CLOCK PERFORMANCE AS A CRITICAL PARAMETER IN NAVIGATION SATELLITE SYSTEMS

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ABSTRACT

The high performance of available oscillators has permitted the development of invaluable navigation and geodetic satellite systems. However, still higher performance oscillators would further improve the accuracy or flexibility of the systems.

INTRODUCTION

Oscillator performance is a critical factor in the operation of the Navy Navigation Satellite System (NAVSAT) and of the NAVSTAR Global Positioning System (GPS). It is also an important element in the geodetic applications of these systems. The NAVSAT system is based on Doppler observations of satellites at 1000 km altitude. While the GPS system is based on simultaneous range observations to four satellites at altitudes of 20,000 km, it is useful to think of the computation of the ephemerides of the satellites as being based upon Doppler data also. The reason for choosing this interpretation stems from the fact that it is desirable to base the computation of the ephemeris on several days of observations in order to minimize the uncertainties in the computed orbit period and solar radiation parameters. Over a five day period, an error of one part in 10^{-13} in oscillator frequency would produce an error in time of 43 ns, or 12 m in range, since the range is based on the measured travel time of signals propagating at the speed of light. As will be shown below, analysis of Doppler data during the five day period would give range to the satellite which is accurate to better than a meter using the same oscillator. It is possible to use the range information directly while still accounting for the oscillator instability either in a sequential processor by introducing process noise or in a batch processor by using a correlated weight matrix. While these alternatives are mathematically more rigorous than the conversion of range data to Doppler data, the techniques fundamentally weaken the accuracy of relative range measurements made at widely spaced times, tending to approach the Doppler interpretation of the data.

TIME TAGGING

Both the NAVSAT and GPS systems require time tagging of observations and of ephemeris data to sufficient accuracy to allow interpolation in the relative positions of the satellite and observer to the desired accuracy. Since the relative accuracy of the satellite and observer is about 5 km/sec, an uncertainty in the time tag of 0.2 ms would produce a relative position error of one meter. In both navigation systems, one or more ground oscillators is adopted as a standard, the satellite oscillator is calibrated against the standard, and other ground clocks are calibrated against the satellite clock. Therefore, the satellite oscillator must be sufficiently stable to maintain the desired accuracy of the clock epochs over the time period of several days used for the clock rate determination and prediction. A 0.2 ms accuracy objective over a five day period requires an oscillator stability of 5 parts in 10^{10}.

RANGE MEASUREMENTS

The most stringent requirement on oscillator performance arises from ground measurements of the time of arrival of signals generated from oscillators in the GPS satellites. The GPS system is based on ranges computed by multiplying the travel time of the signals by the velocity of light. The effect of oscillator instability on the computed ranges was referred to in the first paragraph in connection with the determination of the orbits of the GPS satellites. Inverting the calculation, if satellite and ground timing systems were to be maintained to an accuracy corresponding to a one meter range accuracy over a five day time period, oscillator stabilities of eight parts in 10^{15} would be required. The GPS system is able to meet navigation requirements with satellite oscillators which are an order of magnitude poorer because of looser tolerances on range accuracy and shorter fit and prediction intervals for the time signals. The epoch errors of the ground clock are determined each time a navigation fix is obtained by measuring the apparent travel time of signals from four satellites and solving for the clock correction and the three components of the observer's position. Therefore the only requirement on the oscillator in the receiver is to permit interpolation of signals from the satellite to the same epoch for those receivers which do not make simultaneous observations to the four satellites (Hill, 1978). The range computed from the travel time prior to correction of the observer's clock is referred to as a "pseudo-range."

GEOMETRIC DILUTION OF PRECISION

In considering the requirements for oscillator stability, the measurement errors produced by clock uncertainties must be transformed to errors in the position of the observer. Positions based on the pseudo-ranges to four satellites are about a factor of three worse than the measurement errors for the typical geometric configuration of GPS satellites. The ratio of the position error to the measurement error is referred to as the "Geometric Dilution of Precision (GDOP)" (more precisely in this context, "Position Dilution of Precision (PDOP)" (Milliken and Zoller, 1978)). It is simply the average standard error in position corresponding to unit weight for the observations. The GDOP and the effects of oscillator instability on Doppler positioning cannot be summarized as concisely. Before discussing these topics, the conventional interpretation of Doppler data and common terminology will be reviewed.

SENSITIVITY OF DOPPLER DATA TO CLOCK PERFORMANCE

The observed frequency is given to first order by:

$$f = f_s - \frac{f_s}{c}\dot{r}$$

where f_s is the frequency emitted by the satellite, c is the velocity of light, and $\dot{r}$ is the relative velocity of the satellite with respect to the observer. The received frequency is normally mixed with a reference frequency, f_R, in the receiver:

$$f_B = f_R - f_s + \frac{f_s}{c}\dot{r}$$

and the beat cycles are counted over specified time intervals:

$$N_c = \int_{t_1}^{t_2} f_B \; dt = \int_{t_1}^{t_2} (f_R - f_s + \frac{f_s}{c}\dot{r})\, dt$$

so that

$$N_c = (f_R - f_s)(t_2 - t_1) + \frac{f_s}{c}(r_2 - r_1)$$

Some receivers measure $(t_2 - t_1)$ for fixed N_c, some count N_c for fixed $(t_2 - t_1)$, and some count integer N_c in fixed $(t_2 - t_1)$ and read out the clock at the time corresponding to the integer N_c. Many receivers continue counting as the measurements are made and recorded, so that the measurements at the i^{th} data point can be written as $r_i - r_o$ rather than as $r_i - r_{i-1}$. In such cases, the Doppler measurements during a satellite pass can be represented as ranges subject to an unknown range base, r_o, rather than as uncorrelated range differences. Biased range representation yields a better GDOP than uncorrelated range differences, as will be shown later. The offset frequency $f_R - f_s$ varies among receivers. For the Navy Navigation Satellites the offset ratio $(f_R - f_s)/f$ is 80×10^{-6}; for the NAVSTAR Geodetic Receiver (Anderle, 1978c) the offset $(f_R - f_s)$ is 28.75 KHz. To first order, oscillators make two contributions to the range error:

$$\delta_1(r_2 - r_1) = \frac{c}{f_s}(f_R - f_s)\delta t$$

$$\delta_2(r_2 - r_1) = \frac{c}{f_s}(t_2 - t_1)\delta(f_R - f_s)$$

For the above frequency offsets, the first equation establishes the time interval accuracy required per meter precision in range difference as 40 μs for NAVSAT and 200 μs for GPS. The second contribution to the range difference error imposes more severe requirements on the oscillator. The time interval between the first and last time in a satellite pass is about 1000 s for NAVSAT and 30,000 s for GPS. Therefore, the fractional frequency stability required per meter precision over these intervals is $3x10^{-12}$ for NAVSAT and $1.1x10^{-13}$ for GPS.

INFORMATION CONTENT OF A DOPPLER PASS

Direct conversion of the Doppler errors discussed in the previous paragraph to errors in station position is not useful because Doppler data for a single satellite pass does not provide enough information to permit accurate determination of all three components of station position. Therefore, GDOP is usually calculated for the two position components which are well determined. The effects of errors in these two components on the calculated frequency are illustrated in figure 1. On a non-rotating earth, the Doppler frequency (which is proportional to the range difference) is zero when the satellite reaches its point of closest approach to the observer and has the shape shown by the upper curves in the figure. The offset between the satellite and station frequency standards is easily determined since the Doppler frequency, or calculated range difference per unit time, is equal and opposite in sign at the times of rise and set of the satellite above the station horizon. If the satellite position is known, then an error in the observer's position parallel to the satellite velocity vector at closest approach will produce calculated range differences which are displaced in time as shown by the broken curve in the upper left figure, and bell shaped residuals as shown in the lower left figure. This component of station position determined from a pass of Doppler data is referred to as the "tangential" or "along track" component of position. If the assumed station position is closer to, or further from, the satellite at the time of closest approach, the Doppler curve, or range differences, will have a steeper or shallower slope as shown in the upper right hand part of figure 1. The residuals will be anti-symmetric as shown in the lower right hand part of the figure, and define the location of the station alonw the range vector to the satellite at the time of closest approach (the "range" component of station position). A tropospheric refraction bias will also produce anti-symmetric residuals, but the effect will be greatest at the times of rise and set of the satellite and decrease rapidly at the higher elevation angles. The third component of station position is not defined for an emitter on a linear path and a non-rotating earth, since rotation of the receiver about the emitter path at a fixed distance from the emitter will not change the Doppler curve. While the solution for three components of station position is not singular for the curved satellite path and a rotating earth, the standard error for the third component of station position is orders of magnitude larger than those for the tangential and range component of station position in the plane defined by the range vector to the satellite and the relative velocity vector of the satellite at the time of closest approach, providing no useful information for navigation or geodetic applications. Therefore in order to determine three components of station position, the satellite should be observed on a pass to the left and a pass to the right of the station so that the range components can be used to triangulate station height and the horizontal component normal to the satellite track (longitude for the polar Navy Navigation Satellites). In order to determine a navigator's latitude and longitude from a single pass of Doppler data, the height of the observer must be known; nevertheless the longitude is ill-determined for polar satellite passes crossing the station's zenith. Since the angular velocity of GPS satellites is only twice the rate of earth's rotation while the angular velocity of the NAVSAT satellites is ten times the rate of earth's rotation, it is not clear whether the information content of a GPS Doppler pass is so ideally contained in the range/tangential position components of station position as it is for NAVSAT data. Nevertheless, the same interpretation has been applied to GPS data as a result of the availability of the computer programs and the lack of a better diagnostic tool. Actual orbit determinations and geodetic station position calculations are based on a least squares fit of the parameters of the solution to the aggregate of the Doppler data, not to the position components calculated for diagnostic purposes.

EFFECT OF CLOCK PERFORMANCE ON POSITIONS DETERMINED FROM NAVSAT DOPPLER DATA

It was mentioned earlier that Doppler observations from most receivers can be treated as either range difference data or as range data subject to an unknown bias. Figures 2 and 3 show the uncertainty in the determination of the tangential and range components of the position of the observer, respectively, corresponding to a 10 cm random error in range or range difference data. Figure 2 indicates that the GDOP for the tangential component of position varies from one to four for biased range data and from three to ten for range difference data for elevation angles to the satellite at closest approach from 90 to 20 degrees. Figure 3 reveals that the GDOP for the range component of position varies from one half to two for biased range data and from three to seven for range difference data for these elevation angles. The figures are based on the assumption that the tropospheric refraction is known perfectly and the offset in frequency between the oscillators in the satellite and the receiver is completely unknown but stable during the pass. Uncertainties in tropospheric refraction must actually be considered in precise computations. Introduction of a scale bias for refraction does not affect the standard error in tangential position. The effect on the range component of position depends on the relative magnitudes of the random error of the Doppler observations and the uncertainty in the a-priori refraction data; for typical values of the quantities, the standard error in range component based on range difference data is not significantly affected while that for biased range data is increased markedly percentage-wise, although it always remains smaller in magnitude than that for range difference data. Since the random error of measurement for the better receivers is less than 5 cm, the precision of the Doppler receivers is quite good. However, the effects of the instability of oscillators used in the receivers produces larger errors in position. Specifications of the stability of two oscillators used in NAVSAT Doppler receivers are $1x10^{-11}$ and $6x10^{-12}$ for averaging times of interest (30 to 1000 seconds). Simulations of position accuracies attainable with these oscillators and an oscillator with a stability of $2x10^{-13}$ were conducted by Monte Carlo methods. Doppler observations corresponding to frequency variations expected for each of these oscillators and a random error of 3 cm were synthesized for six passes for each of five pass geometries, and the components of station position were computed for each pass. The rms of the six sample errors for the tangential and range components is plotted in figures 4 and 5, respectively, versus the elevation angle to the satellite at closest approach. Note that the position component errors are about 30 times larger than those due to random error for the specifications of the oscillators used with this equipment regardless of whether the data is represented as biased range data or as range differences. The oscillator stability of $2x10^{-13}$ which has been achieved for rubidium oscillators over these averaging times, yields position errors reasonably close to those expected from the random error of measurement. Irregularities in the curves are probably due to sampling errors in this limited Monte Carlo simulation. The rubidium oscillator is inconveniently large in size for use with the portable Doppler receivers in some applications.

EFFECT OF CLOCK PERFORMANCE ON POSITIONS DETERMINED FROM GPS DOPPLER DATA

Results of computations of GDOP for Doppler observations of the GPS satellites for biased range and range difference data are given in figure 6 for the range component of position. The curves for the tangential component of position are similar. Results for various data sampling strategies are given for the range difference representation of data while the curves for biased range data are proportional to the square root of the sampling interval. Note that the GDOP varies from about one to ten for the different cases for pass lengths greater than 15,000 seconds. Shorter pass lengths probably need not be considered due to the spacing of the satellites in the GPS constellation. Since most GPS receivers are designed to achieve 1 cm precision in Doppler data, the curves imply a high precision in position. However, a very high oscillator stability would be required to achieve these precisions. Simulations similar to those conducted for NAVSAT conditions were also conducted for GPS conditions to determine the effect of oscillator stability on the accuracy of station positions. Data were simulated for the oscillator stability corresponding to the curve labeled "Test A" on figure 7. This curve is close to that for a cesium oscillator, just a little poorer than that measured by the Naval Observatory for the cesium oscillator used in the NAVSTAR Geodetic Receiver. The rms of each position component error obtained from the simulated data is given in figure 8. Only pass lengths longer than 15,000 seconds were considered. These errors are five to fifty times worse than those expected from the random error of observation. Attempts to account for frequency variations by introducing a frequency drift parameter produced still larger errors in computed station position. However, this figure illustrates the point made in the first paragraph that the Doppler technique can be used to determine the range to the satellite to better than a meter accuracy for satellite passes separated by any time interval.

RELATIVE STATION POSITIONING

Even considering the effects of oscillator instability, the errors in computed station positions discussed in the previous sections are smaller than the errors in computed satellite positions except for low elevation angle passes. However, the higher receiver accuracy is desirable for geodetic applications since the accuracy of the computation of the relative position of stations observing the satellite simultaneously is not significantly affected by errors in the satellite position if the distance between the stations is small compared to the height of the satellite (Anderle, 1978a). Similarly, errors due to the satellite oscillator can be expected to be cancelled under these circumstances. The potential for the determination of the relative positions of stations to centimeter accuracy has attracted the attention of geophysicists studying crustal motion. Since the determination of relative station position also negates the requirement for accurate times of emission of the ranging signals from the GPS satellites, near-simultaneous pseudo-range measurements from two stations to four satellites can be used to make an interferometric solution for the relative position of the stations (Anderle, 1978b, MacDoran, 1978). However, a high gain antenna or a high redundancy of observations is required to reduce the random range error which is about a meter for a wide beam antenna. In this application, oscillator requirements are modest since accurate time intervals are only required to interpolate non-synchronous but high data rate data.

SUMMARY

The high performance of available oscillators has permitted the development of invaluable navigation and geodetic satellite systems. However, still higher performance oscillators would improve the accuracy of flexibility of the systems. Oscillator requirements per meter position error are listed in figure 9 for the various aspects of navigation systems discussed in this report. A GPS oscillator stability of 10^{-15} over five days would simplify the orbit determination and prediction functions. Highly portable low cost oscillators with a stability of 10^{-14} for averaging times of eight hours would permit monitoring of crustal motion daily with GPS Doppler receivers. Oscillators the same size and reasonably close to the cost of current quartz oscillators but with a stability closer to 10^{-13} at an averaging time of 1000 seconds would allow more rapid determination of relative station postions from NAVSAT data and more accurate orbit determination. Clearly clock performance is a critical parameter in navigation satellite systems.

ACKNOWLEDGEMENTS

Simulations of the effects of oscillator stability on the position accuracy obtainable from Doppler observations of the NAVSAT and GPS satellite systems were conducted by Ronald Smith of the Naval Surface Weapons Center.

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INFORMATION CONTENT OF DOPPLER PASS

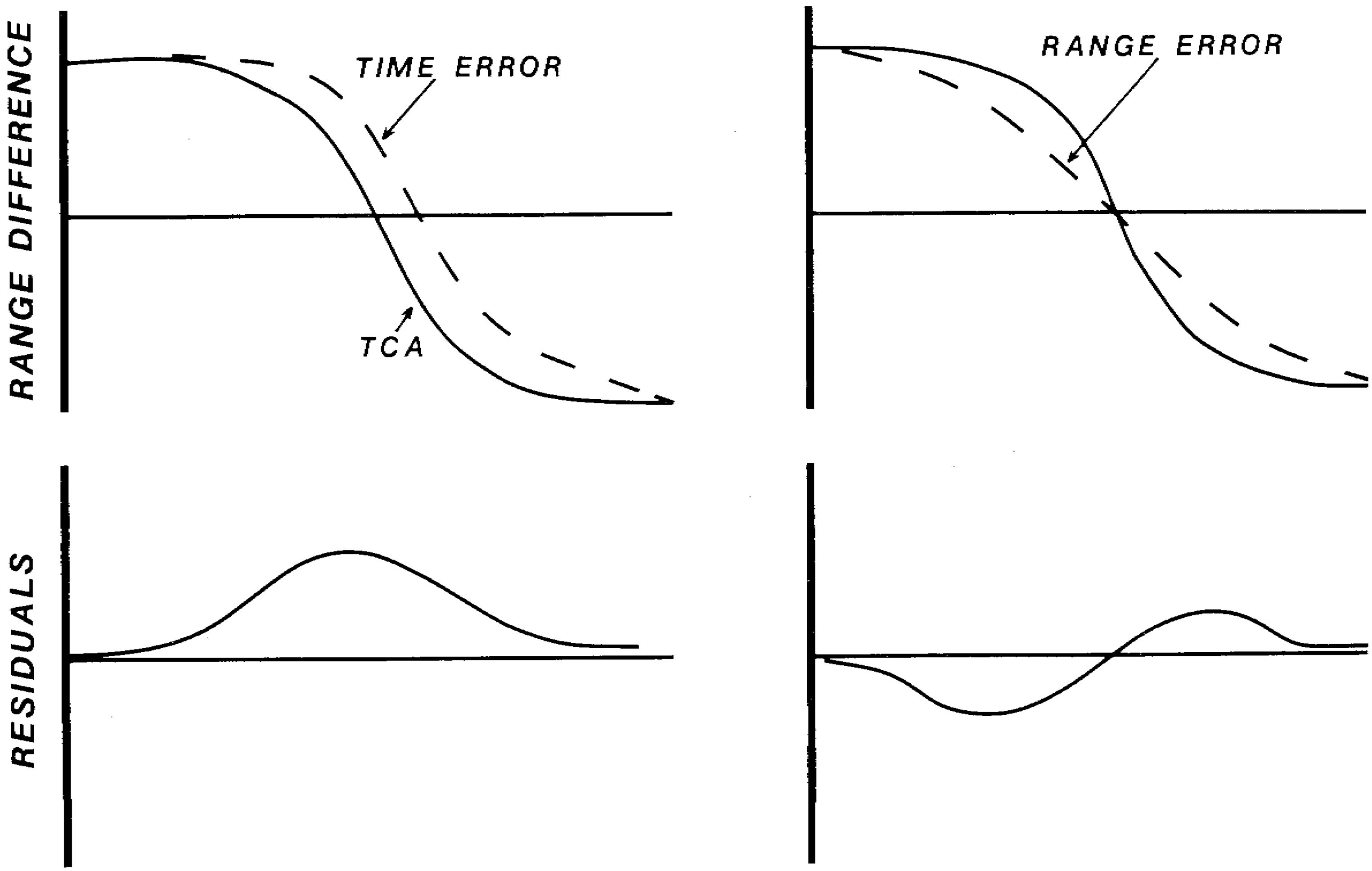

FIGURE 1

STANDARD ERROR IN TANGENTIAL COMPONENT OF POSITION FOR 10 CM RANDOM ERROR OF OBSERVATION NAVSAT PASSES

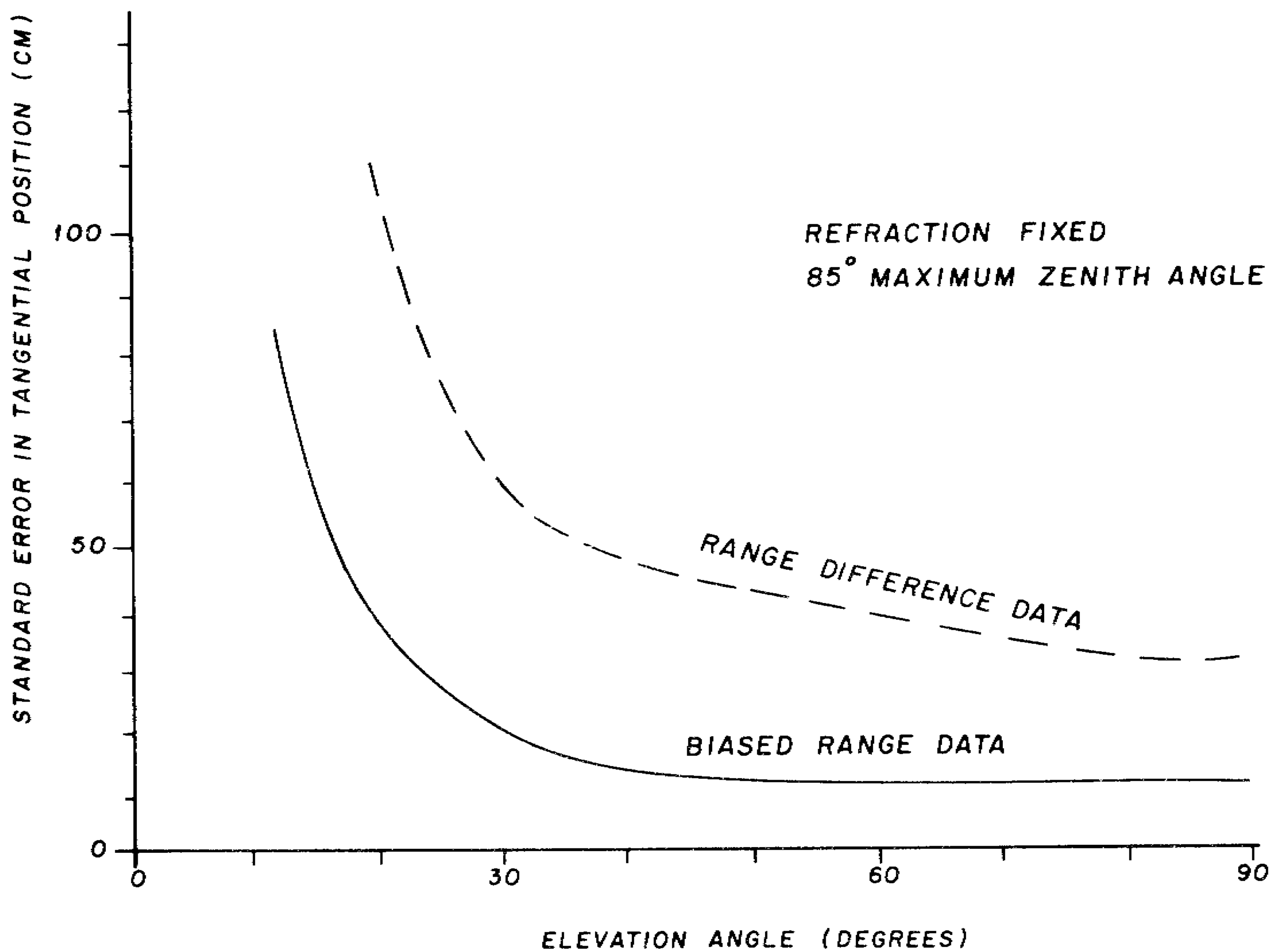

FIGURE 2

STANDARD ERROR IN RANGE COMPONENT OF POSITION
FOR 10 CM RANDOM ERROR OF OBSERVATION NAVSAT PASSES

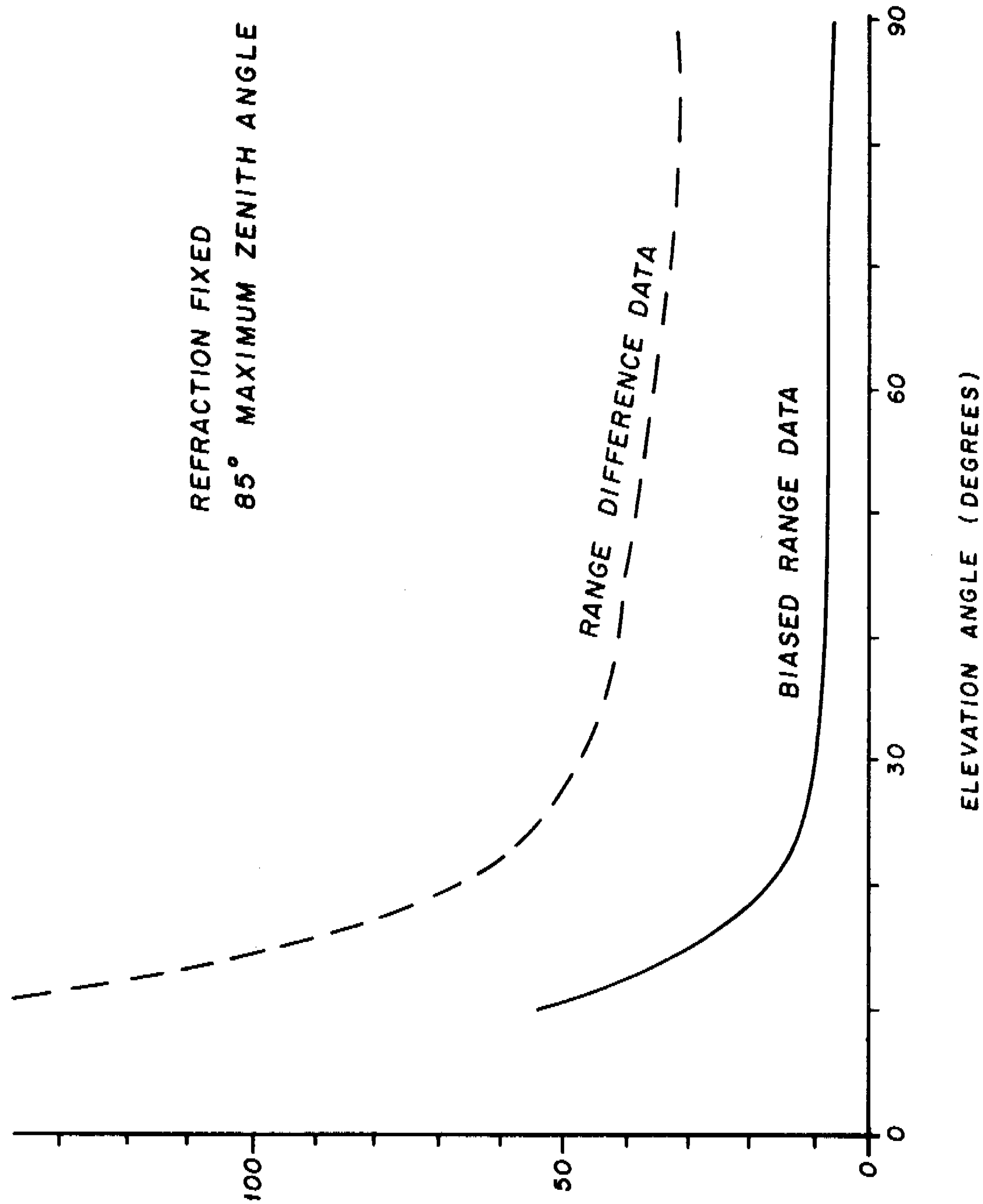

FIGURE 3

EFFECT OF OSCILLATOR ERROR ON TANGENTIAL POSITION COMPONENT

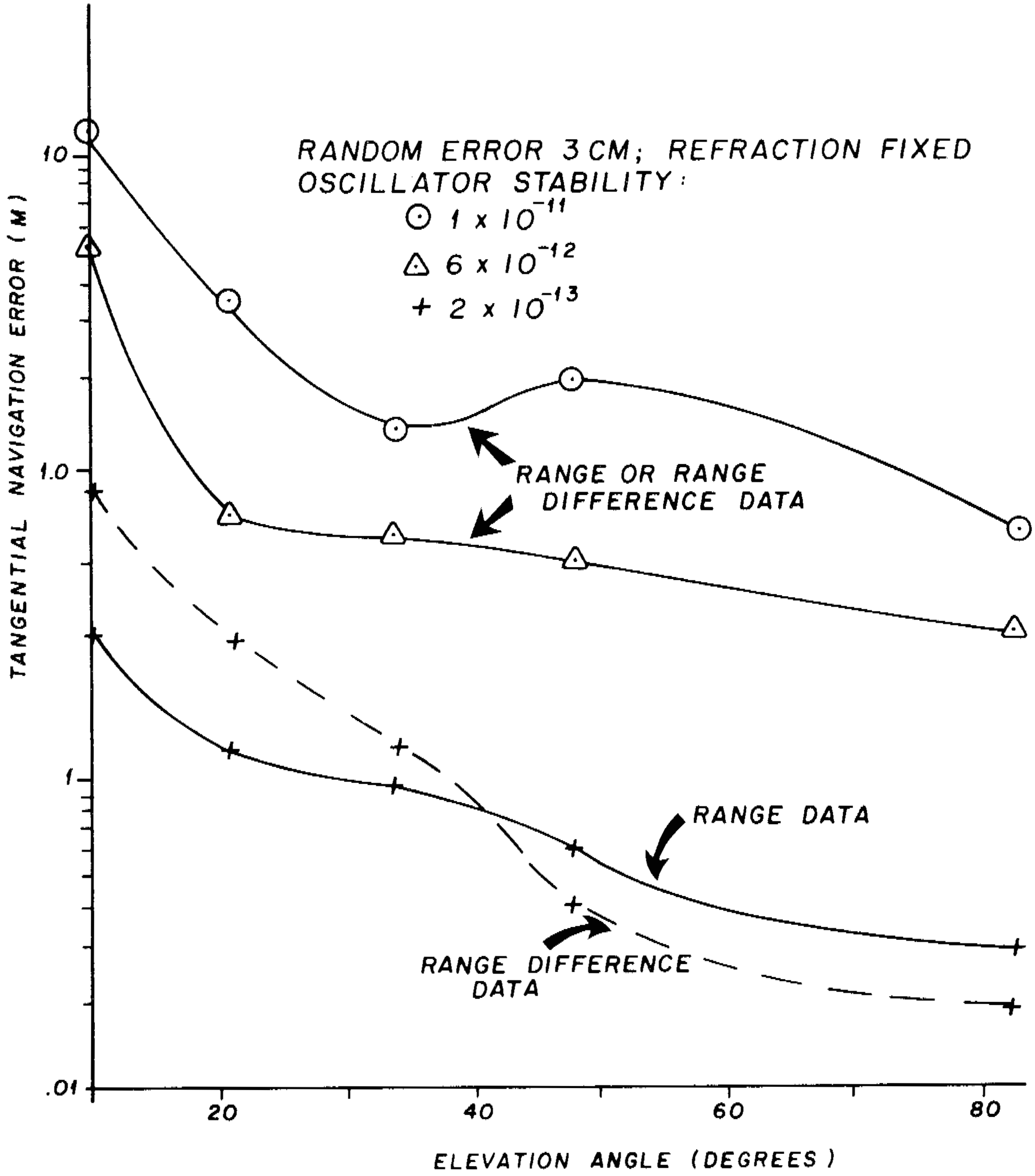

FIGURE 4

EFFECT OF OSCILLATOR ERROR ON RANGE POSITION COMPONENT

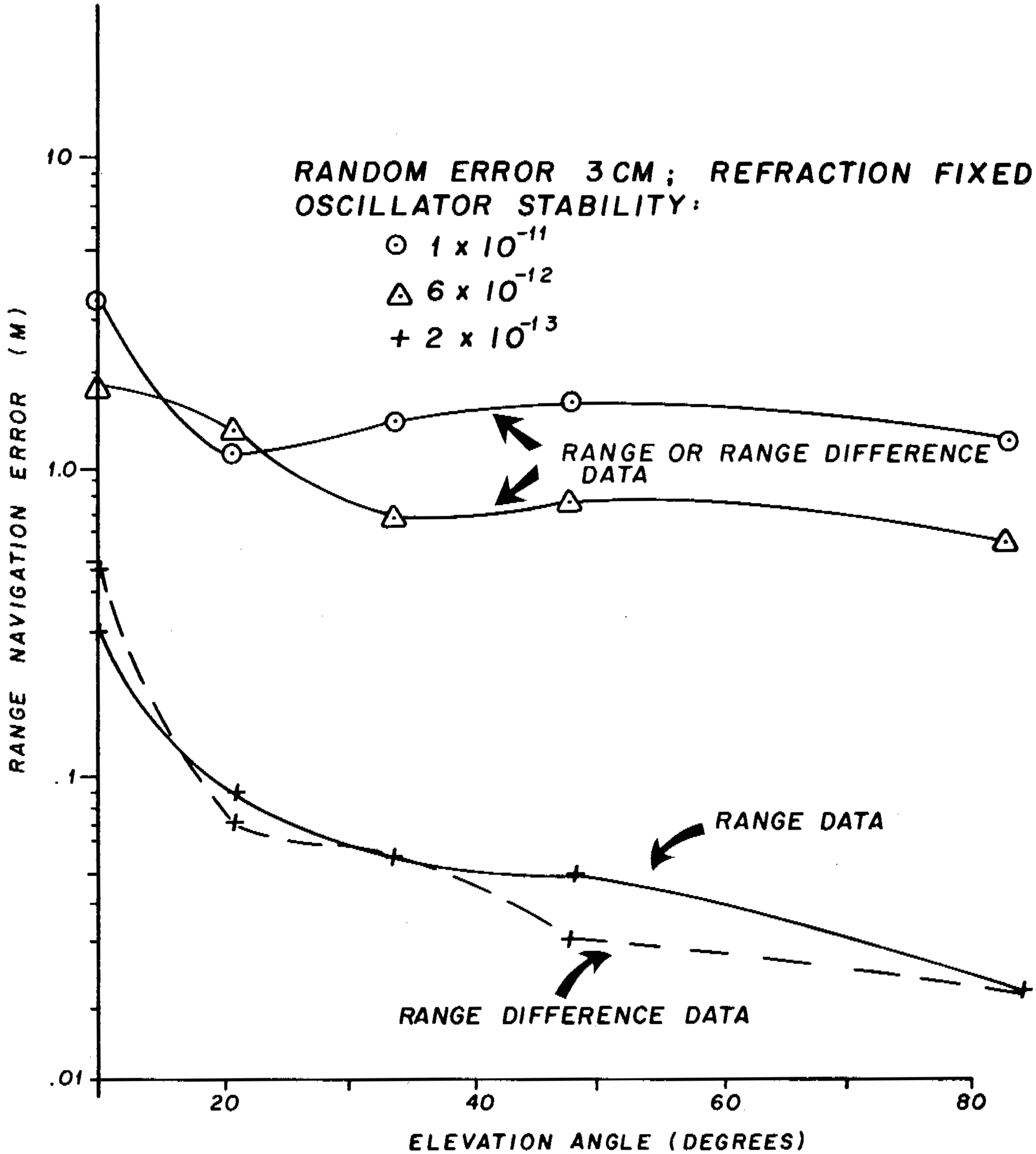

FIGURE 5

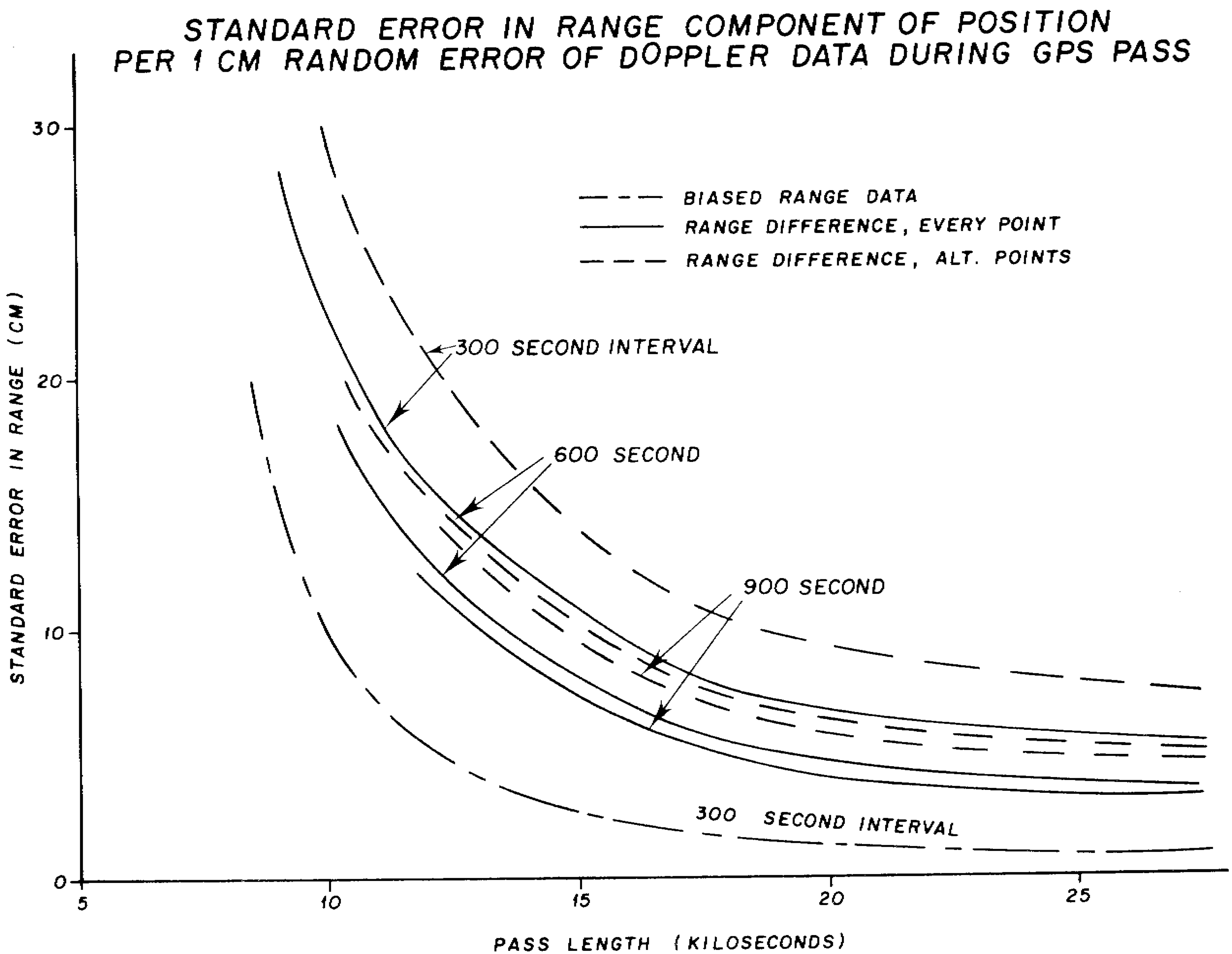
STANDARD ERROR IN RANGE COMPONENT OF POSITION
PER 1 CM RANDOM ERROR OF DOPPLER DATA DURING GPS PASS
BIASED RANGE DATA
RANGE DIFFERENCE, EVERY POINT
RANGE DIFFERENCE, ALT. POINTS
300 SECOND INTERVAL
600 SECOND
900 SECOND
300 SECOND INTERVAL
STANDARD ERROR IN RANGE (CM)
PASS LENGTH (KILOSECONDS)
0
10
20
30
5
10
15
20
25

FIGURE 6

ALLAN VARIANCE

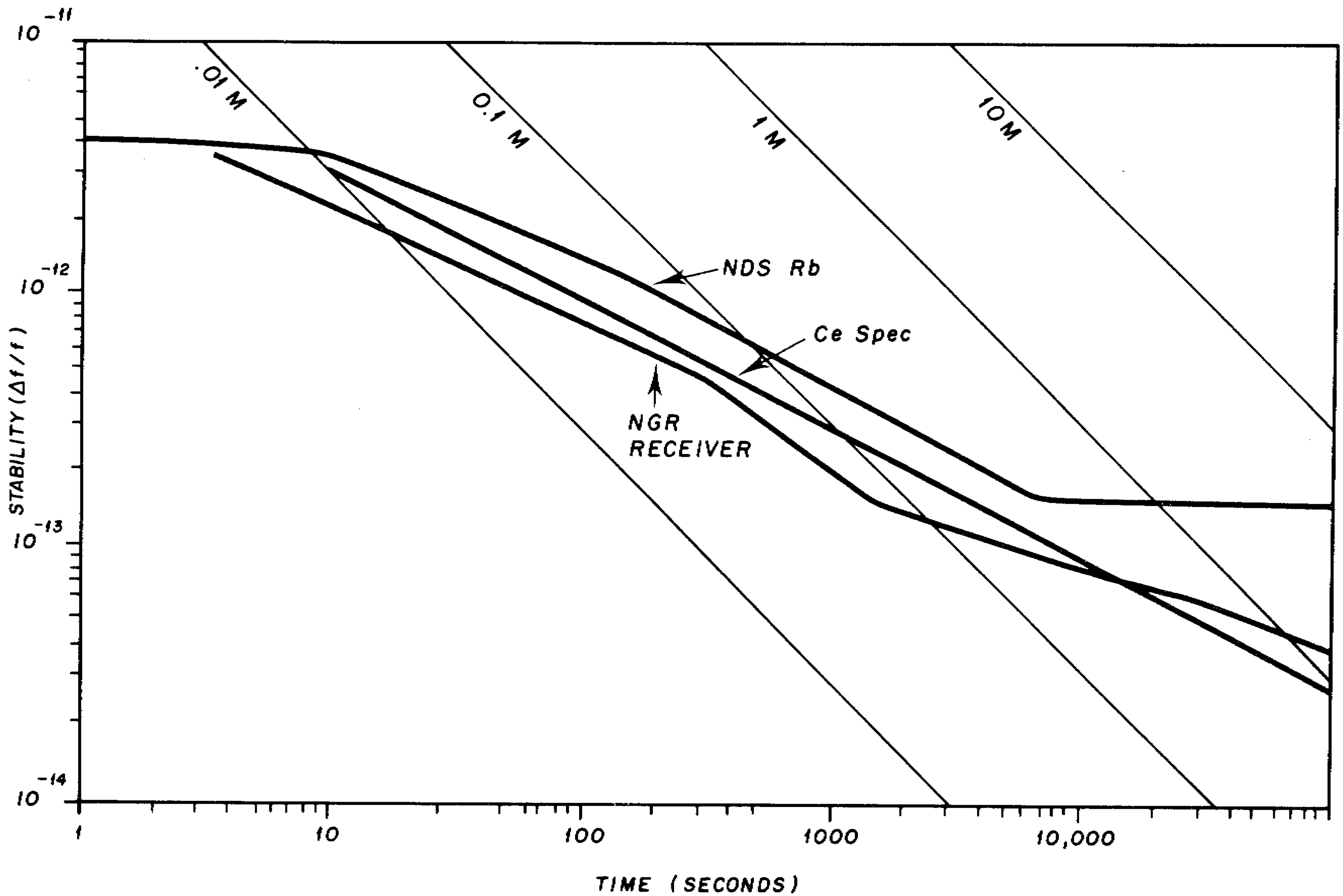

FIGURE 7

RMS POSITION COMPONENT ERRORS (RADIAL/TANGENTIAL) IN CM
DUE TO OSCILLATOR INSTABILITY
(PASSES LONGER THAN 15,000 SECONDS)

	DATA INTERVAL (SECONDS)		
DATA TYPE	300	600	900
RANGE	45/42		44/35
RANGE DIFFERENCE (CONTINUOUS)	36/38	30/32	29/43
RANGE DIFFERENCE (ALTERNATE POINTS)	41/56	48/62	49/73

FIGURE 8

ROLE OF CLOCK PERFORMANCE
IN NAVIGATION SATELLITE SYSTEMS

TIME TAGGING	POSITION ERROR*	CRITICAL T	OSCILLATOR REQUIREMENT PER M POSITION ERROR	
			EPOCH	FREQUENCY
EPHEMERIS DETERMINATION	$V_s \delta\tau$, $V_s \int_o^T \frac{\delta f}{f} dt$	6 hours	100 μs	10^{-9}
SATELLITE - NAVIGATOR		24 hours	100 μs	$2.5\text{x}10^{-9}$
RANGE POSITIONING	$3c\delta\tau$, $3c \int_o^T \frac{\delta f}{f} dt$	24 hours	1 ns	$1.3\text{x}10^{-9}$
DOPPLER POSITIONING				
NAVSAT	$2V_3 \delta\tau$, $2c \int_o^T \frac{\delta f}{f} dt$	.3 hours	50 μs	$1.5\text{x}10^{-12}$
GPS		8 hours	50 μs	$5\text{x}10^{-14}$

*The coefficients "2" or "3" represent typical ratios of position error to measurement error.

FIGURE 9

QUESTIONS AND ANSWERS

DR. VICTOR REINHARDT, NASA Goddard Space Flight Center:

Can you define the term, g-dop?

DR. ANDERLE:

Well, loosely speaking, it is the position accuracy per unit measurement accuracy. The detailed definitions of the GPS are given in the last issue of "Navigation". A number of conditions are involved: horizontal, g-dop, position-dop, a number of those terms, but fundamentally, it is, loosely speaking, position accuracy per unit measurement accuracy.

DR. IVAN NURUR, Ohio State:

In these biased ranges, did you assume that these ranges are independent from each other on a given pass, or did you consider correlations between them?

DR. ANDERLE:

Each measurement I assume is essentially independent. The only common bias is the range bias for the pass, but each biased range is independent of the preceding one.

MR. MIKE MCCONAHY, Johns Hopkins University, Applied Physics Lab:

Would you like to comment, Dick, on what you think the potential of GPS is for geophysical studies, in view of what you now know?

DR. ANDERLE:

I have addressed that in a number of papers and there are a number of ways, I think, of achieving centimeter accuracies in fairly short time spans with better oscillators. This doppler receiver would do it, and with the existing receivers, depending upon how biases work out and depending on averaging times, it is theoretically possible to get a centimeter that way also. There are a number of other proposals that have been made for using GPS in a VLBI mode as another technique. So, there are four or five different approaches to using GPS for geophysical applications. There is a question of what the equipment cost; you know, which one would have the least cost, the fastest operation, how the various system classes would work out in each respective application. I don't have any doubt that one of them will work for centimeter accuracy at some acceptable cost.

Speaker (unheard)

DR. ANDERLE:

I am sorry. When I talk about those accuracies, I am talking about relative positioning; I am not talking about absolute positioning. But, there are two stations equipped with these things, in getting relative positions.

DR. WILLIAM MURPHY, Rockwell:

You might have made this point clear but it wasn't clear to me. When you were speaking about clock performance, were you talking about stability or accuracy?

DR. ANDERLE:

In terms of absolute time tags. I never talked in terms of absolute time tags, absolute epochs because, as I say, we adopt some ground station as a standard and time tags are with respect to that. Is that the kind of question you were asking, or were you asking a deeper question?

DR. MURPHY:

6 parts in 10^{12}, for instance, on this particular oscillator. I was wondering if that was a stability figure or an accuracy figure?

DR. ANDERLE:

It is a figure corresponding to the Allan variance.

DR. MURPHY:

Right.