STEERING OF A TIME SCALE

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ABSTRACT

The atomic time scales can present some large time and frequency errors. The primary cesium standards which are the references allow to correct them. These devices can be used as frequency standards running from time to time or as clocks ; and two approaches to the steering problem appear possible.

The starting point of this study is an independent atomic time scale S computed by the laboratory 1 ; independent means that the scale has no intentional relationship with the time scale of another laboratory m and/or with a reference time and frequency standard . It is computed using only the data coming from a set of atomic clocks running freely and this computation is managed to get a time scale with a long term good stability. Models of such time scales have been developed [1, 2] . Figure 1 shows a purely random simulated time scale over 120 years the stability of which has the following expression :

$$\sigma(\tau) = (324\ \tau^{-2} + 0.25 + 4 \times 10^{-4}\ \tau)^{\frac{1}{2}}$$

where $\sigma(\tau)$ is the square root of the pair variance of the time scale frequency and is expressed in 10^{-13}, the parameter τ being expressed in days. It appears that the time differences between this time scale and the ideal one can be pretty large after a couple of years. Figure 2 gives the mean frequency of the time scale during a ten years interval and shows an apparent frequency decrease of about 0.6×10^{-13} per year.

The only way to avoid so large time and frequency errors is to correct the time scale S using information coming from some references. Several authors have derived filters in order to obtain the best estimate of the frequency of a time scale from series of measurements of this frequency by the primary standards [1, 3, 4]. An empirical use of the results of the filter developed in [4] has been proposed by the Bureau

International de l'Heure (BIH) as an accuracy - stability algorithm for the International Atomic Time scale (TAI). It was shown from simulations that the accuracy could be ensured and the stability of the original time scale S could be maintained or improved [5]. However it is realized that estimation of the frequency and steering of the time scale are different problems, at least theoretically. We have tried to develop a steering filter. The figure 3 shows the general problem : the primary cesium standards are used to compute a correction useful for the present date t ; this one is applied to the original time scale S which has a good stability. It is desired that the corrected time scale S' be accurate and stable at any present date t. Finding a linear predictor filter which allows the corrected time scale S' to be accurate and stable is the problem. Solutions can be developed after some hypothesis are adopted and the choice of assumptions leads to a specific approach of the problem.

The table 1 details two approaches. The first one is the method presently used by the BIH. It is based on the estimation filter [4] and the steering process [5] ; it will be referred as approach 1. It corresponds quite well to the use of the primary standards as frequency generators and research devices ; the standards and/or the calibration measurements can be modified from one test to the other to obtain better results than before. The approach 2 corresponds to the use of the primary standards as clocks ; the standards are considered as metrological devices which must run a long time (several months) without changes.

Several points are common in both studies : the time scale model, the independence of the involved primary standards and the stability criterion. Important differences appear in the calibration models. In approach 1, the calibrations can take place at any time and have any duration, but they are supposed to be affected by white noise and constant error. For the second approach, there is no theoretical restriction concerning the calibrations noise but the calibrations have to be considered as regularly distributed every 60 days in succession without gap. The involved frequencies in this case are mean frequencies over 60 days which is considered as the unit of time interval. If several standards are involved in the computation, they are assumed independent of each other in both cases ; but, in the first approach, they are members of ensembles the statistical properties of which are known. An accuracy filter was derived for the approach 1 and was detailed in [4] . Pratical steering filters were proposed in [5] in this case.

APPROACH 1	APPROACH 2
TIME SCALE MODEL	
purely random time scale with white PM plus flicker FM plus random walk FM	idem
sampling period : 10 d	sampling period : 60 d
CALIBRATION MODEL	
the calibrations occur randomly and their duration is variable.	the calibrations occur periodically every 60 d and last 60 d.
the errors of the calibrations belong to a white noise.	
if several standards are involved, they are independent.	if several standards are involved, they are independent
each standard belongs to an ensemble the statistical properties of which are known.	
CORRECTED FREQUENCY	
It depends on the steering process see [3] .	the original frequency minus a weighted linear combination of the corrections from each standard.
ACCURACY CRITERION	
the weighted linear combination of all the calibration results near to the time scale frequency (in mean square) see [4] .	the corrected frequency near to the best estimate of the frequency of the standards (in mean square).
STABILITY CRITERION	
Pair variance of the corrected frequency has to be minimum .	idem

TABLE 1. Main characteristics of two steering approaches carried out by the Bureau International de l'Heure (BIH).

More details are given now for the approach 2. The corrected frequency is computed at the present date t (which means t times the sampling period 60 days). The data used to compute the correction are the mean frequency differences $Y^i - R^i$ at the date i between the time scale S and one primary standard, where i goes from 0 to $t - \alpha$; α represents the delay of the available data. If they are used in real time , $\alpha = 0$. These data enter a linear filter the impulse response is G ; the output of the filter is the convolution

$$C = (Y - R) \otimes G$$

or more explicitly, at any present time t :

$$C^t = \sum_{i=0}^{t-\alpha} g^i (Y^i - R^i)$$

When several independent primary standards are involved, the total correction is a weighted linear combination of the individual corrections as C^t ; the weight of a standard depends on its qualities. Let us come back to the simpler case of only one standard. The corrected frequency at the date t is :

$$y^t = Y^t - C^t$$

The scale S', represented by the time process y^t , is said accurate if

$$E \left[(y^t - R^t)^2 \right] \text{ is minimum}$$

where E designates an ensemble average . The quantity $y^t - R^t$ can be written :

$$y^t - R^t = Y^t - R^t - \sum_{i=0}^{t-\alpha} g^i (Y^i - R^i)$$

The functional diagram of the figure 4 represents this relation. It appears that the filter G which is looked for and satisfies the accuracy criterion is a pure prediction filter.

Let us see the stability criterion : the best stability of the corrected scale S' is obtained if the pair variance of y^t is minimum. The computation of the pair variance introduces the parameter τ (time interval between two successive data) and it will be possible to obtain the best stability for some specified value of τ . The expression of y^t is

$$y^t = Y^t - \sum_{i=0}^{t-\alpha} g^i (Y^i - R^i)$$

and is represented by the functional diagram of the figure 5. The role of the filter K in the chain is simply to convert the usual Wiener criterion (minimization of the variance of the error) into the new criterion : minimization of the pair variance of the error. The steering filter G which satisfies the stability criterion is a predicting filter which allows the signal Y^t to be extracted from the imput $Y^i - R^i$.

Solutions of both separate problems, i. e. filters G , are found using the Z transform domain [6] which is deduced from the Laplace one by

$$Z = e^{\tau p}$$

where τ is the sampling period (here 60 days). The solutions depend on the noise of the calibrations and this one of the time scale. The table 2 presents, as an example, the steering filters which are obtained when the time scale noise is either a flicker FM or a random walk FM and the calibration noise is white FM. The last hypothesis corresponds to the approach 1 and so the same exponential accuracy filter as in [4] is obtained. The stability filter depends on the parameter τ .

The second approach allows to obtain filters which ensure either accuracy or stability (for a specified value of τ) whatever is the calibration noise . A realistic steering filter will be a compromise between an accuracy filter and a stability filter. Furthermore it will depend on the noise of the primary standards and also on some practical limitations of the involved calibrations due to a possible drift of the time scale S, an improvement of the primary standards and/or of the calibration measurements.

REFERENCES

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TIME SCALE NOISE	FLICKER FM	RANDOM WALK FM
ACCURACY steering filter	● last calibration	● last calibration
	○ exponential type	○ exponential
STABILITY steering filter	● Weighted past values depends on τ	● weighted past values depends on τ
	○ idem	○ idem

Table 2. Steering filters obtained when using the second approach. The symbol ● refers to non delayed data and ○ to delayed data. The calibrations are affected by a white noise FM.

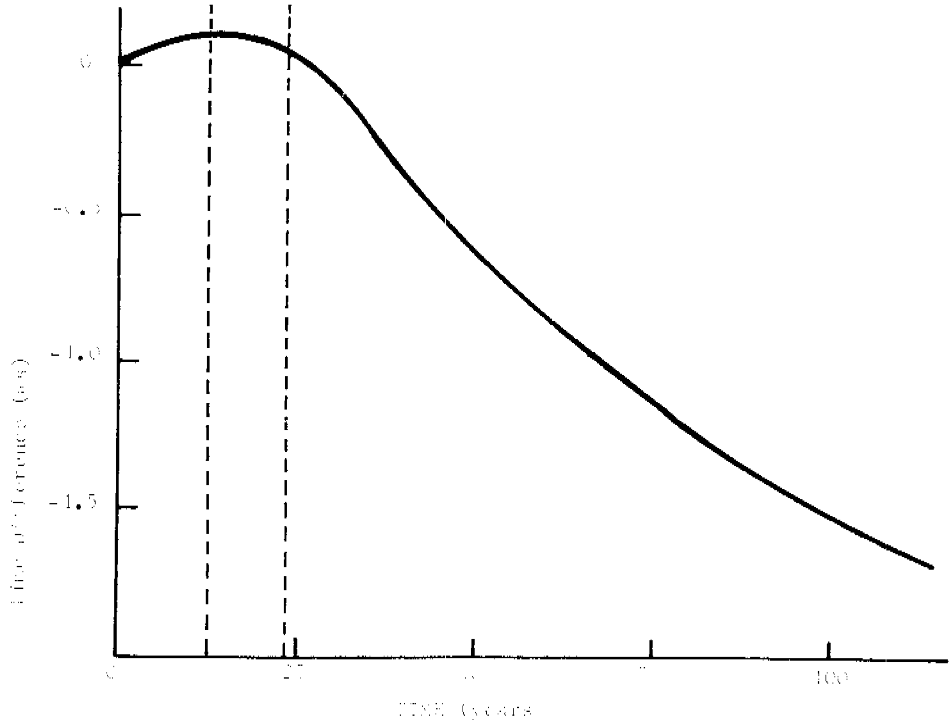

Fig. 1. The difference between a simulated random time scale and the 'ideal' one as a function of time.

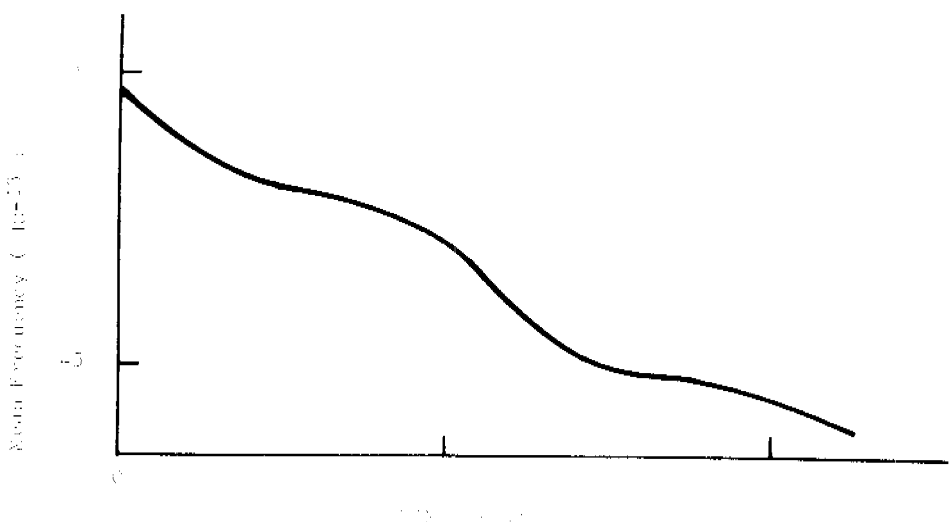

Fig. 2. [illegible] scale during [illegible]

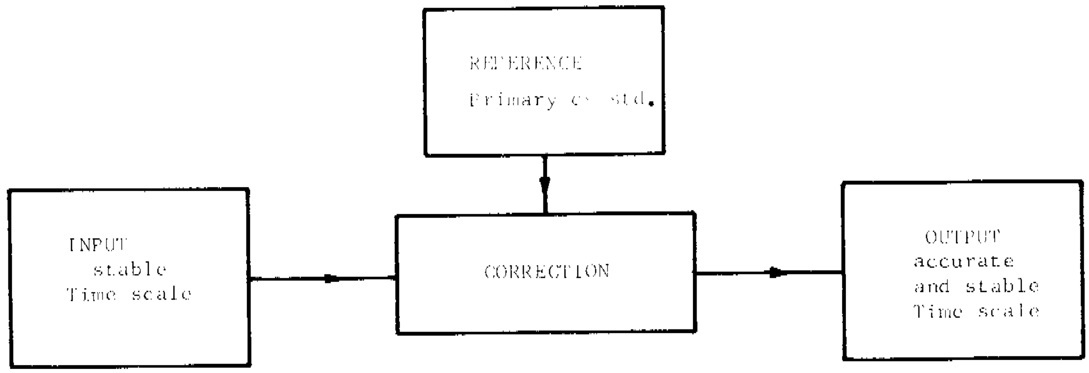

Fig. 3 . Diagram of a corrected time scale.

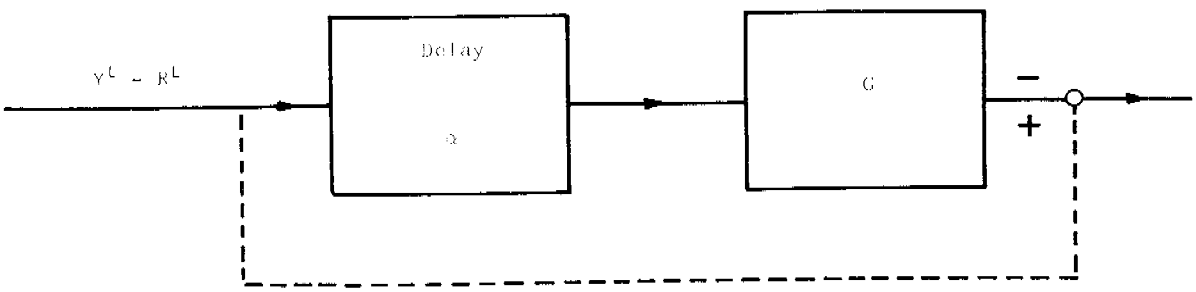

Fig. 4. Functional diagram of the accuracy filter.

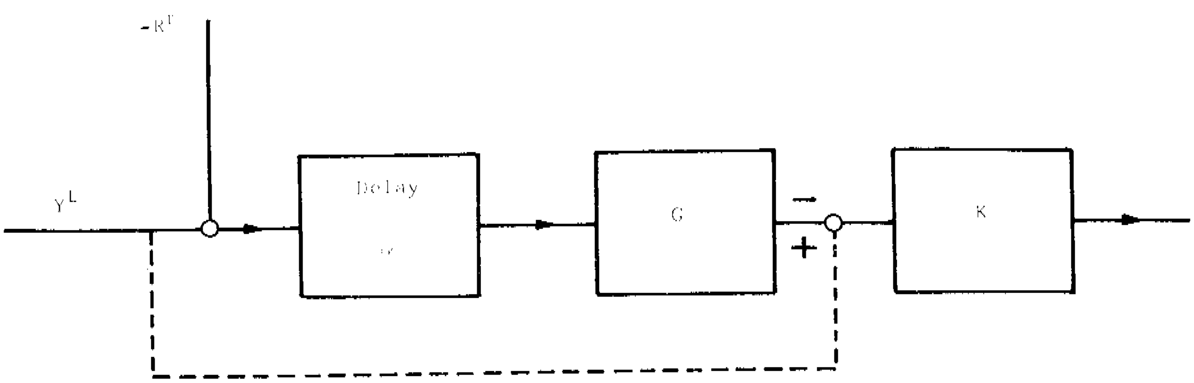

Fig. 5. Functional diagram of the stability filter.

QUESTIONS AND ANSWERS

DR. GERNOT M. R. WINKLER, U. S. Naval Observatory:

To what degree would your results change if you drastically change the assumed models by assuming a very large and overwhelming contribution from just phase noise?

DR. GRANDVEAUD:

It depends on whether you are speaking from a theoretical point of view or from the practical point of view. From a practical point of view, it seemed that choosing an exponential figure and doing some quite good manipulations produced satisfying results. But from the theoretical point of view, I think it is quite different and more complicated.