ANALYSIS OF THE IMPACT OF SATELLITE NAVIGATION SYSTEM RECEIVER OSCILLATOR ERRORS ON USER NAVIGATION PERFORMANCE*

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ABSTRACT

This paper considers a satellite-based passive radio navigation system that is capable of providing highly accurate navigation fixes to properly equipped users. For the user to access such a system he must be equipped with a receiver that includes a local oscillator which maintains a time base and generates frequency information required for signal acquisition and for pseudo-range and pseudo-range-rate measurements. This paper examines the impact of receiver quartz crystal oscillator errors on user navigation performance. A model is developed which describes environmentally-induced oscillator errors. This error model is incorporated into a simulation to assess the impact of environmentally-induced oscillator errors on user navigation performance. The analysis considers both basic oscillator performance and the effect of environmentally induced errors including: oscillator warm-up, temperature, vibration and acceleration sensitivities and the effect of a shock to the oscillator. Methods are presented with which the receiver software may be designed to reduce the impact of the significant error sources.

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INTRODUCTION

Recent advances in technology have demonstrated the feasibility of a satellite-based passive radio navigation system that will be capable of providing highly accurate navigation fixes to properly equipped users (Ref. 1). For the user to access the satellite navigation system he must be equipped with a receiver that includes a local oscillator which maintains a time base and generates frequency information required for signal acquisition and for pseudo-range and pseudo-range-rate measurements. A relative phase offset between the user clock and the satellite clocks will lead to pseudo-range errors while a frequency offset in the user oscillator introduces pseudo-range-rate errors. This paper examines the impact of quartz crystal oscillator errors on user navigation performance for such a satellite navigation system.

In order to properly assess the impact of environmentally-induced quartz crystal oscillator errors on navigation performance a comprehensive model of the oscillator error behavior must be developed. The complete error model will include a description of the intrinsic instability of the oscillator as well as the environmentally-induced errors. The model for intrinsic instability follows directly from a consideration of the frequency stability of a specific oscillator and has been developed previously (Ref. 2). This paper presents the development of an error model which describes the effects of the environment on a quartz oscillator. The error model is then utilized to assess the degradation in navigation performance for an airborne user which arises from the use of the receiver oscillator in a realistic (non-ideal) environment.

QUARTZ CRYSTAL OSCILLATOR ERROR MODELING

The prototype for the quartz crystal oscillator error model to be considered is shown in Fig. 1 with

$\delta\phi(t)$ = total clock phase error in seconds at time t

$\delta\phi(t_0)$ = initial phase error (seconds)

$\delta f(t_0)$ = initial fractional frequency offset (dimensionless)

$\delta\dot{f}(t_0)$ = initial fractional frequency drift or "aging" (seconds^{-1})

q_f = spectral level of white noise driving the flicker noise model (dimensionless)

δf_{env} = environmentally-induced fractional frequency error

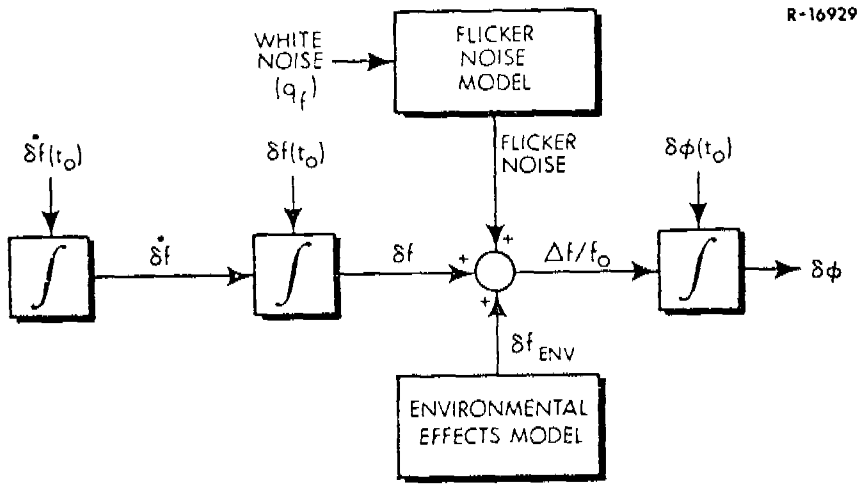

Fig. 1 Prototype Quartz Crystal Oscillator Error Model

As mentioned above, the methodology for constructing the basic oscillator model has been presented in an earlier paper (Ref. 2). This paper will deal only with the development of an environmental effects model for a quartz oscillator. A completely general error model for environmental effects would be prohibitively complex. However, if certain assumptions are made about the form and number of environmental influences, then an environmental effects model can be developed for a specific device. In the following paragraphs the effects of temperature, warm-up, acceleration, vibration and shock on a quartz crystal oscillator will be discussed.

Temperature - A quartz oscillator responds to a change in ambient temperature with a shift in frequency. The relationship between this frequency shift and the temperature change varies from oscillator to oscillator and may be quite nonlinear. When the oscillator is incorporated into the navigation system receiver, a linearized coefficient relating the frequency shift to a temperature change can be obtained.

For temperature excursions about a nominal receiver operating temperature (assumed to be $\approx 60^{\circ}C$) the assumed linear temperature drift coefficient for this study is $K_{tv} = 1 \times 10^{-10}/^{\circ}C$.

The frequency shift, which results from a temperature change, does not occur instantaneously due to a "thermal lag" inherent in the oscillator. The temperature-regulating oven surrounding the quartz crystal is the main contributor to this lag. The typical oscillator thermal response shown in Fig. 2 can be adequately described by a simple first-order system as depicted in Fig. 3. The time constant, τ_{tv}, depends upon the number of ovens surrounding the crystal. For a single oven device τ_{tv} is on the order of 80 minutes.

The error model shown in Fig. 3 represents the response of the crystal oscillator to an arbitrary temperature variation. In order to utilize this model the input temperature variation must be described quantitatively. It is obvious that differing users will encounter vastly different environmental conditions. These dissimilar environmental scenarios will result in mission-dependent temperature fluctuations outside the receiver. These variations are not transmitted instantaneously to the crystal oscillator due to the moderating influence of the receiver thermal mass. Assuming random temperature variations, the ambient temperature surrounding the crystal oscillator is reasonably modeled as a

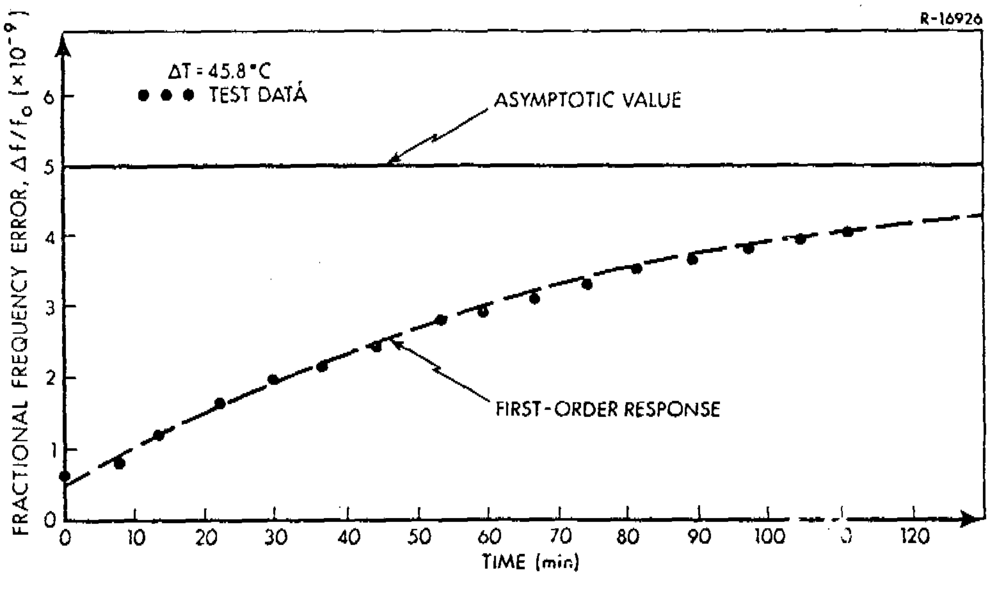

Fig. 2 Response of Crystal Oscillator Frequency to a Step Ambient Temperature Change

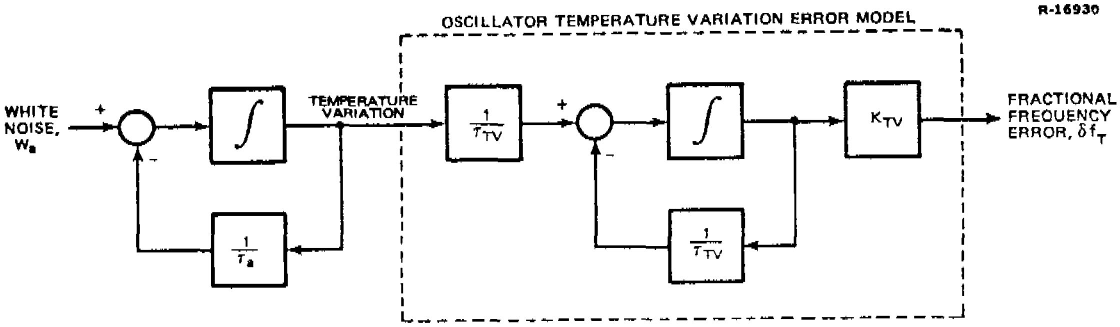

Fig. 3 Quartz Crystal Thermal Error Model

first-order Markov process. The parameters of the process, rms value and time constant, must be chosen to reflect the temperature variations outside the receiver and the moderating effect of the receiver. For a typical receiver the rms value of the temperature fluctuations near the crystal oscillator is expected to be on the order of 3°C with a time constant (τ_a) of 100 minutes.

Warm-up - Figure 4 shows the fractional frequency error versus warm-up time characteristics of three quartz crystal oscillators. The three oscillators exhibit vastly different behavior which is not amenable to direct analysis. The dashed line in the figure describes an exponentially decaying envelope (the plot is semi-log) which can be viewed as a statistical bound on the frequency error. The warm-up fractional frequency error variance ($\sigma^2_{\delta f w}$) for this exponential model is

$$\sigma^2_{\delta f w} = \sigma^2_w \, e^{-2t/\tau_w}$$

with $\sigma^2_w = 1 \times 10^{-10}$ and $\tau_w \simeq 100$ seconds.

Acceleration - A constant acceleration introduces a frequency offset in a quartz oscillator proportional to the magnitude of the acceleration, i.e., $\delta f_a = K_g |\underline{a}|$. The coefficient of proportionality, K_g, is dependent upon the orientation of the acceleration vector with respect to the crystallographic axes of the quartz crystal. A worst case value is $K_g = 1 \times 10^{-9}/g$.

Vibration - Vibration introduces sidebands (at the vibration frequency) into the spectrum of the signal from a quartz crystal oscillator. These sidebands produce a frequency error which is dependent upon the oscillator sensitivity at the vibration frequency. Figure 5 presents the results of measurements of the fractional frequency error (averaged over a 10-second interval) arising from the lateral (sinu-

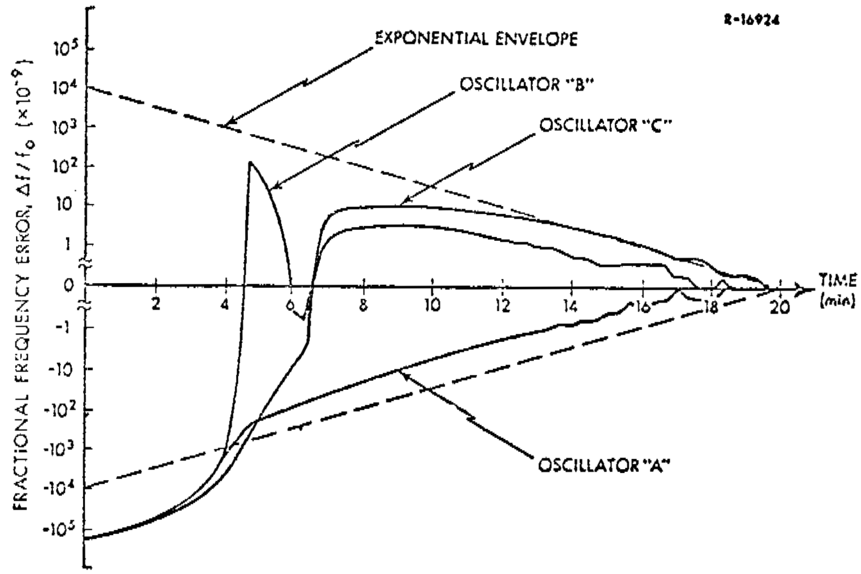

Fig. 4 Warm-up Characteristics of Three Quartz Crystal Oscillators

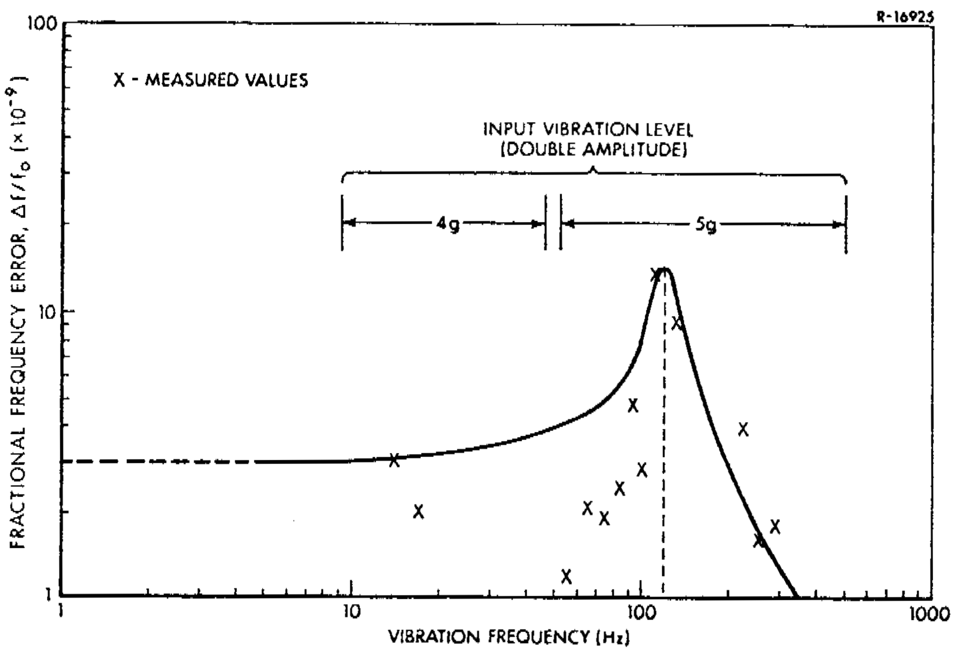

Fig. 5 Frequency Error as a Function of Vibration Frequency

soidal) vibration of a quartz crystal oscillator. The worst case response is adequately modeled by the resonant second-order curve which is superimposed upon the data. In lieu of a well-defined spectrum for the vibration in an operational environment the baseline model will assume a white noise input with spectral level

$$q_v = 0.02\ g^2\ \text{sec.}$$

The model representing the frequency error due to vibration, δf_v, is shown in Fig. 6. The parameters of the model are obtained from Fig. 5. The resonant curve in the figure corresponds to a natural frequency, ω_n, of 754 rad/sec and a damping ratio, $\zeta = 0.1$. The constant K_v is the lateral vibration drift coefficient with a value of 6×10^{-10}/g from Fig. 5. As expected, the vibration coefficient K_v is very close to the g-sensitive drift coefficient K_g.

Shock - When a quartz crystal oscillator is subjected to a shock, a permanent frequency offset results. No specific values are available (at this time) for the magnitude of this offset during the shock but step fractional frequency shifts on the order of 1×10^{-9} have been observed following shocks of 15g to 60g. A reasonable model for the effect of a shock of short duration ($\approx$1 msec), occuring at time t_s, is a step fractional frequency shift ($\delta f_s|_{t_s}$) of 1×10^{-9}.

In the previous paragraphs models were developed to describe the frequency errors which arise from perturbations in the oscillator's environment. The total fractional fre-

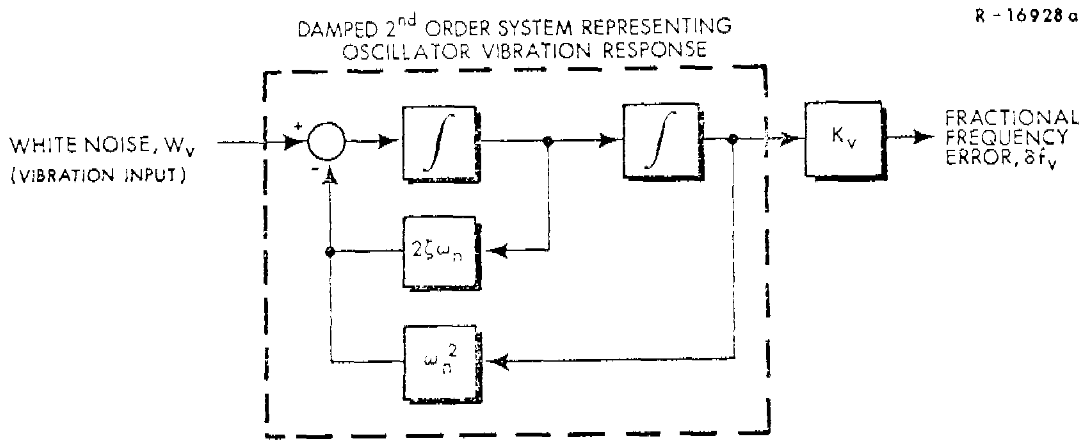

Fig. 6 Crystal Oscillator Vibration Error Model

quency error due to environmental effects, δf_{env}, is the sum of the individual errors resulting from temperature changes, oscillator warm-up, acceleration, shock and vibration effects, viz,

$$\delta f_{env} = \delta f_t + \delta f_w + \delta f_a + \delta f_s|_{t_s} + \delta f_v$$

The environmental effects model can be combined with the intrinsic instability model of the quartz oscillator as shown in Fig. 1. The error state vector for the quartz crystal oscillator error model is given in Table 1. Two error sources are not included explicitly in the state vector. The error arising from the acceleration of the oscillator (i.e., $\delta f_a = K_g|\underline{a}|$) is more appropriately modeled as an additive quantity. The second error source, which will remain unmodeled, is the effect of an impulsive shock to the oscillator. Although it is unmodeled, the impact of such an unanticipated shock will be examined in this paper.

TABLE 1

QUARTZ CRYSTAL OSCILLATOR
ERROR MODEL STATE VECTOR

STATE	SYMBOL	DESCRIPTION
1	$\delta\phi$	Phase Error
2	δf	Frequency Error
3	$\delta\dot{f}$	Frequency Rate Error
4	x_{1f}	Flicker Noise
5	x_{2f}	Intermediate Flicker Noise State
6	δf_t	Frequency Error (Thermal Model)
7	x_t	Temperature Variation
8	δf_w	Frequency Error (Warm-up Model)
9	δf_v	Frequency Error (Vibration Model)
10	x_v	Intermediate Vibration Error State

USER EVALUATION SCENARIO AND ERROR MODELS

The analysis of the airborne user performance employed a simulation program developed at TASC in order to investigate the effects of environmentally-induced errors. The airborne user (helicopter) trajectory is illustrated in Fig. 7. The user initially processes one minute of measurements while stationary to calibrate the receiver clock. After takeoff the helicopter makes turns at a maximum of 15 deg/sec, accelerates at a maximum of 2.5 mps/sec and changes altitude at a maximum of 3.0 mps.

An inertial navigation system (INS) is included in the user formulation to provide accurate short-term navigation between measurements. A barometric altimeter is also included to bound the vertical channel of the INS. The measured altitude is used in a closed-loop fashion to damp the altitude errors. For the airborne user performance analyses considered in this paper, pseudo-range and pseudo-range-rate measurements are obtained simultaneously from four of the satellites in view every 10 seconds.

Two error models are required for the generation of user performance projections. The first is a model describing the "real world" errors. This model would ideally be a complete description of all significant error sources which impact upon user navigation performance. The second model, which is a deliberately simplified description of the "real world" errors, is implemented in the user Kalman filter.

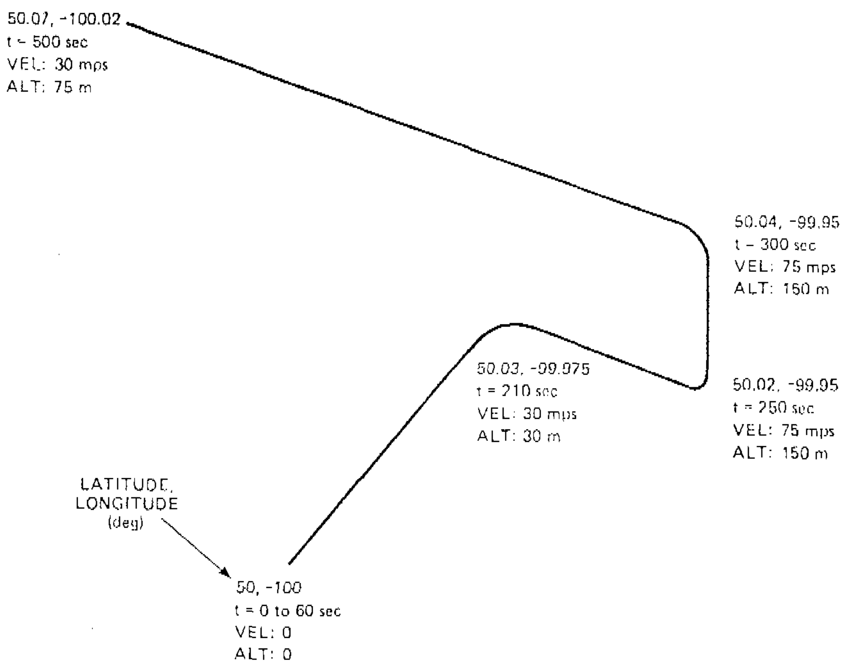

Fig. 7 Airborne User Trajectory

The "real world" error model includes a description of the satellite navigation system errors, the user INS errors and a user clock error model. The satellite navigation system errors, as implemented in the simulation program, include satellite clock and ephemeris errors, signal propagation delays (ionospheric, tropospheric and multipath), and receiver noise and quantization.

The user clock error model was developed above. The 10-state clock error model includes a description of the effects of temperature, warm-up and vibration. Errors which arise from acceleration and shock are not explicitly included in the error state vector (see Table 1). However, these two error sources are included in the description of the "real world" and their effects on user navigation performance will be evaluated.

The user Kalman filter (Ref. 3) estimates the 20 user navigation and clock errors summarized in Table 2. The user position and velocity errors are referenced to a locally level [north (n), east (e) and down (d)] coordinate system located at the instantaneous user position. As indicated in Table 2, INS errors are characterized by gyro drift (with bias and random walk components) and accelerometer bias error terms. This model will provide a reasonable description of INS errors during the short (≃8 minute) mission under consideration. Since the purpose of this paper is to discuss the effect of user clock errors, a more detailed INS error model would be undesirable. The user filter <u>does not</u> include a model of any environmentally-induced errors in the user clock. In addition, the state modeling the effects of aging has been deleted in the filter model since the total mission duration is shorter than the time required for the effects of aging to become significant.

The user filter formulation assumes that the user is equipped with a multichannel receiver to allow the simultaneous processing of pseudo-range and pseudo-range-rate information from four of the satellites in view. The filter-assumed receiver measurement noise levels are 7.5 m (rms) and 0.15 mps (rms) for pseudo-range and pseudo-range-rate, respectively. Errors due to the ionospheric and tropospheric effects are assumed (in the filter) to be compensated to within the measurement noise and are not correlated with the other error sources.

TABLE 2

STATE DESCRIPTIONS FOR 20-STATE AIRBORNE USER FILTER

STATE NUMBER	SYMBOL	DESCRIPTION
1	δR_n	North Position Error
2	δR_e	East Position Error
3	δh	Altitude Error
4	δV_n	North Velocity Error
5	δV_e	East Velocity error
6	δV_d	Down Velocity Error
7	ψ_n	Platform Misalignment About North
8	ψ_e	Platform Misalignment About East
9	ψ_d	Platform Misalignment About Down
10	δs_a	Barometric Altimeter Error
11	ε_n	North Gyro Drift
12	ε_e	East Gyro Drift
13	ε_d	Down Gyro Drift
14	μ_n	North Accelerometer Bias Error
15	μ_e	East Accelerometer Bias Error
16	μ_d	Down Accelerometer Bias Error
17	$\delta\phi$	Phase Error
18	δf	Fractional Frequency Error
19	x_{1f}	Flicker Noise
20	x_{2f}	Intermediate Flicker Noise State

USER PERFORMANCE PROJECTIONS

This section will present user performance projections which predict the impact of environmentally-induced receiver clock errors on user navigation performance. The results are given in the form of user estimation errors and the rms estimation error envelope (standard deviation boundary) predicted by the user Kalman filter. Each error source modeled above is introduced into the "real world" error model separately and its effect on user navigation performance is assessed by comparing the resultant navigation errors with those of a baseline run in which environmentally-induced errors were deleted. The following paragraphs discuss the impact of the various error sources.

Baseline - The baseline performance of the airborne user is indicative of the navigation errors which arise from navigation system errors (satellite ephemeris and clock errors, signal propagation errors, etc.) and user dependent errors (INS error sources and the nonenvironmentally-induced clock errors). This performance is illustrated in Fig. 8 for the trajectory of Fig. 7. The peak in horizontal velocity error at the first turn is due to a residual heading error which couples the acceleration experienced during the turn into the horizontal velocity error. This makes the heading error highly observable, such that pseudo-range-rate measurements during the turn calibrate the heading error.

Temperature and Vibration - The impact of the oscillator temperature and vibration sensitivities on user navigation performance was minimal for the assumed levels of random inputs. In both cases the projected user performance was indistinguishable from the baseline values. It should be noted, however, that a strong sinusoidal vibration near 120 Hz (see Fig. 5) would result in a large frequency error and could lead to a degradation in user navigation performance.

Warm-up - The impact of quartz crystal oscillator warm-up on user navigation performance is considered next. It was assumed that the oscillator had been turned on 12 minutes prior to the start of the flight. The 12-minute period was chosen to be compatible with a 5-minute coarse gyrocompass and 420-second fine-align of the INS.

Figure 9 illustrates the effects of oscillator warm-up when it is not modeled in the user filter. The frequency offset present during warm-up appears directly as the predominant contributor to frequency estimation error. This large frequency offset results in an increase in velocity error since

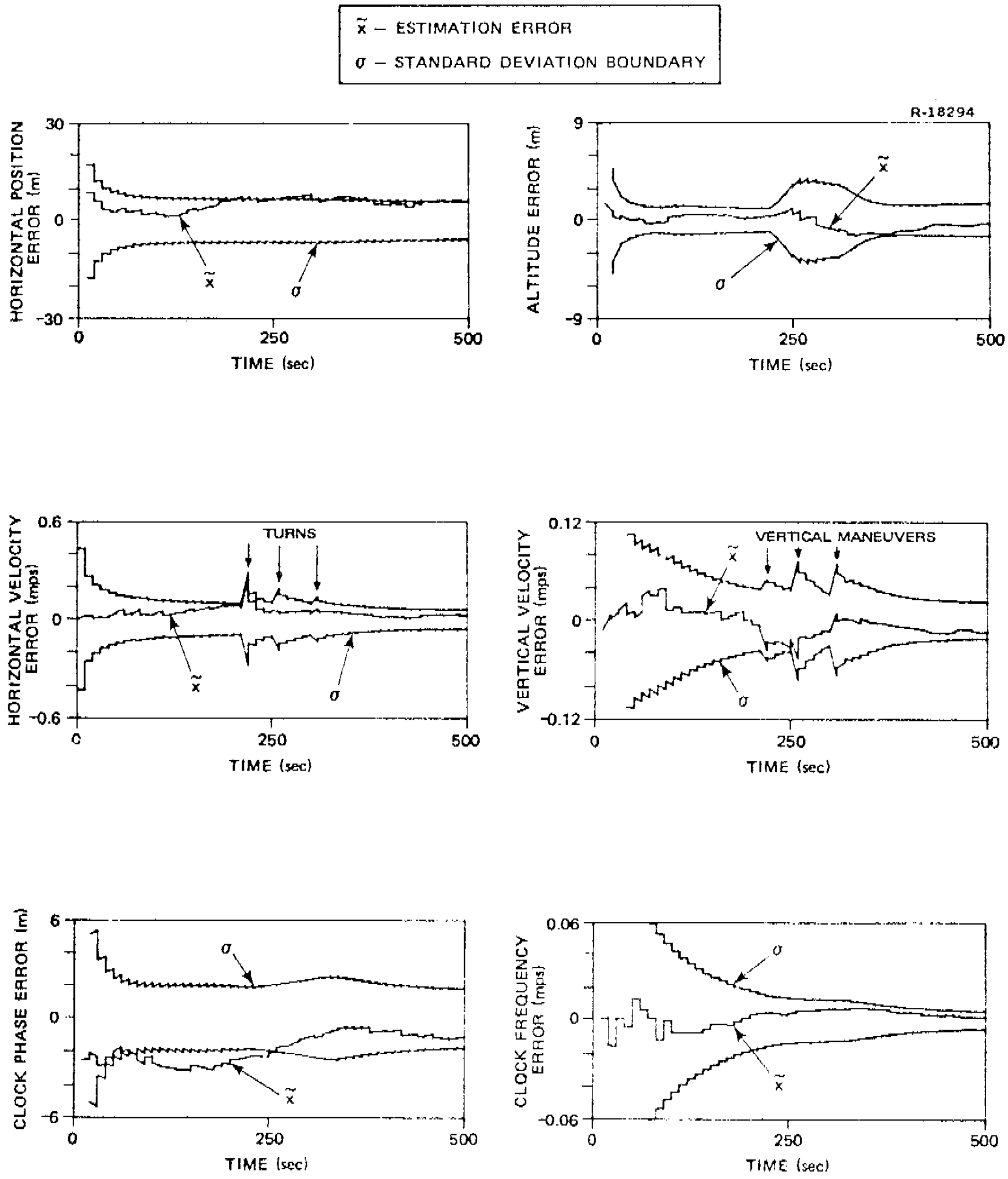

Fig. 8 Airborne User Errors - Baseline

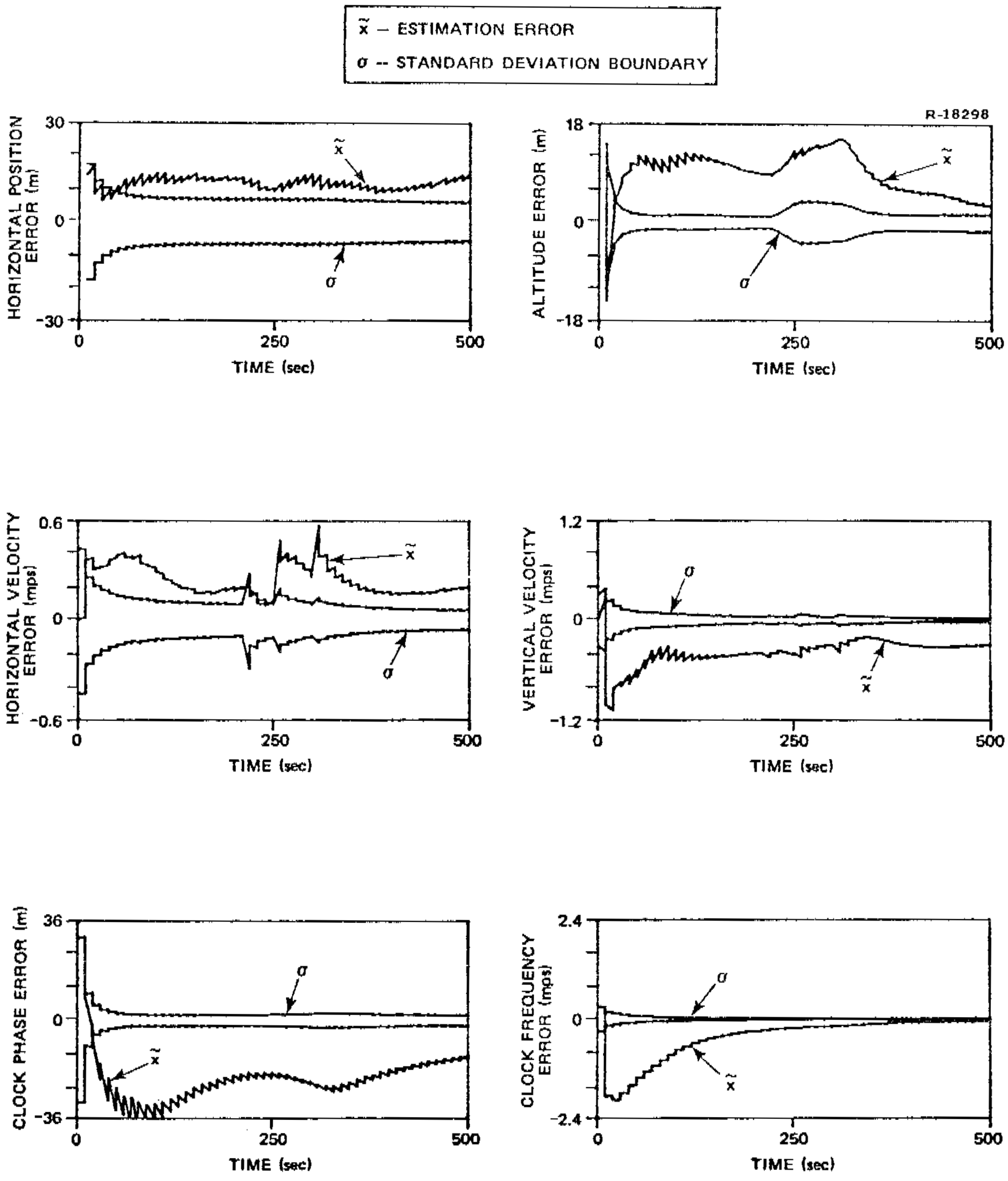

Fig. 9 Airborne User Errors - Warm-up (Not Modeled in User Filter)

the filter cannot properly allocate the large pseudo-range-rate measurement residual. The clock phase error also increases due to the warm-up induced frequency offset and consequently user position errors are increased.

The use of warm-up initial conditions at 12 minutes after oscillator turn-on is arbitrary; however, it is consistent with the time frame imposed by the calibration of the INS. If the oscillator were turned on at a point in time closer to the start of the flight, the frequency offset would be larger and consequently user navigation performance would be degraded. Conversely, if a longer warm-up period were allowed, user navigation errors would tend to be closer to their baseline values.

Acceleration - The effects of acceleration of the user clock on navigation performance are shown in Fig. 10. The frequency error arising from the g-sensitivity of the quartz oscillator is readily apparent as the airborne user traverses the trajectory of Fig. 7. The impact of this frequency error is manifest in the horizontal and vertical velocity errors. When the user resumes straight and level flight following the third turn, the frequency error is reduced through processing of pseudo-range-rate measurements. Since the filter model does not contain any information about the acceleration sensitivity, the filter gains are small at this point and do not weight the pseudo-range-rate measurements heavily enough to remove the residual frequency error entirely. One consequence of this residual frequency error is the large residual velocity error which remains after straight and level flight is resumed.

The clock phase error grows to a maximum value of 200 m as the user proceeds through the turns and begins to recover during the straight and level portion of the flight. However, since the filter gains are small (the filter has no information about the large clock phase error), the phase error is not reduced rapidly. The large horizontal velocity error ($\simeq$1 mps) drives the horizontal position error to 60 to 70 m. The altitude error results from the misallocation (by the filter) of the large pseudo-range measurement residual which is due to the clock phase error (these errors are highly correlated).

Shock - The effects on user navigation performance of a shock to the receiver crystal oscillator are shown in Fig. 11. The sharp jump in frequency error is quite evident as is the rapid growth in clock phase error due to this frequency shift. These errors are subsequently reduced through

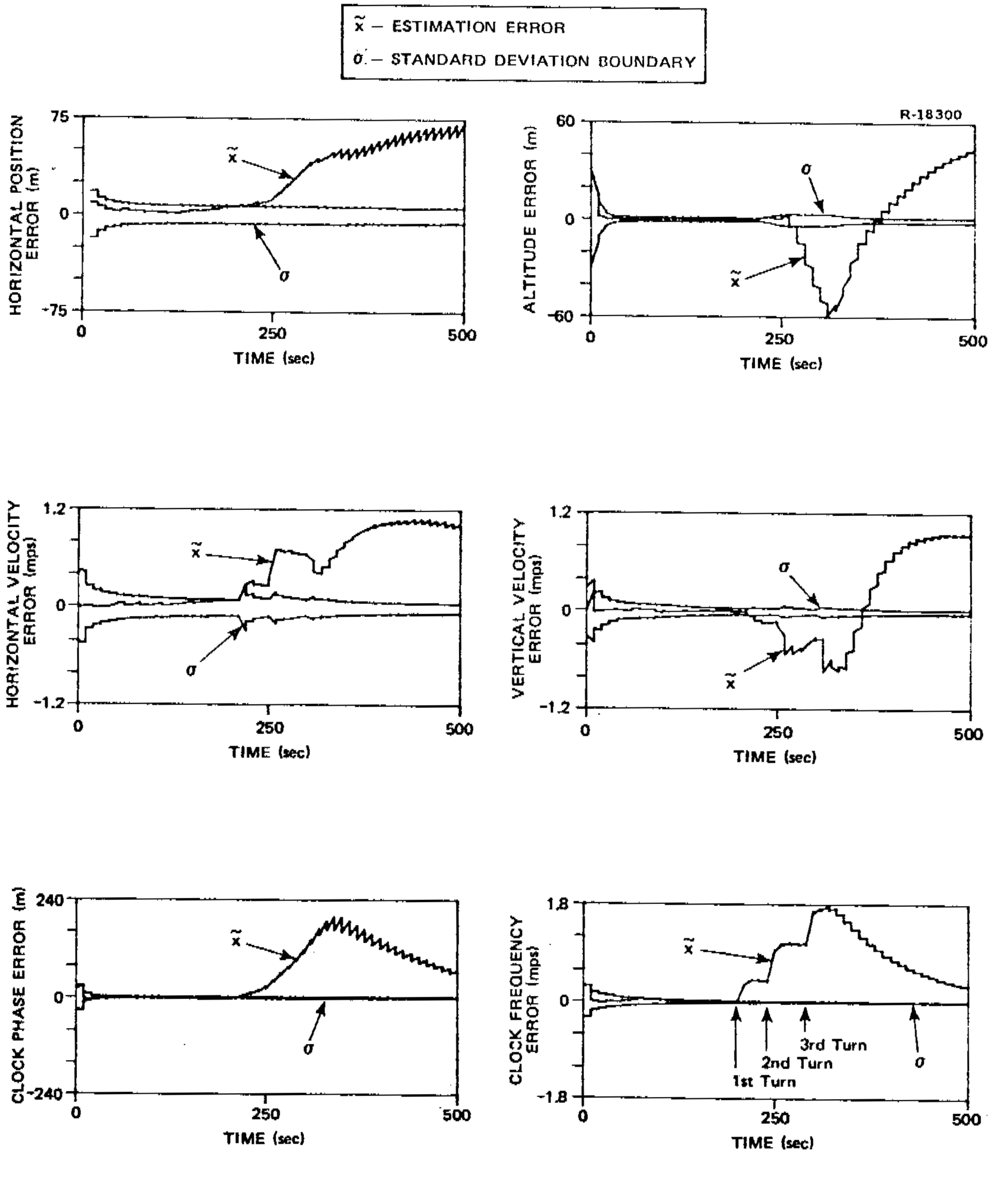

Fig. 10 Airborne User Errors - Acceleration (Not Modeled in User Filter)

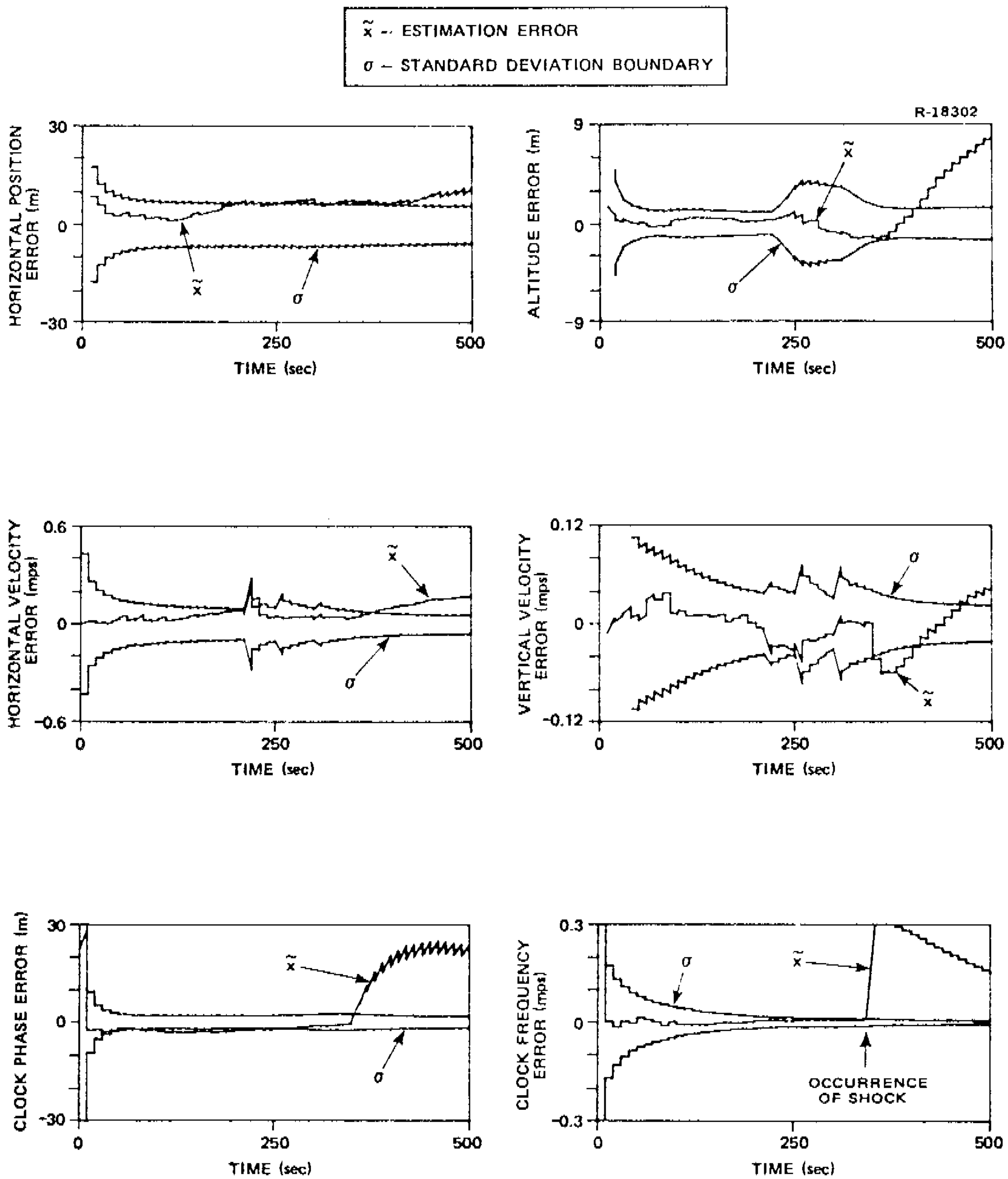

Fig. 11 Airborne User Errors - Shock (Not Modeled in User Filter)

processing of pseudo-range and pseudo-range-rate measurements; however, the recovery is not rapid since the filter gains are small during this latter portion of the flight, and the position and velocity errors appear to diverge after t = 350 seconds.

The performance projections presented above have graphically demonstrated that a significant degradation in user navigation performance can result from environmentally-induced oscillator errors. Rather than attempt to further isolate the oscillator from its environment as a means to reduce the navigation errors, it should be possible to reformulate the user Kalman filter such that it "acknowledges" the simultaneous presence of all the environmentally-induced error sources. The dominant error sources which must be considered in the filter design are warm-up, acceleration and shock.

In order to compensate for the impact of oscillator warm-up the decaying exponential model developed above may be included in the user filter. This approach is not applicable in the case of either acceleration or shock since, for the acceleration sensitivity of the oscillator, any explicit model would require knowledge of vehicle acceleration relative to the crystal orientation within the oscillator - a much too complex situation for a real time filter. Similarly, for the case of a shock to the oscillator, a priori knowledge of the time of occurrence of a shock is unlikely and such behavior cannot be described by a Gauss-Markov random process model.

In light of this one simple method of improving the filter's response to unknown accelerations or an unanticipated shock is to add white process noise to the filter to increase the variance of the frequency error state. This results in an increased uncertainty associated with this filter state, making it less sensitive to unmodeled errors.

Thus, modifying the user filter to include the model of oscillator warm-up [decaying exponential with initial variance of $(3.05 \text{ mps})^2$] and the additional white noise driving the frequency error [spectral level, $q = 1.83\times10^{-3}(\text{mps})^2/\text{sec})$], should provide adequate user navigation performance under all environmental conditions. This performance is indeed realized as shown in Fig. 12. For this run all of the environmental error sources discussed individually in the preceding paragraphs were included in the "real world" clock model. The horizontal position and velocity errors are indistinguishable from their values in the baseline run (in

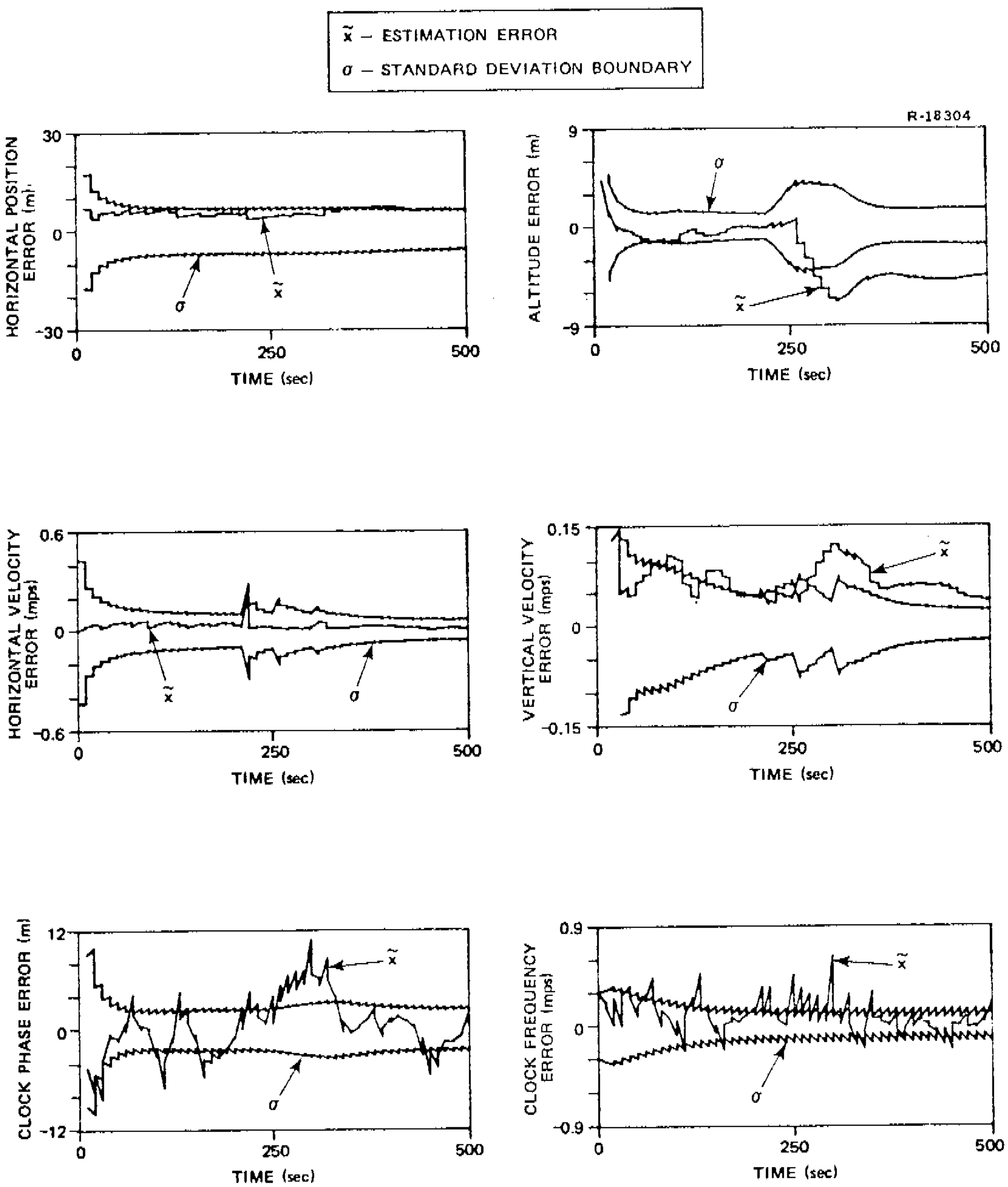

Fig. 12 Airborne User Errors - Modified User Filter
(All Environmental Error Sources Present)

which it was assumed that no environmentally-induced errors existed). The increase in altitude error during the second turn is a result of the unmodeled phase error growth in the turn. Due to the high correlation between altitude and clock phase errors, the filter cannot distinguish between the large phase error and the altitude errors, thus causing an increase in the altitude error at the pseudo-range update. Following the turn, the altitude error settles out at 4 to 5 m. A similar situation arises between vertical velocity and clock frequency errors. The vertical velocity error increases at the pseudo-range-rate measurement as a result of the jumps in frequency error but is reduced to about 0.05 mps during straight and level flight. The altitude and vertical velocity errors are much smaller with the modified filter than they would be with the original filter (compare with the errors resulting from acceleration of the oscillator prior to the filter modification, shown in Fig. 10). Although filter design was not the objective of this study, it should be possible to further reduce these errors by including white process noise driving the altitude and vertical velocity states in the filter. This alternative was not pursued here.

It should be noted that this approach does have its disadvantages. In a relatively benign environment (e.g., small accelerations, no unanticipated shocks, etc.), the inclusion of additional process noise in the user filter will lead to a minor degradation in user navigation performance. The tradeoff between improved filter performance in the presence of environmental disturbances must be weighed against any performance degradation which may arise in a benign environment when a filter design study is undertaken.

CONCLUSIONS

This paper has presented the development of a model for environmentally-induced errors in a satellite navigation system receiver quartz crystal oscillator. The error sources, which proved to be dominant in an airborne user scenario, were acceleration, shock and oscillator warm-up. The temperature and vibration sensitivities of the oscillator had minimal impact on user navigation errors for the environment considered in this study.

The environmental error sources have direct impact upon the oscillator phase and frequency errors. Due to the high correlation between clock errors and vertical channel errors, large unmodeled phase and frequency errors tend to increase altitude and vertical velocity errors at filter updates, i.e., the filter cannot properly attribute the large pseudo-range and pseudo-range-rate measurement residuals to the clock states alone and thus "divides" the large residual between clock and vertical channel errors. The horizontal position and velocity errors are insensitive to all but extreme environmentally-induced errors (e.g., acceleration).

A simple modification of the user filter to model oscillator warm-up and the effects of acceleration and shock substantially improves user navigation performance if these error sources are present. However, in the absence of these errors the modified filter introduces some degradation in user navigation performance. Careful "tuning" of the user filter or an adaptive filtering mechanization could reduce this sensitivity.

ACKNOWLEDGEMENT

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QUESTION AND ANSWER PERIOD

MR. RUEGER:

Where did you come by the acceleration numbers? Are those experimentally determined by your people?

MR. MEALY:

No, we don't do it. Dr. Hafner at ECOM provided some of it.

MR. RUEGER:

I was going to say that our experience has been factors of 4 higher than that in the coefficients that you might use. I don't know that it doesn't create a problem.

MR. MEALY:

It is a factor of 4 higher, or a factor of 4 worse is what it amounts to, so we haven't treated those.

DR. REINHARDT:

In your first slide, you seemed to indicate that you had a model for generating flicker noise from white noise. Was that true?

MR. MEALY:

Yes.

DR. REINHARDT:

And what kind of model is that?

MR. MEALY:

It is described in a paper I gave last May at the Frequency Control Symposium. It is a transfer function that I can discuss with you if you would like. It does work fairly well. It is a shaping filter that works in the frequency domain, such that you shape the spectrum of the white noise to follow $\frac{1}{f}$ behavior.

DR. REINHARDT:

Does it work in a finite domain?

MR. MEALY:

This was all generated using discrete time. It is a continuous formulation, but can be converted to a discrete time formulation.

DR. REINHARDT:

But does it work in a finite or infinite frequency domain?

MR. MEALY:

Oh, finite frequency range, but the limits are arbitrary.