FREQUENCY STABILITY MEASUREMENT PROCEDURES

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ABSTRACT

This paper is intended to be a tutorial review of established techniques for workers new to this field and as a reference for those who must make these measurements infrequently. Enough background is included to remove most of the mystery, leaving rigorous mathmatical justifications to be found in the references. Special techniques for pushing measurement precision to the known limit are only mentioned briefly. It is recognized that any single application of a precision oscillator will have system requirements for which a limited range of these procedures will apply.

The measurements covered are phase modulation sidebands from 0.01 Hz to 50 kHz, and short term stability by Allan variance from 0.1 millisecond to 10^4 seconds, including the effects of non-random components such as spurs or bright lines.

Applications of a new frequency stability analyzer system are discussed in some detail also. This system is particularly suited for characterizing VHF frequencies and above.

INTRODUCTION

As an introduction to the subject of frequency stability, the first paragraphs review some basic concepts which set a context for understanding the measurement methods and results. Frequency measurements are concerned with describing changes in the phase of the output signal of an oscillator. This can be viewed as observing how uniformly in time the zero crossings occur. There are two parameters which have little to do with frequency stability as it affects a user system. These are amplitude modulation and harmonic distortion. These points will be explained next to remove them as possible confusion factors.

Amplitude Modulation

Most measurement methods and specifications ignore amplitude noise, that is unwanted amplitude modulation. This is because amplitude noise can be stripped off in an inexpensive limiter stage. Many systems use such a limiter to interface with a frequency standard input for this reason, as well as to provide gain and/or to standardize the signal level for the stages which follow. Even when not specified, AM noise sidebands in most quality sources are comparable to or better than phase noise.

Harmonic Distortions

The square wave output of a symmetrical limiter mentioned above contains large odd order harmonics which can be filtered off if the system would be perturbed by them. Where a limiter is used, odd order harmonic distortion on the output of the oscillator is of no consequence. Of more concern is even order harmonic distortion because it causes the positive half-cycles to have a different shape from the negative half cycles. This asymmetry can cause unbalanced operation in some frequency doubler circuits.

Most applications are not sensitive to moderate amounts of harmonic content. Because of this, even the highest quality sources have harmonic distortion specifications around 30 to 40 dB down. This performance is easily measured directly on a spectrum analyzer, or selective voltmeter.

To summarize: any distortion in the shape of the wave which remains unchanged from one cycle to the next, shows up as harmonic distortion. This is easily measured, and has no effect on frequency or phase stability.

FREQUENCY, PHASE, TIME

Examination of the frequency standard output waveform will now concentrate on the variations in the time of occurance of the zero crossings (1). Consider again a frequency standard signal having passed through a hard limiter. The only information remaining is the time of the zero crossings. It is here that all the stability of frequency and phase, or time, is defined. This suggests the technology of digital logic where waveshape and amplitude receive little attention, but edge timing is critical. An illustration of these points exists in the fact that Schottky T^2L logic circuitry can be used to process the output of a high quality source with comparable results to well designed discrete analog circuitry. Also, the action of logic circuitry meets the definition of a hard limiter.

Frequency Counters

The most familiar method of frequency measurement is the frequency counter instrument. When it is used for a stand-alone measurement (without use of an external reference standard) its internal time base oscillator accuracy will range from 10^{-8} for a top quality instrument on a monthly calibration schedule, to 10^{-6} for a low cost counter receiving annual checks.

In order to make frequency measurements with accuracies better than 10^{-8}, a carefully maintained house standard is usually the reference of choice in a laboratory environment. In a field or system environment, an atomic resonance stabilized oscillator can, even with significant environmental effects, still offer 10^{-11} absolute accuracy.

The resolution of the frequency counter itself represents a measurement precision limit separate from the reference source. For example a modern high performance counter may actually count a 500 MHz internal clock for a large number of periods of the input signal to be measured. Such a counter then divides the latter by the former to display a frequency result. The resolution is simply the total number of cycles of 500 MHz counted. If the operator can afford to wait 1000 seconds for a reading, then the resolution is one out of 5×10^{11} or 2×10^{-12} per increment of the least significant digit.

Frequency From Phase Rate

The difference between two frequencies can be measured by triggering an oscilloscope with one while observing the other and timing the rate of phase drift between the two with a stop watch. For instance, if during 10 seconds, four cycles pass across the center line of the graticule, then the frequencies differ by 0.4 Hz.

It is more common to time the passage of a zero-crossing of the signal as it moves across a major division of the oscilloscope screen for two reasons: First, for small frequency errors, many minutes or even days might elapse before a complete cycle had passed. Second, frequency errors and tolerances are usually expressed in normalized or fractional notations as $\Delta f/f$. If the 0.4 Hz error above were on a 10 MHz signal, then its fractional frequency error or $\Delta f/f$ would be 4×10^{-8}.

As an example of this method, conditions near the practical limit of its use are: a 10 nanosecond phase change during a 100 second elapsed time. This is referred to as a $\Delta t/t$ of 1×10^{-10}. Since a $\Delta t/t$ measured in this way equals $\Delta f/f$ directly, computation is minimal and simple. It is useful to note that this method does not involve knowledge of the carrier frequency. This can be especially convenient when the carrier frequency is a cumbersome non-integer, or for comparing errors among branches of a system where the carrier frequencies differ. A coherent synthesizer will probably be needed to serve as a trigger reference in these cases.

Phase Meter, Strip Chart Recorder

Several more orders of magnitude of resolution down to $\Delta f/f = 1 \times 10^{-14}$ can be obtained by using a phase meter with some means of recording its output. The simplest arrangement to achieve this is a vector voltmeter with its dc phase output connected to a strip chart recorder. This can resolve 0.1 nanosecond and, in less than three hours, or 10^4 seconds, the resolution becomes $\Delta t/t = 1 \times 10^{-14}$.

A time interval counter can also be used to measure phase, and can reach fractional nanosecond resolution with time interval averaging, or interpolation. Recording data from a counter can be handled with a printer, a digital to analog converter and strip chart recorder, or an interface bus to a calculator and peripherals.

A comment on the $\Delta t/t$ method is in order: The fractional frequency difference measured with this method is the average (mean) during the measurement interval t.

These techniques are recommended for the measurement of absolute frequency error, long term aging, effect of environmental changes, and house standard monitoring. Two beat frequency methods which yield extreme resolution in very short measurement times are described in HP Application

Note 52-2, page 3 (1), and, by David W. Allan, "Report on NBS Dual Mixer Time Difference System (DMTD)..." (2).

FREQUENCY STABILITY

An oscillator's inherent instabilities, other than those induced by environmental effects, can be grouped conveniently into three classes of frequency changes: monotonic, periodic, and random.

Aging

Monotonic drifts in frequency over time ranging from days upward are called frequency aging and are measured by repeated application of techniques described above for absolute frequency measurement. (3) Non-monotonic drifts also occur and can be measured by these techniques.

Periodic FM

Periodic changes in frequency amount to frequency modulation (whether or not intentional) by a sine wave and its harmonics. This is a typical problem in many applications and will be discussed along with random frequency variations and their measurements. It is worth noting at this early point that frequency modulation or phase modulation are no more than different ways of measuring the same signal. For example, should one attempt to analyze the signal coming from a black box emitting a 1.0 MHz carrier, whose frequency swings sinusoidally from 0.999 MHz to 1.001 MHz at a 1.0 kHz rate, it would be found to have FM/PM sidebands, spaced at 1.0 kHz multiples from the carrier, of amplitudes indicating a modulation index of 1.0. Correspondingly a phase meter would show that the carrier phase was swinging sinusoidally at a 1.0 kHz rate with peak excursions of 1.0 radian.

RANDOM FREQUENCY VARIATIONS, TIME DOMAIN

Possibly the most familiar measure of the randomness or scatter of any variable is the standard deviation or rms, the usual symbol is a lower case sigma, σ, and a formula is:

$$\sigma_x = \sqrt{\frac{1}{N-1} \sum_{j=1}^{N} (x_j - \bar{x})^2} \qquad (1.$$

This means that N measurements, each an x_i, are averaged to find $\bar{x}$, then σ is computed. This was applied to characterize frequency sources until it was found that some of the non-white noise processes commonly present caused σ to vary depending on the choice of N. This is even worse because, for larger N, σ can increase instead of converging. Further detail is in NBS Technical Note 669, p. 7, (4).

For these reasons, in the measurement of frequency sources, a special definition of σ, which avoids the divergence problems, is in universal use. This measure is called the Allan variance after its developer. It may be recalled that a variance is the square of a standard deviation. This measure is defined by the formula:

$$\sigma_y^2(\tau) \sim \frac{1}{2(M-1)} \sum_{k=1}^{M-1} (y_{k+1}-y_k)^2 \qquad (2.$$

where M measurements are made of $\Delta f/f$, each of which is a y_k. On sigma, the subscript y means that sigma is a measure of the scatter in fractional frequency difference, $y = \Delta f/f$. The "approximates" symbol, $\sim$, is used instead of "equals", =, because this is a measure of a random phenomemon by a finite number of samples, M. This means that some scatter must be expected in measurements of $\sigma_y(\tau)$, amounting to several percent for M=100. (15) The time duration of each measurement is τ, also called averaging time or sample time. The significance of the parameter τ appears when we measure $\sigma_y(\tau)$ for different averaging times, in effect varying τ stepwise, then plot the results. This is the familiar frequency standard specification plot of $\sigma_y(\tau)$ versus τ. Since τ is the independent variable, these are called TIME DOMAIN measurements and specifications, and also SHORT TERM STABILITY.

In order to measure the $\sigma_y(\tau)$ of all higher quality sources. which includes quartz oscillators well below $1000, some special arrangement is required beyond a simple frequency counter to attain sufficient resolution. The basic method which has seen the widest use is the heterodyne or beat frequency method. This is diagrammed in Figure 1 and requires that the two oscillators to be compared be offset by the desired difference frequency, f_D, usually between 1.0 and 10 hertz. Proper choice of f_D will make the counter read directly in $\Delta f/f$ scaled by a convenient power of 10. The relationship is:

$$\frac{\Delta f}{f} = \frac{f_D^2 \Delta\tau}{f_o} \qquad (3.$$

where $\Delta f/f$ is the fractional frequency difference indicated by an increment in the period count of $\Delta\tau$. The nominal carrier frequency is f_o and the beat frequency f_D. For example, if $f_o = 10^7$ Hz, $f_D = 10$ Hz, and the least digit of the period count is $\Delta\tau = 10^{-7}$ second, then the resolution of each measurement is 10^{-12}. This gives the scale of the counter readings so that they may be entered into equation (2. above directly as the y_k measurements. The averaging time of the measurement is the period which is counted, at the minimum, $\tau = 1/f_D$, and multiple periods can be counted for longer τ.

In Figure 1, the block labeled "Mixer, Low Pass Filter, Amplifier" must be designed with some care to achieve a low noise interface between the mixer and the counter, and to set the noise bandwidth of the system to the desired value, conventionally 100 kHz. When measuring in a region of τ where $\sigma_y(\tau)$ shows white noise phase modulation, the measurement result will be proportional to the square root of the system bandwidth. There is a commercial product which serves these needs, the HP 10830A mixer/amplifier shown in the 5390 System (5), Figure 2.

Another parameter which can affect the measurement is the reset interval of the counter, also called dead time. In this arrangement the counter must ignore one period between the completion of one measurement and the start of another, while it outputs data and resets. This dead time biases the measurement result, requiring corrections which are called bias functions. These are tabulated in NBS Monograph 140, pages 190 - 204 (6) and HP Application Note 174-7 (7). An abbreviated table is included in Appendix D.

In order to choose the proper bias function to apply to a data point, the slope of the data at that point must be known. This means that $\sigma_y(\tau)$ must be measured at least two different values of averaging time τ. If only a few values of τ are used, they should be separated by ratios of about two or three. Measurement uncertainty would lead to a large slope uncertainty for a pair of closely spaced points. If a pair of data points were spaced a decade apart in τ, the slope could be changing significantly. The best policy is to plan to take as many data points as circumstances allow. More detailed discussion of which slopes may be expected is located in the section headed "Conversion of Data Between Time and Frequency Domains."

If a 10 Hz offset between the unit under test and the reference can be achieved, then this heterodyne method can be used to measure $\sigma_y(\tau)$ conveniently for τ from at least 1000 seconds down to 1/10 second. Efforts to increase the offset frequency, to measure at shorter times, must be applied with care, because such conversion techniques as synthesizing and mixing can easily add more noise than that which was to be measured.

If a signal from a third oscillator of similar or better quality to the unit under test is available, then two more sets of data can be taken, pairwise among the three. These three measurements then can be combined to find the performance of each individual unit (8).

$$\sigma_a = \sqrt{\frac{1}{2}\left(\sigma_{ab}^2 + \sigma_{ac}^2 - \sigma_{bc}^2\right)} \tag{4.}$$

The subscripts refer to units a, b, and c. Once the reference unit is calibrated, then future measurements of unknowns can be computed more simply:

$$\sigma_a = \sqrt{\sigma_{ab}^2 - \sigma_b^2} \tag{5.}$$

Clearly, all measurements combined must be at the same τ.

For nearly all stable sources, their short term stability in the region of approximately one millisecond to one second shows white noise of phase as the dominant process. Direct measurement of this performance requires an additional level of complexity beyond the basic methods of this paper.

Fortunately there is a basic method which, though indirect, gives a more detailed characterization of performance. This method is the phase noise, $\mathcal{L}(f)$, measurement in the frequency domain. Most stable sources exhibit a flattening-out of their phase noise spectrum in the 100 Hz to 10 kHz region. The asymptote of this measurement can be converted using the white phase equation for $\sigma_y(\tau)$ in Table 1 to give the corresponding Allan variance in the time domain.

The improved detail comes from the fact that the phase noise measurement will show the frequency and amplitude of any discrete spurious sidebands. The peak effect of these on $\sigma_y(\tau)$ is calculated by:

$$\sigma_y(\tau) = \frac{\sqrt{8}}{\pi f_o} \sqrt{\text{antilog}\ (\mathcal{L}_{PM}\ \text{dB}/10)}\ \tau^{-1} \tag{6.}$$

where f_0 is the nominal carrier frequency and $\mathcal{L}_{PM}$ is the single sideband to carrier ratio in decibels of the spur. A derivation of this formula appears in Appendix A. Note that this result is not a function of the sideband frequency.

The above points are illustrated by actual data plots in Figure 3. The $\sigma_y(\tau)$ data was taken with an HP 5390A Frequency Stability Analyzer and the $\mathcal{L}(f)$ data was taken with the arrangement in Figure 4, using an HP 10534A Mixer, 3581 Wave Analyzer and 7035B X-Y Recorder.

Incidentally for τ less than one millisecond, the frequency domain measurement is still the better choice for the same reasons. In some sources, a bandpass filter is included which causes the phase noise or $\mathcal{L}(f)$, curve to break downward at a slope of f^{-2} starting at some frequency above 10 kHz. This is random walk of phase or white FM and has a time domain slope of $\tau^{-\frac{1}{2}}$. Whenever this performance is encountered in a frequency domain measurement, its asymptotic slope can be converted to $\sigma_y(\tau)$ by using the white frequency equation in Table 1.

Turning to the region of τ=1 second and greater, the situation reverses, and direct measurements of $\sigma_y(\tau)$ become simpler. The following section gives details on phase noise measurement.

FREQUENCY DOMAIN, PHASE NOISE MEASUREMENT

To minimize confusion,this paper deals with frequency domain measurements in terms of only one of the measures available. The choice of single sideband phase noise to carrier ratio, $\mathcal{L}(f)$, was made because this measure is currently in almost universal use on oscillator data sheets, when frequency domain specifications are offered. This situation may change due to strong efforts to standardize on $S_\phi(f)$ to replace $\mathcal{L}(f)$. If this occurs it will proceed slowly and apace with the desires of the user community. Accordingly it is the reader's prerogative to make his preferences known to his vendors in order to cast his vote in the matter. In the comments and equations to follow, $\mathcal{L}(f)$ may be replaced with any of the other measures by including the scaling coefficients tabulated in Appendix B. Some discussion of the uses of the symbol f is in Appendix C.

The basic method of measuring phase noise on signals makes use of the doubly balanced mixer as a phase detector as shown in Figure 4. Two recent publications describe extensions of this approach; NBS Technical Note 679 by David Howe (9), and HP Application Note 207 (10).

This system operates with both the reference and the unit under test at the same frequency. When the two signals into the mixer are in phase quadrature, the mixer's average dc output voltage will be zero and phase fluctuations will be translated to voltage fluctuations about zero. In order to keep the input phases near quadrature where the mixer's phase sensitivity is greatest and linear, and where its amplitude sensitivity is very small, there is a feedback path to the EFC (electronic frequency control) input of the reference oscillator. This constitutes a phase locked loop and functions as a convenience to the operator by helping maintain quadrature. Its operation is not part of the measurement of phase noise; in fact, care should be taken that the time constant of this loop is at least a tenth of a second if phase noise is to be measured as low as 5 Hz, so that the loop will not track and reduce the phase variations to be measured.

The network following the mixer/phase detector in Figure 4 is a low pass filter. The output of the mixer will contain both the sum and difference of the two frequencies at its inputs. Since the two frequencies are the same, f_o, then the mixer output is dc with phase information plus a 2 f_o component of amplitude about 6 dB below the smaller of the two inputs. The low pass attenuates the 2 f_0 signal to avoid overload or dynamic range problems with the input stage of the analyzer which follows. The purpose of the 51Ω resistor is to provice a matched load for the 2 f_0 signal from the mixer.

This is desirable because it helps the R and L ports of the mixer perform as specified. The 3900 pf capacitor allows the 2 f_0 signal to pass, above 0.5 MHz, while blocking the thermal noise of the resistor to keep it from adding to the signal to be measured. Without the capacitor, if the signal were 6 dB above the resistor noise, the measured value would be made 1 dB worse by the resistor noise. This capacitor also blocks the signal frequencies to be measured and prevents the resistor from loading the mixer at these lower frequencies. This, with the high imput impedance of the analyzer yields 6 dB more signal level than if the mixer were terminated at low frequencies.

The 82μH inductor and 0.039 μF capacitor form a two pole low pass section which is flat within 0.1 dB to 50 kHz, is 3 dB down at 100 kHz and has skirts more than 40 dB per decade down above 100 kHz. When constructed of an inexpensive choke coil and mylar capacitor this circuit has reached -60 dB at 2.0 MHz and stayed below -60 dB will past 10 MHz.

The 100 kΩ resistor and 0.1 μF capacitor form a 10 millisecond time constant low pass to assure the slowness of the phase lock loop and help isolate the signal path from the EFC path where an oscilloscope or chopper stabilized sensitive dc voltmeter might be connected.

Calibration Factors Using Double-Balanced-Mixer Phase Detector

For small deviation phase modulation, the total signal instantaneous voltage is given by (11):

$$V(t) = \underbrace{A_c \cos \omega_c t}_{\text{carrier}} - \underbrace{\tfrac{1}{2}A_c\beta \cos (\omega_c - \omega_m)t + \tfrac{1}{2}A_c\beta \cos (\omega_c + \omega_m)t}_{\text{sidebands}} \tag{7.}$$

where A_c is the peak amplitude in volts of the carrier component sine wave alone, ω_c is the carrier frequency and ω_m the modulation frequency both in radians per second, t is time, the independent variable, in seconds, and $\beta \equiv \Delta\phi$, the peak phase excursion in radians, also defined as $\Delta\omega/\omega_m$ or $\Delta f/f_m$, the peak frequency excursion divided by the modulating frequency.

By using the appropriate terms from (1., the ratio of single sideband-to-carrier power can be expressed as:

$$\mathcal{L}(f) = \left(\frac{\tfrac{1}{2}A_c\beta}{A_c}\right)^2 = \frac{\beta^2}{4} \tag{8.}$$

When a phase noise measurement system, as shown in Figure 4, is calibrated by offsetting the frequency of one of the inputs to the double-balanced mixer phase detector, the voltage, V_{cal}, of the sine wave out of the mixer at the difference frequency is usually measured by an rms indicating instrument. This gives the desired mixer transfer coefficient if scaled as follows:

$$\frac{\partial V}{\partial \Delta\phi} \frac{\text{volts peak}}{\text{radians peak}} = \left(V_{cal} \text{ volts rms}\right) \left(\sqrt{2}\ \frac{\text{volts peak}}{\text{volts rms}}\right) \tag{9.}$$

This is because the sinusoidal waveform of the mixer output has a slope, as it crosses the zero axis, expressed in volts per radian, equal to the peak amplitude of the wave in volts. Recall that the sine function has a peak value of one and also a slope at the origin of one.

Then, when a phase noise reading is taken, the indication will represent the actual phase excursions and again will be rms, that is V_{DSB} volts rms and:

$$V_{DSB} \text{ volts peak} = V_{DSB} \text{ volts rms} \left(\sqrt{2} \text{ volts peak/volts rms}\right) \tag{10.}$$

and the phase modulation is related to the mixer output by its transfer coefficient:

$$\beta \text{ radians peak} = \frac{V_{DSB} \text{ volts peak}}{\frac{\partial V}{\partial \Delta\phi} \frac{\text{volts peak}}{\text{radian peak}}} \qquad (11.$$

Substituting (9. and (10. into (11.:

$$\beta \text{ radians peak} = \frac{\sqrt{2}\, V_{DSB} \text{ volts rms}}{\sqrt{2}\, V_{cal} \text{ volts rms}} \qquad (12.$$

Then substituting (12. into (8.:

$$\mathcal{L}(f) = \left(\frac{V_{DSB}}{V_{cal}}\right)^2 \frac{1}{4}$$

$$\mathcal{L}(f) \text{ dB} = 20 \log \frac{V_{DSB}}{V_{cal}} - 6 \text{ dB} \qquad (13.$$

Changes in attenuator or gain control settings between calibration and measurement phase must also be taken into account. The signal level applied to the L port of the mixer should be as large as possible, within the mixer specification and should not be changed for either calibration or measurement phases. If it is planned to change the R port signal level between calibration and measurement, then during the calibration phase the R level should be run up to the highest level to be used, while monitoring the changes in V_{cal} to verify that the mixer is in a linear range. On the other hand the operator may choose to operate the mixer at high levels, where several dB of compression are occurring, to get the best system noise performance. This is valid provided that no signal level changes occur between calibration and measurement at the L and R ports of the mixer.

After offsetting the two oscillators to calibrate the system, they must be returned to the same frequency and the EFC loop locked. The mixer dc average output voltage should be checked to make sure it is near zero, preferably within a few millivolts, before and after making the phase noise measurement.

Thus far all the points discussed apply equally to a discrete spectral component (bright line) as well as to the random noise phase modulation contained in a specified noise bandwidth. Since most of the measuring instruments which might be used for these tests are designed to measure and are calibrated for discrete spectral components, equation (13. applies directly for them. Examples of these instruments are wave analyzers, spectrum analyzers, and tuned voltmeters.

In the case of random phase modulation, if the bandpass filtering function of the measuring instrument were followed by a true rms detector, followed by linear low-pass smoothing or averaging, then equation (13. would still apply. However, in order to give a reading linear in decibels, many analyzers utilize logarithmic IF amplifiers followed by an average-responding amplitude detector.

If the measurement system has a logarithmic conversion followed by an average detector before the final averaging or smoothing (which may be by visual estimate), then a factor of +2.5 dB must be applied to (13. above when measuring random noise (10, 12, 13, 14). Also the resolution bandwidth control setting is narrower than the actual noise bandwidth effective in the measurement so that a factor of -0.8 dB must also be applied to (13. above when measuring random noise (13). For ultimate accuracy, this factor should be checked, because many IF bandwidths are specified as ±10%. The resulting formula for random noise is then:

$$\mathcal{L}(f)\ \text{dB} = 20 \log \frac{V_{DSB}}{V_{cal}} - 4.3\ \text{dB} \qquad (14.$$

In the HP 3580A (and 3581A) Spectrum Analyzer, following the ac log amp and average-responding detector, there is a dc output to the rear panel for an external recorder. Equation (14. applies for this output because it is proportional to the detector output. However this signal is also fed into an analog-to-digital converter for internal display and storage. This converter operates in a time window during which its conversion algorithm increments the digital word whenever an excursion of the analog input exceeds the converted value. This has the effect of "peak grabbing" and seems to give a displayed result which is about one to two sigma of the detected signal above its long-term mean, a source of error from 2 dB for Gaussian to about 6 dB for non-Gaussian noise in the HP 3580A, when measuring random noise. The only defense against this is the use of maximum smoothing to minimize the variability of the signal being converted, during the conversion time window.

Under many desirable sweep and bandwidth conditions, the use of maximum smoothing will cause the "ADJUST" lamp to light. For the measurement of random noise having a gently sloping spectrum this is of no consequence. The meaning of the lamp is that the instrument will be sweeping past any discrete signals more rapidly than the smoothing circuit can respond so that their indicated amplitude will be depreciated and uncalibrated. A reasonable set-up procedure seems to be the adjustment of the bandwidth, sweep width and rate with the smoothing set at minimum, so that the most expeditious settings are obtained while keeping the adjust lamp off; then switching to maximum smoothing, which brings the adjust lamp on. Should bright lines be present, they will be almost as noticeable above the noise with maximum smoothing as without. If their presence is detected, then a measurement of their true amplitude can be made using settings which extinguish the adjust lamp. The only subtlety discovered in testing this technique occurs whenever a large number of closely spaced discrete lines can, in a wide sweep, masquerade as random noise. An example of this is 60 Hz and its harmonics viewed in a 10 kHz wide sweep. Harmonics through the 50th, to 3 kHz have been seen. In this example, a 10 Hz bandwidth was little help in resolving the true situation. This trap existed with both minimum and maximum smoothing but maximum smoothing makes such situations even less suspicious in appearance. The tactic for testing against this trap is to narrow the sweep width to no more than 100 times the bandwidth and use a sweep rate and smoothing which extinguish the adjust lamp. If, under these conditions the spectrum appears smooth, then there are no discrete lines separated by more than the bandwidth setting.

When measuring a rapidly sloping spectrum it is important to keep the measurement bandwidth narrow enough so that nearby components at a higher level do not come in through the skirt response of the IF and bias the measurement upward.

A measurement worksheet format, included with the figures, has been found useful in unifying the results of different workers. The one-tenth decade frequency steps facilitate the use of linear scaled graph paper, for later plotting, on which integer slopes are more readily recognized and manipulated. The steps are also 1/3 octave so that octave steps starting at any point and proceeding in either direction are pre-computed. This is also true of half decade points as well as five-per-decade points. The most important aspect of the worksheet is that it encourages the worker to record "Raw Readout" so that the scaling of the data may be questioned, reconstructed, and either vindicated or modified at a later date without repeating the measurement.

Individual Oscillator Phase Noise Characterization

The single sideband phase noise, $\mathcal{L}(f)$, of an individual oscillator at a particular frequency f can be deduced from pairwise measurements among three. The approach used here is analogous to one for $\sigma_y(\tau)$ developed by Gray and Allan (8). For $\mathcal{L}(f)$ expressed in decibels, the measured values will obey the relationship:

$$\text{Measurement} = 10 \log\left(\text{antilog}\ (\mathcal{L}_1(f)/10) + \text{antilog}\ (\mathcal{L}_2(f)/10)\right) \qquad (15.$$

For example: Measurements among three sources would yield the results depicted in the following examples:

	INDIVIDUAL UNIT PERFORMANCE	PAIR-WISE MEASUREMENT RESULT
UNIT 1	-90 dB	
		-89.6 dB
UNIT 2	-100	
		-97
UNIT 3	-100	
		-89.6
UNIT 1	-90	
UNIT 4	-90	
		-89.6
UNIT 5	-100	
		-89.6
UNIT 6	-90	
		-87
UNIT 4	-90	

	INDIVIDUAL UNIT PERFORMANCE	PAIR-WISE MEASUREMENT RESULT
UNIT 7	-94	
		-92.2
UNIT 8	-97	
		-95.2
UNIT 9	-100	
		-93
UNIT 7	-94	

Note that in units 1 through 6 the individual unit performance numbers are simple and repetitious. Now consider the measurement results to discern how these results indicate the particular equalities and differences which exist between the individual units. In particular, the fact that $\mathcal{L}_{12}$ equals $\mathcal{L}_{13}$ means that units 2 and 3 are equal. Then each alone must be 3 dB better than the measured result for $\mathcal{L}_{23}$. Once this is known, equation (15. can be used to find the individual performance of unit 1.

The example with units 4, 5 and 6 is offered to allow the reader to check the understanding gained from the first trio.

The third example with units 7, 8 and 9 is more representative of the range of performance typically encountered. Note that in this last example, the differences from the median measurement of -93 dB are only +0.8 and -2.2 which could have been brushed aside as experimental error or scatter. This would lead to the interpretation that all three sources are substantially equal, and therefore about -93 -3 = -96 dB. This would be an unfortunate misuse of the data because it ignores significant differences among the three sources of four times the noise power in the sidebands of the -94 dB source relative to the -100 dB source.

Equation (15. and the examples are based on simple addition of the individual power levels of uncorrelated noise. Expressed in units of watts, this can be stated as:

$$P_{12} = P_1 + P_2 \qquad (16.$$

where P_{12} is a measurement of signal 1 versus signal 2. Given three measurements, the individual source performance can be computed. A formula for this is derived as follows:

$$P_{ab} = P_a + P_b \tag{17.}$$

$$P_{bc} = P_b + P_c \tag{18.}$$

$$P_{ac} = P_a + P_c \tag{19.}$$

then:

$$P_a = -P_c + P_{ac}, \quad P_c = -P_b + P_{bc},$$

$$P_b = -P_a + P_{ab}$$

and by substitution:

$$P_a = + P_b - P_{bc} + P_{ac}$$

$$P_a = - P_a + P_{ab} - P_{bc} + P_{ac}$$

$$2P_a = P_{ab} + P_{ac} - P_{bc}$$

$$P_a = \frac{P_{ab} + P_{ac} - P_{bc}}{2} \tag{20.}$$

and similarly for P_b and P_c:

$$P_b = \frac{P_{bc} + P_{ab} - P_{ac}}{2} \tag{21.}$$

$$P_c = \frac{P_{ac} + P_{bc} - P_{ab}}{2} \tag{22.}$$

When these noise sideband powers are expressed in dB power ratio relative to the carrier, equation (20. becomes:

$$\mathcal{L}_a(f)\mathrm{dB} = 10 \log\left\{\frac{1}{2}\left[\mathrm{antilog}(\mathcal{L}_{ab}/10) + \mathrm{antilog}(\mathcal{L}_{ac}/10) - \mathrm{antilog}(\mathcal{L}_{bc}/10)\right]\right\} \tag{23.}$$

and again the form is the same for equations (21. and (22. with appropriate subscript changes.

For development work, after an individual oscillator of good performance has been characterized by this procedure, it can serve as a measurement reference for testing oscillators as much as 3 dB better. Only a single measurement is required, the result being interpreted in the light of the known performance of the reference oscillator by the use of equation (15. This relationship is plotted in Figure 5 so that a measurement against a known reference can be converted graphically to the $\mathcal{L}(f)$ of the unknown. The plot also makes it convenient to notice the uncertainty range of the result versus the uncertainty range of the measurement (respectively, 3 to 1 for $\mathcal{L}_{DUT} - \mathcal{L}_{REF} = -3$ dB).

The function plotted in Figure 5 can be derived as follows. The desired expression has the form:

$$\mathcal{L}_{DUT} - \mathcal{L}_{REF} = G\left(\mathcal{L}_{MEAS} - \mathcal{L}_{REF}\right)$$

because $\mathcal{L}_{DUT}$ (device under test) is the desired result and $\mathcal{L}_{REF}$ (reference) and $\mathcal{L}_{MEAS}$ (measurement data) are the known quantities and G represents the function to be derived. Starting with a version of equation (15:

$$\mathcal{L}_{MEAS} = 10 \log \left(\text{antilog } (\mathcal{L}_{REF}/10) + \text{antilog } (\mathcal{L}_{DUT}/10)\right) \quad (24.$$

Rearranging:

$$\mathcal{L}_{DUT} = 10 \log \left(\text{antilog } (\mathcal{L}_{MEAS}/10) - \text{antilog } (\mathcal{L}_{REF}/10)\right) \quad (25.$$

Subtracting $\mathcal{L}_{REF}$ from both sides:

$$\mathcal{L}_{DUT} - \mathcal{L}_{REF} = 10 \log \left(\text{antilog } (\mathcal{L}_{MEAS}/10 - \text{antilog } (\mathcal{L}_{REF}/10)\right) - 10 \log \left(\text{antilog } (\mathcal{L}_{REF}/10)\right)$$

$$= 10 \log \frac{\text{antilog } (\mathcal{L}_{MEAS}/10) - \text{antilog } (\mathcal{L}_{REF}/10)}{\text{antilog } (\mathcal{L}_{REF}/10)}$$

$$\mathcal{L}_{DUT} - \mathcal{L}_{REF} = 10 \log \left(\text{antilog } \left((\mathcal{L}_{MEAS} - \mathcal{L}_{REF})/10\right) - 1\right) \quad (26.$$

It is important to keep in mind that a measurement result represents the combined performance of the two units. The simplest accurate rule to remember is that the noise powers contributed by the two units add. There has been a prevalent practice of assuming the two devices under test to have equal performance since little if any other information existed. This rationale is used to justify scaling the measurement result down by 3 dB, then ascribing this performance to both units. Since no other facts exist this assumption is usually allowed to stand unquestioned.

Closer examination of the assumption can begin by hypothesizing a population of units whose performances scatter by +2 dB and -2 dB for one-sigma or standard deviation about the mean for all units. For any realistic distribution shape, it should seem highly unlikely that any two units would exhibit equal performance, even within one decibel. If the purpose of the measurement was to determine how good either of the units might be, then the most pessimistic assumption is 3 dB below the measurement. However if a purpose of the measurement was to determine how poorly a unit might perform, then the assumption of 3 dB below the measurement is not only the most optimistic but also the least likely.

If an assumption must be made, it may be much more supportable engineering judgement to assume a "two source correction factor" between one and two decibels below the measurement, as indicated on Figure 6 (a replot of Figure 5). However this is not recommended practice. It is more informative to report the measurement with no correction.

CONVERSION OF DATA BETWEEN TIME & FREQUENCY DOMAINS

The procedure for converting frequency domain data into time domain or vice versa can be approached in a number of different ways, all valid. This is mostly a matter of personal preference just as any two people may go about solving a given algebraic equation with minor differences but both will agree on the result. The object here is to present a sequence which carries a mnemonic thread of logic, possibly at the expense of brevity, by not "skipping steps" or combining them.

Step one is the choosing of a single portion of the data under consideration which can be represented by a straight line on a dB-log frequency or log-log plot. This will be repeated until all portions of interest are covered. The logic or assumption applied to this step is: "If this straight line represented the total (broad range) and only performance characteristic of the oscillator, it can be converted to the other domain to see what it would look like there."

From this logic follows the most fundamentally important point to be considered in interpreting the results: As each portion of an actual performance curve is approximated by a straight line segment, converted and re-plotted, it has been handled by the mathematics as if it were independent of the other segments. This places the burden on the user to combine the segments into a new curve through a reasonable and logical interpretation. Step three will expand on this later.

Next comes the detailed technique of matching a straight line segment to a smoothly curving and/or randomly scattered plot: A basic constraint is the fact that conversion formulas only exist for particular slopes, and for only five different slopes. In the frequency domain for $\mathcal{L}(f)$ in decibels, these range from flat, white noise of phase or f^{0}, to f^{-4}, random walk of frequency, or -40 dB per decade of frequency, with the intervening slopes occuring with integer steps of the exponent on f, which correspond to 10 dB steps per decade. A "map" of the slopes is shown in Figure 7. It is only reasonable to arbitrarily choose a "convertible" slope. Align parallel rules, or the edge of a triangle opposite a reference straight edge, versus the graduations on the axes of the plot, to the chosen slope, then slide it into the region of the data while maintaining the slope constant. If a region of the data can be found which seems asymptotic to the trial slope, that is within three decibels over a decade of frequency, then an accurately representative straight line segment has probably been found.

Visual averaging of measured data exhibiting scatter or randomness which has been plotted on a logarithmic scale must be done with special consideration of the illusionary effect of the log scale. For example, imagine a linear plot of data points scattered symmetrically about their true mean, that is no skew in their distribution. In this case the mean and the median would very likely coincide quite closely and could be discerned visually with an accuracy of a tiny fraction of a standard deviation. Now imagine this same ensemble of data being replotted on a log scale. The scatter of the values larger than the mean will be compressed by the logarithmic action. Correspondingly, the scatter of the values less than the mean will be expanded. The net effect is to impart a visual skew to the distribution so that if a mean is visually estimated on the log plot, it will be low by a major fraction of a standard deviation.

A defense against this misinterpretation is found in the coincidence of the true mean and the median. This is accomplished by disciplining the eye to estimate a line through the data points such that half the points lie on each side without regard to how far away they may be. The importance of this technique becomes very apparent whenever the total scatter of the data approaches one-half decade where interpretation errors of 30% can occur.

Any straight line is completely specified by its slope and an intercept. The most convenient intercepts to use in frequency stability analysis are one hertz and one second. (Please do not infer any general correspondence between one hertz and one second performances. Experience shows this to be coincidental at best.)

The completion of the first step is writing down the slope and intercept of the line through the data.

The second step is the choosing of the conversion formula, from Table 1, corresponding to the data domain (frequency or time) and slope, then plugging in the intercept and "turning the crank" to calculate the result. Each of the conversion formulas contains both f and τ variables raised to powers appropriate to both cancel the slope of the incoming slope-intercept data as well as establish the slope of the result. The presence of these terms gives rise to the convenience of choosing them equal to unity, then converting only the intercept point, while the slope conversion becomes obvious by inspection of the exponents on f and τ.

The second step is completed by writing down the computed intercept value and slope of the converted line. This result can be assumed to specify a straight line on a log-log plot in the new domain. In the time domain, the slopes change in steps of $\tau^{1/2}$, and range from τ^{-1} for white noise of phase to $\tau^{+1/2}$ for random walk of frequency, with one exception and an additional case.

The exception appears when converting from $\mathcal{L}(f)$ of slope f^{-1}, flicker of phase, to $\sigma_y(\tau)$, which will yield a plot having a slope very near -0.95. This one case is less obvious from the form of the equation, but being unique, is easily remembered and recognized. Also in this case, the simple slope-intercept interpretation of the conversion result sacrifices some accuracy and should be avoided until the user has enough familiarity with the results versus his application to determine whether the error may be negligible. The preferred method is the computation of three values for $\sigma_y(\tau)$, widely separated in τ, over the range of interest, using the conversion formula.

The additional case is $\sigma_y(\tau)$ data having τ^{+1} slope. This does not appear in conversions from $\mathcal{L}(f)$, frequency domain, but may be seen in actual $\sigma_y(\tau)$ data for $\tau \geq 100$ seconds for quartz oscillators. It can be shown that constant-rate aging drift of frequency results in τ^{+1} slope of $\sigma_y(\tau)$. Slowly changing ambient thermal effects can be responsible for τ^{+1} slope below 100 seconds.

The third step in the conversion process is the plotting of the converted straight lines on log-log axes of the new domain. This begins with locating the intercept points, then drawing a line through each point with the corresponding slope. This slope is indicated by the exponent in the conversion formula for the independent variable, the horizontal axis, frequency or time.

Interpretation of the straight lines in order to combine them into a single smooth curve is quite similar to Bode plot work. The final curve is, at each point, the sum of the component straight lines. From this follows the fact that where two lines intersect, the smooth curve is 3 dB higher for $\mathcal{L}(f)$ or 1.414 for $\sigma_y(\tau)$. This is because noise powers add, and variances, $\sigma_y^2(\tau)$, also add.

Since the equations of each of the straight lines are the results of step two above, these equations can be programmed into a desk calculator with a plotter and the final result curve (and a tabulation of numerical results) can be obtained directly. The relationships to be computed are:

$$\mathcal{L}(f) = \mathcal{L}_1 + \mathcal{L}_2 + \cdots \qquad (27.$$

For $\mathcal{L}(f)$ expressed as a power ratio. To convert from decibels:

$$\mathcal{L}(f)_{\text{power ratio}} = \text{antilog}\left(\frac{\mathcal{L}(f)\ \ \text{dB}}{10}\right) \qquad (28.$$

And for $\sigma_y(\tau)$:

$$\sigma_y(\tau) = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots} \qquad (29.$$

Since the slope of a group of data points is of similar importance to their magnitude, it is helpful to standardize the choice of scales for plotting data. This fosters visual familiarity which increases with experience. The alternative is to use a variety of scales for plotting, which presents the eye/brain with an assortment of optical illusions.

The recommended standards are:

1. Plot σ_y vs. τ on log log scales chosen so that a decade of each variable is the same length. Further, that on both axes linear subdivisions within decades be used and labeled with their logarithmic values.

2. Plot $\mathcal{L}$ vs. f on scales linear in dB for $\mathcal{L}$ and log for f such that the length of 20 dB of $\mathcal{L}$ equals the length of one decade of f. Further, that linear subdivisions be used for both $\mathcal{L}$ and f, and that the subdivisions within the decades of f be labeled with their logarithmic values.

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APPENDIX A

Effect of Sinusoidal Phase Modulation on $\sigma_y(\tau)$

Short term stability of frequency sources is usually characterized by the square root of Allan variance, $\sigma_y(\tau)$, where y is fractional frequency deviation, and τ is the averaging time or measurement interval. To measure an estimate of $\sigma_y(\tau)$, a series of M measurements of $\bar{y}$ are made and entered into:

$$\sigma_y{}^2(\tau) \sim \frac{1}{2(M-1)} \sum_{k=1}^{M-1} \left(\bar{y}_{k+1} - \bar{y}_k\right)^2 \qquad \text{(A1.}$$

where $\bar{y}$ is the average y over an interval τ long.

The worst case effect of sine wave frequency modulation of modulating frequency f_m will occur when τ is equal to τ_p, the period of one half-cycle of f_m, and the measurement intervals are phased relative to f_m to catch the maximal excursions.

The fractional frequency deviation y can be stated:

$$y = \frac{\Delta f_o}{f_o} \qquad \text{(A2.}$$

where f_o is the carrier frequency.

For one half-cycle of f_m, the average to peak ratio of a sinusoid is $2/\pi$ and :

$$\bar{y} = \frac{2}{\pi} \frac{\Delta f_o \text{ peak}}{f_o} \qquad \text{(A3.}$$

and the next half-cycle $\bar{y}$ result would have the same magnitude and opposite sign.

Substituting into (A1,simplifying and letting M=2 because the result is independent of the number of measurements:

$$\sigma_y{}^2(\tau_p) \sim \frac{1}{2}\left(\left(\frac{2}{\pi} \frac{\Delta f_o \text{ peak}}{f_o}\right) - \left(-\frac{2}{\pi} \frac{\Delta f_o \text{ peak}}{f_o}\right)\right)^2$$

$$\sigma_y{}^2(\tau_p) = \frac{8}{\pi^2} \left(\frac{\Delta f_o \text{ peak}}{f_o}\right)^2 \qquad \text{(A4.}$$

$$\sigma_y(\tau_p) = \frac{\sqrt{8}}{\pi} \; \frac{\Delta f_o \text{ peak}}{f_o} \qquad \text{(A5.}$$

From modulation theory, the peak phase deviation or modulation index β is:

$$\beta = \frac{\Delta f_o \text{ peak}}{f_m} \qquad \text{(A6.}$$

rearranging:

$$\Delta f_{o \text{ peak}} = \beta \; f_m \qquad \text{(A7.}$$

A bright line of phase modulation whose level is specified as the ratio between a single sideband and the carrier as $\mathcal{L}_{PM}$ (in the same way as random noise) is indistinguishable from FM and:

$$\mathcal{L}_{PM} = \frac{\beta^2}{4} \qquad \text{(A8.}$$

Solving for β with $\mathcal{L}_{PM}$ expressed in decibels:

$$\beta = \sqrt{4 \text{ antilog } (\mathcal{L}_{PM} \text{ dB}/10)} \qquad \text{(A9.}$$

Substituting into (A7:

$$\Delta f_{o \text{ peak}} = f_m \sqrt{4 \text{ antilog } (\mathcal{L}_{PM} \text{ dB}/10)} \qquad \text{(A10.}$$

Substituting into (A5:

$$\sigma_y(\tau_p) = \frac{2\sqrt{8}}{\pi} \; \frac{f_m}{f_o} \sqrt{\text{antilog } (\mathcal{L}_{PM} \text{ dB}/10)} \qquad \text{(A11.}$$

And as was stated earlier:

$$\tau_p = \frac{1}{2 f_m} \qquad \text{(A12.}$$

A more rigorous derivation of (A11. shows that the general case includes a term of the form $(\sin \pi \tau)/(\pi \tau)$. This causes the function to have lobes whose peaks fall off as τ^{-1}. Incorporating a τ^{-1} term and adjusting coefficients to agree with (A11. gives:

$$\sigma_y(\tau_p) = \frac{\sqrt{8}}{\pi f_o} \sqrt{\text{antilog } (\mathcal{L}_{PM} \text{ dB}/10)} \quad \tau^{-1} \qquad \text{(A13.}$$

which, given $\mathcal{L}_{PM}$, is a worst case predictor of $\sigma_y(\tau)$. Conversely, given $\sigma_y(\tau)$, equation (A13. yields a minimum value for $\mathcal{L}_{PM}$. Solving for $\mathcal{L}_{PM}$:

$$\mathcal{L}_{PM} \text{ dB} = 10 \log \left(\frac{\tau \pi f_o \; \sigma_y(\tau)}{\sqrt{8}} \right)^2 \qquad \text{(A14.}$$

In the case where a frequency source output is displayed on a spectrum analyzer so that the upper and lower sidebands can be examined separately, if they are found to be asymmetrical, that is different amplitudes above versus below the carrier, then this indicates that both am and pm exist, of comparable modulation index, and correlated.

APPENDIX B

Interrelationships of Various Frequency Domain Stability Measures

$S_\phi(f)$ $\quad$ rad^2/Hz

$\equiv$ spectral density of variance of phase fluctuations per hertz of bandwidth at sideband frequency f in $V(\tau) = \cos(2\pi f_0 \tau + \phi)$

$= S_{\delta\phi}(f)$ (used in some publications 1971-1974)

$= 2\,\mathcal{L}(f)$

$$= \frac{\text{power in both upper and lower phase modulation sidebands per hertz of bandwidth at sideband frequency } f}{\text{carrier power}}$$

(Some definitions of $S_\phi(f)$ are, in effect, the above ratio by virtue of their use of a mixer as a phase detector. This definition holds only for broadband $\Delta\phi_{peak} < 0.1$ radian.)

$$= \frac{f_0^2}{f^2} S_y(f)$$

$$= \frac{\beta^2}{2} = \left(\frac{\Delta f_{rms}}{f_m}\right)^2 = \left(\frac{\Delta\omega_{rms}}{\omega_m}\right)^2$$

= mean square modulation index

$S_\phi(f)$ $\quad$ dB (re 1 rad^2/Hz)

$= 10 \log S_\phi(f)$

$= \mathcal{L}(f)\ \text{dB} + 3$

$\mathcal{L}(f)$ dimensionless power ratio

$$\equiv \frac{\text{power in phase modulation single sideband per hertz of bandwidth at sideband frequency } f}{\text{carrier power}}$$

(defined only for $\Delta\phi_{peak} < 0.1$ radian)

$$= \frac{1}{2} S_{\phi}(f)$$

$$= \frac{c_o^2}{2f^2} S_y(f)$$

$$= \frac{\beta^2}{4}$$

$\mathcal{L}(f)$ dB, dBc (re carrier)

$$= 10 \log \mathcal{L}(f)$$

$$= S_{\phi}(f) \text{ dB} - 3$$

APPENDIX C

Comments on Symbology

There has been some effort to restrict the use of the symbol f to mean only Fourier frequency (in the context of oscillator stability, at least). It is the author's view that the symbol f is well established as a quite general symbol for frequency (with units of hertz strongly implicit due to convention). Accordingly, any specialization or limitation of meaning will only be accomplished by the use of a subscript or a different symbol entirely. Any attempt to the contrary flies in the face of a universe of deeply ingrained convention.

One specific problem with the restriction of f to mean sideband frequency is that f_0 could no longer be used for carrier frequency without confusion. The next alternative for a carrier frequency symbol would, by usage, be ν. This is very familiar and not confusing to physicists. However, the major number of people who will be trying to decipher the things we are writing today are not physicists, and the confusion between ν and v make its choice questionable.

The signals whose phase instabilities we are treating may be modeled in voltage versus time as:

$$V(t)=\cos\left(\omega_0 t + \phi(t)\right) \qquad \text{(C1.}$$

The analysis of the ϕ term versus frequency is often discussed as Fourier frequency. These become the sideband frequencies when the total spectrum of an actual signal is discussed. It seems much preferable to use the specific term "sideband frequency" when that is specifically what is being discussed, and reserve the use of the term "Fourier frequency" for occasions when its implicit generality is intended to be part of the concept being communicated.

The use of the term Fourier frequency could call to the reader's mind two or three pictures (definitions), only one of which is correct and intended by the author if he actually means sideband frequency. These are shown in Figure C1.

APPENDIX D

Time Domain Measurement Bias Functions

Definition: $\sigma_y(\tau)$ implies N=2 and T=τ, no dead time.

```
| Measurement Time | Dead Time | Measurement Time |
|                  |           |                  |
|-------- τ -------|           |-------- τ -------|
|                              |
|------------- T --------------|
```

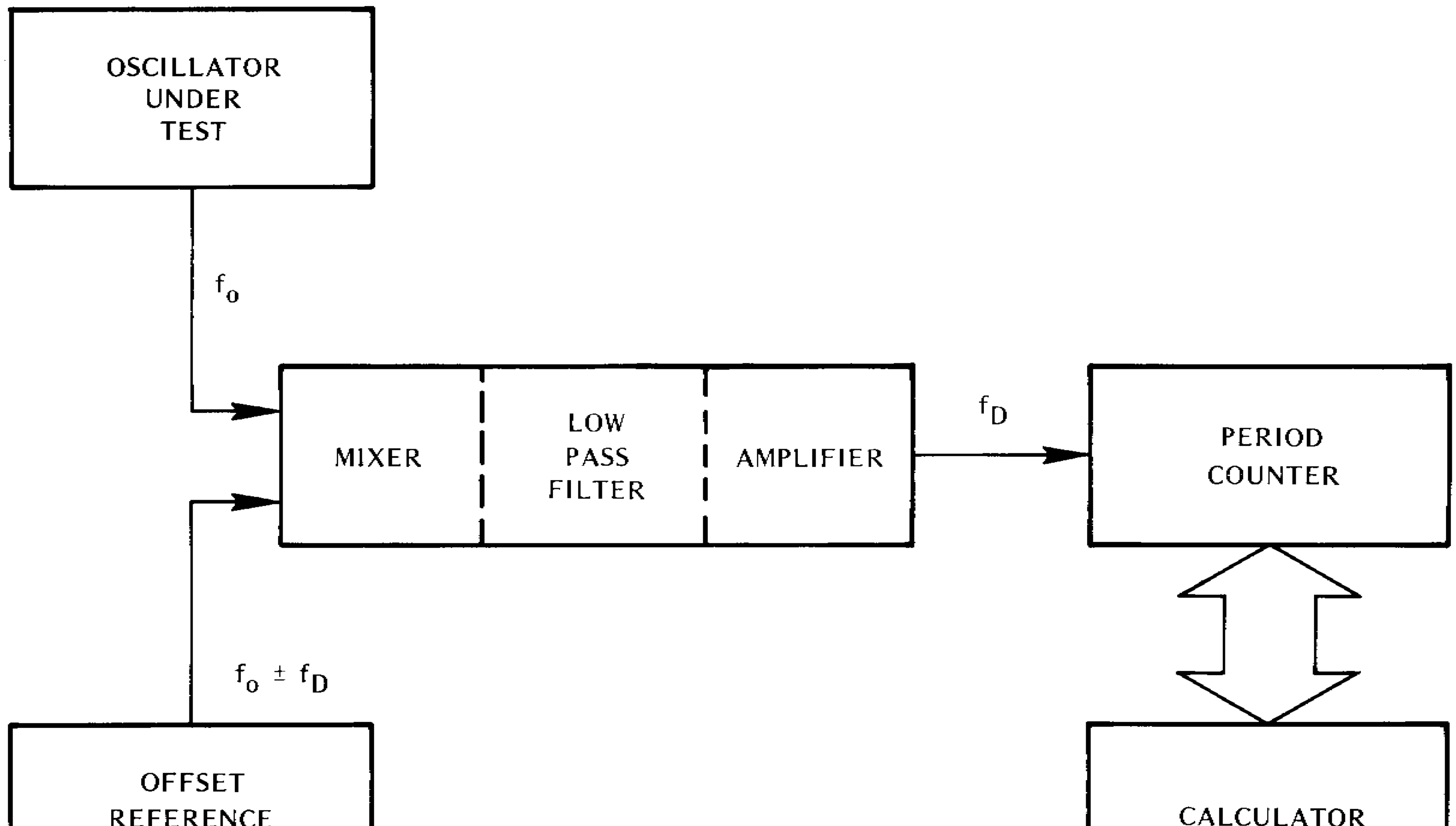
$$r = T/\tau$$

$$B_2 = \frac{\sigma_y^2(2,T,\tau)}{\sigma_y^2(\tau)} \qquad \sigma_y(\tau) = \frac{\sigma_y(2,T,\tau)}{\sqrt{B_2}}$$

$B_2(r,\mu)$

	μ, Slope of Allan Variance, $\sigma_y^2(\tau) \propto \tau^{\mu}$				
r	-2	-1	0	1	2
1.00	1.00	1.00	1.00	1.00	1.00
1.01	0.67	1.00	1.01	1.015	1.02
1.10	0.67	1.00	1.09	1.15	1.21
2.00	0.67	1.00	1.57	2.50	4.00
4.00	0.67	1.00	2.08	5.50	16.00
8.00	0.67	1.00	2.58	11.50	64.00
16.00	0.67	1.00	3.08	23.50	256.0
32.00	0.67	1.00	3.58	47.50	1024.

NOTE: There is no dead time small enough to be negligible for white noise of phase, unless a 15% error in $\sigma_y(\tau)$ is tolerable.



$$\frac{\Delta f}{f} = \frac{f_D^2 \, \Delta\tau}{f_o}$$

Figure 1. **Heterodyne Frequency Measurement Method**

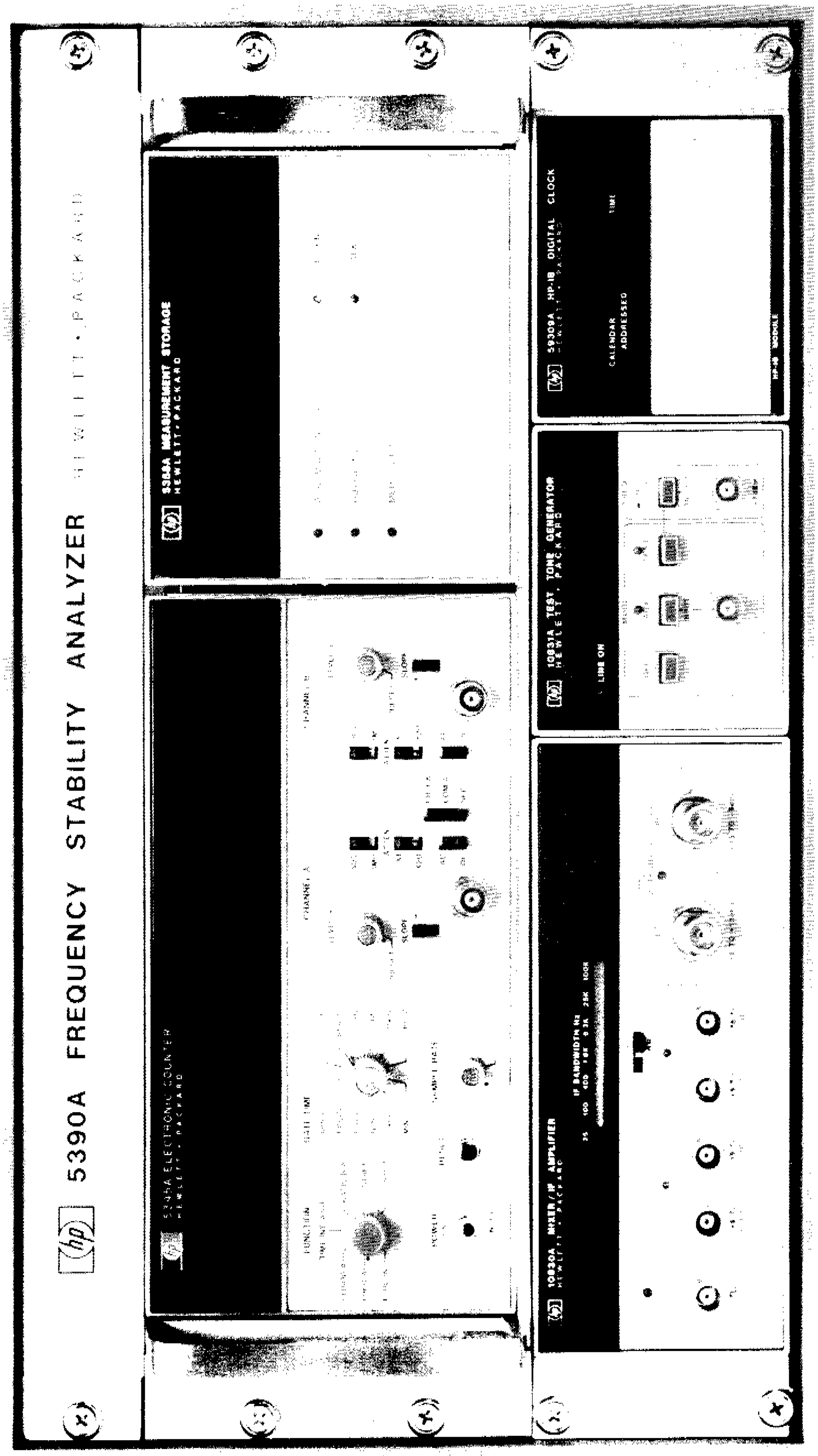


Figure 2. 5390A FREQUENCY STABILITY ANALYZER

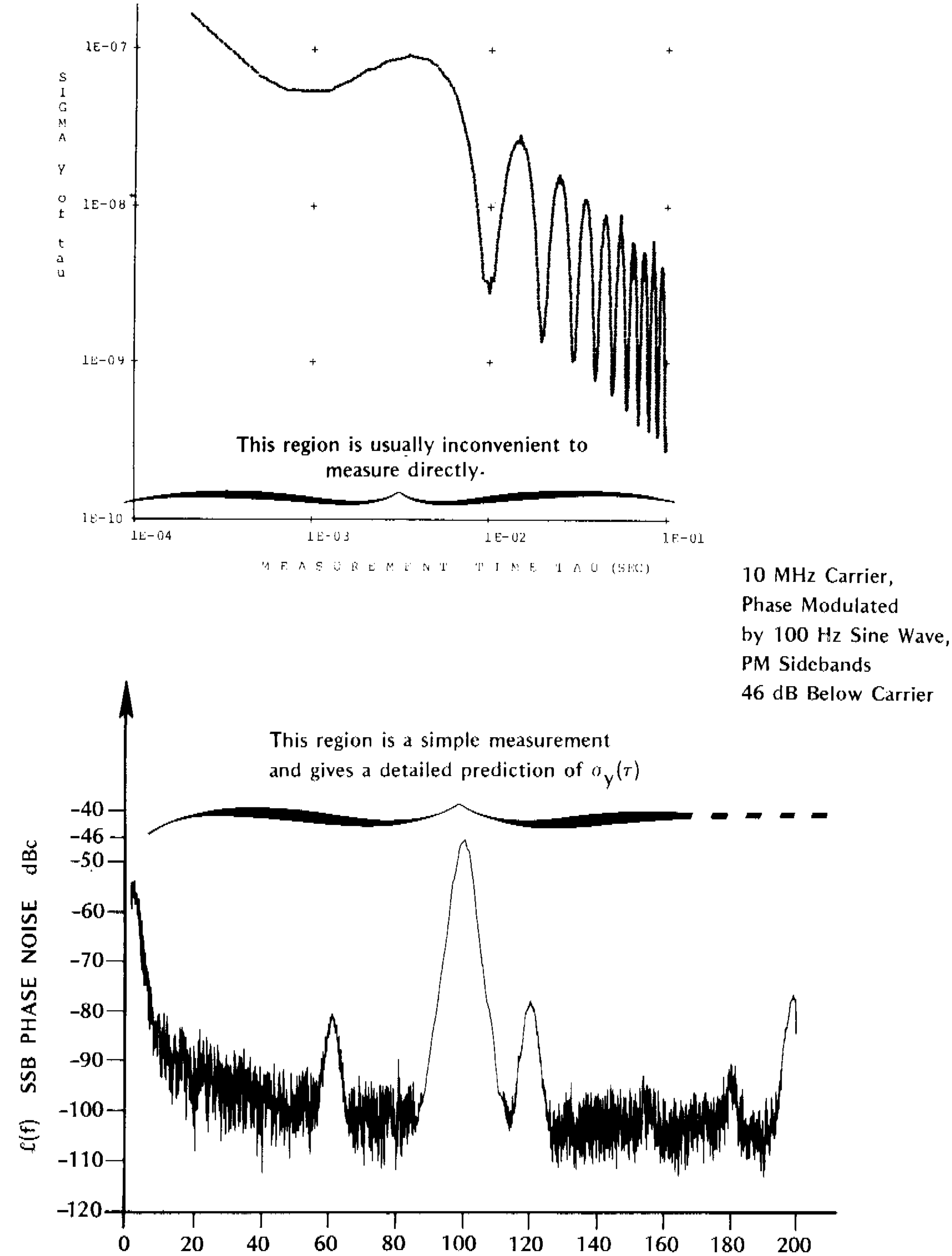


Figure 3. Comparison of σ_y & $\mathcal{L}(f)$ Data

TABLE 1

	$\mu =$	$\sigma_y(\tau) =$	$\mathcal{L}(f) =$	$\beta =$
WHITE PHASE	−2	$\sqrt{\mathcal{L}(f)\ 6\ f_h} \big/ 2\pi\ f_o\ \tau$	$(\sigma_y(\tau)\tau\ 2\pi\ f_o)^2 \big/ 6\ f_h$	0
FLICKER PHASE	−1.9	$\sqrt{\mathcal{L}(f)\ f\ (1.269 + \ln\ (2\pi\ f_h\ \tau))/6.58} \big/ f_o\tau$	$6.58(\sigma_y(\tau)\tau\ f_o)^2 \big/ (1.269 + \ln\ (2\pi\ f_h\ \tau))f$	−1
WHITE FREQ.	−1	$\sqrt{\mathcal{L}(f)\ f^2} \big/ f_o\ \tau^{1/2}$	$(\sigma_y(\tau)\tau^{1/2}\ f_o)^2 \big/ f^2$	−2
FLICKER FREQ.	0	$2\sqrt{\mathcal{L}(f)\ f^3\ \ln\ 2} \big/ f_o$	$(\sigma_y(\tau)\ f_o)^2 \big/ 4\ (\ln\ 2)\ f^3$	−3
RANDOM WALK FREQ.	+1	$\sqrt{\mathcal{L}(f)\ f^4}\ 2\pi\ \tau^{1/2} \big/ \sqrt{6}\ f_o$	$0.75\ (\sigma_y(\tau)\ \tau^{-1/2}\ f_o/\pi)^2 \big/ f^4$	−4

τ = measurement time, $y = \Delta f_o/f_o$, f_o = carrier, f = sideband frequency, f_h = measurement system bandwidth

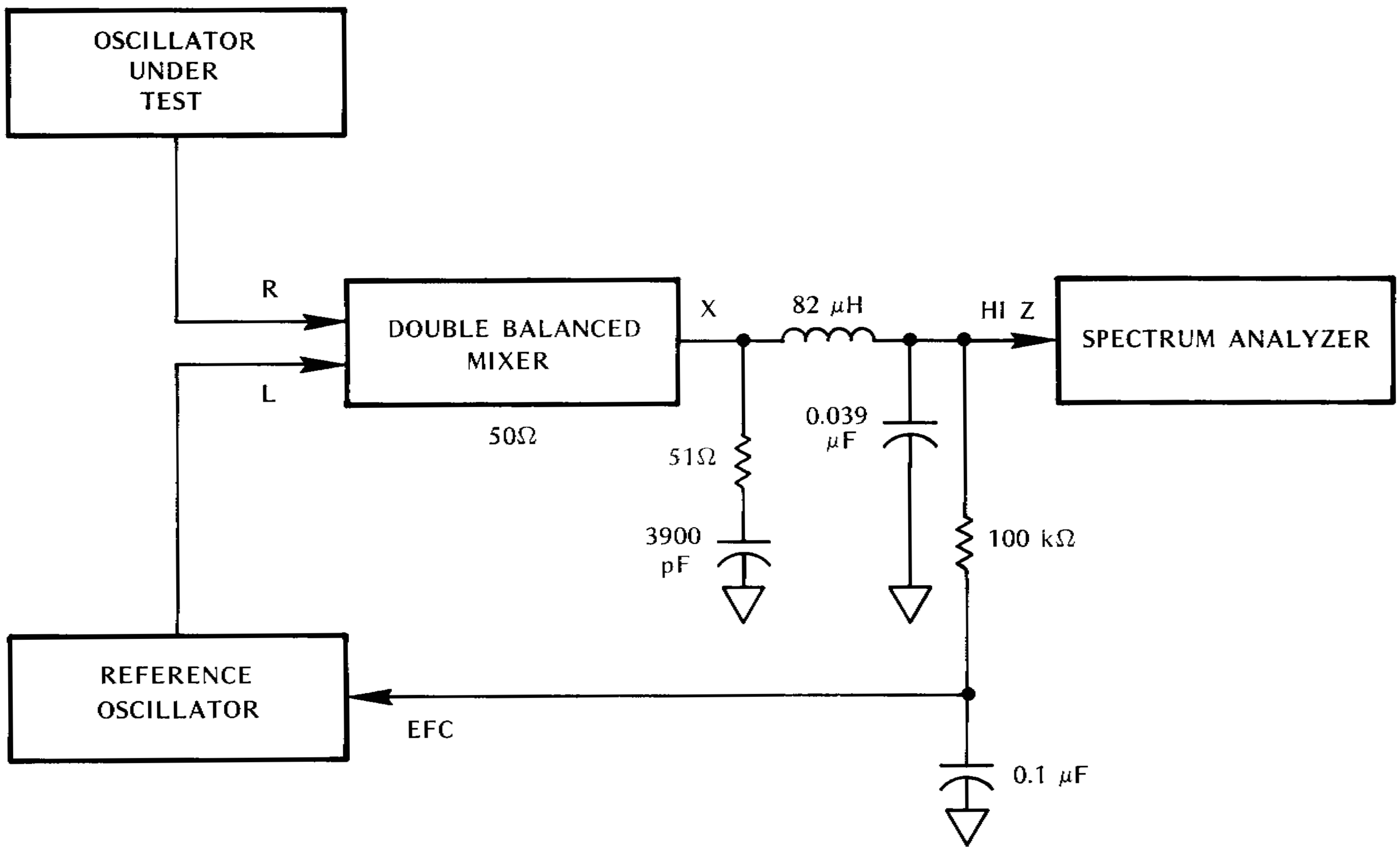


Figure 4. Phase Noise Measurement Method

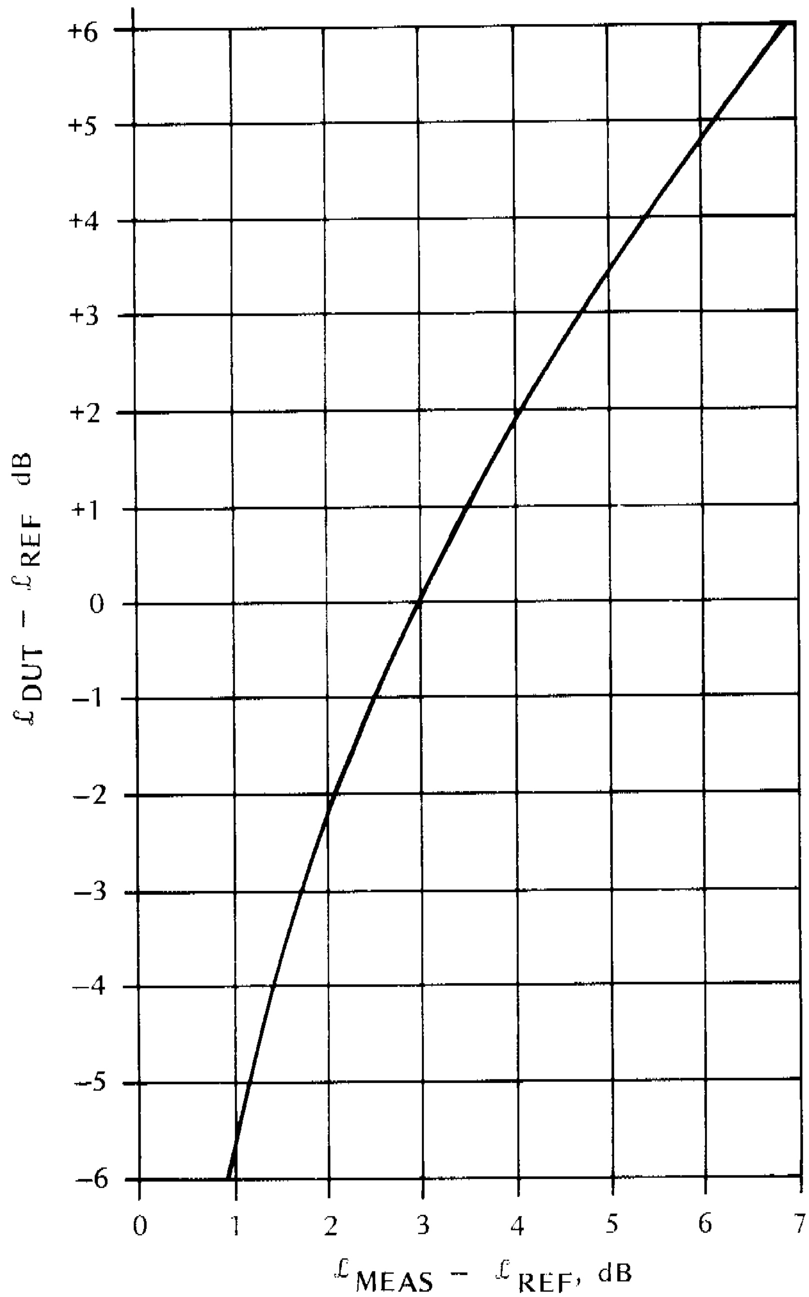


Figure 5. Combined Effect of Two Noise Sources on a Measurement

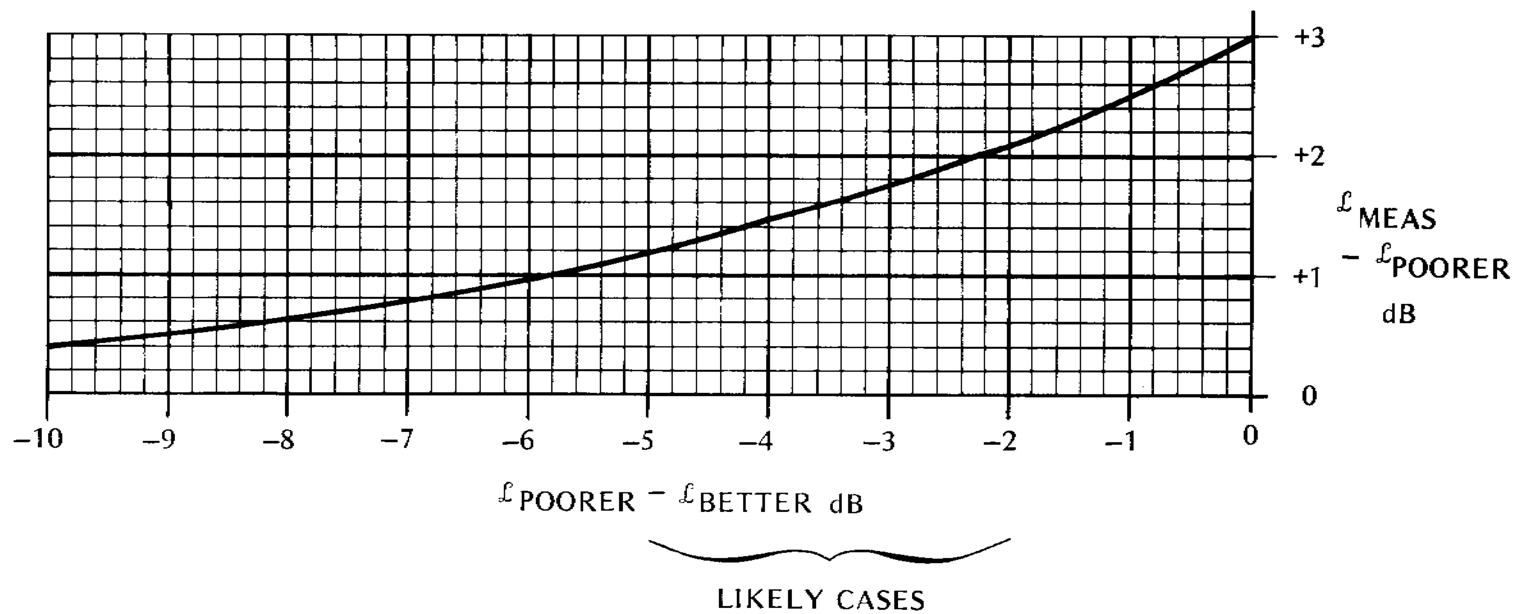


Figure 6. Combined Effect of Two Noise Sources on Measurement

Single Sideband Phase Noise Measurement Worksheet
Using a Double-Balanced Mixer and HP 3580A or 3581A

L port signal level should be the maximum available, up to the mixer specs, and must remain constant for both calibrate and measure.

R port signal level must be varied before calibration to verify mixer linearity.

Freq. Hz	Raw Readout dB	-(Syst. Resp.) dB	+(Scale Factor) dB	=(Meas. Data) dB
1.0				
1.3				
1.6				
2.0				
2.5				
3.2				
4.0				
5.0				
6.3				
8.0				
10				
13				
16				
20				
25				
32				
40				
50				
63				
80				
100				
130				
160				
200				
250				
320				
400				
500				
630				
800				
1.0k				
1.3k				
1.6k				
2.0k				
2.5k				
3.2k				
4.0k				
5.0k				
6.3k				
8.0k				
10k				
13k				
16k				
20k				
25k				
32k				
40k				
50k				
63k				
80k				
100k				

TEST CONDITIONS AND SCALE FACTOR COMPUTATION

Calibrate Freq ( ) Hz

CALIBRATE
-( ) dB RAW READOUT
+( ) dB SYST. RESP. @ CAL. FREQ.
+( ) dB R PORT ATTEN
-( ) dB INPUT SENSITIV

MEASURE
-( ) dB R PORT ATTEN
+( ) dB INPUT SENSITIV
-6 dB RADIANS TO SSB

[ ] dB BRIGHT LINE SCALE FACTOR

"ADJUST" LAMP? ( )

-( ) dB=10 log(Hz BW)

+1.7 dB +2.5 logging
-0.8 Gauss BPF

[ ] dB RANDOM NOISE SCALE FACTOR

SMOOTHING? ( )

UNIT UNDER TEST:

REFERENCE SIGNAL SOURCE:

MIXER:

AC AMP:

DC AMP:

NAME

DATE

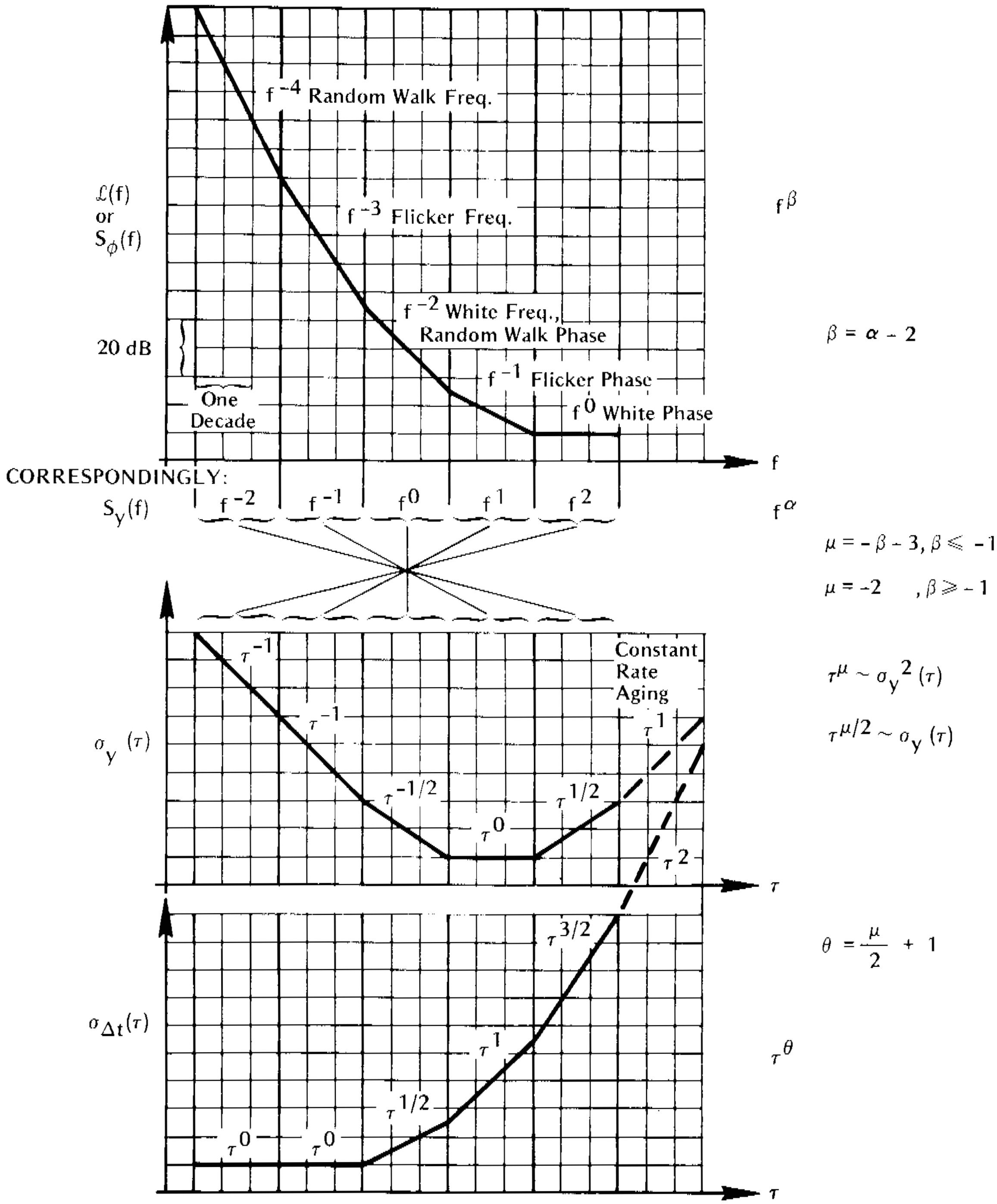


Figure 7. Interrelationships of Various Random Frequency Instabilities

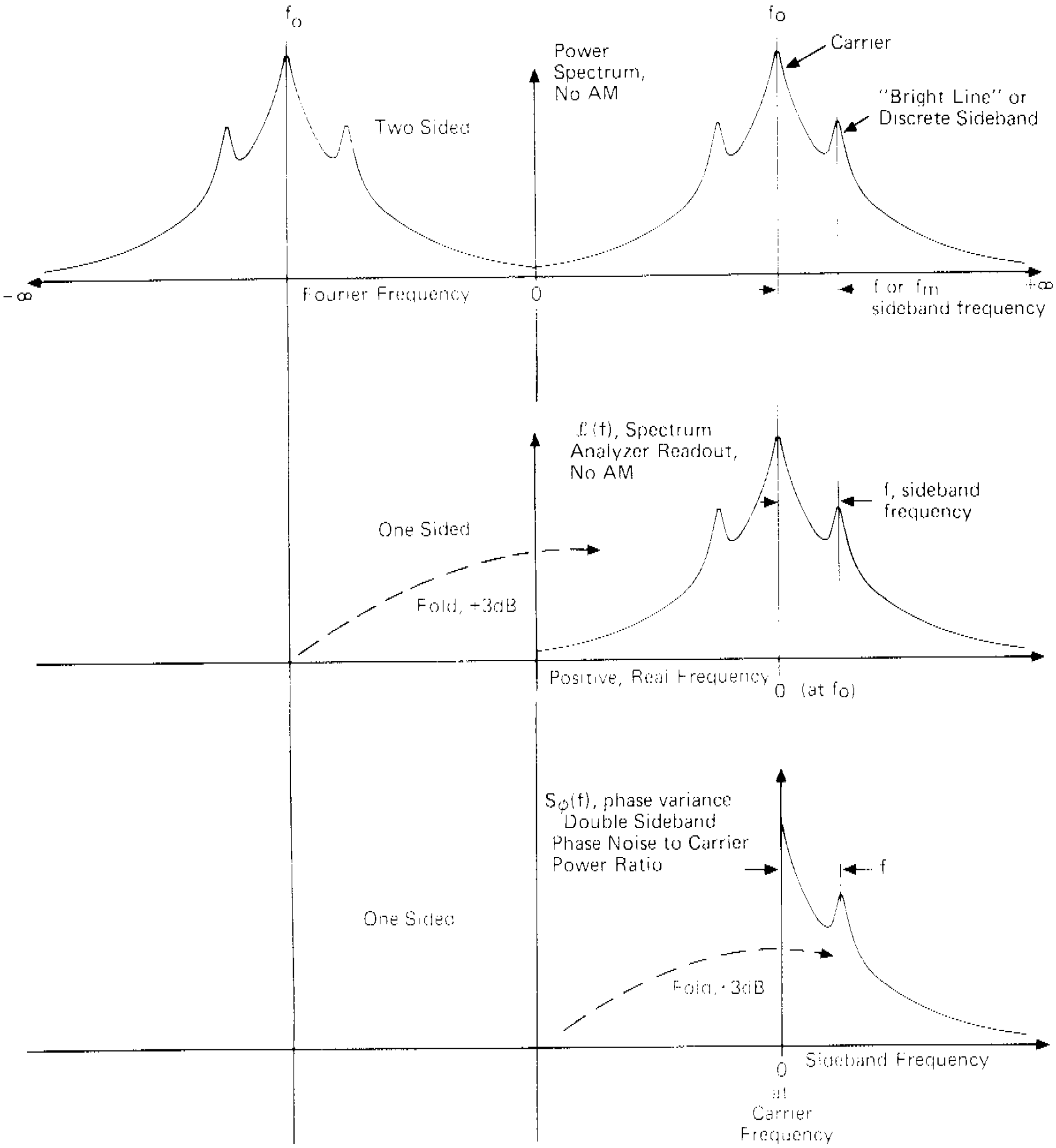


Figure C1. Graphic Comparison of Some Frequency Domain Measures, Definitions and Symbols.