A NEW CALIBRATION METHOD FOR TIME DELAY STANDARD AND ITS APPLICATION

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ABSTRACT

This paper presents a new method which is used to measure time delay accurately by using a Type 900-LB Slotted Line. The accuracy for calibrating time delay of a precision coaxial air line Type 900-L is about $\pm(0.4-0.6)$ps, and for coaxial cables with VSWR less than 1.5 and time delay t less than 50ns is about $\pm(3-5)$ps.

Theoretical analysis and mathematical derivation of microwave networks in cascade are given in this paper. Methods to eliminate the errors which are caused by the discontinuities and the error analysis of the measuring system are presented.

In the last part of this paper, skin effect analysis of the transient characteristic of coaxial transmission line are discussed in detail. Methods to eliminate the errors which are resulted from using the calibrated time delay standard to calibrate time interval measurement instruments are presented. The estimation of errors and formulae for correction of those errors are described.

1. INTRODUCTION

Owing to the rapid growth of the requirement of measuring time delay accurately nowadays, a lot of commercial precision electronic instruments such as time delay generators with an accuracy ±1ns, electronic counters for time interval measurement with an accuracy of ±100ps, and coaxial time delay standard with

an accuracy of ±50ps have been developed and manufactured. Since
frequency f is the reciprocal of period T and there is no dif-
ficulty to set a frequency within the accuracy of ±1 × 10⁻⁹, it is
possible to measure the time delay of a coaxial transmission line
accurately by means of frequency domain measurement method. By
using the method described in this paper a coaxial cable with VSWR
less than 1.5 and time delay t less than 50ns has been measured.
The accuracy of measuring the time delay of that cable is about
±(3-5)ps. Skin effect analysis of the transient characteristic of
coaxial transmission line have been discussed in detail. The
effect of the inductive component and resistive component of the
coaxial line on pulse transient characteristic have been analyzed
and estimated also. This paper indicates how to correct the
error which is caused by skin effect, in this consequence measur-
ing and transfer accuracy has been increased.
When the calibrated time delay standard is used to calibrate a
precision time measurement counter, the total calibration uncer-
tainty is about ±10ps.

II Principle and Method of Measurement

2.1 The principle of measurement is based on the method of measur-
ing the displacement of the minimum of the standing wave pattern as
shown in Fig.1.
When the device under test (DUT) is disconnected and connected
between the Slotted Line and Short, the displacement of the mini-
mum of the standing wave pattern is Δd. The time delay t of the
device under test at measuring frequency is known as

$$t = \frac{n}{2}\frac{1}{f} \pm \frac{\Delta d}{c} \qquad (1)$$

Where c is the velocity of the electromagnetic wave and n is any
positive integer.

2.2 Method of Measurement

There are two methods to measure the time delay of coaxial trans-
mission line accurately by using a Type 900-LB Slotted Line.

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They are the periodic and the nonperiodic method.

Periodic Method

When the time delay of the DUT is exactly n/2 multiples of the
period of the measuring frequency, the Δd in eq.(1) equals zero.
Through properly adjusting the measuring frequency the displacement
of the minimum of the standing wave pattern Δd should approach zero,
when the DUT is disconnected and connected between the Slotted Line
and Short. Then the time delay of the DUT at measuring frequency is

$$t = \frac{n}{2}\frac{1}{f}$$

This method is called "Periodic Method". It is suitable to measure
the time delay of two port devices with small VSWR.
If S_{11} S_{22} and S_{21} are the scattering parameters of the reversible
two ports DUT then the maximum measured phase and time delay
errors can be represented as

$$\Delta \phi \simeq \pm \sin^{-1}\left(\frac{|S_{11}| + |S_{21}|^2 |S_{22}|}{|S_{21}|^2} \right) \qquad (degree)$$
$$\Delta t = \frac{\Delta \phi}{720} \cdot \frac{1}{f_{GH_z}} \quad (ns) \qquad\qquad (2)$$

Nonperiodic Method

If the VSWR of the DUT is rather large, the measured phase error
$\Delta \phi$ in eq.(2) will be quite large and it is difficult to adjust
the measuring frequency properly to make the displacement of mini-
mun of the standing wave pattern Δd approach zero, so that the
measured time delay error Δt would be rather large. In such con-
dition   it is impossible to acquire an accurate time
delay measurement by making measurement at one frequency only; the
nonperiodic method which makes time delay measurement with a series
of measuring frequencies is required.

2.3 Example of measurement

Use the nonperiodic method to measure the time delay of a piece of
coaxial cable with N connectors.
In order that the coaxial cable can be inserted into the Type

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900-LB Slotted Line system, the coaxial cable with Type N connectors has to be connected with the Type 900-QNJ adaptors as shown in Fig. 2.

The system of connection is as in Fig. 1. When the DUT is inserted in system at terminal 1-1, the minimum of the standing wave pattern would shift. The method, which evaluate the time delay of the DUT by means of measuring the displacements of minimum and the measuring frequencies, is called nonperiodic method, and eq. (1) is suitable for this method to calculate the time delay.

The measurement procedure is as follows.

1. According to a certain test frequency f_i , there is a corresponding displacement of minimum Δd_i , and the time delay (including the two Type 900-QNJ adaptors in this example) can be evaluated.

2. Repeat a series of time delay measurement by changing the test frequency in fixed frequency intervals, and obtain a set of time delay data which would be changing periodically with frequency.

3. As shown in Fig.2. the measuring terminal is changed owing to the adding of Type 900-QNJ adaptors. They are shifted from terminals T_1 and T_4 to terminals 1-1 and 2-2 respectively. There would be discontinuities at terminal T_1 , and T_2 owing to the mismatch between the coaxial cable and Type N connectors. In general the VSWR of conventional coaxial cables are about 1.3 to 1.5 and the VSWR of Type 900-QNJ adaptors are 1.1016 at 3 GHZ so that it can be neglected as compared with the coaxial cable.

There are rapid periodic variations in the time delay data, owing to the discontinuity at T_1 , and the error of measurement can be minimized by taking the arithmetical mean value of the maximum and minimum time delay values in the frequency range of interest. But the discontinuity at T_2 would introduce unnoticed systematic error if the terminal T_2 is connected directly to the Short Type 900-WNC. In our experimental system terminal T_2 is connected through a Type 900-QNJ adaptor to the Short Type 900-WNC. The electric length of the Type 900-QNJ adaptor is 5Cm, so that there are slow periodic variations (about 3000MHZ) superimposed on the

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rapid periodic variations in the time delay data. In order to increase the electric length a piece of Type 900-L30 Air Line is added between terminal 2-2 and the Type 900-WNC Short, then the frequency of slow periodic variation becomes 428.6 MHZ and it is acceptable in this experiment. By using the method described above, a set of time delay data with two kinds of periodic variations would be present; take the arithmetical mean value of the maximum and minimum time delay values in the frequency range of interest as the required time delay datum. Then the systematic error owing to the discontinuities at Terminal T_1 and T_2 can be eliminated.

4. Why are there two kinds of periodic variation in the time delay data? There is a $e^{-j2\beta(\ell + \ell_{20})}$ term in eq.(4) (section III), which is introduced by the reflection at the M network in Fig 3. Let $\ell_1 = \ell + \ell_{20}$ then $e^{-j2\beta(\ell + \ell_{20})}$ changes into $e^{-j2\beta\ell_1}$. The condition for producing a periodic variation is $2\beta\ell = 2n\pi$, therefore the frequency interval for a rapid periodic variation is

$$\Delta f = \frac{c}{2\ell_1}$$

(3)

For example $\ell_1 = \ell + (\ell_{20} + \ell_{30}) = \ell + \ell'_{20} = 267.5\,cm$

ℓ --electric length of the coaxial cable with Type N connectors

ℓ_{20} --electric length of Type 900-QNJ Adaptor

ℓ_{30} --electric length of Type 900-L30 Air Line

ℓ'_{20} --electric length of Type 900-QNJ Adaptor and 900-L30 Air Line.

put the value of ℓ_1 into eq. (3) and obtain $\Delta f_1 = 56$ MHZ

In the same way the slow periodic variation which is caused by the reflections at N network in Fig 3. can be analyzed.

If the Type 900-L30 Air Line is connected in the measuring system the frequency interval for it would become

$\Delta f_2 = 428.6$ MHZ

The calculated values of Δf_1 and Δf_2 are well matched with the experimental curve shown in Fig. 6.

5. Evaluate the time delay of the coaxial cable under test.

In this paper a Type N connector coaxial cable with a Type 900-QNJ

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adaptor at each end is used as a device under test(DUT). Take the arithmatical mean value of the maximum and minimum time delay value to obtain the accurate time delay value of the DUT.

$$t=\frac{1}{2} \ (t_{max} + t_{min})=8209.6ps.$$

The time delay of the air line has been discounted from this value.
III Theoretical Analysis of Microwave Networks in Cascade
Let ℓ represent the electric length of the coaxial cable, ℓ_{10} and ℓ_{20} the electric length of Type 900-QNJ adaptors, ℓ'_{20} is the electric length of Type 900-QNJ adaptor andType 900-L30 Air Line as shown in Fig.3. Terminal 1-1 is connected to the output terminal of the 900-LB Slotted Line and Terminal 2-2 (or 2'-2') is connected to the Type 900-WNC Short.

Use m_{11}, m_{21}, m_{12} and m_{22} to represent the scattering matrix elements of the connector pair M and n_{11}, n_{21}, n_{12} and n_{22} to represent the scattering matrix elements of the connector pair N. Use S_{11}, S_{21}, S_{12} and S_{22} to represent the scattering matrix elements of the network between the terminals T_1 and T_2 and S'_{11}, S'_{21}, S'_{12} and S'_{22} to represent the scattering matrix elements of the network between the terminals 1-1 and 2-2 (or 2'-2').

When terminal 2-2 (or 2'-2'), is connected to the Type 900-WNC Short, the reflection coefficient Γ'_1 at terminal 1-1 can be represented as (see appendix I)

$$\Gamma'_1 \simeq -m_{12}n_{12}m_{21}n_{21}e^{-j2\beta(\ell+\ell_{10}+\ell_{20})}.(1+2\delta)\{1- \frac{m_{11}e^{j2\beta(\ell+\ell_{20})}}{m_{12}m_{21}n_{12}n_{21}}(1-2\delta)- \frac{n_{11}e^{j2\beta\ell_{20}}}{n_{12}n_{21}}(1-\delta)-n_{22}e^{-j2\beta\ell_{20}}-$$

$$-n_{12}n_{21}m_{22}e^{-j2\beta(\ell+\ell_{20})}.(1+\delta)\} \qquad (4)$$

The $m_{12}\ n_{12}\ m_{21}\ n_{21}\ e^{-j2\beta(\ell+\ell_{10}+\ell_{20}).(1+2\delta)}$ term in eq.(4) represents the required quantity for the time delay measurement of the DUT, and the other four terms in the bracket represent the rapid and slow periodic variations of the time delay measurement data. If there is no ℓ_{20}, n_{11} and n_{22} would introduce an unnoticed systematic error. If ℓ_{20} is very short, the frequency of the slow periodic variation would be very high (as described in Secrion II it is 3000MHZ). After adding

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a 30 Cm precision air line the frequency decreases to 428.6MHZ.
In the point of view of our experiment it is acceptable and easy
to be realized.

The phase angle $\beta(\ell+\ell_{10}+\ell_{20}')$ in eq.(4) corresponds to the time delay
between terminals 1-1 and 2'-2', from which the time delay of the
DUT can be evaluated by subtracting 1.0012 ns, the time delay of
the precision air line Type 900-L30.. If the time delay of the two
900-QNJ adaptors are subtracted, the time delay of the coaxial ca-
ble with Type N connector can be obtained.

It doesn't introduce systematic error in time delay measurement
when the DUT has loss, but it increases the random error as the
sharpness of the minimum of the standing wave pattern decreases.

Proceed the error analysis according to eq.(4) as follows
1. Let the VSWR of the DUT be 1.5, then $|S_{11}'| = |S_{22}'| = 0.2$, $|m_{22}| = |n_{11}|$
$=0.1$ so that $|\delta| = 0.01$. Under the worst phase combination
it corresponds to a phase angle of 1.15°. When the measuring
frequency is 4 GHz, it just introduces an error of 0.4 ps.

2. The effect of m_{12} n_{12} m_{21} n_{21} .
Evaluate m_{12} m_{21} as shown in Fig. 4.
Evaluate n_{12} n_{21} by the same way.
If the transmission line has an impedance discontinuity at a certain
interface, we use network M to represent this discontinuity and
let m_{11}, m_{21}, m_{12} and m_{22} represent the elements of the M matrix. For
convenience use normalized impedance in Z_1 and Z_2. Let $Z_1 = 1$,
$Z_2 = 1 + x + jy$ There is the following relation

$$m_{12} m_{21} = \frac{1}{1 + \frac{(X+JY)^2}{4(1+X+JY)}} = 1 + \delta' \qquad \text{where } \delta' = \frac{-(X+JY)^2}{4(1+X+JY)}$$

since $|S_{11}'| = |S_{22}'| = 0.2$ x $= 0.1$, y $= 0.1$ so that $\delta' \le 0.01$ In the same
way n_{12} n_{21} $= 1 + \delta'$ then m_{12} m_{21} n_{12} n_{21} $\approx 1 + 2\delta'$ (5)
From eq.(5) it introduces a 0.4 ps time delay incertainty, but in
precision time delay measurement, this uncertainty can be elimin-
ated by interchanging the input and output of the DUT in two mea-
sure-ments and taking the mean value.

3. Analysis about the effects of the other four terms in the bracket

Consider the $\dfrac{m_{11} e^{j2\beta(l+l_{20})}}{m_{12} m_{21} n_{12} n_{21}} (1-2\delta)$ term and $n_{12} n_{21} m_{22} e^{-j2\beta(l+l_{20})} \cdot (1+\delta)$

term first. Neglect δ as a small quantity, and assume the M matrix is lossless, then the extrema of the expression

$$1 - \frac{m_{11} e^{j2\beta(l+l_{20})}}{m_{12} m_{21} n_{12} n_{21}} - n_{12} n_{21} m_{22} e^{-j2\beta(l+l_{20})} \qquad (6)$$

are $1+g+jh$ and $1-g-jh$, they correspond to the phase angles

$$\theta_1 = \tan^{-1} \frac{h}{1+g} \quad , \quad \theta_2 = \tan^{-1} \frac{-h}{1-g} \qquad (7)$$

Then the expression for the phase error $\Delta\theta$, while taking the mean value of the extrema as the time delay, is

$$\Delta\theta \simeq \frac{1}{2} \tan^{-1}(-2gh) \qquad (8)$$

In the same way, the effect of the two terms $\dfrac{n_{11} e^{j2\beta l_{20}}}{n_{12} n_{21}} (1-\delta)$

and $n_{22} e^{-j2\beta l_{20}}$ can be considered.

Since $|S_{11}'| = |S_{22}'| = 0.2$, $g \leq 0.14$ $h \leq 0.14$
then $\Delta\theta/2 = 35'$ it corresponds to a 0.4ps time delay uncertainty.
When taking the mean value of the extrema as the time delay of the DUT, the four terms in the bracket would introduce an uncertainty of 2×0.4ps.

IV Measurement Errors
There are eight kinds of errors source for the time delay measurements of two ports DUT in the Type 900 system.

1. The frequency instability of the signal generator introduce the error Δt_1

$$\Delta t_1 = \frac{\Delta f}{f} \cdot t$$

in example above $\Delta t_1 = \pm 0.4$ps.

2. The inaccuracy of the slotted line scale introduces the error Δt_2
The inaccuracy of the slotted line scale is 0.1mm, (with the micrometer it can be decreased to 0.01 mm).
so that $\Delta t_2 = \pm 0.33$ ps.

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3. The Type 900-L30 precision Air Line introduces time delay error Δt_3 In the measuring system, a Type 900-L30 Precision Air Line is added to increase the electric length of ℓ_{20}, and its time delay is 1001.2 ± 0.4 ps, so that it introduce the error Δt_3.

$$\Delta t_3 = \pm \ 0.4 \ \text{ps}.$$

4. The M and N networks of the connector pairs would introduce the error Δt_4.
This error comes from the δ term in eq. (4) and it has been described in part III $\Delta t_4 = \pm \ 0.4 \ \text{ps}.$

5. The test frequency interval can't be infinitesimal, it would introduce the error Δt_5.
Since a series of measured time delay data of the required measuring frequencies are required to obtain the extrema, the smaller the measuring frequency interval, the more accurate extrema can be obtained. In the example described in Part II, the measuring frequency interval is 10MHZ, and a frequency interval of two to three MHZ can be used in the neighborhood of the extrema. According to the data analysis, the error Δt_5 is less than 1ps.

6. The reflection of the Type 900-BT connector would introduce the error Δt_6.
Type 900-BT connectors are used in both the terminal 1-1 and 2-2 of the measuring system, and the time delay error introduced by them is less than 0.8 ps.

7. While taking the mean value of the extrema to evaluate the time delay it would introduce the error Δt_7 and Δt_8.
As described in part III, $\Delta t_7 = \pm \ 0.4$ ps. $\Delta t_8 = \pm \ 0.4$ps.
From Δt_1 to Δt_8 the total r.m.s. error is about 1.6 ps.

V   Skin Effect Analysis of the Transient Characteristic of Coaxial Transmission Line.
The time delay of a coaxial transmission line changes with frequency Since the series inductance changes with frequency owing to skin effect. For example if the time delay of a piece of coaxial cable

is 4996.64 ps at 1000MHZ it would change to 5000 ps at 3000MHZ.
So that, when it is operated in a pulse condition, the risetime of
the output pulse would be degenerated owing to the frequency res-
ponse of time delay. If an input pulse with 1ns risetime is pass-
ing through a coaxial cable with 50ns time delay the rise time of
the output pulse would change to 1.14ns. On the other hand the
changing of series resistance owing to skin effect degenerates the
risetime of the output pulse too. The magnitude of degeneration is
a function of the attenuation of the coaxial cable and the risetime
of the input pulse. If an input pulse with 1ns risetime is passing
through a coaxial cable with 0.4 dB attenuation, the risetime of
the output pulse would change to 1.03ns.

5.1 The relation between the frequency response of the series in-
ductance and the pulse time delay and transient characteristic.
As described above, the series inductive component increases as
the frequency decreases so that the wave propagation speed through
the cable decreases and the time delay increases. For this reason
the calibrated time delay standard should be used at the cali-
brated frequency. If it is used on the other frequencies, correc-
tion should be taken. According to the relation between the elec-
tromagnetic wave propagation velocity and the inductive component,
the formula of time delay as a function of frequency can be deriv-
ed. (see appendix II)

$$t = t_o(1 + \frac{b}{\sqrt{f_{GHz}}}\%) \text{ ns.} \qquad (9)$$

As the existence of skin effect, the phase frequency characteristic
is no longer linear. When working in a pulse condition, since the time
delays are different for each of the harmonic components of the
pulse, the resultant output waveform would change and the risetime
degenerates as shown on Fig. 5.
This means, the frequency response of the inductive component
would also degenerate the pulse output waveform besides the fre-
quency response of the resistive component but, when using the
calibrated time delay standard to calibrate time measurement in-

struments the effect of the inductive component is very small and can be neglected, provided the calibrating frequency of the time delay standard and the bias voltage of the calibrated equipment are properly selected.

$$\text{Let} \quad U_1(t) = \frac{E}{2} + \frac{2E}{\pi} \sum_{\nu=0}^{\nu=\infty} \frac{\sin(2\nu+1)\Omega t}{2\nu+1} \tag{10}$$

where $\Omega = \frac{2\pi}{T}$ —fundamental angular frequency

E -- pulse (or square wave) amplitude

If the coaxial transmission line has ideal linear phase frequency characteristic and the module of the transmission coefficient is constant throughout the passband and equal to zero outside the passband, then the output can be represented as

$$U_2(t) = \frac{KE}{2} + \frac{2KE}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \frac{\sin(2\nu+1)\Omega(t-t_c)}{2\nu+1} \tag{11}$$

where K -- the module of transmission coefficient

t_c -- delay time

Since the phase frequency characteristic is nonlinear owing to skin effect, the delay time is no longer constant and should be expressed as

$$t_x = t_0 \left(1 + \frac{b}{\sqrt{f_{x\,GH_z}}}\%\right) \tag{12}$$

where t_0 -- delay time of zero depth of penetration

t_x -- delay time at frequency $f_x = x \cdot f_s$

f_s -- the highest harmonic frequency of the input pulse

$f_s \approx 0.35/\delta_0$

b -- correction coefficient

under this condition eq. (11) should change to

$$U_2(t) = \frac{KE}{2} + \frac{2KE}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \frac{\sin(2\nu+1)\Omega(t-t_x)}{2\nu+1} \tag{13}$$

When the bias voltage equals to KE/2, there is a delay time, $t=t_A$ which satisfies the following formula (14)

$$\sum_{\nu=0}^{\nu=\nu_m} \frac{\sin(2\nu+1)\Omega(t_A-t_x)}{2\nu+1} = 0 \tag{14}$$

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$$t_A - t_x = t_o b \left(\frac{1}{\sqrt{A}} - \frac{1}{\sqrt{X}} \right) \frac{1}{\sqrt{f_o \, GH_z}} \% \qquad (15)$$

$$f_A = A \cdot f_o \qquad (16)$$

Substituting formulae (15) and (16) into formula (14) and taking integrating we get

$$A \simeq 0.3 \qquad (17)$$

so that $\quad t_A = t_o \left(1 + \frac{b}{\sqrt{0.3 \, f_o \, GH_z}} \% \right) \qquad (18)$

According to the preceding analysis, when KE/2 is used as the bias voltage, $0.3 \, f_0$ should be taken as the calibrating frequency for the time delay standard. For example, if the input pulse E=0.5V and the pulse risetime τ_o =0.1ns, then f_o =3.5GHZ. When use 0.25V as the bias voltage, the time delay data should be taken at the measuring frequency f =1.05GHZ as the standard time delay value for this specified condition.

Besides, from eq. (12) and Fig. 5. it is evident that the time of the lower part of the leading edge of the delayed pulse corresponds to delay time of its higher harmonics. So that, when the risetime and the amplitude of the input pulse are rather large KE/10 (or smaller) may be used as the bias voltage, and f_0 is taken as the calibrating frequency for the time delay standard. When using the calibrated time delay standard to calibrate time measurement instruments the error which is caused by the frequency response of the inductive component, can be eliminated by using the method described above.

5.2 The distortion of pulse transient characteristic owing to the skin effect of resistive component.

When a pulse passes through a coaxial cable, the output waveform would be degenerated owing to the changing of the high frequency loss of the coaxial cable. But in general the dielectric loss is much smaller than the resistive loss.

1. The expression of resistive component in coaxial cable may be expressed as

$$R = \frac{\rho_b}{2\pi \delta_b \, b_o} + \frac{\rho_a}{2\pi \delta_a \, a_o} \qquad (19)$$

where δ_b , δ_a are the skin depths of the inner and outer conductors ρ_b , ρ_a are the resistivities of the materials of the inner and outer conductors.

b_o , a_o are the radius of the inner and outer conductors

It shows that the changing of the resistive component owing to skin effect of the coaxial cable increases with frequency. It doesn't introduce time delay measurement error when the standard is calibrated at a single frequency but it would degenerate the output pulse. Since the higher harmonics have larger attenuation so the risetime of the output pulse would increase. If the bias voltage used is not zero, time delay measurement error is introduced. The larger the bias voltage used, the larger the time delay measurement error that would be introduced, but by using the correction formula (22) given below, this error term can be reduced by a factor of 10.

2. Correction formula for reducing the time delay measurement error owing to the resistive component

Considering the resistive loss of the coaxial cable, the increment of the risetime can be evaluated as (see appendix IV)

$$\Delta\delta' = \delta_o \, a_s$$

where $\qquad a_s = \frac{2}{3}\propto_{f_o} - \frac{1}{4}(\propto_{f_o})^2 + \frac{1}{15}(\propto_{f_o})^3 - \cdots \qquad (20)$

$\propto_{f_o}$ -- the attenuation nB at the highest frequency $f_o = 0.35/\delta_o$
δ_o -- the risetime of the input pulse

As described above if 0.1KE or 0.5KE is used as bias voltage the correction value should be

$$\Delta\delta_1 = 0.1 \, \delta_o \, a_s \qquad \text{for 0.1KE bias voltage}$$
$$\Delta\delta_2 = 0.5 \, \delta_o \, a_s \qquad \text{for 0.5KE bias voltage} \qquad (21)$$

So that in a pulse condition the effective standard delay time of the calibrated time delay standard should be

$$t_s = t_h + 0.1\,\delta_o\,a_s \simeq t_h + \frac{\alpha_{f_o}}{15}\,\delta_o$$

$$t'_s = t'_h + 0.5\,\delta_o\,a_s \simeq t'_h + \frac{\alpha_{f_o}}{3}\,\delta_o$$

(22)

where t_s--effective standard delay time of the calibrated time delay standard, when the input pulse risetime is δ_o and the bias voltage is 0.1KE.

t'_s-- the same as t_s, except 0.5KE is used as bias voltage

t_h--delay time of the calibrated time delay standard at frequency f_o .

t'_h-- the same as t_h, except it is at frequency 0.3 f_o .

δ_o-- the risetime of the input pulse

α_{f_o}-- the attenuation in neper of the time delay standard at frequency f_o .

VI Conclusion

1. This paper presents a new method to calibrate the time delay standard (coaxial cable). The accuracy of calibration is $\pm(3-5)$ps.

2. When we use the calibrated time delay standard to calibrate precision time measurement instruments, we must use formula (22) to evaluate effective standard delay time for correcting the errors owing to skin effect.

3. When we use the calibrated time delay standard to calibrate precision time measurement instruments, two additional errors should be considered as follows.

a. The evaluation of the highest limiting frequency may have a tolerance of 10%, and it introduces an error $\Delta t_1'$.

$$\Delta t_1' = 0.054\,\frac{t_o b}{\sqrt{f_o\,GHz}}$$

when $f_o = 0.35$GHZ, $b=0.0816$, $t_o=8200$ps, $\Delta t_1'=0.61$ps
when $f_o = 0.35$GHZ $b=0.0816$, $t_o=50.000$ps $\Delta t_1'=3.72$ps
It is evident the higher the f_o the smaller the $\Delta t_1'$.

b. 10% tolerance of the bias voltage used may introduce an error $\Delta t_2'$.

$$\Delta t_2' = 0.0067 \alpha_{f_0} \delta_0 \qquad \text{for } 0.1KE \text{ bias voltage}$$
$$\Delta t_2' = 0.033 \alpha_{f_0} \delta_0 \qquad \text{for } 0.5KE \text{ bias voltage}$$

Since α_{f_0} is proportion to $\sqrt{f_0}$, and $\delta_0 \simeq 0.35/f_0$, then $\Delta t_2'$ decreases as δ_0 decreases in the rate of $\sqrt{f_0}$.

In general, for an input pulse with $E \geq 2V$, $\delta_0 \leq 1ns$ or $E \geq 0.5V$, $\delta_0 \leq 0.1ns$, using the calibrated time delay standard to calibrate time measurement instruments, the additonal error owing to skin effect is less than $\pm 5ps$.

So that, when the calibrated time delay standard is used to calibrate a precision time measurement instrument (counter), the total calibration uncertainty is less than $\pm 10ps$.

Appendix I

Formula derivation for the reflection coefficient at the input terminal of the microwave networks in cascade.

As shown in Fig. 3. the microwave networks has the following relation

$$b_1 = m_{11}a_1 + m_{12}A_2$$
$$B_2 = m_{21}a_1 + m_{22}A_2 \tag{1}$$

and
$$A_1 = B_2 e^{-j\beta\ell}$$
$$A_2 = B_1 e^{-j\beta\ell} \tag{2}$$

$$B_1 = n_{11}A_1 + n_{12}a_2$$
$$b_2 = n_{21}A_1 + n_{22}a_2 \tag{3}$$

where β --phase constant, thus

$$S_{11} = m_{11} + \frac{m_{12}m_{21}n_{11}e^{-j2\beta\ell}}{1 - m_{22}n_{11}e^{-j2\beta\ell}}$$

$$S_{22} = n_{22} + \frac{n_{12}n_{21}m_{22}e^{-j2\beta\ell}}{1 - m_{22}n_{11}e^{-j2\beta\ell}} \tag{4}$$

$$S_{21} = \frac{m_{21}n_{21}e^{-j\beta\ell}}{1 - m_{22}n_{11}e^{-j2\beta\ell}}$$

$$S_{12} = \frac{n_{12}m_{12}e^{-j\beta\ell}}{1 - m_{22}n_{11}e^{-j2\beta\ell}}$$

Since the Type 900-QNJ Adaptors have electric length the S' matrix of the network between terminal 1-1 and 2-2 becomes

$$S_{11}' = S_{11}e^{-j2\beta\ell_{10}}$$

$$S_{22}' = S_{22}e^{-j2\beta\ell_{20}} \tag{5}$$

$$S_{12}' = S_{12}e^{-j\beta(\ell_{10}+\ell_{20})}$$

$$S_{21}' = S_{21}e^{-j\beta(\ell_{10}+\ell_{20})}$$

Let $\delta = m_{22}\,n_{11}e^{-j2\beta\ell}$, eq. (5) can be rewritten as

$$S_{11}' \simeq m_{11}e^{-j2\beta\ell_{10}} + m_{12}m_{21}n_{11}e^{-j2\beta(\ell+\ell_{10})}.(1+\delta)$$

$$S_{22}' \simeq n_{22}e^{-j2\beta\ell_{20}} - n_{12}n_{21}m_{22}e^{-j2\beta(\ell+\ell_{20})}.(1+\delta) \tag{6}$$

$$S_{12}' \simeq n_{12}m_{12}e^{-j\beta(\ell+\ell_{10}+\ell_{20})}.(1+\delta)$$

$$S_{21}' \simeq m_{21}n_{21}e^{-j\beta(\ell+\ell_{10}+\ell_{20})}.(1+\delta)$$

According to theory of the network analysis, the expression for the reflection coefficient at the input terminal of the S' network is

$$\Gamma' = S'_{11} + \frac{S'_{12} S'_{21} \Gamma_L}{1 - S'_{22} \Gamma_L}$$

where Γ_L is the reflection coefficient of the termination of the network. When this S' network is terminated a Short, $\Gamma_L = -1$

$$\Gamma' = S'_{11} - \frac{S'_{12} S'_{21}}{1 + S'_{22}} \tag{7}$$

Substituting eq. (6) into eq. (7) and neglecting δ^2 and higher order terms of the small quantities the expression for the reflection coefficient at terminal 1-1 becomes

$$\Gamma'_1 = -m_{12} n_{12} m_{21} n_{21} e^{-j2\beta(\ell+\ell_{10}+\ell_{20})} (1+2\delta) \left[1 - \frac{m_{11} e^{j2\beta(\ell+\ell_{10})}}{m_{12} m_{21} n_{12} n_{21}} (1-2\delta) - \frac{n_{11} e^{j2\beta\ell_{20}}}{n_{12} n_{21}} (1-\delta) - \right.$$

$$\left. - n_{12} n_{21} m_{22} e^{-j2\beta(\ell+\ell_{20})} (1+\delta) - n_{22} e^{-j2\beta\ell_{20}} \right] \tag{8}$$

This is the eq. (4) in this paper.

Appendix II

Formula derivation for the relation between delay time and measuring frequency.

In lossless coaxial transmission line the propagation velocity of the electromagnetic wave is

$$V = \frac{1}{\sqrt{LC}} \tag{9}$$

where   L --inductance per unit length

C --capacitance per unit length

The delay time for a piece of transmission line is inversely proportional to the propagation velocity

$$t \propto \frac{1}{V} \tag{10}$$

Thus $t \propto \sqrt{L}$ (11)

the expression of inductance for zero skin depth is

$$L_o = 2 \ln \frac{D_o}{d_o} \times 10^{-9} \; H/cm \tag{12}$$

 where D_o, d_o --the inner diameter of the outer conductor and the outer diameter of the inner conductor respectively.

In those frequencies, where skin effect can not be neglected, the effective diameters of the inner and outer conductors of the co-axial transmission line are no longer their mechanical dimensions d_o and D_o. In this case, the effective diameter of the outer conductor increases with the amount of skin depth, on the contrary, the effective diameter of the inner conductor decreases with the amount of the skin depth. So that, the value of inductance is no longer expressed as eq. (12) it should change to

$$L = 2 \ln \frac{D}{d} \times 10^{-9} \; H/cm \tag{13}$$

where D, d --the effective diameters of the outer and inner conductors considering the skin effect.

From eq. (11), (12) and (13) we get

$$\frac{\Delta t}{t_o} = \frac{\sqrt{L} - \sqrt{L_o}}{\sqrt{L_o}} \% \tag{14}$$

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From eq. (14) we can express the time delay as the function of measuring frequency

$$t = t_o(1 + \frac{b}{\sqrt{f_{GHz}}} \%)$$ (15)

where b -- correction coefficient for the inductive component variation owing to skin effect.

Coefficient b is the function of the diameter of the coaxial transmission Line, and the resistivity of the conductors. Here are some examples for the Type 900-LZ Reference Air Line b is 0.0284, and for the Type SYV-50-5 and SYV-50-2-2 coaxial cables made in China b is 0.0816 and 0.1711 respectively.

Appendix III

The relation between the risetime of the output pulse and the bandwidth.

Let the expression for input pulse (square wave)

as $\quad U_1(t) = \dfrac{E}{2} + \dfrac{2E}{\pi} \sum_{\nu=0}^{\nu=\infty} \dfrac{\sin(2\nu+1)\Omega t}{2\nu+1}$ \hspace{1cm} (16)

where $\quad \Omega = \dfrac{2\pi}{T}$ ——fundamental angular frequency

$\quad$ E —— amplitude of the input pulse (square wave)

When the input pulse has passed through an ideal network which has linear phase frequency characteristic and constant transmission coefficient inside the pass band and zero transmission coefficient outside the pass band, the output pulse becomes

$$U_2(t) = \dfrac{KE}{2} + \dfrac{2KE}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \dfrac{\sin(2\nu+1)\Omega(t-t_c)}{2\nu+1} \hspace{1cm} (17)$$

where $\quad$ K——transmission coefficient

$\quad t_c$ —— delay time

$\quad \nu_m$—— the maximum value which depends on the highest order $(2\nu_m+1)$ of harmonics which can pass through the network.

It is evident that $\quad (2\nu_m+1)\,\Omega \leq \omega_0$ \hspace{1cm} (18)

where ω_0——the highest angular frequency of the passband.

Differentiate with respect to eq. (17)

$$\dfrac{d\,U_2(t)}{dt} = \dfrac{2KE\,\Omega}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \cos[(2\nu+1)\Omega(t-t_c)] \hspace{1cm} (19)$$

when $\quad t = t_c$, $\qquad \left[\dfrac{d U_2(t)}{dt}\right]_{max} = 2KE f_0$ \hspace{1cm} (20)

where $\quad f_0$——the highest frequency of the passband.

So that the risetime of the output pulse can be expressed

as $\qquad \delta_0 = \dfrac{KE}{\left[\dfrac{d U_2(t)}{dt}\right]_{max}} = \dfrac{0.5}{f_0}$ \hspace{1cm} (21)

In practice, when $\omega > \omega_0$, the transmission coefficient is not zero, but it decreases rapidly as the frequency increases. So that in general the following formula is used instead of (21)

$$\delta_0 \simeq \dfrac{0.35}{f_0} \hspace{1cm} (22)$$

Appendix IV

Formula derivation for the output pulse risetime increment owing to the variation of coaxial transmission line loss.

Assume that the coaxial transmission line has linear phase frequency characteristic, When an input pulse expressed as eq. (16) passes through a piece of coaxial transmission line with loss, the amplitude frequency characteristic of the output pulse can be expressed as

$$U_2(t) = \frac{E}{2} + \frac{2E}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \frac{\sin(2\nu+1)\Omega(t-t_c)}{2\nu+1} e^{-\alpha} \qquad (23)$$

where α --attenuation of coaxial transmission line in neper

$$\alpha = \frac{\alpha'}{8.68} \ell \quad (Neper) \qquad (24)$$

For conventional coaxial cable

$$\alpha' = 2.98 \times 10^{-9} \sqrt{f} \; \frac{1}{a_o} \left(\frac{a_o}{b_o} + 1 \right) \frac{\sqrt{\varepsilon_r}}{\ell n \frac{a_o}{b_o}} \quad (dB/cm) \qquad (25)$$

Where b_o, a_o --the radius of the inner and outer conductors respectively

ε_r --relative dielectric constant of the dielectric medium

f -- frequency

According to eq. (25), can rewrite eq. (24) to

$$\alpha = K\sqrt{f} \qquad (26)$$

Differentiate with respect to eq. (23)

$$\frac{dU_2(t)}{dt} = \frac{2E\Omega}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \cos(2\nu+1)\Omega(t-t_c)e^{-\alpha(f)} \qquad (27)$$

since $\alpha_{f_x} = \sqrt{x}\,\alpha_{f_o}$, make transformation $x = \nu/\nu_m$ and integrate when $t = t_c$ (assumed t_c is constant), there is

$$\left[\frac{dU_2(t)}{dt} \right]_{max} = \frac{E\omega_o}{\pi} \left(1 - \frac{2}{3}\alpha_{f_o} + \frac{1}{4}(\alpha_{f_o})^2 - \frac{1}{15}(\alpha_{f_o})^3 + \cdots \right) = 2Ef_o a \qquad (28)$$

$$a = 1 - \frac{2}{3}\alpha_{f_o} + \frac{1}{4}(\alpha_{f_o})^2 - \frac{1}{15}(\alpha_{f_o})^3 + \cdots$$

$$a_s = 1 - a = \frac{2}{3}\alpha_{f_o} - \frac{1}{4}(\alpha_{f_o})^2 + \frac{1}{15}(\alpha_{f_o})^3 - \cdots \quad (28)$$

where α_{f_o} --attenuation in neper at the highest frequency f_o

f_o -- the highest harmonic frequency of the input pulse

take $f_o = 0.35/\delta_o$

δ_o --risetime of the input pulse

thus, the increment of the output pulse risetime can be obtained

$$\Delta\delta' = \delta_o a_s \quad (29)$$

Reference

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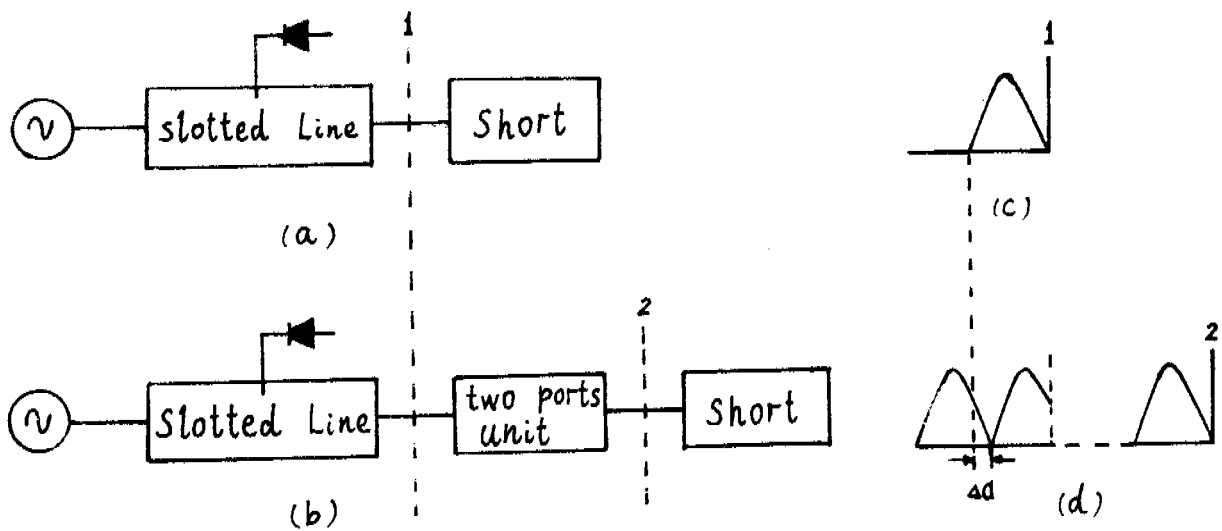
Fig. 1   Principle   diagram   for time   delay   measurement
with   microwave   system
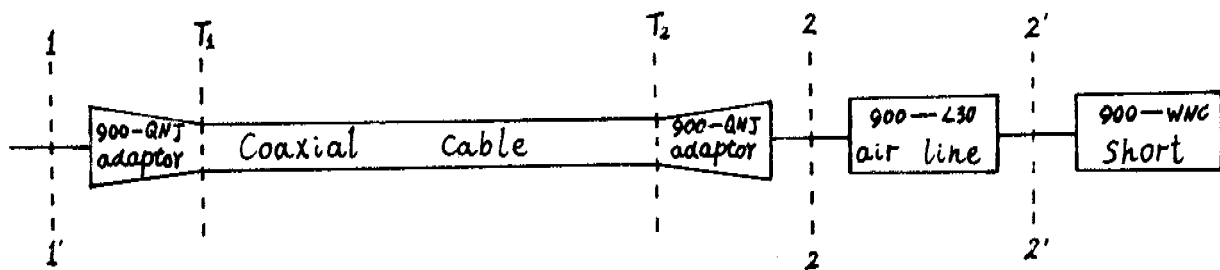

Fig. 2   Using   the   nonperiodic   method   to   measure   coaxial
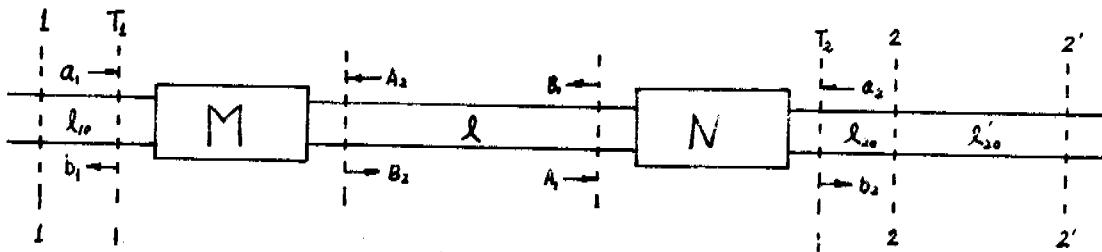Cables   with   Type N   connectors

Fig.3   Theoretical analysis of microwave networks
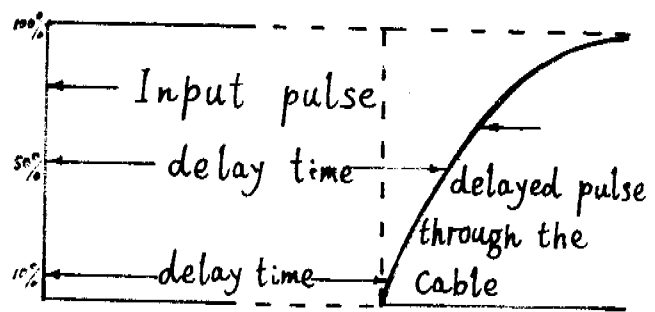
in   cascade

Fig. 4

Fig. 5

Fig.4   Analysis at the discontinuity of network M

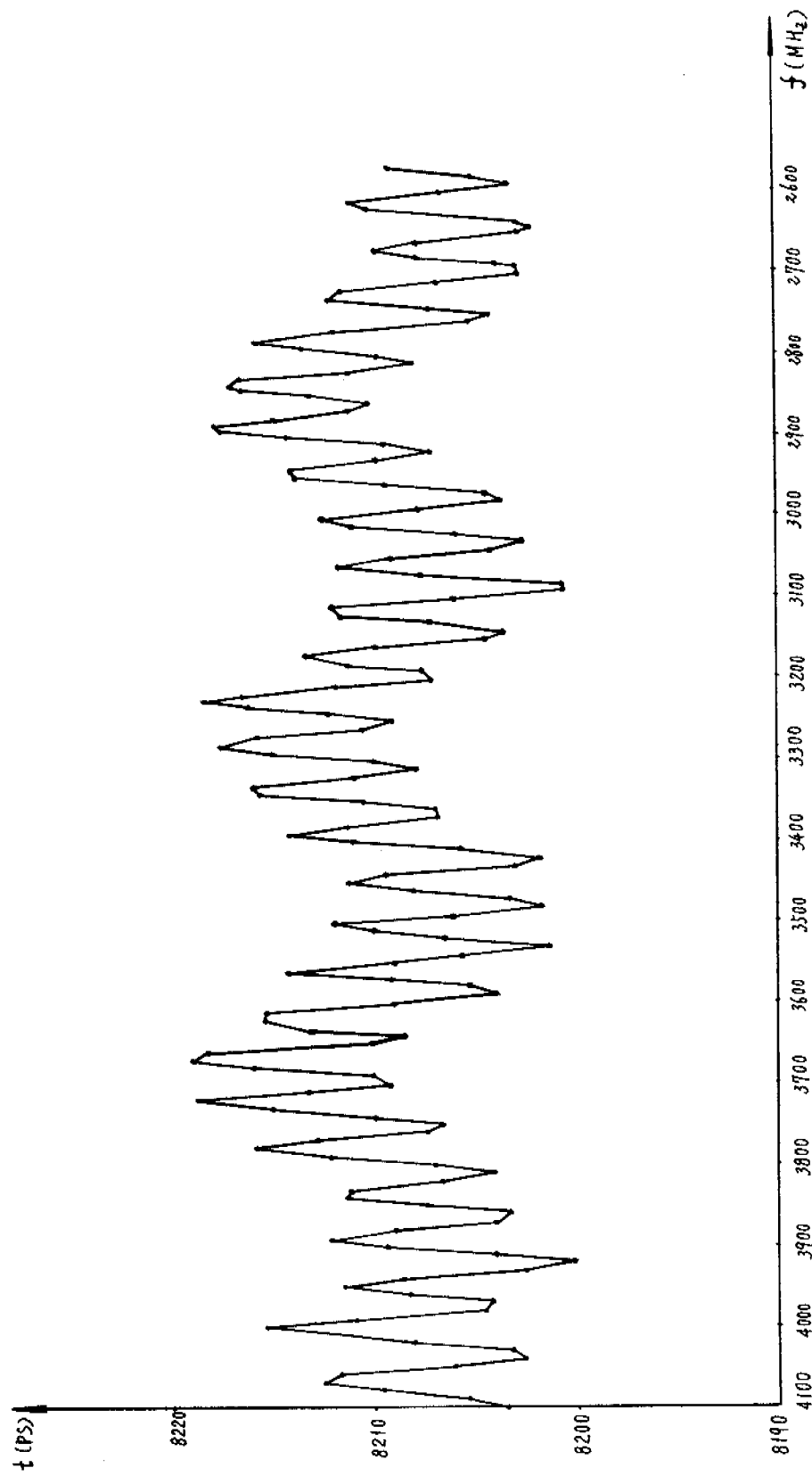Fig.5   Step response distortion owing to non-linear phase frequency characteristic of skin effect

Fig.6 Delay Time versus frequency

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QUESTIONS AND ANSWERS

None for Paper #8