THE MECHANICS OF TRANSLATION OF FREQUENCY STABILITY MEASURES BETWEEN FREQUENCY AND TIME DOMAIN MEASUREMENTS

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ABSTRACT

The mechanics of translation of the frequency stability measures from the frequency domain to the time domain and vice versa will be presented and discussed. The discussion will be based on the model proposed by Barnes, et al, of the Subcommittee on Frequency Stability of the Technical Committee on Frequency and Time of the IEEE's Group on Instrumentation and Measurement.

Examples based on published frequency stability data of various oscillators will be presented and examined.

1.0 INTRODUCTION

Theoretical models on frequency stability of oscillators have been developed and tested by various latoratories. Excellent reviews from different approaches and view points on the subject matter have also been published in the last two years. In addition, extensive references are given in these review papers.

This paper deals with the mechanics of translation of frequency stability measures from frequency domain to time domain and vice versa. The technique is to use discrete data points to approximate the characteristics of each noise source in the domain to which the data are translated. The difficulties encountered with this approach are often due to lack of complete data. In particular for flicker phase data hard decisions have to be made. The merit in using this technique is in the simplicity of translation and as such, it provides a quick handle for the users to examine the performance characteristics of oscillators of interest in terms of the requirements. If, however, accurate data on the performance characteristics of oscillators are needed, the simplest approach is still the direct approach by measuring the frequency stability in the domain of interest.

2.0 DISCUSSION OF THEORY

A theoretical model which is in wide use to describe the characteristics of frequency stability is the one recommended by the Subcommittee on Frequency Stability of the IEEE. This model is based on the assumptions that; (a) the instantaneous frequency fluctuations of the output signal of an oscillator is due to independent noise sources in the oscillator system; (b) as many as five noise sources may be needed to characterize the frequency instability observed in most oscillators; and (c) each independent noise source can be modeled by a power law. Since each noise source is independent of the other they may be combined by superposition.

2.1 FREQUENCY DOMAIN

In frequency domain measurement, the spectral density of a noise source in an oscillator that perturbs the frequency stability of the oscillator system can be considered as the average contribution of all individual perturbations of noise components each having a frequency, f, ranging from minus infinity to plus infinity. Thus the total noise power is the sum of the contributions of all of the noise components. In practical terms this frequency range is folded and covers only the range of frequencies from zero to positive infinity. The spectral density of a signal of only positive frequency components is called the one sided spectral density of frequency and is represented by $S_y(f)$ where f is the variable frequency component, also called Fourier frequency, and the subscript, y, is the fractional frequency change from the nominal frequency, ν_0 of the oscillator. Thus the power law noise model may be written as

$$S_y(f) = h_\alpha f^\alpha \tag{1}$$

where h_α is a constant and is characteristic of an oscillator

α is the characteristic value of a noise source and takes the values of -2, -1, 0, 1 and 2.

Cutler and Searle have shown that

$$S_{\dot{\phi}}(f) = 4\pi^2 \nu_0^2 S_y(f) \tag{2}$$

$$S_\phi(f) = (\nu_0^2/f^2) S_y(f) \tag{3}$$

$$S_x(f) = (1/4\pi^2 f^2) S_y(f) \tag{4}$$

where the subscripts denotes phase ϕ, $\dot{\phi} = d\phi/dt$ and $x = \phi/2\pi\nu_o$. X is phase measured in units of time and is often written as phase-time. Substituting equation (1) into equations (2), (3) and (4) one obtains the power law noise model for $\dot{\phi}$, ϕ and X.

$$S_{\dot{\phi}}(f) = 4\pi^2\nu_o^2 h_\alpha f^\alpha \equiv h_{\dot{\phi}} f^\alpha \tag{5}$$

$$S_\phi(f) = \nu_o^2 h_\alpha f^{\alpha-2} \equiv h_\phi f^\beta \tag{6}$$

$$S_x(f) = \frac{1}{4\pi^2} h_\alpha f^{\alpha-2} \equiv h_x f^\beta \tag{7}$$

where

$$h_{\dot{\phi}} = 4\pi^2\nu_o^2 h_\alpha \tag{8}$$

$$h_\phi = \nu_o^2 h_\alpha \tag{9}$$

$$h_x = \frac{1}{4\pi^2} h_\alpha \tag{10}$$

$$\beta = \alpha - 2 \tag{11}$$

Since the power law model is based on five independent noise sources and if they are all present in the oscillator, the one-sided spectral density becomes:

$$S_y(f) = h_{-2} f^{-2} + h_{-1} f^{-1} + h_0 f^0 + h_1 f^1 + h_2 f^2 \tag{12}$$

2.2 TIME DOMAIN

In the time domain measurement, frequency perturbations of a signal due to all frequency components of a noise source are measured collectively as a function of time and are averaged over a sampling time period. Thus the frequency stability characteristics are represented by variance or standard deviation of the measured frequency fluctutations based on statistical calculations. The expected value of the variance of fractional frequency fluctuations, y, from the nominal frequency of an oscillator is:

$$\langle \sigma_y^2(N,T,\tau) \rangle = \langle \frac{1}{N-1} \sum_{n=1}^{N} (\bar{y}_n - \frac{1}{N} \sum_{k=1}^{N} \bar{y}_k)^2 \rangle \tag{13}$$

where N is the number of data points

T is the time interval between the beginnings of adjacent measurements

τ is the sampling time within T

$\bar{y}$ is the average fractional frequency fluctuations over a sampling time

< > is a symbol denoting an infinite average

In statistical analysis, N is desired to be large approaching infinity and the noise source is a random process. In real life N is finite and noise source is not always random. In addition, it may not even be stationary, thus corrections and approximations are needed in statistical analysis in order to match or to check the theory.

In the simplest case N = 2 and T = τ, Allan has shown that

$$<\sigma_y^2(2,\tau,\tau)> \equiv \sigma_y^2(\tau) = <\tfrac{1}{2}(\bar{y}_{k+1} - \bar{y}_k)^2> \tag{14}$$

and Allan and Vessot have shown that

$$<\sigma_y^2(\tau)> \sim |\tau|^{\mu} \tag{15}$$

This variance and its square root, the standard deviation, have been used to describe the frequency stability characteristics of oscillators in the time domain. The variance is known as the Allan variance.

2.3 CLOCK ERROR

Barnes has shown that to predict a clock reading error, X(t), one may define a time stability by a time variance:

$$\sigma_x^2(\tau) = \tau^2 \sigma_y^2(\tau) \tag{16}$$

3.0 TRANSLATION

The translation relationship between the frequency and time domain measures based on equations (12) and (13) was derived by Cutler and is given by

$$<\sigma_y^2(N,T,\tau)> = k_\alpha C_\alpha \tau^{\mu} \tag{17}$$

where $\mu = -\alpha - 1$

$-3 < \alpha < 1$

and C_α is a transfer function given by

$$C_\alpha = \frac{N}{(N-1)\pi^{\alpha+1}} \int_0^\infty du\, u^\alpha \frac{\sin^2 u}{u^2}\left[1 - \frac{\sin^2 Nru}{N^2 \sin^2 ru}\right] \tag{18}$$

in equation (18) $u = \pi f \tau$

$r = T/\tau \quad (r = 1 \text{ if } T = \tau)$

In the case N = 2, r = 1

Allan variance becomes:

$$\langle \sigma_y^2(\tau) \rangle = h_{-2}\left(\frac{4\pi^2}{6}\right)\tau + h_{-1}(2\ln 2) + h_0\left(\frac{1}{2}\right)\frac{1}{\tau} + h_1\left[\frac{1.038 + 3\ln(2\pi f_h \tau)}{4\pi^2}\right]\frac{1}{\tau^2} + h_2\left(\frac{3f_h}{4\pi^2}\right)\frac{1}{\tau^2} \qquad (19)$$

Where f_h is the high frequency cutoff of an idealized infinitely sharp cutoff low pass filter.

Equation (19) may be written also in the form of

$$\langle \sigma_y^2(\tau) \rangle = \sum_{\alpha=-2}^{2} h_\alpha C_\alpha \tau^\mu \qquad (20)$$

so that

$$C_{-2} = \frac{4\pi^2}{6} \qquad (21)$$

$$C_{-1} = 2\ln 2 \qquad (22)$$

$$C_0 = \frac{1}{2} \qquad (23)$$

$$C_1 = \frac{1.038 + 3\ln(2\pi f_h \tau)}{4\pi^2} \qquad (24)$$

$$C_2 = \frac{3f_h}{4\pi^2} \qquad (25)$$

Thus the six basic power law equations from Equations (5), (6), (7), (12), (16) and (20) may be collected together for convenience given below:

$$\left.\begin{aligned} S_y(f) &= h_{-2}f^{-2} + h_{-1}f^{-1} + h_0 + h_1 f + h_2 f^2 \equiv \sum h_\alpha f^\alpha \\ S_{\dot\phi}(f) &= 4\pi^2\nu_0^2 \sum h_\alpha f^\alpha \\ S_\phi(f) &= \nu_0^2(h_{-2}f^{-4} + h_{-1}f^{-3} + h_0 f^{-2} + h_1 f^{-1} + h_2) \equiv \nu_0^2 \sum h_\alpha f^\beta \\ S_x(f) &= \frac{1}{4\pi^2}\sum h_\alpha f^\beta \end{aligned}\right\} \qquad (26A)$$

$$\left.\begin{aligned} \sigma_y^2(\tau) &= h_{-2}C_{-2}\tau + h_{-1}C_{-1} + h_0 C_0 \tau^{-1} + h_1 C_1 \tau^{-2} + h_2 C_2 \tau^{-2} \\ \sigma_x^2(\tau) &= h_{-2}C_{-2}\tau^3 + h_{-1}C_{-1}\tau^2 + h_0 C_0 \tau + h_1 C_1 + h_2 C_2 \end{aligned}\right\} \qquad (26B)$$

Comparing term by term for each noise source between equations (26A) and 26B) one can derive the translation relationship. For example the first terms for the random walk of frequency noise are given by:

$$S_y(f) = h_{-2} f^{-2}$$

and

$$\sigma_y^2(\tau) = h_{-2}\left(\frac{4\pi^2}{6}\right)\tau$$

thus

$$S_y(f) = \frac{6}{4\pi^2}\left(\frac{1}{\tau}\sigma_y^2(\tau)\right)\frac{1}{f^2}$$

Figure 1 shows the log-log plots of frequency stability characteristics of oscillators in frequency and time domains based on the power law model of five noise sources. The equations to translate the frequency stability measured in the frequency domain to the time domain and vice versa are given in Table 1. The terms in the bracket are the measured quantitites. The subscript y for $S_y(f)$, $\sigma_y^2(\tau)$ and $\sigma_y(\tau)$ is often deleted in most texts without confusion if a subscript is always used for $\dot{\phi}$, ϕ and X. $\dot{\phi}$ is not listed in Table 1 because it is not used often; however, a general formula can be easily derived as shown in Table 1.

Since the theory in the frequency domain is modeled by a power law it is reasonable to assume the flicker phase noise is independent of the sampling time in the frequency domain. This assumption gives a starting point for the translation. If the flicker phase noise is translated from the frequency domain into the time domain the flicker phase noise becomes dependent on the sampling time. Flicker phase noise is generally lower in level than the white phase noise and becomes indistinguishable from the white phase noise in the time domain unless it is specially tested. The time dependence of the flicker phase noise in the time domain is therefore a major problem for translation of frequency stability from time domain to frequency domain.

4.0 EXAMPLES OF TRANSLATION FROM FREQUENCY DOMAIN TO TIME DOMAIN

4.1 DATA

Brandenberger et al published in 1971 an excellent set of data of frequency stability measurement of a very high quality 5 MHz quartz crystal controlled oscillator. The spectral density of phase per oscillator measured in the frequency domain is shown in Figure 2 and is given for the frequency range between 0.1 and 10^4 Hz by:

$$S_\phi(f) = 1.58\times10^{-12} f^{-3} + 3.16\times10^{-13} f^{-1} + 3.98\times10^{-15} f^{0} \tag{27}$$

The time domain measurements of the same oscillator are shown in Figure 3.

Using the coefficients of equation (27) one can plot the flicker frequency noise, flicker phase noise and white phase noise in Figure (2) by straight lines correspondingly proportional to f^{-3}, f^{-1} and f^{0} respectively. These data points are given in Table 2.

4.2 FLICKER FREQUENCY

The calculations for the translation of the flicker frequency, for an example, from frequency domain to time domain is given below:

$$\sigma^2(\tau) = \frac{C_4}{\nu_0^2}\left(f^3 S_\phi(f)\right)\tau^0$$

$$= \frac{2\ln 2}{25\times10^{12}}\left(1^3\times1.58\times10^{-12}\right)$$

$$= 8.76\times10^{-26}$$

$$\sigma(\tau) = 2.96\times10^{-13}$$

If the calculated Allan variance is for one oscillator, the data plotted in Figure 3 are for both the test and reference oscillator. If this is true, a factor of 2 should be multiplied.
Thus

$$2\,\sigma^2(\tau) = 1.75\times10^{-25}$$

and $$\sqrt{2}\,\sigma(\tau) = 4.18\times10^{-13}$$

4.3 FLICKER PHASE

Using Table 1 one obtains the translation equation to be

$$\sigma^2(\tau) = \frac{C_1}{\nu_o^2} \left(f\, S_\phi(f) \right] \tau^{-2}$$

$$= \frac{1}{4\pi^2\nu_o^2}\left[1.038 + 3\ln(2\pi f_h\tau)\right]\left(f\, S_\phi(f)\right)\tau^{-2}$$

for $f = 1\ Hz,\ S_\phi(1) = 3.16\times 10^{-13},\ \nu_o = 5\ MHz,$ and $f_h = 1\ kHz$

$$\sigma^2(\tau) = \left(6.5516 + 3\ln f_h + 3\ln\tau\right]\left(\frac{S_\phi(1)}{4\pi^2\nu_o^2}\right)\tau^{-2}$$

$$= \left[8.7327\times 10^{-27} + 9.6052\times 10^{-28}\ln\tau\right]\tau^{-2}$$

The calculated $\sigma^2(\tau)$ and $\sigma(\tau)$ for flicker phase noise for three values of τ are given in Table 3.

4.4 WHITE PHASE

Again using Table 1 one obtains the translation equation to be

$$\sigma^2(\tau) = \frac{C_2}{\nu_o^2}\left(f^0 S_\phi(f)\right]\tau^{-2}$$

$$= \frac{3f_h}{4\pi^2\nu_o^2}\left(f^0 S_\phi(f)\right]\tau^{-2}$$

for $S_\phi(f) = 3.98\times 10^{-15},\ \nu_o = 5\ MHz,$ and $f_h = 1\ kHz$

$$\sigma^2(\tau) = \frac{3\times 10^3}{4\pi^2\nu_o^2}\left(3.98\times 10^{-15}\right)\tau^{-2}$$

$$= 1.2098\times 10^{-26}\,\tau^{-2}$$

$$\sigma(\tau) = 1.1\times 10^{-13}\,\tau^{-1}$$

The calculated values of $\sigma^2(\tau)$ and $\sigma(\tau)$ for white phase noise are also given in Table 3.

4.4 COMPARISON OF RESULTS

For comparison purposes a summary of the time domain data for the crystal oscillator translated from the frequency domain is shown in Table 4 together with the measured time domain data taken from Figure 3. One can see the agreement is generally good except for the flicker frequency noise. The difference is 2.2 db which is within the measured accuracy. This difference could be explained if the data either in Figure 2 or in Figure 3 is plotted for both the test and reference oscillators instead of for only one oscillator.

One can see the dependence of the flicker phase noise on the sampling time. The error it can introduce in translation by a straight line approximation from the time domain to the frequency domain should be obvious. In the first place the "-1" slope data of the sigma-tau plot in the time domain is the sum of the white phase noise and flicker phase noise power. In the second the level of the flicker phase noise is lower than the level of the white phase noise and is also lower for shorter sampling time.

5.0 EXAMPLES OF TRANSLATION FROM TIME DOMAIN TO FREQUENCY DOMAIN.

Figures 4 and 5 show the frequency stability of a high performance cesium beam tube oscillator measured in time domain and frequency domain respectively. The straight lines are the translated or predicted values using the power law model for N = 2 and $T = \tau$. These data are based from publications by Babitch and Oliverio and private communications from M. Fischer.

Using the measured data points in Figure 4 one can translate the identified noise sources from the time domain to the frequency domain by using the relationship given in Table 1. The results are given in Table 5. The calculated spectral density of phase for the flicker phase noise from the time domain data are based on f_h = 50 KHz and ν_o = 5 MHz.

Sample calculation of translating the flicker phase noise from time domain to frequency domain are given below:

$$S_\phi(f) = \frac{\nu_0^2}{C_1}\left[\tau^2\sigma^2(\tau)\right]f^{-1}$$

$$C_1 = \frac{1}{4\pi^2}\left[1.038 + 3\ln(2\pi f_h \tau)\right]$$

$$= \frac{1}{4\pi^2}(39.011 + 3\ln\tau)$$

Thus
$$S_\phi(f) = \frac{4\pi^2\nu_0^2}{39.011+3\ln\tau}\left[\tau^2\sigma^2(\tau)\right]f^{-1}$$

For $\tau = 10^{-1}$ sec.

$$S_\phi(f) = 3.0743\times10^{11}\left[\sigma^2(\tau)\right]f^{-1}$$

For $\tau = 10^{-2}$ sec.

$$S_\phi(f) = 3.9172\times10^{9}\left[\sigma^2(\tau)\right]f^{-1}$$

a. Use measured data points in Figure 4 for $\tau = 10^{-1}$ and $\tau = 10^{-2}$ seconds respectively

$$\sigma(10^{-1}) = 4.0\times10^{-12}$$

$$\sigma(10^{-2}) = 2.5\times10^{-11}$$

Thus for $\tau = 10^{-1}$ sec., $\sigma(10^{-1}) = 4.0\times10^{-12}$

$$S_\phi(f) = 4.919\times10^{-12}f^{-1}$$

$$10\log S_\phi(10) = -123.1 \text{ db}$$

$$10\log S_\phi(10^2) = -133.1 \text{ db}$$

For $\quad \tau = 10^{-2}$ sec., $\sigma(10^{-2}) = 2.5 \times 10^{-11}$

$$S_\phi(f) = 2.448 \times 10^{-12} f^{-1}$$

$$10 \log S_\phi(10) = -126.1 \text{ db}$$

$$10 \log S_\phi(10^2) = -136.1 \text{ db}$$

b. Use the translated data (straight line) in Figure 4 also for $\tau = 10^{-1}$ and 10^{-2} seconds.

$$\sigma(10^{-1}) = 5.6 \times 10^{-12}$$

$$\sigma(10^{-2}) = 5.0 \times 10^{-11}$$

Thus for $\quad \tau = 10^{-1}$ sec., $\sigma(10^{-1}) = 5.6 \times 10^{-12}$

$$S_\phi(f) = 9.641 \times 10^{-12} f^{-1}$$

$$10 \log S_\phi(10) = -120.2 \text{ db}$$

$$10 \log S_\phi(10^2) = -130.2 \text{ db}$$

For $\quad \tau = 10^{-2}$ Sec., $\sigma(10^{-2}) = 5.0 \times 10^{-11}$

$$S_\phi(f) = 9.793 \times 10^{-12} f^{-1}$$

$$10 \log S_\phi(10) = -120.1 \text{ db}$$

$$10 \log S_\phi(10^2) = -130.1 \text{ db}$$

It can be seen from Table 5 for the flicker phase noise the translated results from the time domain to the frequency domain show a better agreement when the actual measured data in the time domain are used. This is to be expected since the straight line data in (b) are translated from the frequency domain. Further, the results in (a) are better for lower values of tau.

6.0 DISCUSSION

Historically the frequency stability measurements as most people have pointed out were made in a larger number of oscillators, in the time domain. This is because of the readily available electronic counters in the laboratories for making the period measurements. In addition, the sigma-tau plot of frequency stability in the time domain seems to be easier to communicate the concept of the characteristics of the noise model based on the power law. The difficulties are in the data analysis and in the identification of noise processes. For example, the flicker frequency noise in Figure 4 could be deleted if the dashed lines were used. In this case only one data point is discarded (i.e., t = 2 sec). Another question can be raised. "Is white phase noise present in Figure 4 in the time domain measurement?" The answer is "negative" based on the spectral density of phase measurement in the frequency domain in the frequency range measured. Obviously there is no answer to the question if only time domain data is available. Frequency stability measurements in the frequency domain are becoming more available as improved design and computer operated spectrum analyzers are becoming available in laboratories. Thus, the interest in the relating of the measured data in the two domains are continuously on the increase.

Figures 6 and 7 show the relative frequency stability of typical oscillators measured in the time domain and frequency domains respectively. These data are based on the results of Fischer and Vessot. It is to be noted that the apparent random walk of frequency (lines with +1/2 slope in Figure 6) is normally not shown in frequency stability of sigma versus tau plots. This is because the random walk of frequency noise is likely to be the net result of the noises in the electronic circuit of the frequency locked loop, the white phase of the quartz crystal oscillator, and the flicker phase of the cesium beam tube. The random walk of frequency behavior (less than the servo loop time constant) is therefore a characteristic of the system design of a manufacturer.

7.0 CONCLUSION

The mechanics of translation of frequency stability from frequency domain to time domain and vice versa have been shown. The procedure is simple and the translation relationships are given in Table 1. While the mechanics of translation are straight forward the same cannot be said for the preservation of the accuracy after translation. This is particularly true for the translation of the white phase and flicker phase noises from the time domain to the frequency domain. Additional errors may be introduced if the data collection and data analysis are not on a common base. Those who are interested in the validity of translation and theoretical limitations of the power law model of frequency stability are referred to the original papers and review papers given in the reference.

8.0 ACKNOWLEDGEMENT

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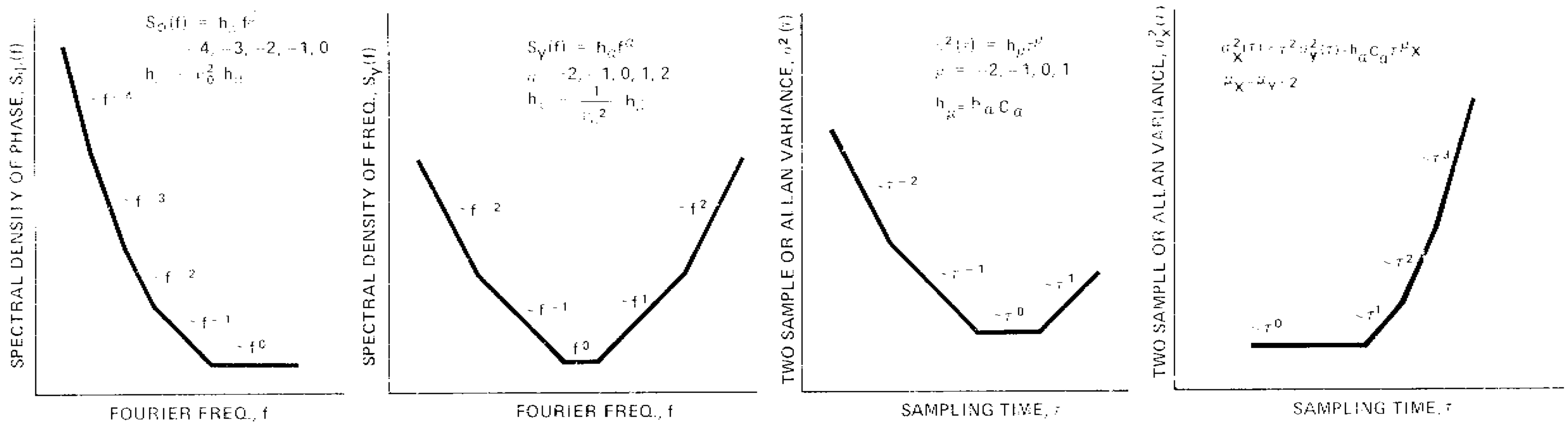


DESCRIPTION OF NOISE PROCESSES	POSSIBLE ORIGIN
RANDOM WALK OF FREQUENCY	CAVITY PULLING IN A MASER DUE TO THERMAL ORIGIN
FLICKER OF FREQUENCY (1/f NOISE)	UNKNOWN
WHITE FREQUENCY (RANDOM WALK OF PHASE)	SHOT NOISE – RANDOM ARRIVAL OF PARTICLES AT A DETECTOR
FLICKER OF PHASE	UNKNOWN
WHITE PHASE	ADDITIVE NOISE DUE TO THERMAL ORIGIN

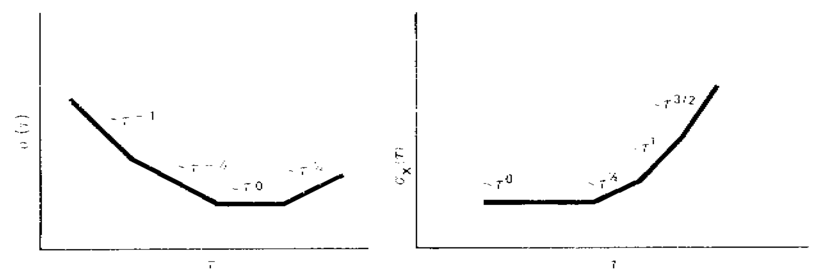


Figure 1. Power law noise models that contribute to frequency instability in oscillators.

Table 1. Translation equations for frequency stability measured in frequency domain to time domain and vice versa.

DESCRIPTION OF NOISE SOURCE	FREQUENCY DOMAIN TO TIME DOMAIN								TIME DOMAIN TO FREQUENCY DOMAIN			
	$\sigma_y^2(\tau)$						$\sigma_x^2(\tau)$		$S_y(f)$	$S_y(f)$	$S_x(f)$	$S_x(f)$
	α	FREQUENCY	μ_y	PHASE	μ	PHASE - TIME	μ_x	PHASE - TIME	FREQUENCY	PHASE	PHASE - TIME	PHASE - TIME
GENERAL FORMULA		$C_\alpha \tau^{-\alpha} S_y(f) f^{\mu_y}$ [illegible]		[illegible]		$4\pi^2 C_\alpha$ [illegible]		$4\pi^2 C_\alpha$ [illegible] $S_x(f) f^{\mu_x}$	$\frac{1}{C_\alpha}$ [illegible]	[illegible]	$\frac{1}{4\pi^2 C_\alpha}$ [illegible]	$\frac{1}{4\pi^2 C_\alpha}$ [illegible]
RANDOM WALK FREQ.	2	[illegible]	1	[illegible]	4	[illegible]	3	[illegible] $S_x(f)$ [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
FLICKER FREQUENCY	1	[illegible]	0	[illegible]	3	[illegible]	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
WHITE FREQUENCY	0	[illegible]	1	[illegible]	2	[illegible]	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
FLICKER PHASE	1	[illegible]	2	[illegible]	1	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
WHITE PHASE	2	[illegible]	2	[illegible]	0	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

EQUATIONS USED IN THE TABLE

[illegible]

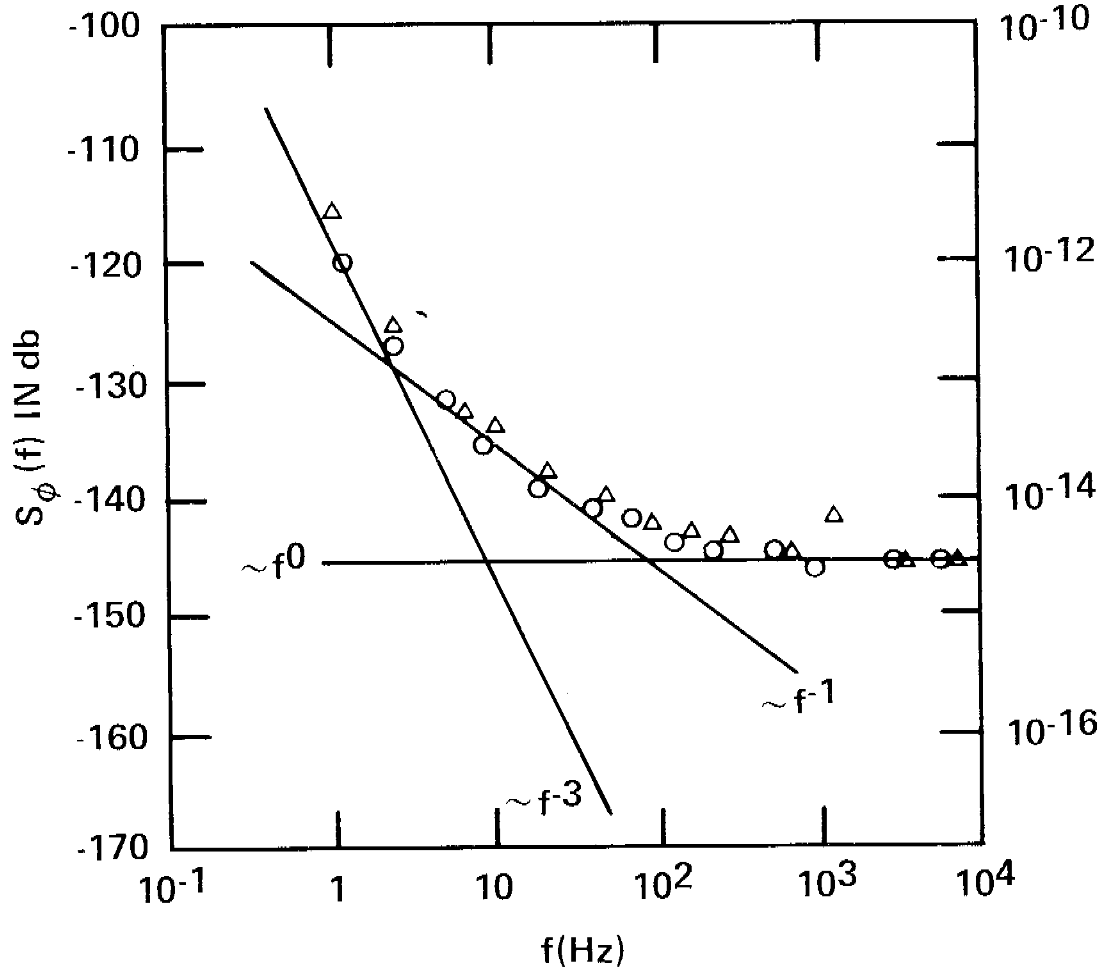


Figure 2. Spectral density of the phase of a very high quality 5 MHz quartz crystal oscillator.

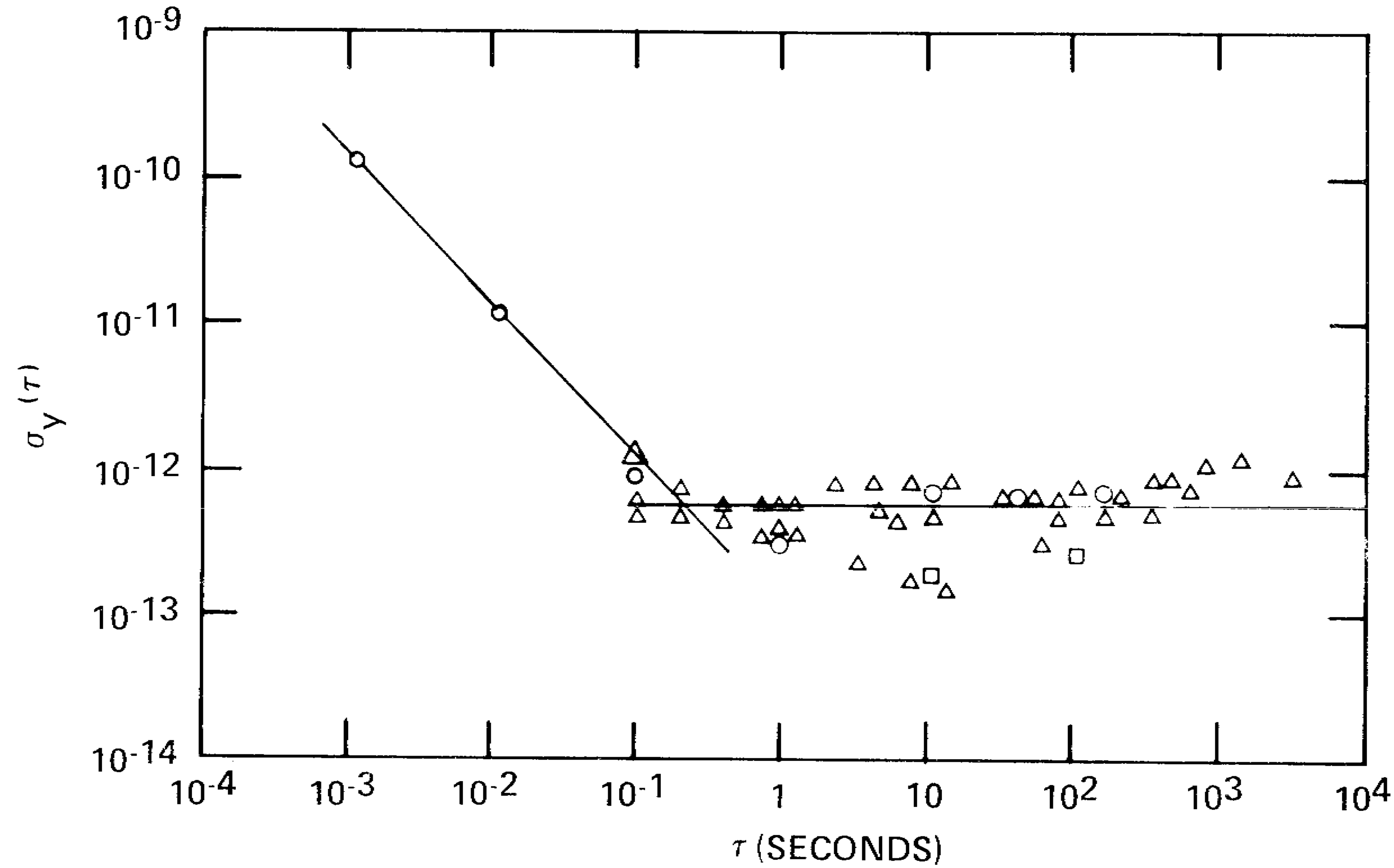


Figure 3. Frequency stability in time domain, $\sigma_y(\tau)$ vs τ, 5 MHz very high quality quartz crystal oscillator.

TABLE 2 DATA USED TO PLOT THE THEORETICAL NOISE SOURCES IN FIGURE 2

f	FLICKER FREQ. $S_\phi(f)$	FLICKER FREQ. $10\log S_\phi(f)$	FLICKER PHASE $S_\phi(f)$	FLICKER PHASE $10\log S_\phi(f)$	WHITE PHASE $S_\phi(f)$	WHITE PHASE $10\log S_\phi(f)$
10^0	1.58×10^{-12}	-118 db	3.16×10^{-13}	-125 db	3.98×10^{-15}	-144 db
10^1	1.58×10^{-15}	-148 db	-	-	3.98×10^{-15}	-144 db
10^2	-	-	3.16×10^{-15}	-145 db	3.98×10^{-15}	-144 db

TABLE 3 CALCULATED DATA FROM FREQUENCY DOMAIN TO TIME DOMAIN

τ	FLICKER PHASE $\sigma^2(\tau)$	FLICKER PHASE $\sigma(\tau)$	WHITE PHASE $\sigma^2(\tau)$	WHITE PHASE $\sigma(\tau)$	FLICKER + WHITE $\sigma^2(\tau)$	FLICKER + WHITE $\sigma(\tau)$
10^{-1}	6.52×10^{-25}	8.1×10^{-13}	1.21×10^{-24}	1.1×10^{-12}	1.86×10^{-24}	1.91×10^{-12}
10^{-2}	4.31×10^{-23}	6.6×10^{-12}	1.21×10^{-22}	1.1×10^{-11}	1.64×10^{-22}	1.76×10^{-11}
10^{-3}	2.10×10^{-21}	4.6×10^{-11}	1.21×10^{-20}	1.1×10^{-10}	1.42×10^{-21}	1.56×10^{-10}

TABLE 4 COMPARISON OF $\sigma(\tau)$ BETWEEN CALCULATED FROM FREQUENCY DOMAIN AND MEASURED IN TIME DOMAIN

τ	CALCULATED FLICKER FREQUENCY	CALCULATED FLICKER PHASE + WHITE PHASE	MEASURED FLICKER FREQUENCY	MEASURED FLICKER PHASE + WHITE PHASE
-	2.96×10^{-13}	-	5×10^{-13}	-
10^{-1}	-	1.9×10^{-12}	-	1.6×10^{-12}
10^{-2}	-	1.8×10^{-11}	-	1.6×10^{-11}
10^{-3}	-	1.6×10^{-10}	-	1.6×10^{-10}

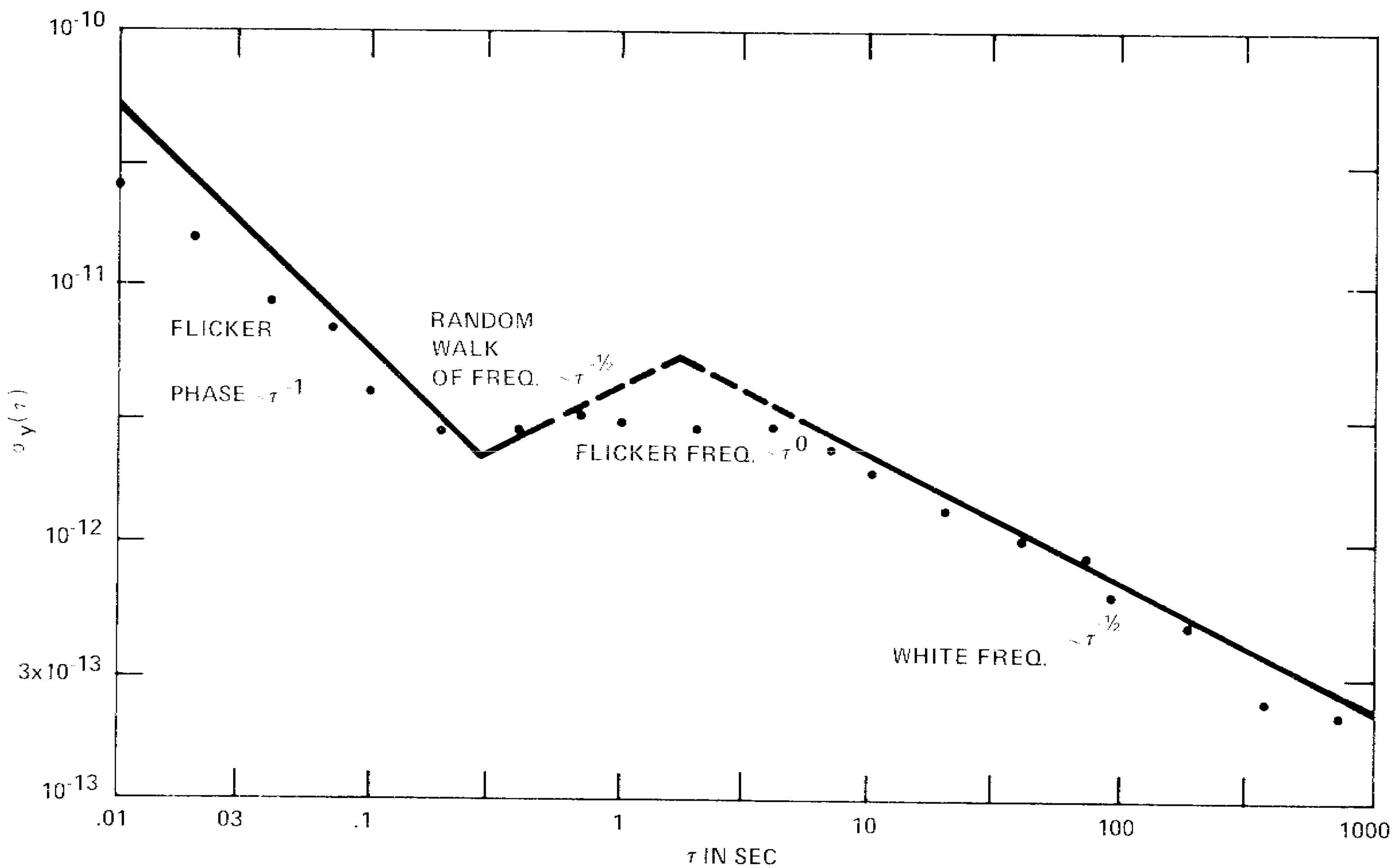


Figure 4. Frequency stability of a high performance cesium beam tube oscillator measured in the time domain.

TABLE 5 CALCULATED DATA FROM TIME DOMAIN TO FREQUENCY DOMAIN

	DATA				CALCULATED RESULT			MEAS.
	$\mu^{1/2}$	τ	$\sigma(\tau)$	$S_y(f)$	f	$S_\phi(f)$	$10\log S_\phi(f)$	$10\log S_\phi(f)$
WHITE FREQ.	-1/2	10	2.3×10^{-12}	$2.64\times10^{-9} f^{-2}$	10^{-2}	2.64×10^{-5}	-45.7db	-46db
FLICKER FREQ.	0	1	3.0×10^{-12}	$1.62\times10^{-10} f^{-3}$	10^{-1}	1.62×10^{-7}	-67.9db	-68db
RANDOM WALK FREQUENCY	1/2	0.3	2.2×10^{-12}	$6.08\times10^{-11} f^{-4}$	1	6.08×10^{-11}	-102.2db	-102.4db
FLICKER PHASE	-1	10^{-1}	a 4.0×10^{-12}	$4.92\times10^{-12} f^{-1}$	10	4.92×10^{-13}	-123.1db	-129.6db
			a 2.5×10^{-11}	$2.45\times10^{-12} f^{-1}$		2.45×10^{-13}	-126.1db	
			b 5.6×10^{-12}	$9.64\times10^{-12} f^{-1}$		9.64×10^{-13}	-120.2db	
a = MEAS.			b 5.0×10^{-11}	$9.79\times10^{-12} f^{-1}$		9.79×10^{-13}	-120.1db	
b = TRANSL.		10^{-2}	a 4.0×10^{-12}		10^{2}	4.92×10^{-14}	-133.1db	-136.8db
			a 2.5×10^{-11}			2.45×10^{-14}	-136.1db	
			b 5.6×10^{-12}			9.64×10^{-14}	-130.2db	
			b 5.0×10^{-11}			9.79×10^{-14}	-130.1db	

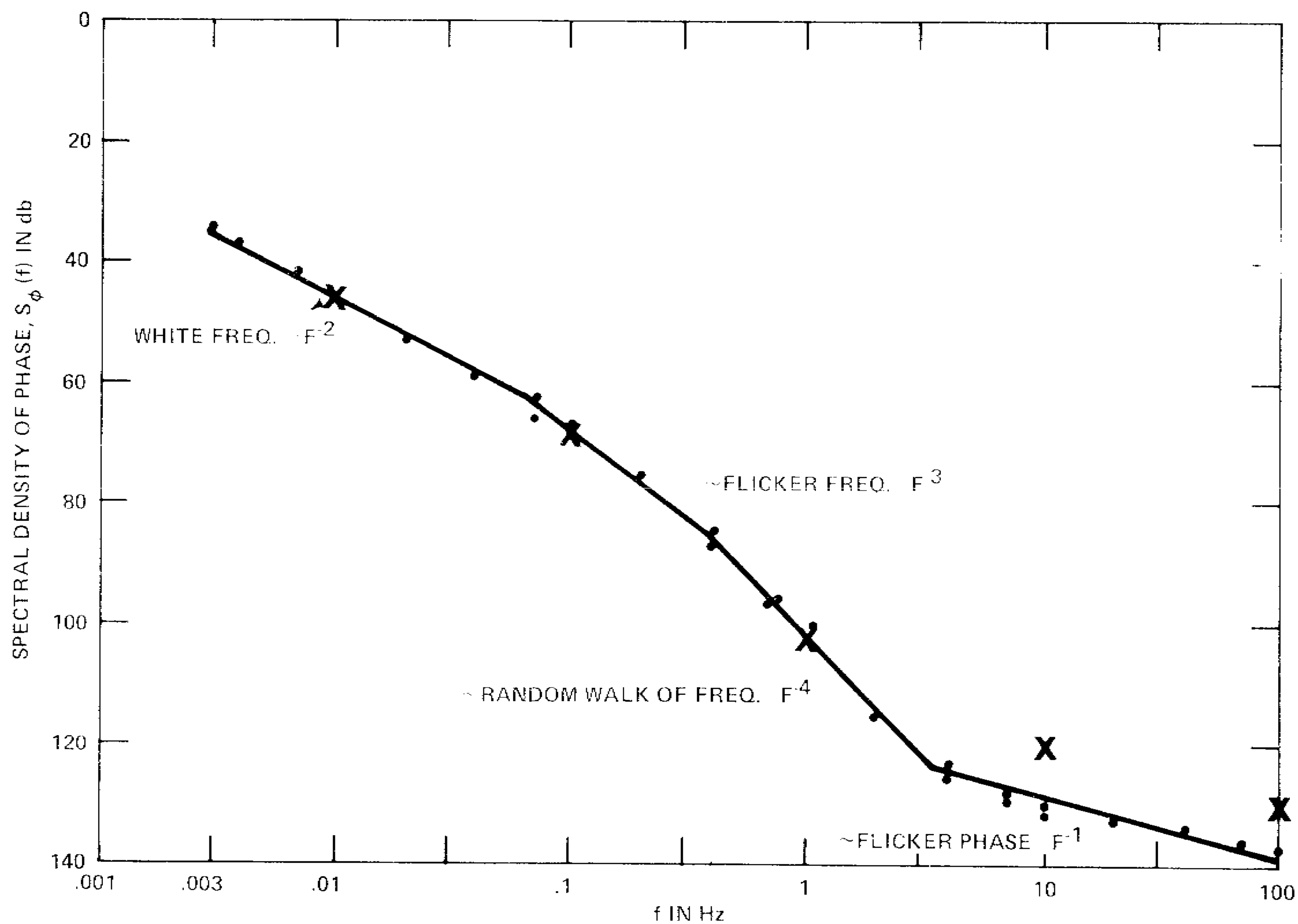
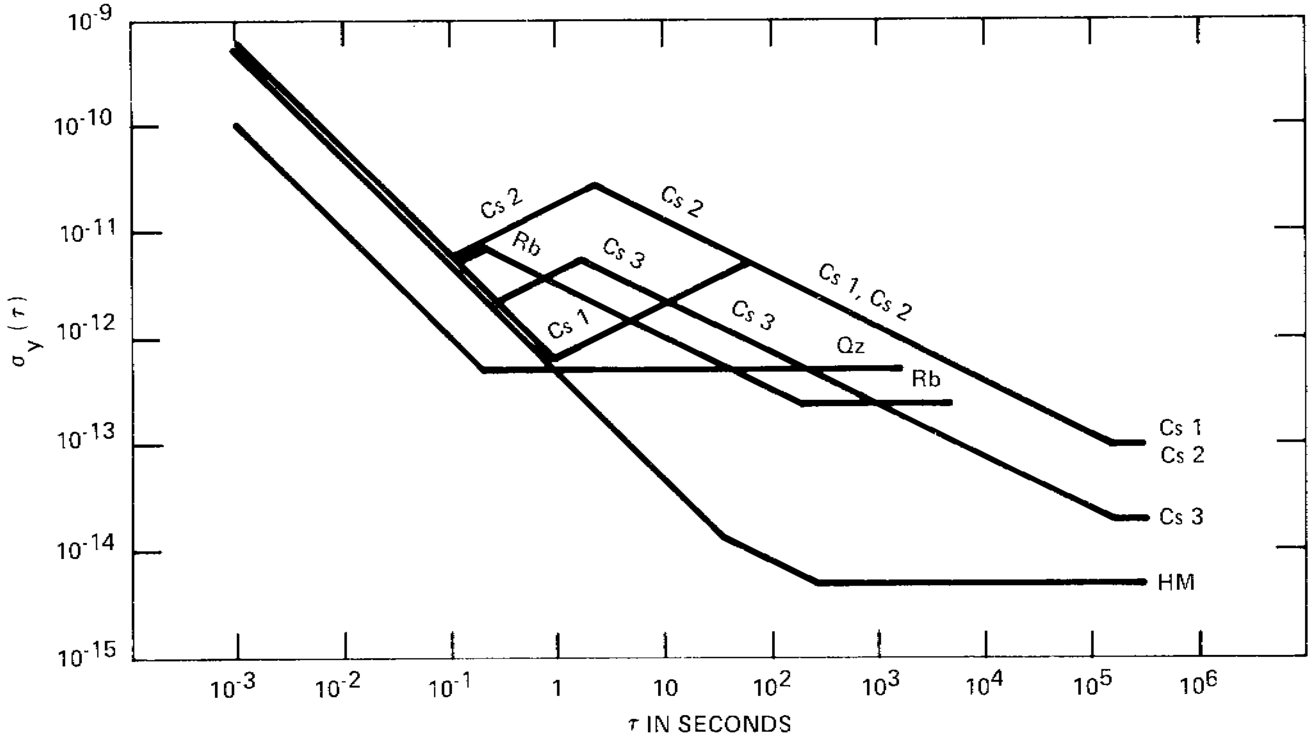
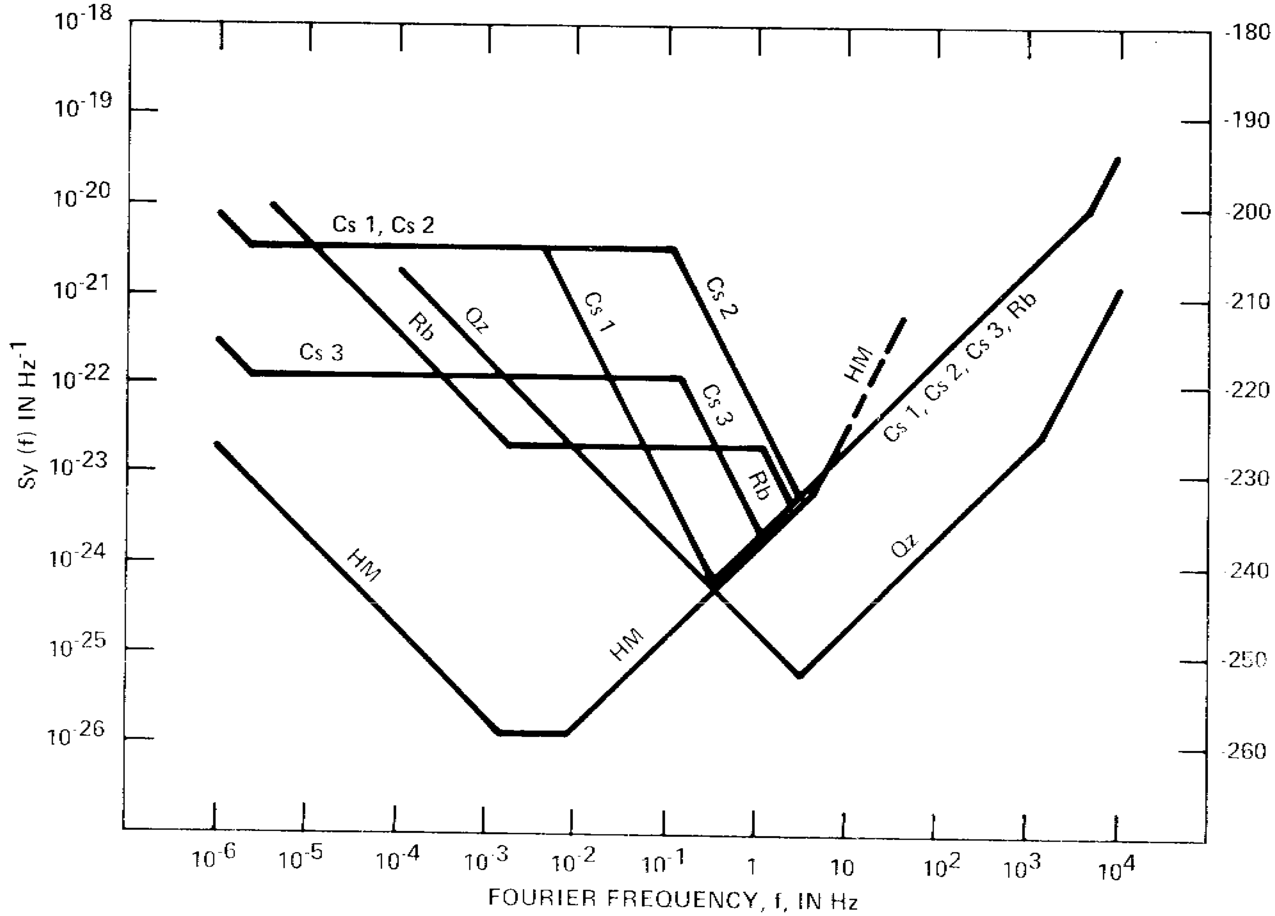


Figure 5. Spectral density of phase of a high performance cesium beam tube oscillator.
The "Crosses" are points translated from the time domain for $f_h = 5 \times 10^4$ Hz $\nu_0 = 5$ MHz.



OSC	SERVO LOOP TIME CONST (SEC)
Cs 1	60
Cs 2	2
Cs 3	2
Rb	0.3
Qz	–
HM	–

Figure 6. Frequency stability of typical atomic frequency standards and a selected quartz crystal controlled oscillator in the time domain (The positive slope portion of the curve is due to the electronic design of the frequency locked loop.)



OCS	f_h (Hz)
Cs 1	10^4
Cs 2	10^4
Cs 3	10^4
Rb	10^4
Qz	10^4
HM	10

Figure 7. Spectral density of frequency of typical atomic frequency standards and a selected quartz crystal controlled oscillator in the frequency domain.

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QUESTIONS AND ANSWERS

DR. JACQUES VANIER, Laval University:

There is a question where you have the transition in the phase lock or frequency lock loop. There I think the table you have shown for transferring from the time to the frequency domain cannot be applied directly like that. And you cannot identify the kind of noise because you have a transition, and this may change the slope quite a lot. This also depends on the damping factor or the time constant in the servo loop.

MR. CHI: Correct.

DR. VANIER:

So, what I wanted to mention is that at some places we have a flat portion in the sigma-tau curve. This is not flicker noise of frequency, but just a question of the servo loop.

MR. CHI:

Within the servo loop, of course, it is partly a parameter for the manufacturers who are designing that system. He would have to tell what is the optimum for his own system.

DR. JAMES A. BARNES, National Bureau of Standards:

I would like to make one comment that is perhaps obvious, but I think it is worth mentioning anyway. One is using these examples, models, where you have super positions of noise. The noise laws that one talks about are the asyntotic behaviors. When you superimpose them, actually you don't make a sharp transition at a sharp corner from a slope minus 1, say, to a slope 0. It is a smooth transition, and when you actually go through the process of filling it in, you get smooth curves, not corners. And when you try to fit the sharp cornered curves, there is a tendency to put in more terms than necessary. The modeling can be done much more straightforward.

DR. GERNOT M. R. WINKLER, U. S. Naval Observatory:

My own opinion is that one really shouldn't make these conversions but that one should measure in that domain in which one needs to have the values.