# CALCULATION OF OMEGA PROPAGATION GROUP DELAY AND APPLICATION TO LOCAL TIME STANDARD MONITORING

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## ABSTRACT

The OMEGA navigation system now has seven transmitting sites strategically located throughout the world. All transmissions are derived from cesium-beam standards, and each station transmits time-multiplexed coherent bursts at 10.2, 11 1/3, and 13.6 kHz. Thus, an observer at some distant location has an opportunity to track the phase of three coherent precision transmissions rather than just a single frequency as is usually the case (e.g., WWVB, Ft. Collins). It is shown that by using the phase information received on all three frequencies, the observer can compute a synthetic group delay referred to any arbitrary frequency in the 10-14 kHz range. By coincidence, it works out that the group velocity (and thus group delay) at 12.5 kHz is about the same for nominal day and nighttime conditions. Thus, the group delay at this frequency has a natural insensitivity to diurnal variations. This invariance to diurnal shifts is demonstrated with actual OMEGA data.

In a monitoring application, it is suggested that there might be an advantage in compensating for propagation lag with group delay rather than the usual predicted phase delay. Most of the low-frequency diurnal error is eliminated in the synthetically-formed group delay, leaving only relatively high-frequency components to be filtered in the residual error. This, of course, simplifies the filtering problem. It is shown that complementary filter theory can be applied to advantage in this application.

## INTRODUCTION

The OMEGA navigation system now has seven transmitting sites strategically located throughout the world. When the eighth station (Australia) commences operation, the system will be fully operational with world-wide coverage [1]. In addition to its primary purpose as a navigation system, it also provides the world with a common precision time/frequency reference system. All transmissions are derived from cesium-beam standards, and each station transmits time-multiplexed coherent bursts at 10.2, 11-1/3, and 13.6 kHz. This makes OMEGA unique as a time reference system, because the observer at a remote location has an opportunity to track three coherent transmissions within a narrow frequency range, rather than the usual single frequency (e.g., WWVB, Ft. Collins). The availability of phase information on multiple frequencies enables the observer to compensate for propagation variations on-line, if he so chooses. For example, the well-known diurnal shift is due to a change in the effective height of the ionosphere from day to night. The same mechanism that causes the velocity of propagation to change also causes different phase shifts at different frequencies. So, one can reverse the reasoning and infer something about the change of velocity of propagation from the measured phase shifts on two or more frequencies.

A number of on-line OMEGA compensation schemes have been proposed, but it is not clear as yet which is to be preferred [2]. On-line compensation (in contrast to prediction) is especially attractive in the navigation application because it has the potential of mitigating unusual situations, such as sudden ionospheric disturbances (SID), as well as the usual diurnal shift. It is suggested here that some of these compensation ideas might be applied to advantage in the precise time/frequency application.

Before proceeding, a simple example should help put the precise timing problem in perspective. Obviously, the observer at a remote location would like to have the equivalent of an expensive cesium-beam standard in the form of a simple radio receiver. Unfortunately, though, the propagation delay is somewhat "rubbery" and relatively large errors can occur over short time periods. To illustrate this, consider tracking a 10 kHz single-frequency source. For long paths, a total phase shift from day to night of one full cycle would not be unusual; and, if this took place over a span of two hours, the apparent frequency error during this period would be about one part in $10^8$ -- a totally monsterous error when dealing with precision systems. Obviously, if one leaves the diurnal shift uncorrected, very long averaging times are needed for precise work. The culprit, of course, is the "rubberiness" of the propagation medium. Would it not be nice to be able to "stiffen" the medium someway? The remainder of this paper will be directed toward online (in contrast to predictive) methods of accomplishing this.

## VLF Wave Propagation

Wave propagation in the VLF range is usually explained in terms of waveguide theory, with the earth's surface and the ionosphere forming the waveguide boundaries. For simple waveguide modes, the phase and group velocities vary with frequency as shown in Fig. 1 [3]. In the

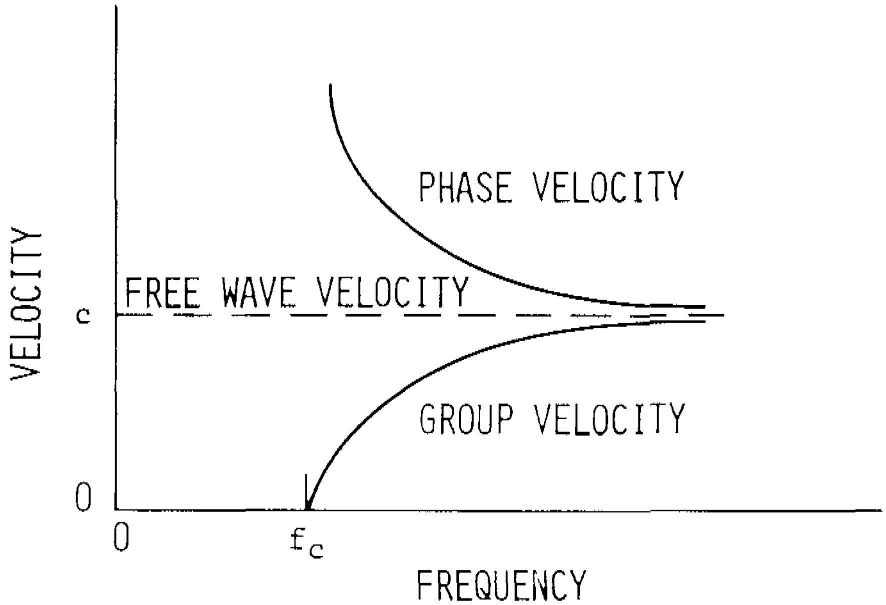


Fig. 1. Phase and group velocities for parallel-plane waveguide.

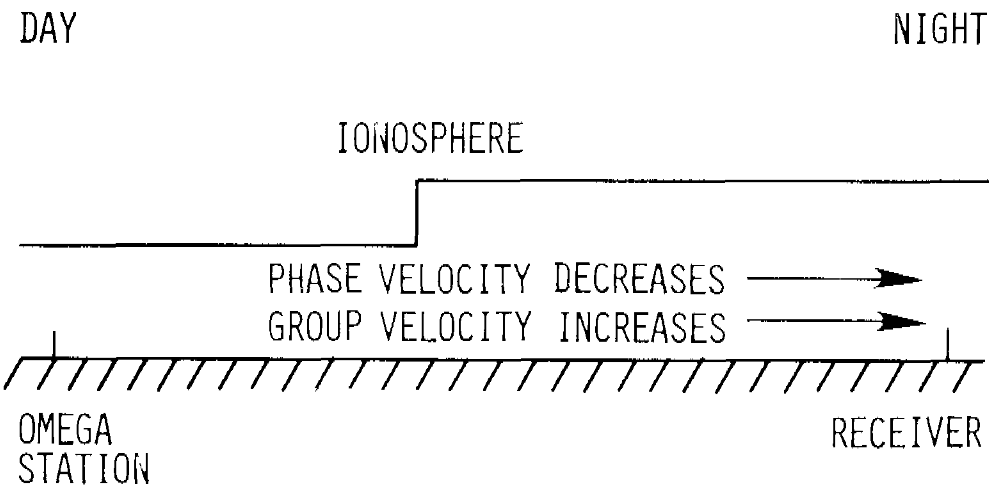


Fig. 2. Flat earth model.

OMEGA case, the waveguide height dimension is considerably greater than the wavelength, so the lowest order mode is considerably above the cutoff frequency, $f_c$. Thus, the phase velocity is only slightly greater, and the group velocity slightly less, than the free wave velocity. The change in ionospheric height that occur from day to night (typically, from 70 to 90 km) do, however, cause a shift in cutoff frequency that results in changes in the phase and group velocities of the order of a few tenths of a percent. The sketch of Fig. 2 shows a simplified "flat-earth" model illustrating the transition from day to night. It should be apparent that if the phase and group velocities change by equal amounts in opposite directions, then the average of the group and phase delays would be invariant from day to night. Compensation for the transit time from transmitter to receiver with a blend of phase of group delays should then eliminate the diurnal variation. This method of on-line compensation was first suggested by J. A. Pierce [4] and is now known as composite OMEGA. Without going into all the details here, two phase measurements on nearby frequencies are needed to accomplish the desired compensation.

A number of variations on Pierce's original compensation scheme have been proposed recently [5,6,9]. These have been necessary because the original idea of an equal blend of phase and group delays did not take into account the curvature of the earth as shown in Fig. 3. It can

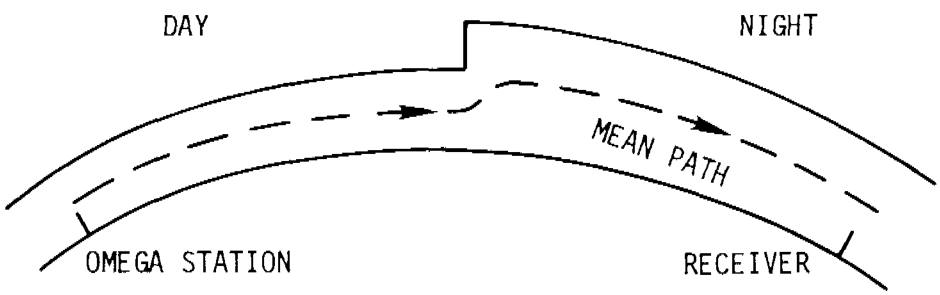



Fig. 3. Curved earth model.

be seen that even though the phase and group velocities change in opposite directions, the average delay is still not invariant because the mean path length increases as the ionospheric height increases. This suggests giving group delay more weight than phase delay in the blending, and this is borne out by recent investigations by Mactaggart [6]. Carrying this line of reasoning a bit further, there might exist a condition where the increase in path length in going from day to night would be exactly proportional to the increase in group velocity. This is confirmed by theoretical curves of group velocity vs. frequency given by both Hampton [7] and Watt [8] which are reproduced in Figs. 4 and 5. Note that these plots indicate that the day and nighttime group velocities should be the same for a frequency somewhere in the 12.5 to 13.0 kHz range. This crossover phenomenon is unique with group velocity (in contrast to phase velocity) and only occurs at one frequency. Quite by coincidence, this crossover frequency occurs within the spectral range of the OMEGA system. This has obvious implications in terms of eliminating the diurnal shift. In principle, all one need do is observe the envelope of a modulated wave at say 12.5 kHz and its transit delay should be relatively invariant from day to night. This is easier said than done, though.

There is very little direct experimental evidence supporting the theoretical curves shown in Figs. 4 and 5. No doubt this is due to the difficulty in making precise envelope time-of-arrival measurements in

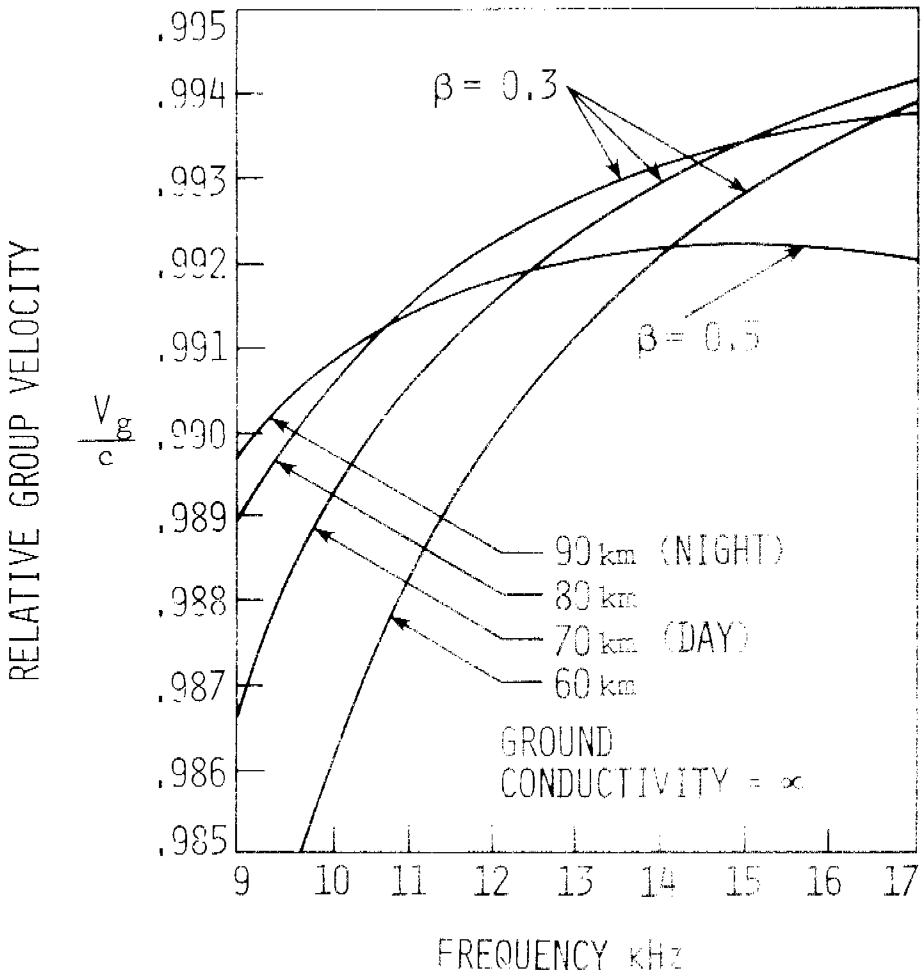



Fig. 4. Variation of group velocity with frequency (Hampton [7]).

the VLF range. However, by making simultaneous phase measurements on three or more coherent transmissions on nearby frequencies, one can infer <u>indirectly</u> the group delay referred to any desired frequency. The procedure for doing this will now be illustrated for the three-frequency case. The extension to more than three frequencies is obvious.

## Computation of Group Delay from Phase Delays

As a matter of review, the phase velocity of a traveling wave is the speed at which the fine structure (individual cycles) appears to move. It is given by the equation

$$v_p = \frac{\omega}{\beta} \tag{1}$$

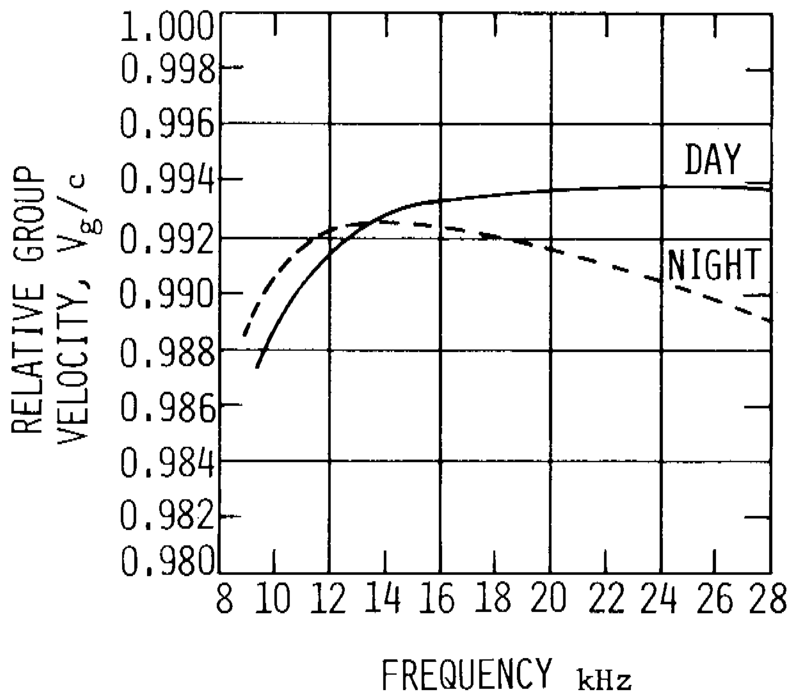



Fig. 5. Variation of group velocity with frequency (Watt [8]).

where $\omega$ is frequency in radians/sec and $\beta$ is the phase shift constant. On the other hand, the group velocity is the speed at which the envelope (modulation) appears to travel, and it is given by the inverse slope of the $\beta$ versus $\omega$ curve, i.e.,

$$v_g = \frac{d\omega}{d\beta} \tag{2}$$

Now assume we have three phase delay measurements, $T_1$, $T_2$, $T_3$, corresponding to the three OMEGA frequencies, $\omega_1$, $\omega_2$, $\omega_3$. Each of these time delays represents a ratio of total phase shift to frequency, i.e.,

$$T_1 = \frac{\phi_1}{\omega_1} = \frac{\beta_1 d}{\omega_1} = \frac{d}{\omega_1/\beta_1} = \frac{d}{v_{p_1}} \tag{3}$$

$$T_2 = \frac{\phi_2}{\omega_2} = \text{etc.} \tag{4}$$

$$T_3 = \frac{\phi_3}{\omega_3} = \text{etc.} \tag{5}$$

where d is the distance from transmitter to receiver. It is tacitly assumed from here on that lane ambiguities (whole number of wavelengths)

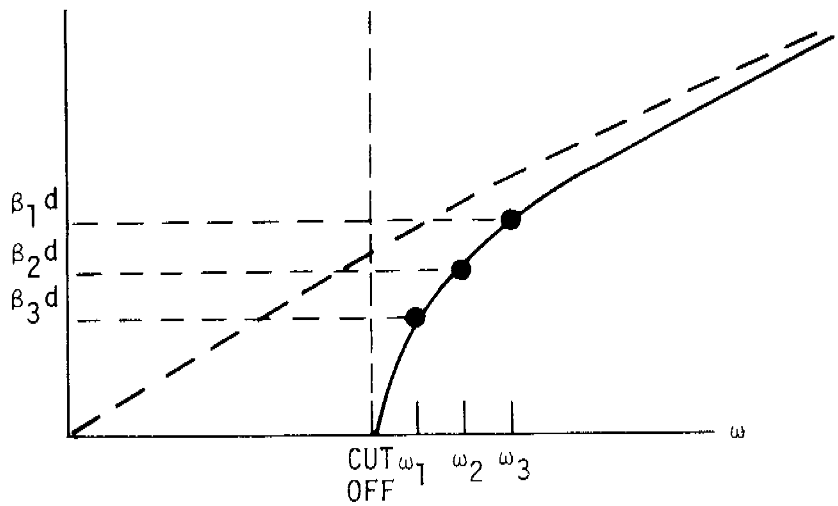



Fig. 6. Phase shift versus frequency.

for all three frequencies have been resolved. In effect, the three phase measurements give us three points on the $\beta d$ vs. $\omega$ curve as shown in Fig. 6. Now assume that $\beta d$ can be approximated as a quadratic function of $\omega$ over a reasonable range of $\omega$; i.e., let

$$\beta d = C_2\omega^2 + C_1\omega + C_0 \tag{6}$$

Next, we will choose the coefficients $C_0$, $C_1$ and $C_2$ such that $\beta d$ goes through the measured $\phi_1$, $\phi_2$, $\phi_3$ points as shown in Fig. 6. Thus, the coefficients are determined by

$$C_0 + C_1\omega_1 + C_2\omega_1^2 = \phi_1 \tag{7}$$

$$C_0 + C_1\omega_2 + C_2\omega_2^2 = \phi_2 \tag{8}$$

$$C_0 + C_1\omega_3 + C_2\omega_3^2 = \phi_3 \tag{9}$$

Omitting the algebra, it is obvious that Eqs. (7), (8), and (9) can be solved explicitly for $C_0$, $C_1$, and $C_2$ in terms of the measurements $\phi_1$, $\phi_2$, and $\phi_3$.

Returning now to Eq. (6), the group delay can be found as

$$T_g = \frac{d}{v_g} = \frac{d}{\left(\frac{d\omega}{d\beta}\right)} = \frac{d(\beta d)}{d\omega} = 2C_2\omega + C_1 \tag{10}$$

If the solutions for $C_1$ and $C_2$ from Eqs. (7), (8), (9) are substituted into Eq. (10), and if the frequencies $\omega_1$, $\omega_2$, and $\omega_3$ are assumed to be in the exact ratio 9:10:12, the following equation results

$$T_g = \left(60 \frac{\omega}{\omega_2} - 66\right)T_1 + \left(-100 \frac{\omega}{\omega_2} + 105\right)T_2 + \left(40 \frac{\omega}{\omega_2} - 38\right)T_3 \quad (11)$$

Equation (11) enables one to compute a group time delay, referred to any arbitrary frequency $\omega$, in terms of the three measured phase delays, $T_1$, $T_2$ and $T_3$. Note that $T_g$ in a <u>linear</u> function of $T_1$, $T_2$ and $T_3$ and that the sum of the coefficients (weight factors) is unity.

Also note that the measured phase delays $T_1$, $T_2$ and $T_3$ are simply the measured phases, including the appropriate multiples of $2\pi$, divided by the frequencies (i.e., Eqs. 3, 4, and 5). However, phase must be measured with respect to some local reference, so an unknown constant will appear in each term on the right side of Eq. 11. The sum of the coefficients is unity, so this same additive constant will appear with $T_g$. This additive term will be assumed to be constant for the moment, but, in any event, it certainly is not dependent on the propagation medium.

Returning now to Eq. (11), it is of special interest to look at variations in the coefficients of $T_1$, $T_2$, and $T_3$ with frequency. These three coefficients will be designated as $K_1$, $K_2$, and $K_3$ (i.e., $T_g = K_1T_1 + K_2T_2 + K_3T_3$), and they are plotted in Fig. 7. Note that in the 12.0 - 12.5 kHz range none of the coefficients exceeds 6. Purely random errors in the phase delay measurements do, of course, get "amplified" by the coefficients, so large values are undesirable. How undesirable, though, is a matter of degree, but certainly factors of 4 or 5 are not unreasonable. Pursuing this further, if one assumes the three measurement errors associated with $T_1$, $T_2$, and $T_3$ to be independent and each having unity variance, the resultant rms error in $T_g$ would be as shown in Fig. 8. It should be apparent that the best choice of reference frequency involves a compromise between the induced measurement noise error shown in Fig. 8 and the diurnal-shift error. This will not be pursued further from a theoretical viewpoint. Instead, we will proceed directly to some experimental results that demonstrate these concepts.

## Experimental Examples

A formula for computing a synthetic group delay at any desired frequency in the OMEGA range was derived in the previous section. This, along with experimental phase measurement data on 10.2, 11-1/3, and 13.6 kHz, should provide a means of verifying the theoretical curves of Hampton [7] and Watt [8], reproduced in Figs. 4 and 5. These curves were worked out for a single mode with idealized boundary conditions, so we should not expect exact correspondence. Qualitatively, though, we would expect to find the group delay to be greater during the day than at

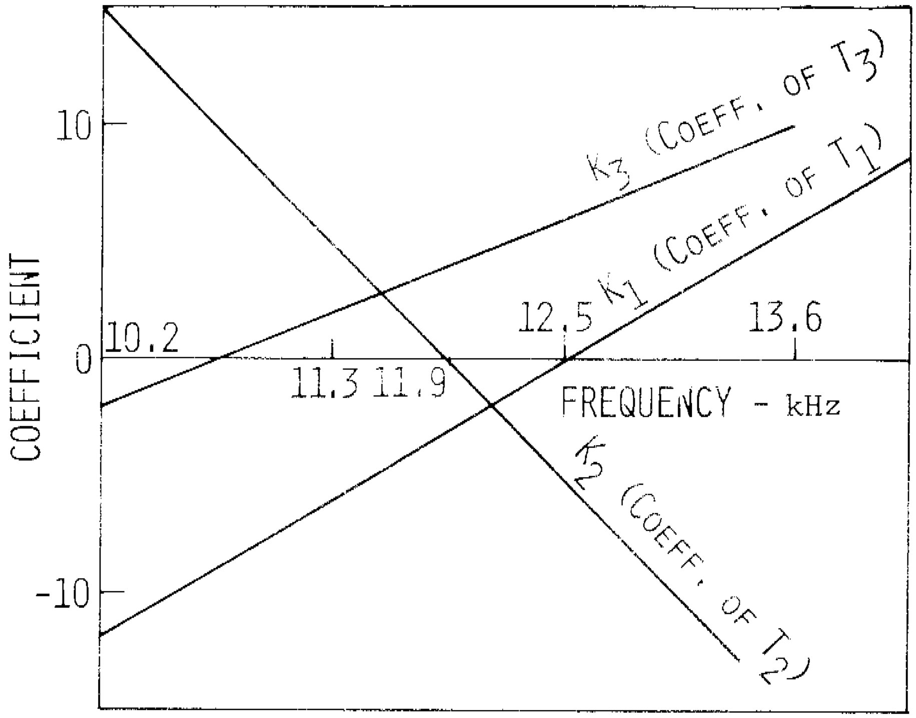



Fig. 7. Variation of $K_1$, $K_2$, $K_3$ coefficients with frequency.

$$(T_g = K_1T_1 + K_2T_2 = K_3T_3).$$

night for frequencies less than the crossover, and then the day and night delays should approach each other as the frequency is increased to the crossover frequency around 12.5 kHz.

A limited amount of experimental data was obtained from the U. S. Coast Guard OMEGA Navigation System Operations Detail (ONSOD), which gathers phase measurement data from various monitoring sites located around the world. Phase difference measurements were in the form of strip-chart recordings and covered the time period from March 10 through March 31, 1975. Two transmission paths, Trinidad to North Dakota and North Dakota to Hawaii were selected as examples for presentation here. In both cases, the monitoring sites were close to the local transmitters, so the recorded phases can be considered as "one-way" phase measurements. Phase data at all three OMEGA frequencies were read from the charts at a rate of one sample per hour, and then these data were used to compute group time delays at various reference frequencies in accordance with Eq. 11.

Results for the Trinidad to North Dakota path (B-D) are shown in Fig. 9. Twenty-two days of data are shown superimposed in each of the four parts of the figure. In order to establish a perspective, the uncompensated phase delay at 10.2 kHz is shown in the upper-left corner.

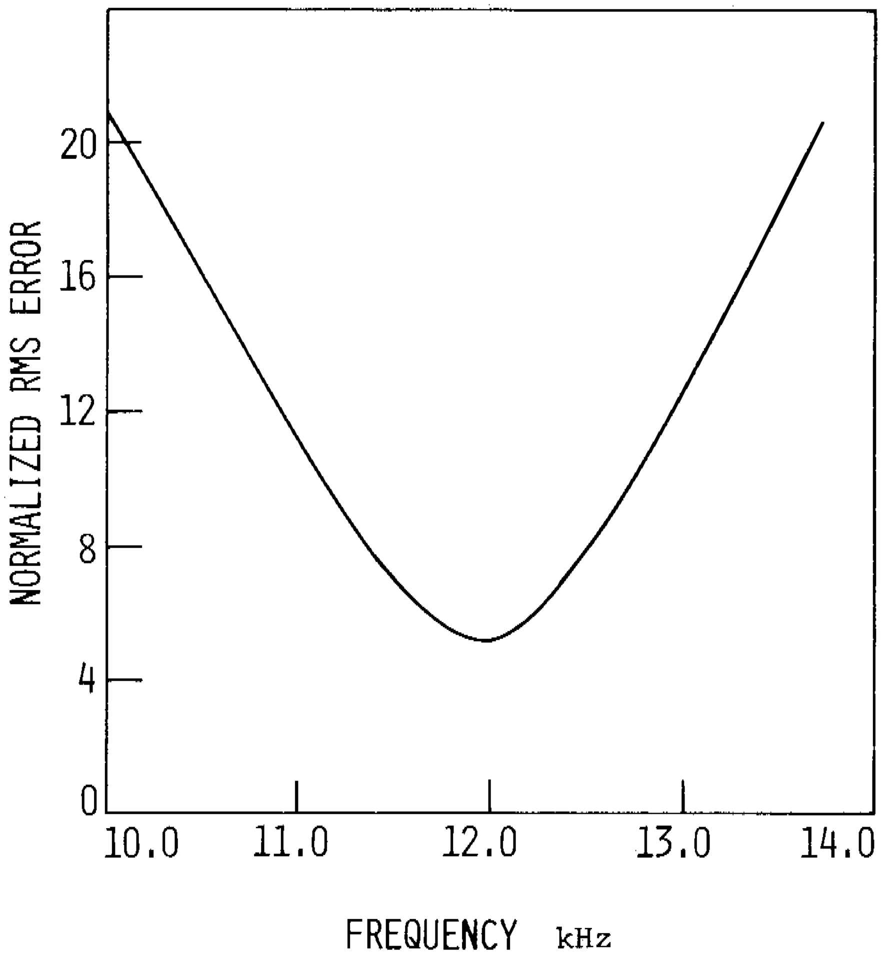



Fig. 8. Normalized RMS error due to measurement noise.

As expected, the diurnal shift is quite large, roughly about 75 microseconds or about 3/4 cycle at 10.2 kHz. The other three parts of Fig. 9 show the group delays computed at 11.5, 12.0 and 12.5 kHz. Note that the average day and night delays do tend to equalize as the reference frequency is increased to 12.5 kHz. The random fluctuations also increase dramatically as the reference frequency is increased, especially at night. It is tempting to explain this as being due to phase measurement errors being amplified by the $K_1$, $K_2$, and $K_3$ coefficients, which do increase somewhat in going from 12.0 to 12.5 kHz. However, the daytime portion of the curves does not show a similar increase in randomness with an increase in reference frequency. Thus, a more reasonable explanation would seem to be the basic instability at night due to modal interference.

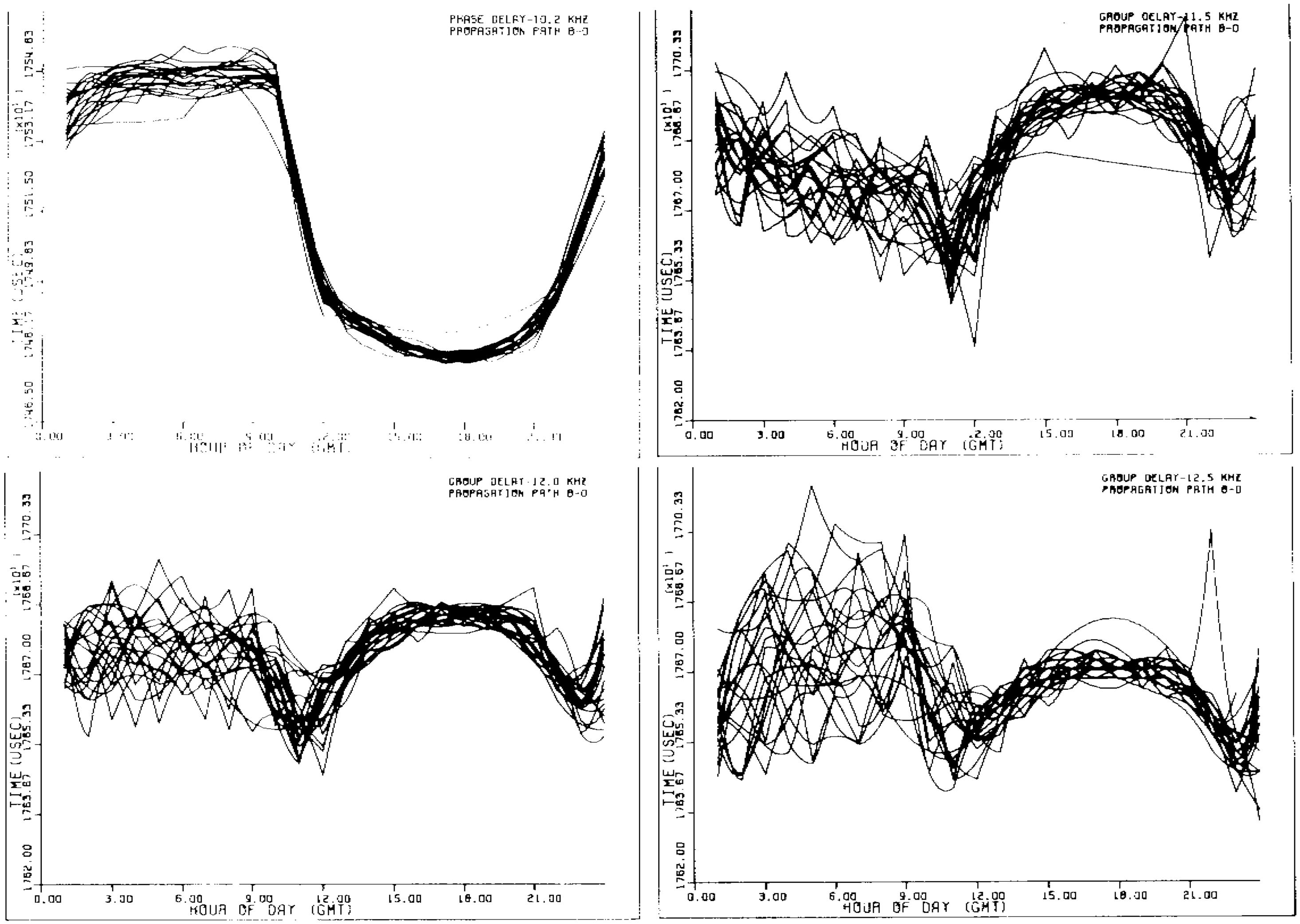



Fig. 9. Phase and group delays for Trinidad-North Dakota path for 10-31 March 1975.

Results from the North Dakota-Hawaii (D-C) path are shown in Fig. 10. The arrangement of the plots is similar to the Trinidad-North Dakota example. The equalization of the day and nighttime delays is not quite as conspicuous in this case, because the path is largely east-west and the transition between day and night is spread over a longer time period than for the Trinidad-North Dakota path.

In both examples, it should be noted that the group delay variations are considerably less than diurnal variation shown by the raw 10.2 kHz phase data. This is to be expected because Hampton's curves (Fig. 4) indicate the day-to-night variation in group velocity should only be about one part in a thousand, whereas we would expect about three times this much variation in phase velocity. Thus, group delay has a natural insensitivity to diurnal variation in the 11.5 to 13 kHz range. For timing purposes, it is important to note that the large 24-hour component error has been virtually eliminated at a reference frequency of 12.5 kHz, leaving only relatively rapidly fluctuating noise. Presumably, this should be easier to filter than the relatively low-frequency 24-hour error, so this will now be pursued further.

Filter Example

In the timing problem under consideration, we will assume that we have a received CW signal from a remote source (OMEGA) and a corresponding signal from a local source. Both will be assumed to be referred to the same nominal frequency via whatever frequency synthesizers and/or dividers are necessary. The local source may be just a simple crystal oscillator, but, in any event, there must exist some local reference to compare with the received OMEGA signal.

The filtering problem here falls into the general category of complementary filtering [10], so a few words are in order about this type of filtering. Figure 11 shows three forms of filtering operating on two independent noisy measurements of the same signal s(t). The contaminating noises are $n_1(t)$ and $n_2(t)$. Note that all three implementations lead to identical end results. The designer's problem is to choose the best Y(s) for the noises present in his particular physical situation. Each of the block diagrams in Fig. 11 lends a slightly different insight into the design problem. The straightforward two-channel version shown in Fig. 11(a) clearly shows the complementary feature of this type of filtering. Note that the signal s(t) passes through the system undistorted and is not affected by the choice of Y(s) in any way. Fig. 11(b) shows that the design problem reduces to a separation of $n_1(t)$ from $n_2(t)$. For example, if $n_1$ is low-frequency noise and $n_2$ is high-frequency noise, the obvious choice for Y(s) is a low-pass filter. This will preserve $n_1(t)$ to some degree of accuracy, and it can then be subtracted from the first measurement to yield an improved estimate of s(t). The feedback version shown in Fig. 11(c) is to be preferred over (a) or (b) in situations where either $n_1$ or $n_2$ is unstable with time. The lin-

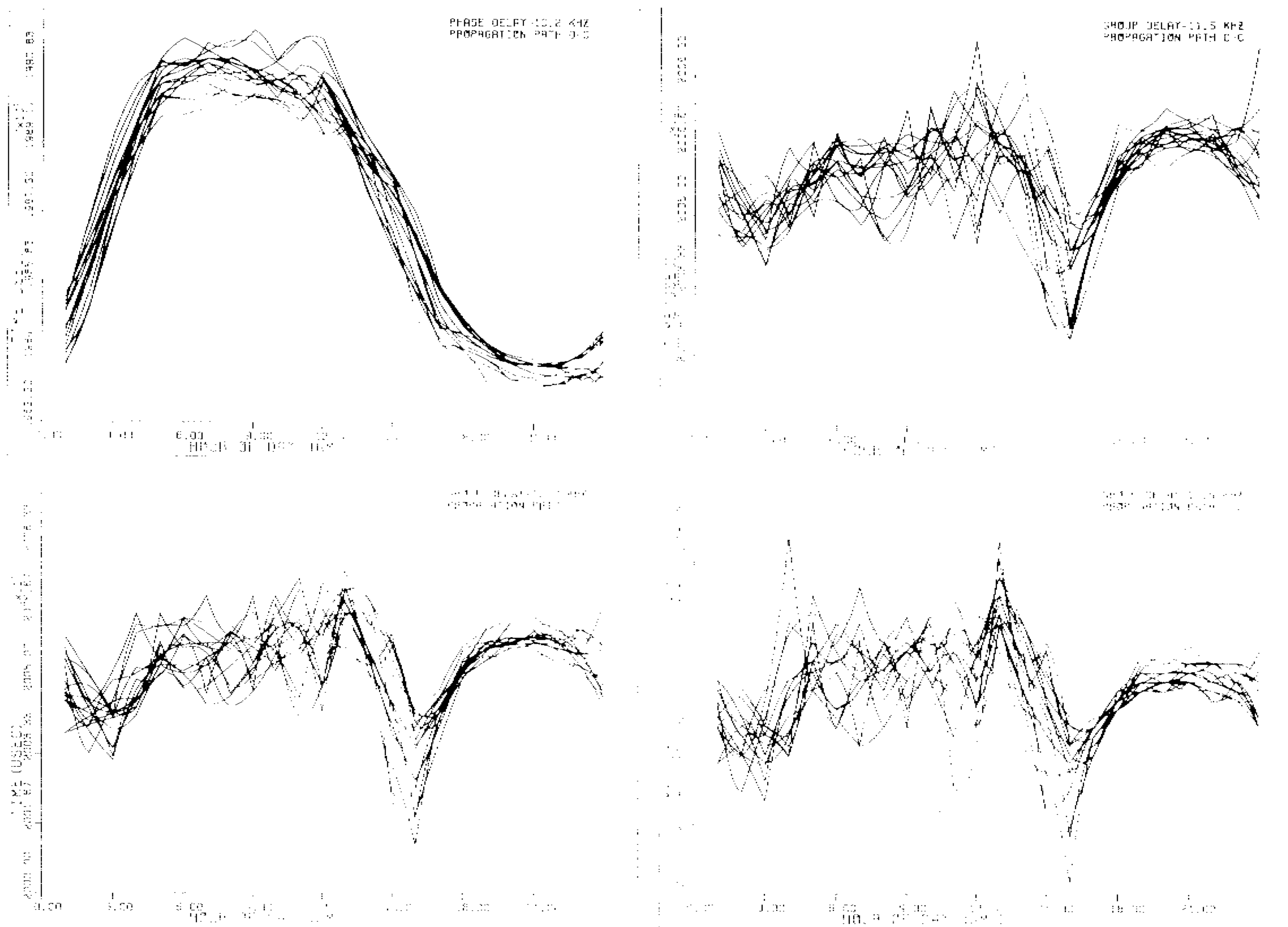



Fig. 10. Phase and group delays for North Dakota-Hawaii path for 10-31 March 1975.

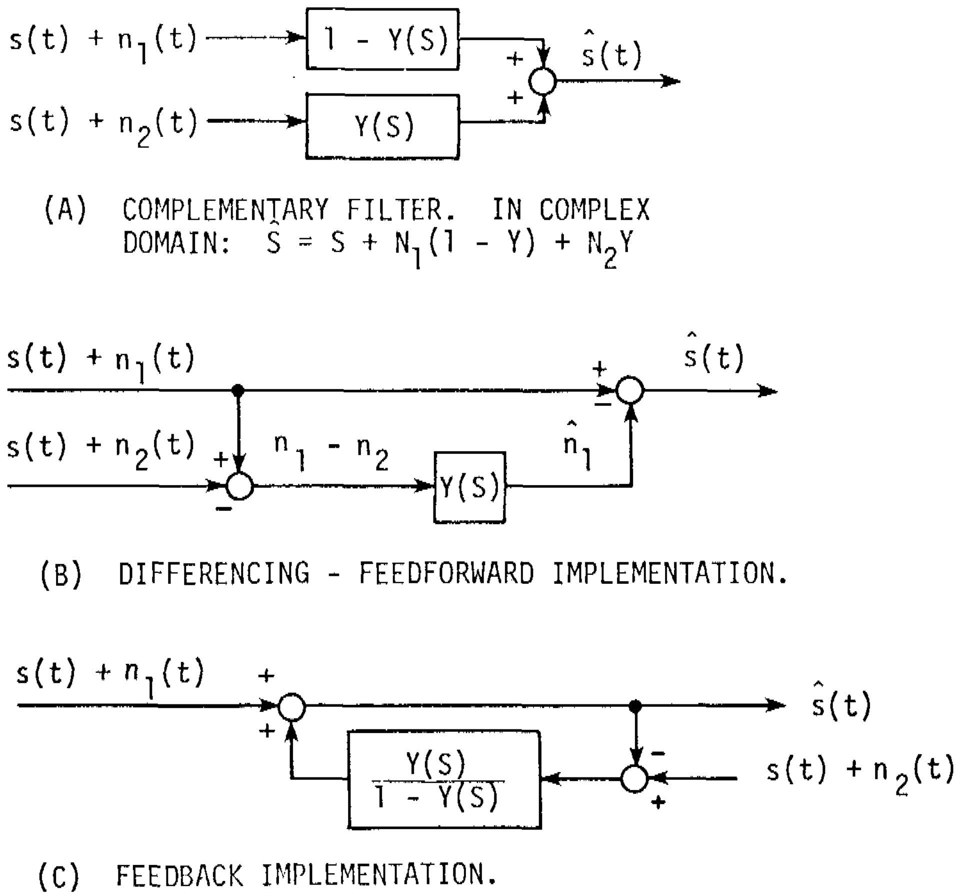
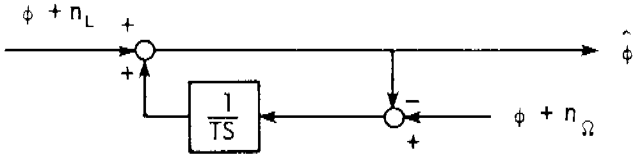



Fig. 11. Three equivalent implementations of a complementary filter.

ear range of the operations indicated in (a) and (b) might be exceeded in this case, but this can be avoided with the feedback implementation shown in (c).

The linearity restriction just mentioned really only applies in analog systems. If the filter is operating on digital data (i.e., numbers) there is virtually no such restriction. Either continuous analog or digital data may be encountered in the timing application. Obviously, with the aid of a digital processor and appropriate interfacing, all sorts of interesting possibilities exist, including computation of group delay referenced to any desired frequency in accordance with Eq. 11. However, in the interest of keeping the discussion brief and simple, we shall be content with a simple analog example.

To illustrate the benefit of the group-delay approach (in contrast to phase delay) consider the observed OMEGA signal to be the beat signal between the 11-1/3 and 13.6 kHz transmissions from a single OMEGA station. We will assume that the electronic circuitry is such that it produces an analog CW signal at the difference frequency of 2-4/15 kHz. This can also be thought of as the envelope of a single modulated wave whose carrier is midway between 11-1/3 and 13.6 kHz, or 12.46---kHz. The envelope travels at group velocity, so the phase of the envelope is delayed by the group delay, in this case referenced to about 12.46 kHz. Note, by coincidence, this is very near the day-night crossover. Thus, the phase-delay characteristics of the beat signal between 11-1/3 and




$n_L$ = LOCAL PHASE ERROR

$n_\Omega$ = OMEGA PHASE ERROR

$$Y(s) = \frac{1}{1 + TS}$$

$$\frac{Y(S)}{1 - Y(S)} = \frac{1}{TS}$$

Fig. 12. First-order filter example.

13.6 kHz should be close to those shown in Figs. 9 and 10 for the 12.5 kHz reference frequency. That is, the major part of the diurnal variation should be eliminated, leaving only the more rapidly varying phase error. The beat signal at 2-4/15 kHz can now be compared directly with the phase of the local source suitably divided down to the same frequency. Thus, in this example, there is no need to "compute" a synthetic group delay, because a sinusoidal wave with essentially no diurnal shift can be obtained directly in analog form.

The filter block diagram for this example is shown in the "feedback" configuration in Fig. 12. In this case we might expect the residual propagation error associated with the OMEGA source to be relatively high frequency noise. On the other hand, one would expect unstable, low-frequency drift error in the local reference. The spectral characteristics of these two error sources are quite different, so we can expect the complementary filter to do a respectable job of separating the two. A low-pass filter is the obvious choice for Y(s). For purposes of illustration let Y(s) be of the form

$$Y(s) = \frac{1}{1 + TS}$$

where T is the time constant of the filter. This is the simplest possible low-pass filter. Some commercial systems are capable of operating with a time constant of about one day, so we will choose this as the time constant T in this example. We shall then compare the complementary filter outputs with raw 10.2 kHz OMEGA as the reference on one hand, and with the 2-4/15 kHz beat signal as the OMEGA reference on the other. Five days of actual B-D OMEGA data for 3-7 March 1975 with a sampling rate of one sample per 10 minutes was used as the remote reference in the simulation. The local reference for this example was assumed to be a relatively high quality source with a drift rate of 1 part in $10^{10}$.

The results for the 5-day simulation are shown in Fig. 13. The first two or three days may be ignored as the transient period, but note as the system approaches steady-state, the simulation with the beat signal as reference has considerably smaller fluctuations than the raw 10.2 kHz phase-reference system.

There was no attempt to optimize the filter in this example. Rather, the filter form and time constant were chosen to conform with current state-of-the-art phase-tracking time/frequency systems. Application of optimal filtering techniques should provide even further reduction of the residual error.

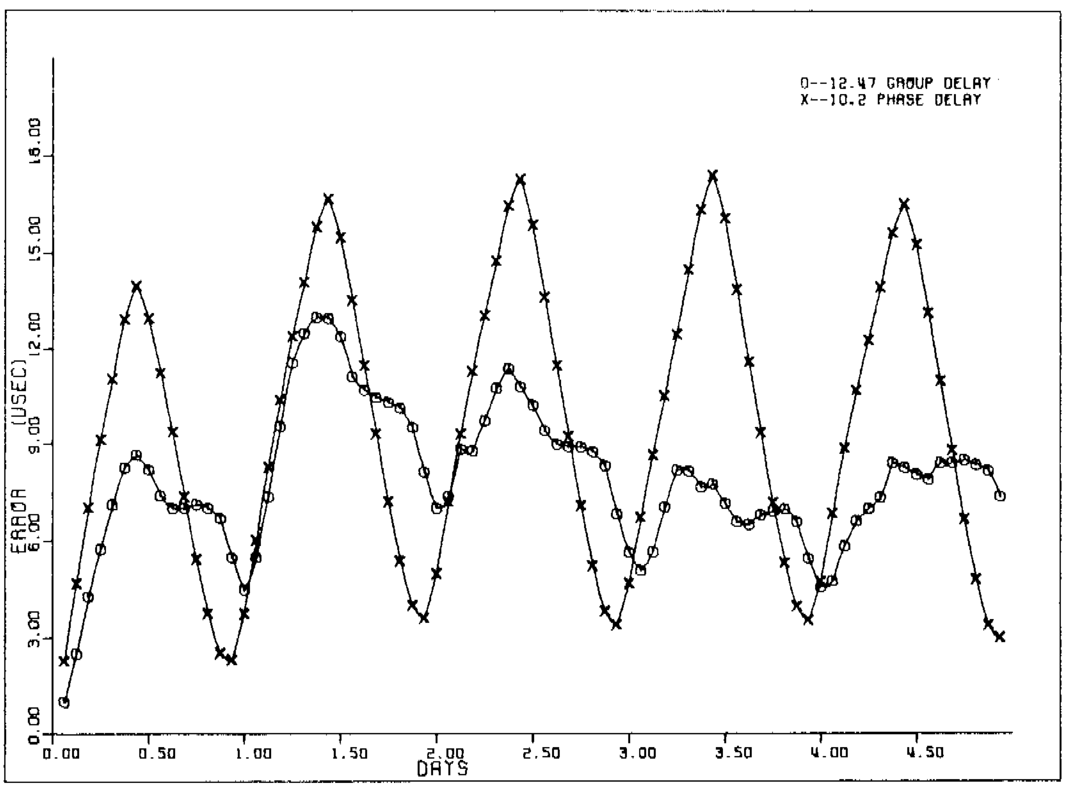



Fig. 13. Comparison of errors for phase- and group-reference systems.

## Summary

It has been demonstrated that group delay in the 12.0 to 12.5 kHz range exhibits much less diurnal variation than the corresponding phase delay. Thus, if the local reference is coupled to the remote reference via group delay rather than phase delay, then the local filtering problem is less severe. Also, the resulting filtering problem was shown to fit within the framework of complementary filter theory. Once this is recognized, a considerable body of both optimization theory and experience can be brought to bear on the problem. Thus, the OMEGA system with its three coherent transmissions shows considerable promise as the long-term reference in a precise time/frequency system.

## Acknowledgements

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