). This average value agreed closely with the projection of the 900-seconds to 1 day. The Allan variance power law solution for this white noise FM process is

$$\sigma_y^2(\tau) = (2.31 \times 10^{-20})(\tau)^{-1.25}.$$

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NAVSTAR GPS
ON-ORBIT
FREQUENCY STABILITY ANALYSIS
FLOW CHART

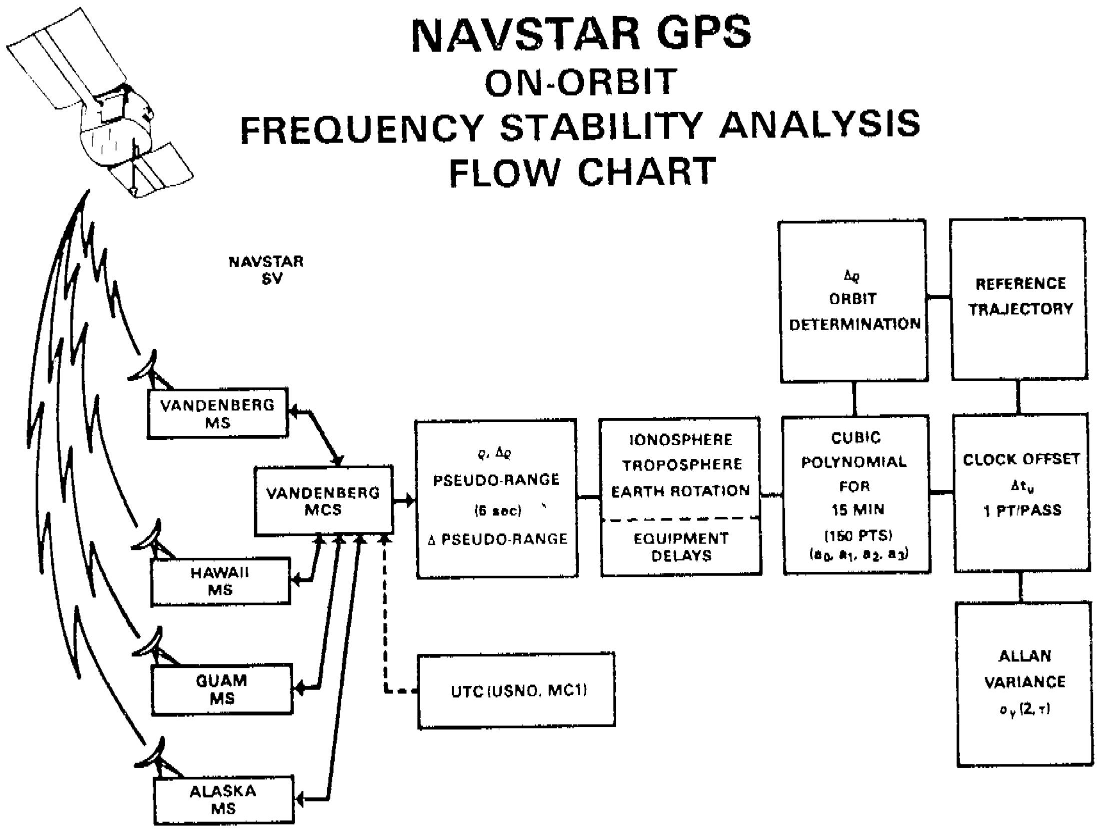


Figure 1

TYPICAL
GPS NAVSTAR SV
PSEUDO-RANGE SIGNATURE

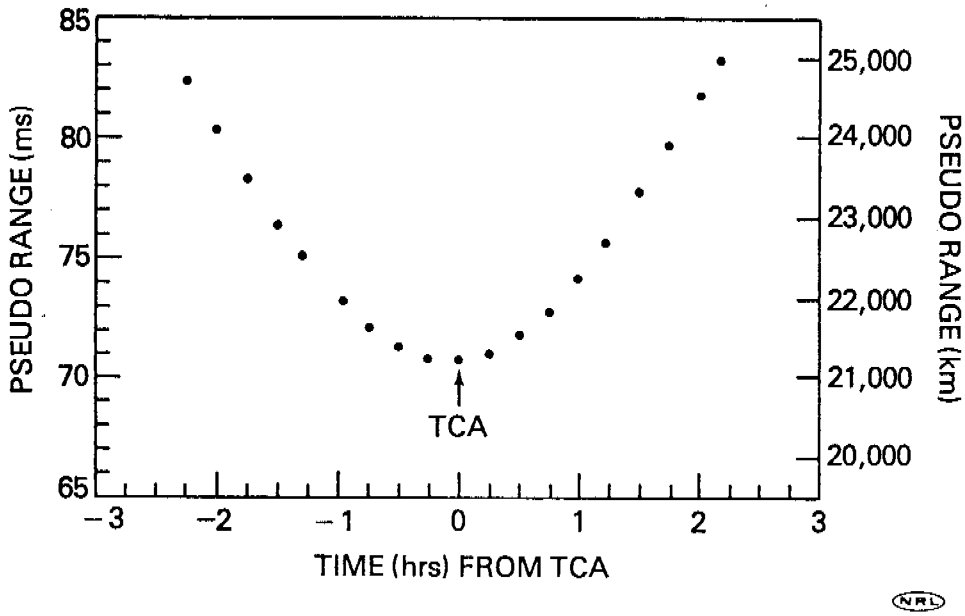


Figure 2

THEORETICAL MODEL OF FREQUENCY STABILITY
FOR GPS CESIUM AND RUBIDIUM CLOCKS

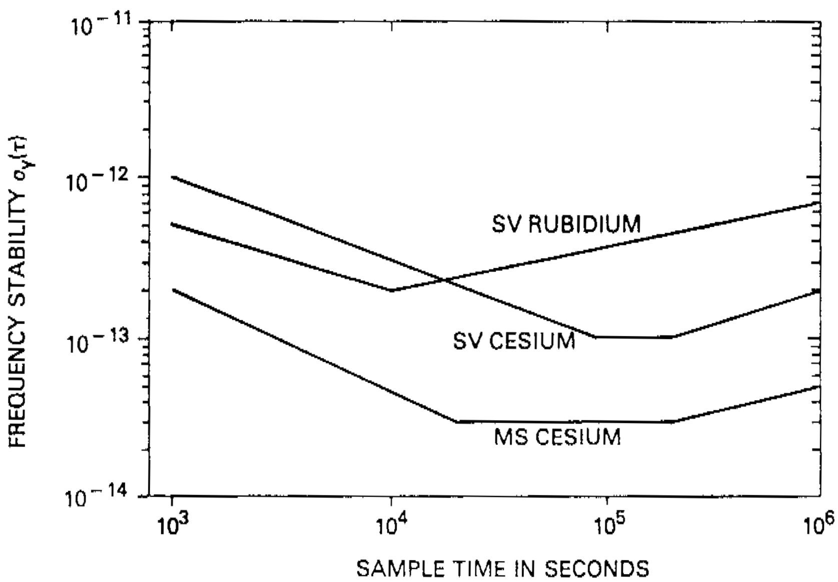


Figure 3

GPS
ON-ORBIT DATA SELECTION PROCEDURE
FOR
ALLAN VARIANCE ANALYSIS

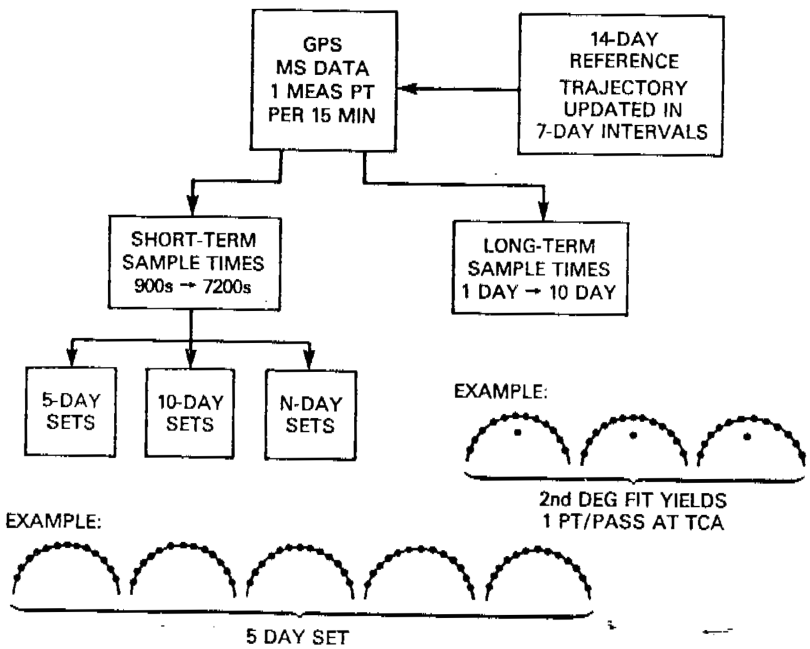


Figure 4

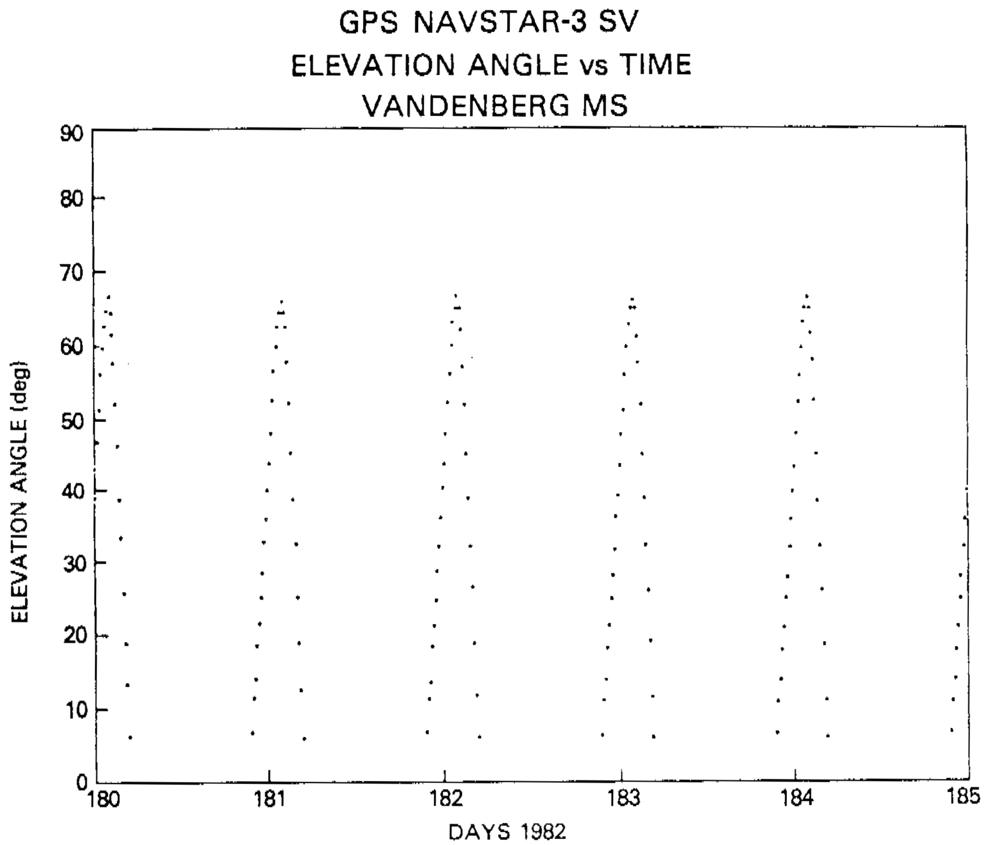

GPS NAVSTAR-3 SV
ELEVATION ANGLE vs TIME
VANDENBERG MS
ELEVATION ANGLE (deg)
90
80
70
60
50
40
30
20
10
0
180
181
182
183
184
185
DAYS 1982

Figure 5

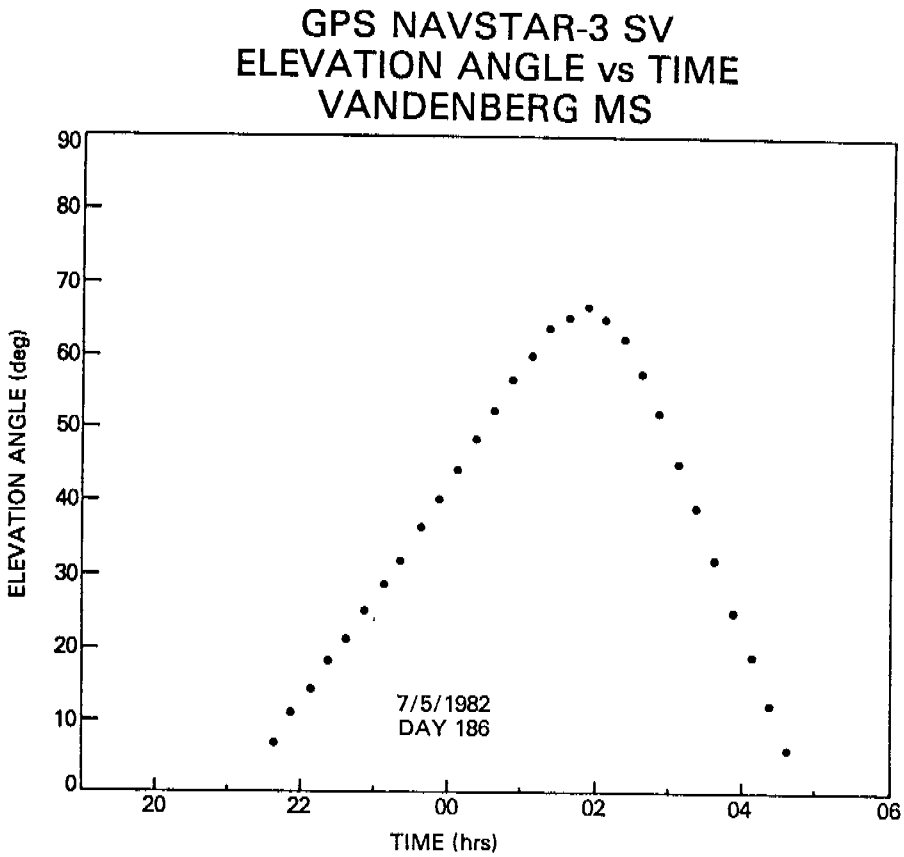

GPS NAVSTAR-3 SV
ELEVATION ANGLE vs TIME
VANDENBERG MS
ELEVATION ANGLE (deg)
90
80
70
60
50
40
30
20
10
0
7/5/1982
DAY 186
20
22
00
02
04
06
TIME (hrs)

Figure 6

GPS CLOCK ANALYSIS
NUMBER OF ALLAN VARIANCE SAMPLES
VS
SAMPLE TIME

SAMPLE TIME	NUMBER OF SAMPLES		ENSEMBLE
τ	5 DAY SET	1 YEAR	4-MS 1-YR
0.25 hrs	115	8280	33120
0.50	105	7560	30240
0.75	95	6840	27360
1.00	85	6120	24480
1.25	75	5400	21600
1.50	65	4680	18720
1.75	55	3960	15840
2.00 hrs	45	3240	12960
1 day		363	1452
2		361	1440
3		359	1436
4		357	1428
5		355	1420
6		353	1412
7		351	1404
8		349	1396
9		347	1388
10 days		345	1380

Figure 7

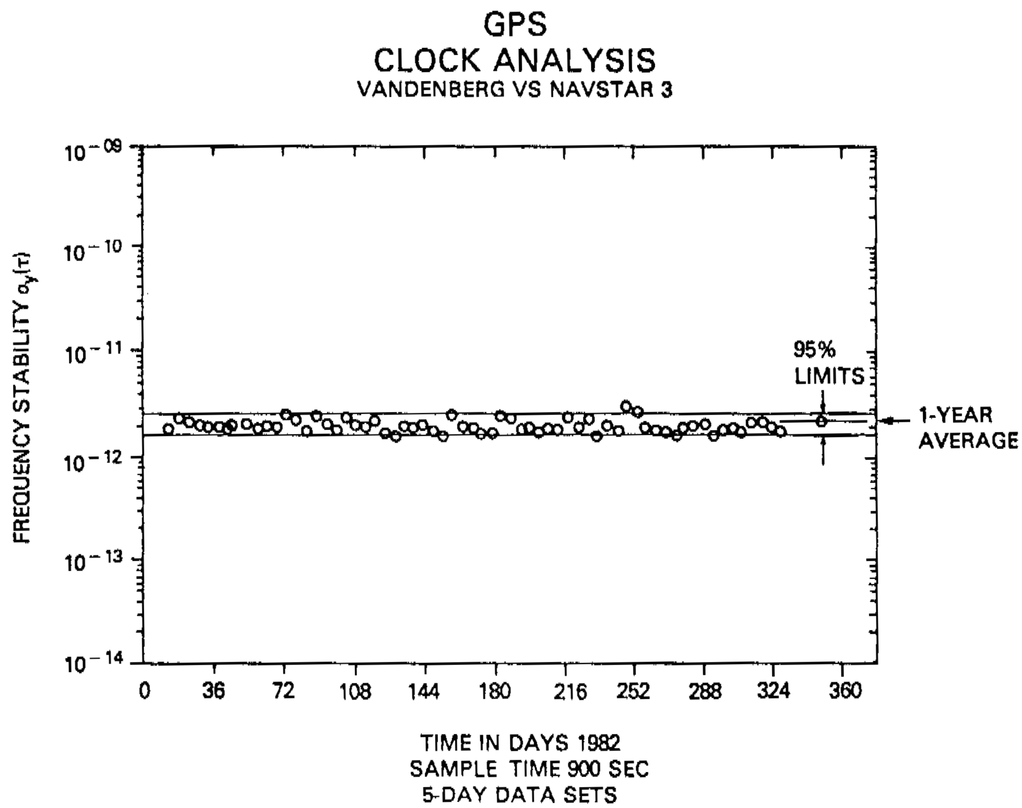

GPS
CLOCK ANALYSIS
VANDENBERG VS NAVSTAR 3
FREQUENCY STABILITY $\sigma_y(\tau)$
10^{-09}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
0
36
72
108
144
180
216
252
288
324
360
95%
LIMITS
1-YEAR
AVERAGE
TIME IN DAYS 1982
SAMPLE TIME 900 SEC
5-DAY DATA SETS

Figure 8

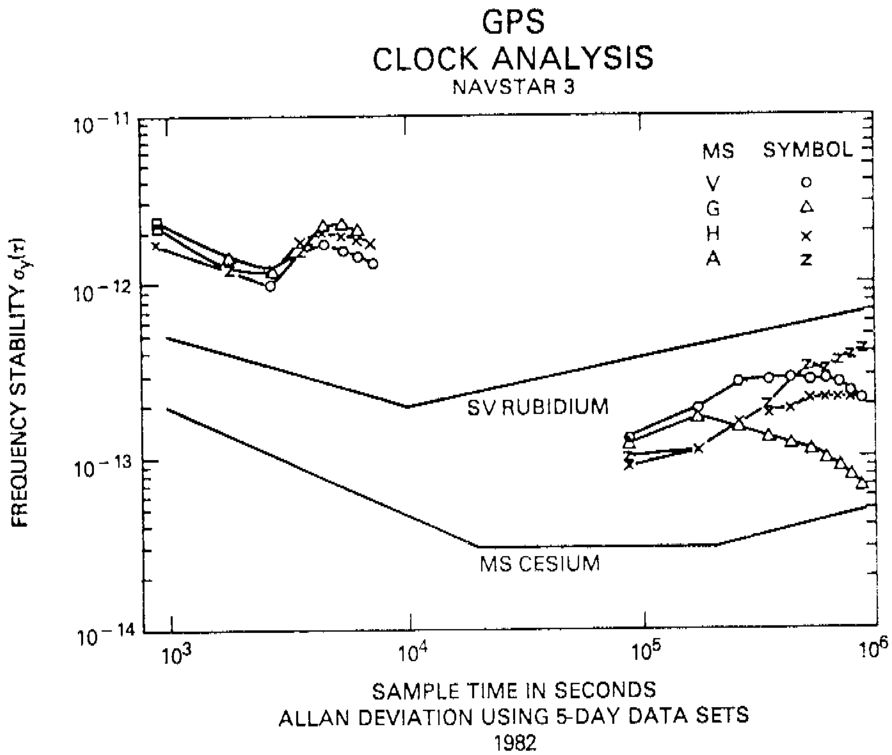

GPS
CLOCK ANALYSIS
NAVSTAR 3
MS SYMBOL
V o
G Δ
H x
A z
SV RUBIDIUM
MS CESIUM
FREQUENCY STABILITY $\sigma_y(\tau)$
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^3
10^4
10^5
10^6
SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 9

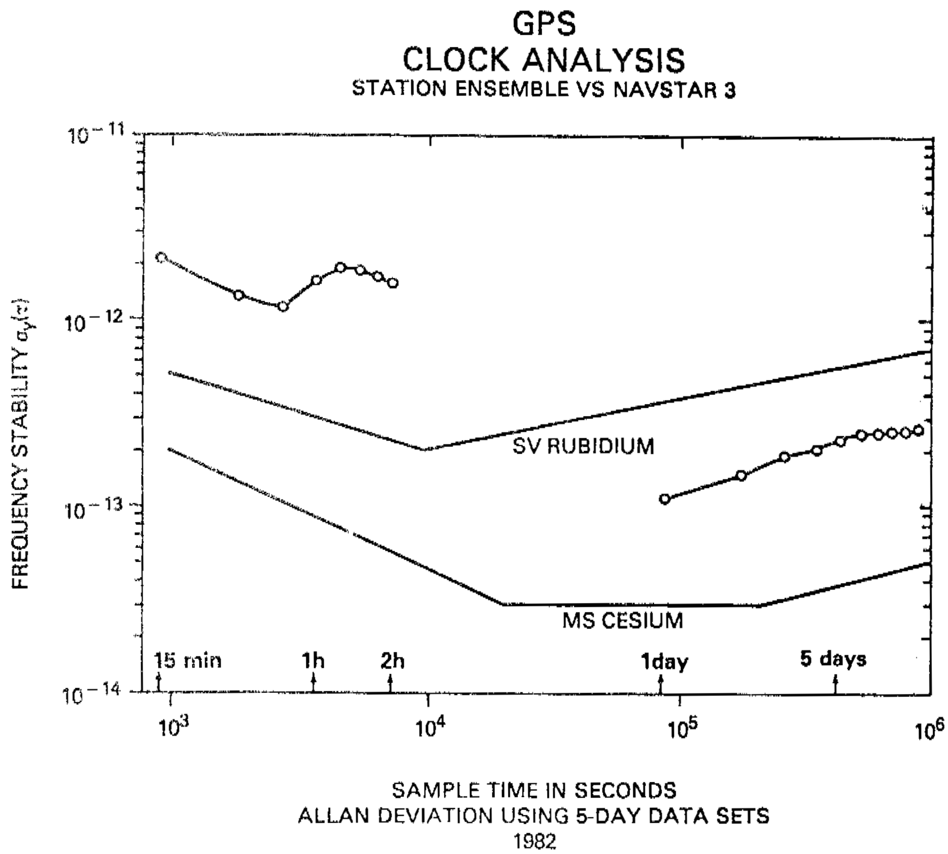

GPS
CLOCK ANALYSIS
STATION ENSEMBLE VS NAVSTAR 3
FREQUENCY STABILITY $\sigma_y(\tau)$
10^{-11}
10^{-12}
10^{-13}
10^{-14}
SV RUBIDIUM
MS CESIUM
15 min
1h
2h
1day
5 days
10^3
10^4
10^5
10^6
SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 10

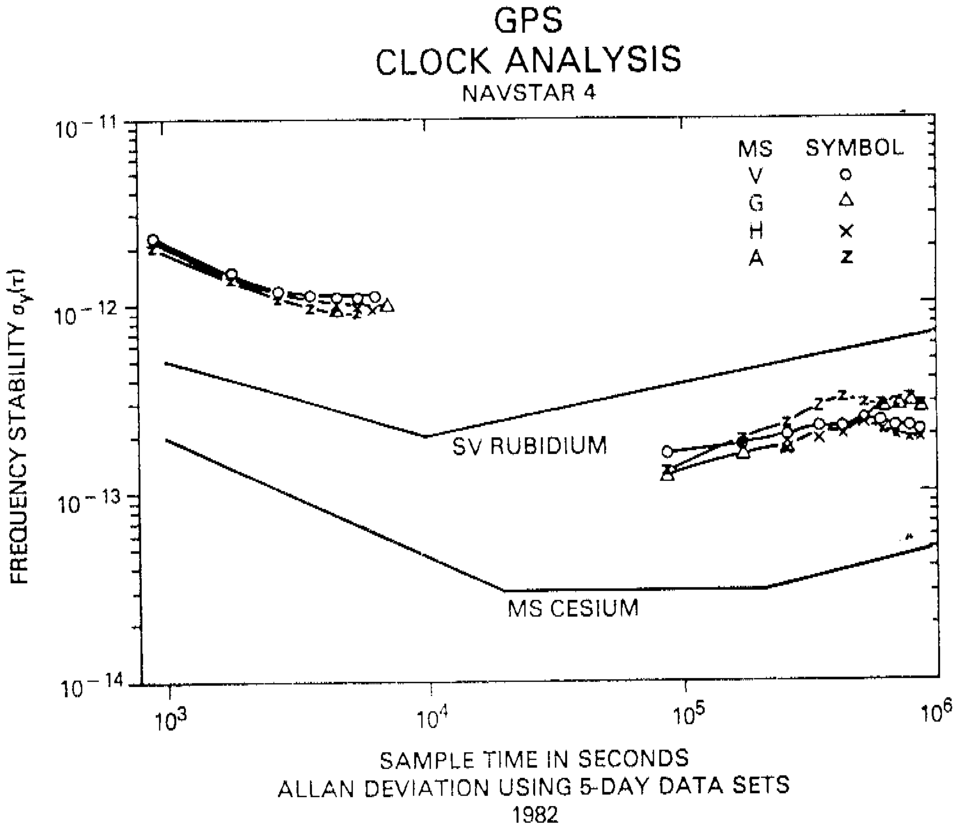

GPS
CLOCK ANALYSIS
NAVSTAR 4
MS SYMBOL
V o
G Δ
H ×
A z
FREQUENCY STABILITY $\sigma_y(\tau)$
10^{-11}
10^{-12}
10^{-13}
10^{-14}
SV RUBIDIUM
MS CESIUM
10^3
10^4
10^5
10^6
SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 11

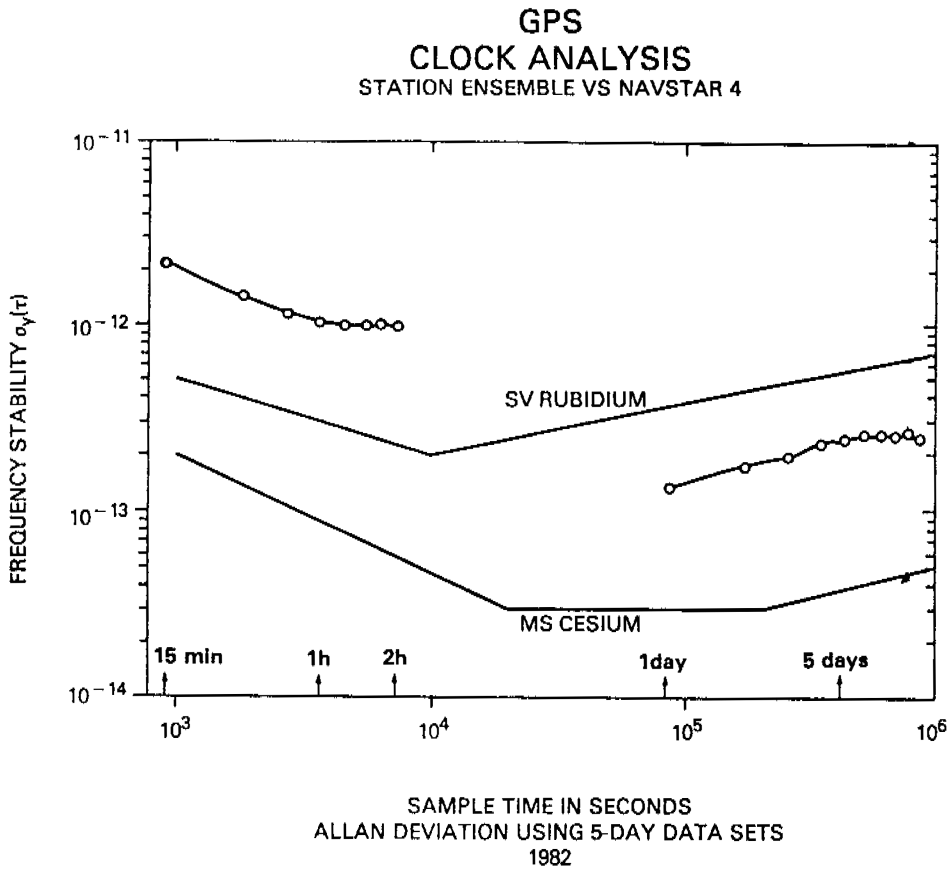
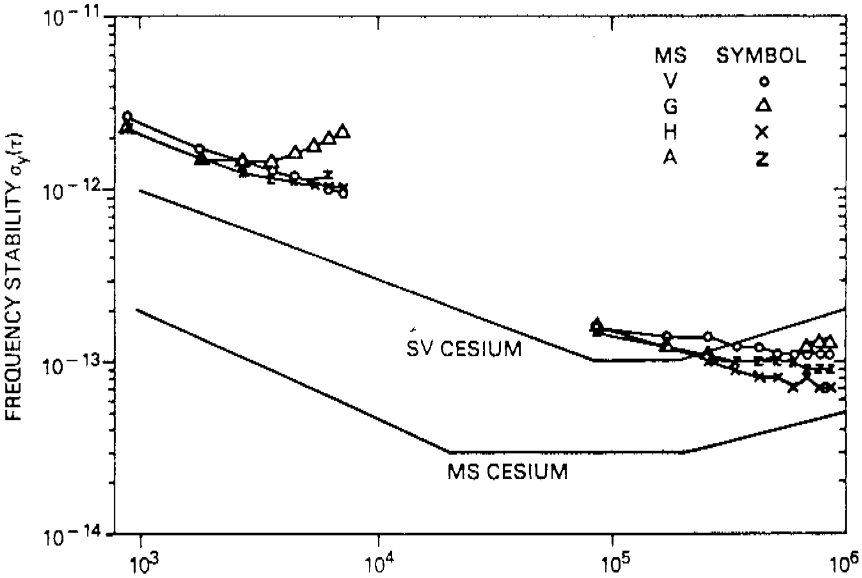

GPS
CLOCK ANALYSIS
STATION ENSEMBLE VS NAVSTAR 4
FREQUENCY STABILITY $\sigma_y(\tau)$
10^{-11}
10^{-12}
10^{-13}
10^{-14}
SV RUBIDIUM
MS CESIUM
15 min
1h
2h
1day
5 days
10^3
10^4
10^5
10^6
SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 12

GPS
CLOCK ANALYSIS
NAVSTAR 5



SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 13

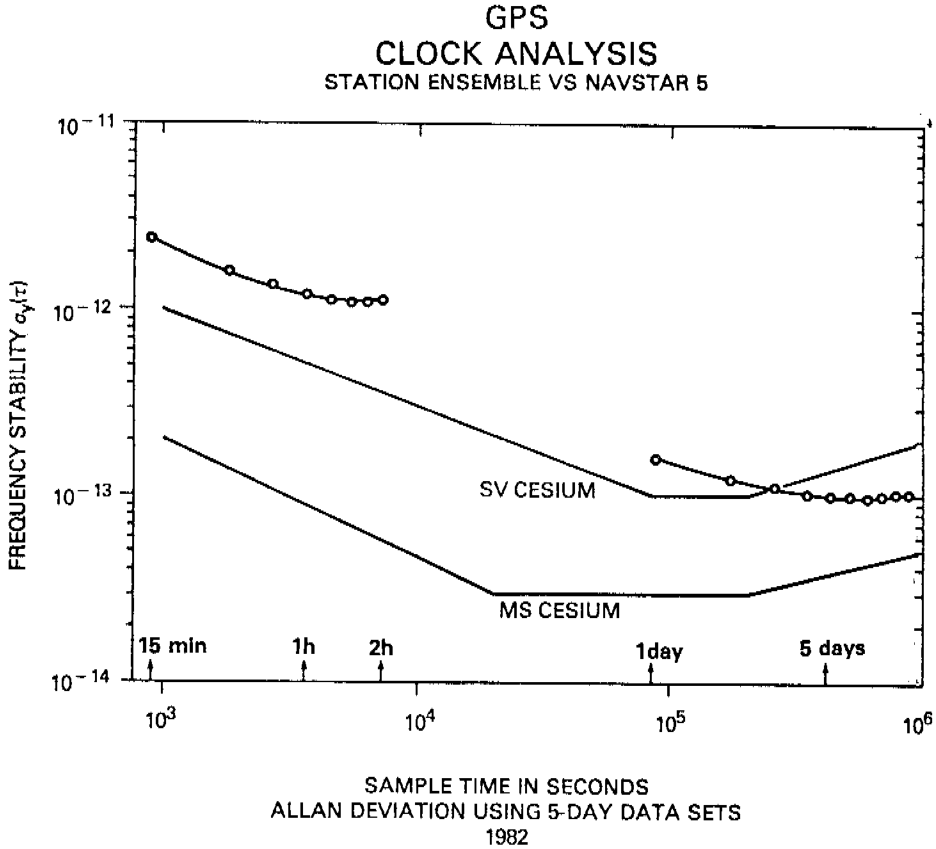

GPS
CLOCK ANALYSIS
STATION ENSEMBLE VS NAVSTAR 5
FREQUENCY STABILITY $\sigma_y(\tau)$
10^{-11}
10^{-12}
10^{-13}
10^{-14}
SV CESIUM
MS CESIUM
15 min
1h
2h
1day
5 days
10^3
10^4
10^5
10^6
SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 14

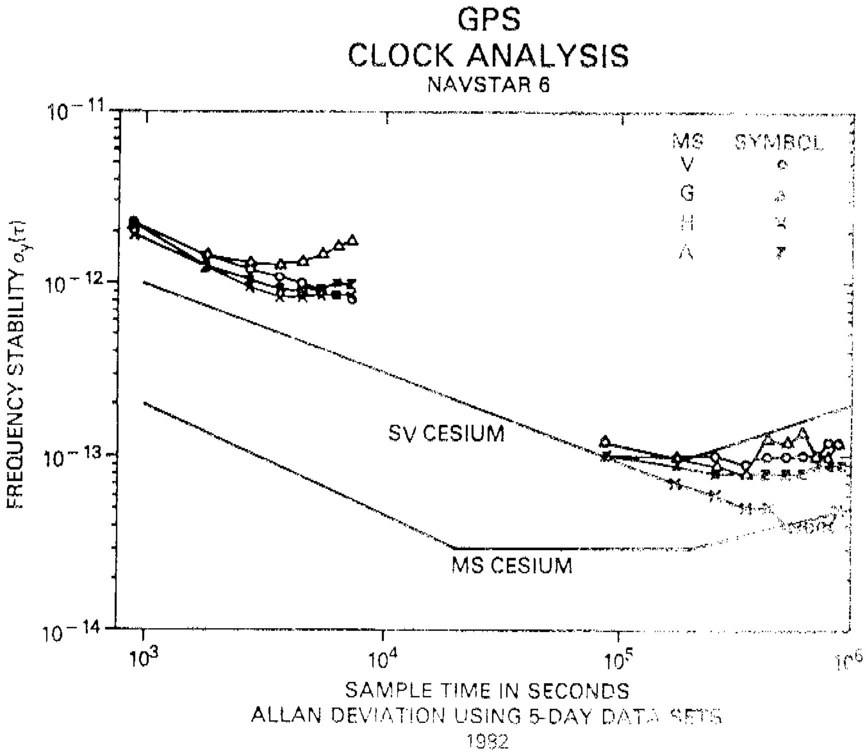

GPS
CLOCK ANALYSIS
NAVSTAR 6
MS SYMBOL
V
G
H
A
SV CESIUM
MS CESIUM
FREQUENCY STABILITY $\sigma_y(\tau)$
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^3
10^4
10^5
10^6
SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 15

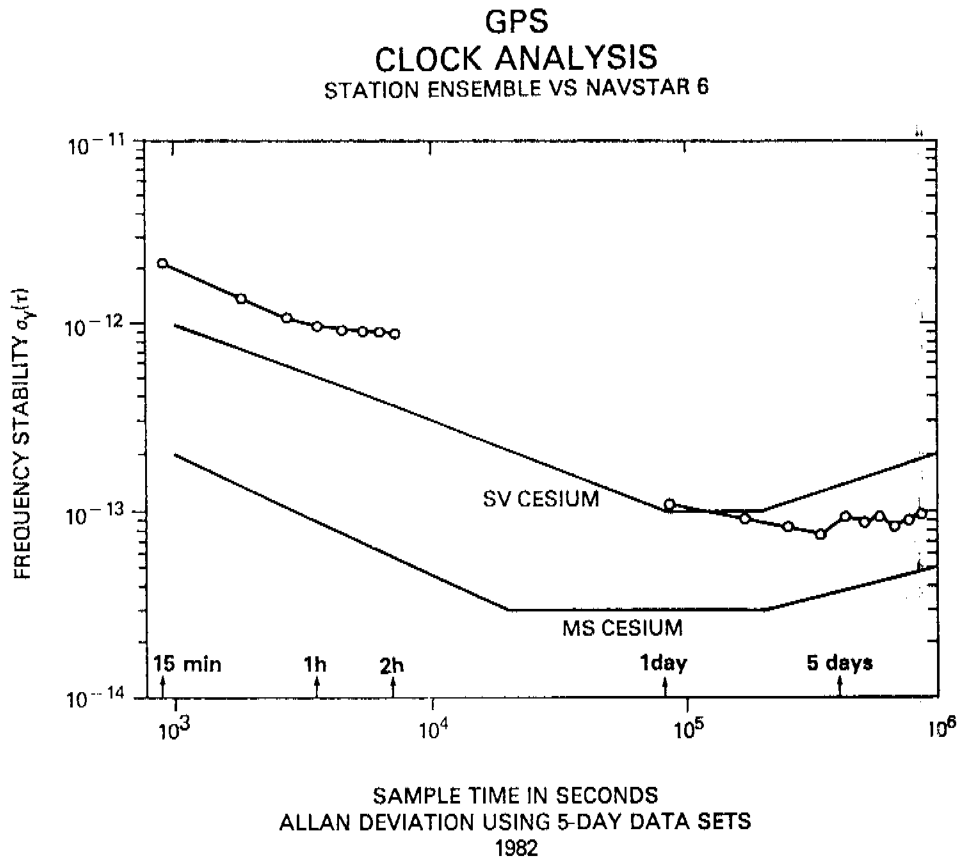

GPS
CLOCK ANALYSIS
STATION ENSEMBLE VS NAVSTAR 6
FREQUENCY STABILITY $\sigma_y(\tau)$
10^{-11}
10^{-12}
10^{-13}
10^{-14}
SV CESIUM
MS CESIUM
15 min
1h
2h
1day
5 days
10^3
10^4
10^5
10^6
SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 16

GPS
CLOCK ANALYSIS
STATION ENSEMBLE vs NAVSTAR 3/4 (RUBIDIUM)

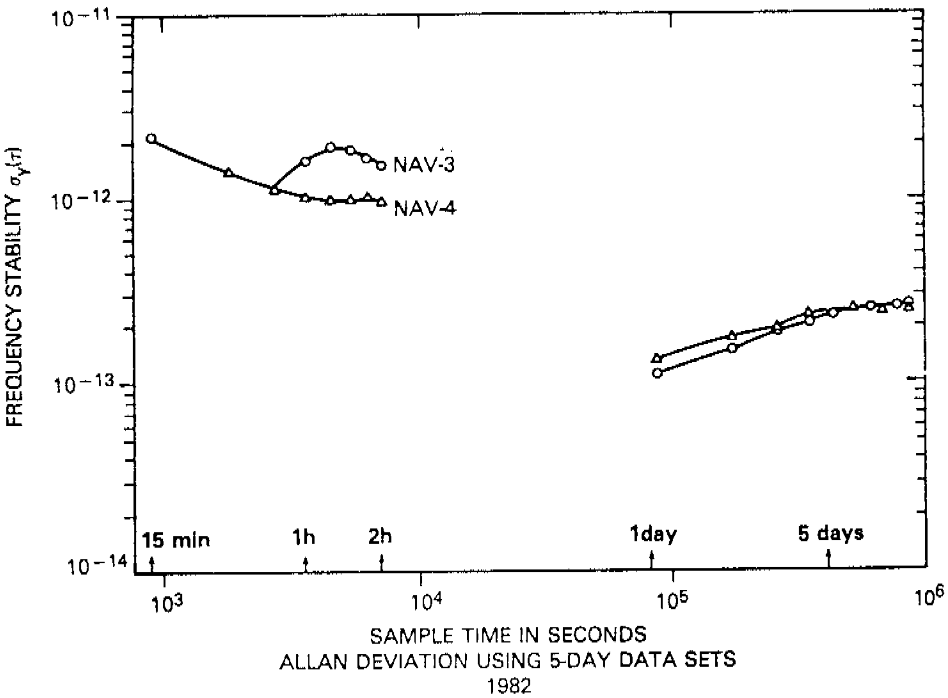


Figure 17

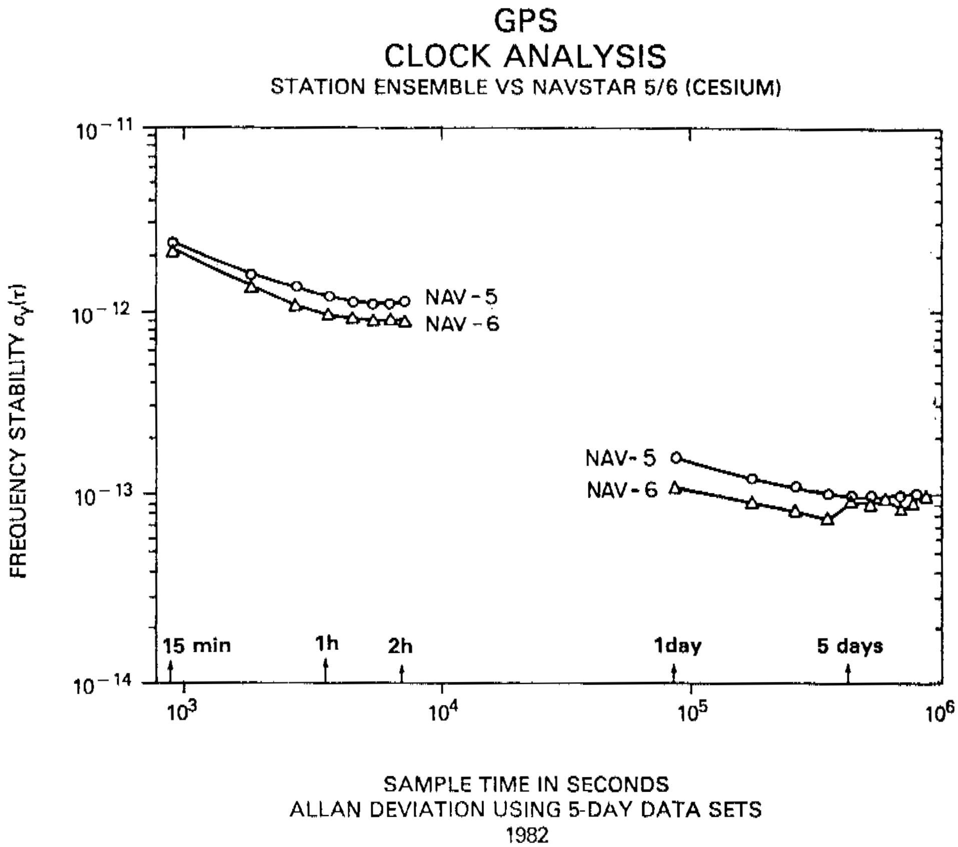

GPS
CLOCK ANALYSIS
STATION ENSEMBLE VS NAVSTAR 5/6 (CESIUM)
FREQUENCY STABILITY $\sigma_y(\tau)$
10^{-11}
10^{-12}
10^{-13}
10^{-14}
NAV - 5
NAV - 6
NAV- 5
NAV - 6
15 min
1h
2h
1day
5 days
10^3
10^4
10^5
10^6
SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 18

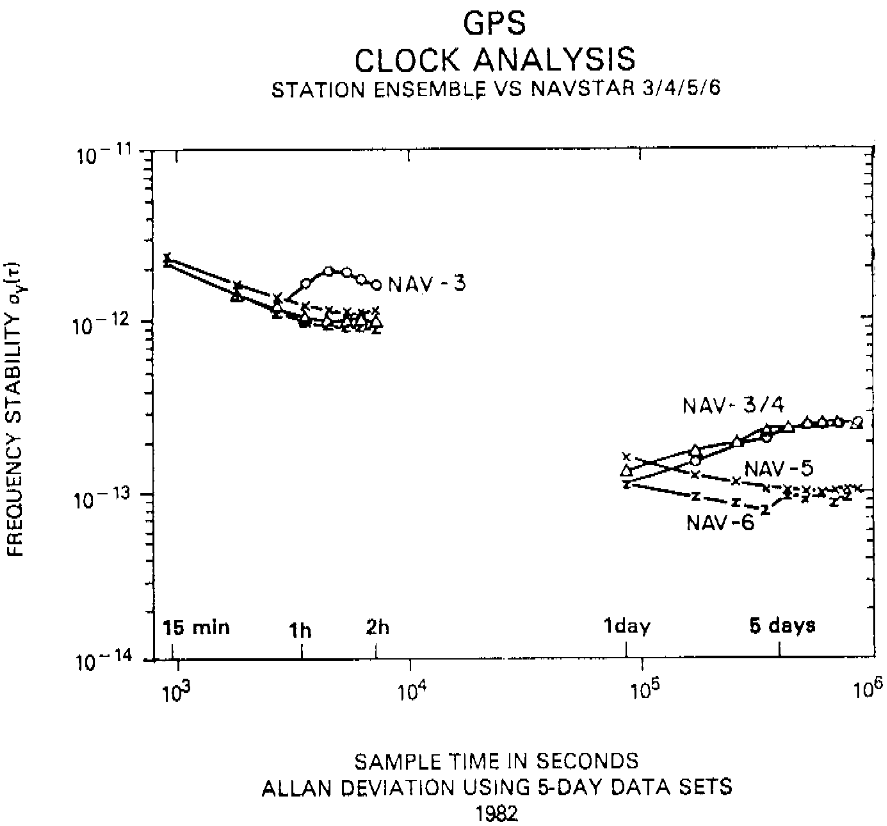

GPS
CLOCK ANALYSIS
STATION ENSEMBLE VS NAVSTAR 3/4/5/6
FREQUENCY STABILITY σy(τ)
10^-11
10^-12
10^-13
10^-14
NAV - 3
NAV- 3/4
NAV -5
NAV -6
15 min
1h
2h
1day
5 days
10^3
10^4
10^5
10^6
SAMPLE TIME IN SECONDS
ALLAN DEVIATION USING 5-DAY DATA SETS
1982

Figure 19

QUESTIONS AND ANSWERS

DR. BARNES:

Jim Barnes, with Austron. A possible suggestion of why you find data that goes as tau to the minus one-half. It might be just the deadtime that you mentioned was in the data. Deadtime can mean that kind of a signature to the allan variance.

MR. McCASKILL:

These data were taken every six seconds continually. So the deadtime corrections for that should be very small. Now, the long term results, we could have possible room for correction there; but I really wouldn't expect much of an effect on short term results.

DR. VESSOT:

Bob Vessot, Smithsonian. On the two hour data, I'm wondering if that anomaly couldn't be the fact that you are using data from a great range of elevation angles; and I think it's rather difficult to estimate the propagation qualities when things are down, say, ten, fifteen degrees. If those enter into the data, then it is likely to cause a wider spread of uncertainties and, hence a larger allan variance.

MR. McCASKILL:

That's a good point, but we do not expect it. We limited the sample time to two hours and if you start at TCA, you would go two hours on one side and two hours on the other, but the typical pass, unless its a short pass, could go up to maybe six hours or more.

So it's possible, if we would have pushed the sample time closer to, let's say, three hours, we could have seen some residual type of results and, of course, it could be there in a two hour sample time, but we just did not expect it at a two hour sample time.

DR. VESSOT:

The prediciton of atmospherics is kind of difficult, and these issues, I think, could increase the probability of having noise. That's the point I wanted to raise.

MR. McCASKILL:

It could be, but one thing--there was one slide missing, and I don't know what happened to it, it showed the number of samples. We have collected data for a year, and when you go across the full station ensemble, the number of samples goes into the order of 20 to 30 thousand, so whatever it is, we have high confidence that there's some type of process that flickered out for the short term results; and at the moment we cannot tell you exactly what it is, except that we believe that it's there and is a well defined measurement.

MR. ALLAN:

Dave Allan, N.B.S., two comments. No. 1. There would be no deadtime, because they're measuring time directly; so, there's no deadtime affect on that data. As relates to the two-hour rise in the sigma tau diagram, the recent work we have done, using the separation of various technique has shown serious problems in the ephemeris prediction and, indeed this may be the problem, and is not as good as perhaps they thought it was.