EM FIELDS IN NONUNIFORM MULTILAYERED STRUCTURES STEADY STATE AND LORAN C PULSE EXCITATION

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ABSTRACT

The steady-state electromagnetic field response from nonuniform layered models of the earth's crust are used to compute the transient response due to LORAN C pulse excitations. Using a full wave approach, the electromagnetic fields are expressed completely in terms of a continuous spectrum of vertically polarized waves (the radiation term) and a discrete set of vertically polarized surface waves (the waveguide modes of the structure). Thus the scattered radiation fields and surface waves due to incident plane waves are computed. The full wave solutions satisfy the reciprocity relationships in electromagnetic theory.

These investigations are relevant to problems of navigation since it is possible to facilitate the derivation of the LORAN grid corrections due to ground effects if the propagation delays of the LORAN pulses can be related to the LORAN pulse distortions. Thus in this work analytical expressions are derived for the propagation delays due to ground effects (compared with the smooth perfectly conducting case) and the dependence of the distortions of the received signal upon the ground parameters along the propagation path is determined for different excitations.

This approach could also be used to determine the effects of a nonuniform stratified model of the ionosphere upon satellite navigation signals.

1. INTRODUCTION

A full wave solution for the steady state electromagnetic fields due to a magnetic line source over a nonuniform stratified model of the earth is the basis for the present investigation [1],[2]. The complete expansion of the electromagnetic fields consists of the radiation term (continuous spectrum of vertically polarized waves) and the surface waves (discrete set of vertically polarized waves). Since these solutions are shown to satisfy the reciprocity relationships in electromagnetic theory the scattered radiation fields and surface waves due to incident plane waves are considered in detail.

The scattered electromagnetic fields were computed earlier for $\exp(i\omega t)$ excitations for various nonuniform models of the earth's crust to determine the feasibility of using a radio wave for geophysical prospecting [3]. It is seen from the formal solution of the problem that considerable physical insight could be gained by determining the electromagnetic response to transient excitations of nonuniform stratified structures [4]. Thus in this work the transient electromagnetic fields due to LORAN C pulse excitations over the earth's surface are evaluated using both analytic methods and Fast Fourier transform techniques.

The principal motivations for this work are twofold.

(a) To determine the distortions an electromagnetic pulse undergoes when it is scattered by a nonuniform stratified model of the earth's crust.

(b) To determine whether the propagation delays of the LORAN pulses due to ground effects can be related to the LORAN pulse distortions.

Thus analytical expressions are derived for the propagation delays due to ground effects (compared with the smooth perfectly conducting case). In addition, the dependence of the distortions of the received signals upon ground parameters along the propagation path is determined for different excitations. This approach could also be used to predict the effects of a nonuniform stratified model of the ionosphere upon satellite navigation signals.

2. FORMULATION OF THE PROBLEM

In this work the scattering of vertically polarized waves only is considered in detail. Thus, the excitations are assumed to be due to a z-directed magnetic line source $\overline{J}_m$ (see Fig. 1), where

$$\overline{J}_m = K\delta(x - x_o)\delta(y - y_o)\overline{a}_z \tag{2.1}$$

The dual problem, scattering of horizontally polarized waves, can be analyzed in a similar manner by considering excitations due to z-directed electric line sources.

The dielectric coefficient of the overburden ($o > y > -h$) is

$$\varepsilon_1 = \varepsilon_o\varepsilon_r(s) \tag{2.2a}$$

where

$$\varepsilon_r(s) = \varepsilon_R + \sigma/s = \varepsilon_R + \varepsilon_I(\omega_o/s) \tag{2.2b}$$

in which ε_R and the conductivity $\sigma = \varepsilon_I\ \omega_o$ are positive real numbers independent of frequency, $s = \alpha + i\omega$ is the complex frequency, ω_o is the radio wave carrier frequency in radians/sec. and ε_o is the permittivity of free space. The permeability of the overburden is assumed to be that of free space μ_o (non-magnetic material), thus the overburden intrinsic impedance is $\eta_1 = (\mu_o/\varepsilon_1)^{\frac{1}{2}}$.

It is convenient to characterize the substratum by a surface impedance

$$Z_s = \sqrt{\mu_o/\varepsilon_2} \tag{2.3}$$

where

$$|\varepsilon_2(\omega_o)/\varepsilon_1(\omega_o)| >> 1$$

and ε_2 is the substratum dielectric coefficient.

The overburden depth is

$$h(x) = h_o + \delta x \qquad -L < x < L \tag{2.4}$$

where h_o is the average overburden depth, $\delta = \tan\Delta$ is the slope of the overburden-substratum interface.

Explicit solutions for the electromagnetic fields scattered by an overburden of nonuniform depth (2.4) have been derived for steady state $(\exp(i\omega t))$ excitations. Since the problem is two dimensional, both the incident and the scattered fields are vertically polarized.

The wave vectors for free space and the overburden are expressed as:

$$\bar{k}_{o,1} = \beta\bar{a}_x + u_{o,1}\,\bar{a}_y \tag{2.5a}$$

in which the longitudinal wave number β is positive real for the radiation term; $0 \leq \beta < k_o$ for the propagating waves, $\beta > k_o$ for evanescent waves, and

$$k_{o,1} = \omega\sqrt{\mu_o\varepsilon_{o,1}} \tag{2.5b}$$

Thus the transverse wave number is

$$u_{o,1} = (k_{o,1}^2 - \beta^2)^{\frac{1}{2}} \tag{2.5c}$$

The square roots in (2.5) are chosen such that $\mathrm{Im}(k_{o,1}) \leq 0$ and $\mathrm{Im}(u_{o,1}) \leq 0$ at the carrier frequency. The wave parameters $u_{o,1}$ and β are also expressed in terms of the complex cosine and sine of the angles θ_o and θ_1. Thus

$$u_{o,1} = k_{o,1}\cos\theta_{o,1} = k_{o,1}C_{o,1},\ \beta = k_{o,1}\sin\theta_{o,1} = k_{o,1}S_{o,1} \tag{2.6}$$

Hence for $0 \leq \beta < k_o$, θ_o is the angle between the wave vector $\bar{k}_o$ and the y axis. The transverse wave impedance is

$$Z(u) = \beta/\omega\varepsilon = \begin{cases} \beta/\omega\varepsilon_o = Z_o, y > 0 \\ \beta/\omega\varepsilon_1 = Z_1, -h < y < 0 \end{cases} \tag{2.7}$$

For $0 > y > -h$, the basis function for the radiation term is

$$\Psi(u,y) = \{\exp[iu_1(y+h)] + R_{21}(u)\cdot\exp[-iu_1(y+h)]\}\Psi(u,-h)/T_{21}(u) \tag{2.8a}$$

where

$$\Psi(u,-h) = \exp(-iu_1h)T_{21}(u)T_{1o}(u)/(2\pi\, Z_o(u))^{\frac{1}{2}}[1-R_{o1}(u)R_1(u)] \tag{2.8b}$$

The reflection coefficients looking in the negative y direction at the air-overburden interface and the overburden-substratum interface are respectively

$$R_o(u) = (K_o - z_1)/(K_o + z_1) \tag{2.9a}$$

$$R_1(u) = R_{21}(u)\exp(-2iu_1h) = [(K_1 - Z_s)/(K_1 + Z_s)]\cdot\exp(-2iu_1h) \tag{2.9b}$$

in which

$$z_1(u) = K_1(Z_s + iK_1 \tan u_1h)/(iZ_s \tan u_1h + K_1) \tag{2.9c}$$

$$K_{o,1} = u_{o,1}/\omega\varepsilon_{o,1} \tag{2.9d}$$

and the two-medium Fresnel reflection and transmission coefficients for the air-overburden interface and the overburden-substratum interface are given by

$$R_{o1}(u) = (K_1 - K_o)/(K_1 + K_o'), R_{21}(u) = (K_1 - Z_s)/(K_1 + Z_s) \tag{2.10a}$$

$$T_{1o}(u) = 1 + R_{1o}(u) = 1 - R_{o1}(u), T_{21}(u) = 1 + R_{21}(u) \tag{2.10b}$$

When the receiver and the source are far from the nonuniform region, ($k_o\rho >> 1$ and $k_o\rho_o >> 1$) (see Fig. 1), the forward scattered radiation field is [3],

$$H_z^f(x,y) = (2\pi/k_o\rho)^{\frac{1}{2}} \exp(-ik_o\rho) \exp(i\pi/4)P(u^f,u^i) \tag{2.11a}$$

where the radiation pattern is given by

$$P(u^f,u^i) = P_o(u^f,u^i)\sum_{p,q}^{\infty} I_{p,q}(u^f,u^i) \tag{2.11b}$$

$$P_o(u^f,u^i) = H_m^i(1-S_1^fS_1^i - z_s^2)T_{21}(u^f)T_{1o}(u^f)T_{21}(u^i)T_{1o}(u^i)/4\pi\sqrt{\varepsilon_r} \tag{2.11c}$$

$$H_m^i = - a_o/[2\pi Z_o(u^i)]^{\frac{1}{2}} \tag{2.11d}$$

and

$$a_o = \tfrac{1}{2} K \exp(i\pi/4)(\omega\varepsilon_o\beta^i/k_o\rho_o)^{\frac{1}{2}} \exp(-ik_o\rho_o) \tag{2.11e}$$

In (2.11c) the incident and scattered waves are in the directions of $\overline{k}_o^i$ and $\overline{k}_o^f$ respectively and H_m^i is the magnitude of the incident magnetic field, H_z, at the origin, excited by the magnetic line source of intensity K (2.1) (Fig. 2). The normalized surface impedance is $z_s = Z_s/\eta_1$. Furthermore

$$\sum_{p,q}^{\infty} I_{p,q}(u^f,u^i) = \frac{1}{S_1^f-S_1^i}\int_{-L}^{L}\Big\{k_oh'(x)\exp[i(\beta^f-\beta^i)x-i(u_1^f+u_1^i)h] \tag{2.12}$$

$$\cdot\sum_{p=1}^{\infty}[R_{o1}(u^f)R_{21}(u^f)\exp(-2iu_1^fh)]^{p-1}$$

$$\cdot\sum_{q=1}^{\infty}[R_{o1}(u^i)R_{21}(u^i)\exp(-2iu_1^ih)]^{q-1}\Big\}dx$$

where

$$h'(x) = dh(x)/dx \tag{2.13}$$

The infinite geometric series expansions of the terms $1/[1-R_{o1}R_{21}\exp(-2iu_1h)]$ used in the above derivations are valid only for $|R_{o1}R_{21}\exp(-2iu_1h)| < 1$. The coefficient $I_{p,q}(u^f,u^i)$ can be integrated by parts [4]. After neglecting the term identified with the edge effect in the theory of scattering by rough surfaces [5],[6], the general expression for the coefficient $I_{p,q}(u^f,u^i)$ for $h = h_o + \delta x, (-L < x < L)$, is found to be

$$I_{p,q}(u^f,u^i) = \frac{[R_{o1}(u^f)R_{21}(u^f)]^{p-1}[R_{o1}(u^i)R_{21}(u^i)]^{q-1}}{(2p-1)C_1^f + (2q-1)C_1^i} \cdot \exp[-i\{(2p-1)u_1^f + (2q-1)u_1^i\}h_o]2k_oL \cdot \operatorname{sinc}[(\beta^f-\beta^i) - \{(2p-1)u_1^f + (2q-1)u_1^i\}\delta] \tag{2.14}$$

where

$$\operatorname{sinc}(v) = \sin(v)/v \tag{2.15}$$

The terms corresponding to $p \neq 1$ are missing from the geometrical optical solution [3].

For the special case $\delta = 0, h = \text{constant}$ (parallel stratified earth) the terms $1/[1 - R_{o1}R_{21}\exp(-2iu_1h)]$ are not functions of x. In this case it is not necessary to expand these factors in a geometric series to facilitate the integration. Thus, for $\delta = 0$, the radiation pattern is

$$P(u^f,u^i) = P_o(u^f,u^i)I_{11}(u^f,u^i)\cdot(1/[1-R_{o1}(u^f)R_{21}(u^f)\exp(-2iu_i^fh)]) \cdot 1/[1-R_{o1}(u^i)R_{21}(u^i)\exp(-2iu_1^ih)] \tag{2.16}$$

Substitute (2.13) for I_{11} into (2.16) to get

$$P(u^f,u^i) = \frac{P_o(u^f,u^i)\exp[-i(u_1^f+u_1^i)h_o]2k_oL\ \operatorname{sinc} L(\beta^f-\beta^i)}{(C_1^f+C_1^i)[1-R_{o1}(u^f)R_{21}(u^f)\exp(-2iu_1^fh_o)][1-R_{o1}(u^i)R_{21}(u^i)\exp(-2iu_1^i)]} \tag{2.17}$$

For the special case $k_o=k_1$ and $S_o^i = S_o^f$ (2.17)

$$P_o(u^f,u^i) = H_m^i(C_1^{i2}-z_s^2)C_1^{i2}/\pi(C_1^i+z_s)^2 = H_m^i\ C_1^{i2}\ R_{21}^i/\pi \tag{2.18}$$

thus

$$H_z^f(x,y) = H_m^i(i/2\pi\ k_o\rho)^{\frac{1}{2}}\exp(-ik_o\rho)2\ k_oLC_1^iR_1 \tag{2.19}$$

Equation (2.19) is the expression for diffuse scattering in the specular direction.

The back scattered radiation field H_z^b at an angle θ_o^b from the y axis can be obtained from (2.11) by replacing β^f by $-\beta^b = -k_o\sin\theta_o^b$.

The modal equation for the surface wave is

$$\frac{1}{R_o} = \frac{1-R_{o1}R_1}{R_1-R_{o1}} = 0 \tag{2.20a}$$

Thus

$$R_{o1}R_1 = R_{o1}R_{21}\exp(-2iu_1h) = 1 \tag{2.20b}$$

It is convenient to express (2.20b) as

$$i\tan u_1h = \left(-\frac{u_o\varepsilon_r}{u_1} - \frac{z_s\sqrt{\varepsilon_r}k_o}{u_1}\right)\Big/\left(1 + \frac{u_o\varepsilon_r}{u_1}\frac{z_s\sqrt{\varepsilon_r}k_o}{u_1}\right) \tag{2.20c}$$

The modal equation (2.20c) is solved numerically for the nth root, $u_{on}(x)$, using the Richmond process [7]. The corresponding values for β_n and u_{1n} are obtained by (2.5).

The solution for the scattered radiation field excited by an incident surface wave is greatly simplified by replacing $\beta_n(x)$ by $\overline{\beta}_n = \beta_n(o)$. The subscript $n = 0$ denotes the dominant surface wave mode.

The surface wave excited by an incident plane wave is [3].

$$H_z^f(x,y) = \exp(-i\overline{\beta}_nx)\Psi(u_n,y)\Psi(u_n,o)(2\pi/\omega\varepsilon_o)P(\overline{u},u^i) \tag{2.21}$$

in which the surface wave basis functions are

$$\Psi(u_n,y) = \Psi(u_n,o)\cdot\begin{cases}\cdot\exp(-iu_oy) & , y > 0\\ \cdot\exp[(-iu_1y)+R_{o1}\exp(iu_oy)]/(1+R_{o1}), & y < 0\end{cases} \tag{2.22a}$$

and

$$[\Psi(u_n,o)]^2 = 2iu_{on}/Z_o\left[1-\left(\frac{u_{on}}{u_{1n}}\right)^2\cdot\left\{1+\frac{Z_s(K_1^2-K_o^2)}{K_o(K_1^2+Z_s^2)} - \frac{iu_1h(K_1^2-K_o^2)}{K_1K_o}\right\}\right] \tag{2.22b}$$

The surface wave basis functions are both explicit functions of y and implicit functions of x through h(x). Thus in (2.21) $\Psi(u_n,y)$ is evaluated at the field point (x,y) and $\Psi(u_n,o)$ is evaluated at the origin. Furthermore

$$P(\overline{u},u^i) = P_o(\overline{u},u^i)\sum_{pq} I_{pq} \qquad p,q=1,2,3\ldots \tag{2.23}$$

where

$$P_o(\overline{u},u^i) = H_m^i(1-S_1^i\overline{S}_{1n}-z_s)T_{21}(u^i)T_{1o}(u^i)T_{21}(\overline{u})/4\pi\sqrt{\varepsilon_r} \tag{2.24a}$$

On neglecting the edge effect

$$I_{pq}(u^i,\overline{u}) = \frac{[R_{21}(u^i)R_{o1}(u^i)]^{p-1}[-R_{21}(\overline{u})]^{q-1}}{(2p-1)C_1^i + (2q-1)\overline{C}_{1n}}$$

$$\cdot\exp[-i\ (2p-1)u_1^i + (2q-1)\overline{u}_{1n}\ h_o]2k_oL$$

$$\cdot \operatorname{sinc}\left[L[\bar{\beta}_n-\beta^i)-\{(2p-1)u_1^i+(2q-1)\bar{u}_{1n}\}\delta]\right] \quad p,q=1,2,3\ldots \tag{2.24b}$$

and H_m^i is given by (2.11d). Equation (2.25d) is valid for

$$|R_{o1}(u^i)R_{21}(u^i)\exp(-2iu_1^ih_o)| < 1 \text{ and } |R_{21}(\bar{u})\exp(-2i\bar{u}_1h_o)| < 1. \tag{2.25e}$$

For the special case $\delta=0$, the closed form solution for H_z^f is

$$H_z^f(x,y) = \exp(-i\bar{\beta}_n x)\Psi(u_n,y)\Psi(u_n,o)(2\pi/\omega\varepsilon_o)P_o(u_n,u^i) \tag{2.26}$$

$$\cdot \frac{\exp[-i(u_1^i+\bar{u}_1)h_o]2k_oL \operatorname{sinc} L(\bar{\beta}_n-\beta^i)}{(C_1^i+\bar{C}_{1n})[i-R_{o1}(u^i)R_{21}(u^i)\exp(-2iu_1^ih_o)][1+R_{21}(\bar{u})\exp(-2i\bar{u}_{1n}h_o)]}$$

The WKB solutions for the surface waves excited by the magnetic line source are given by [3],

$$H_z^f(x,y) = -\frac{K}{2}\Psi(u_n,y_o)\Psi(u_n,y)\exp\left(-i\int_{x_o}^{x}\beta_n dx\right) \tag{2.27a}$$

in which the basis function $\Psi(u_n,y_o)$ is evaluated at the source point (x_o,y_o), (2.22).

The steady state response $H_z(\omega)$ is derived for time harmonic excitations. For arbitrary time varying excitations the instantaneous expression for the magnetic line source is

$$\bar{J}_m(\bar{r},t) = \operatorname{Re} f_s(t)\delta(x-x_o)\delta(y-y_o)\bar{a}_z \tag{2.27b}$$

The excitation function $f_s(t)$ is taken to be complex for convenience. The Fourier transform of $f_s(t)$ is

$$F(\omega) = F[f_s(t)] = \int_{-\infty}^{\infty} f_s(t)\exp(-i\omega t)dt \tag{2.28}$$

and $F(\omega) = F(-\omega)^*$ only when $f_s(t)$ is real. The asterisk denotes the complex conjugate. Thus on applying Fourier transform techniques and noting that $H_z(\omega) = H_z^*(-\omega)$, the transient response can be given by

$$h(\bar{r},t) = \operatorname{Re} F^{-1}[H_z(\omega)\cdot F(\omega)] = \operatorname{Re}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} H_z(\omega)F(\omega)\exp(i\omega t)d\omega\right)$$
$$\equiv \operatorname{Re} h_s(\bar{r},t) \tag{2.29}$$

3. FAST FOURIER TRANSFORM (FFT)

The fast Fourier transform (FFT) algorithm [8] is used to perform the numerical integration required to evaluate either a Fourier transform or an inverse Fourier transform. The number of samples used in subroutine FFT is $N = 2^m$ where m is an integer. For this choice of N the FFT algorithm is most efficient [8]. Let the symbols k(k= 1,2...N) and n = 1,2...N denote the sample number in the time and frequency domains respectively. The summations performed to evaluate the transform and the inverse transform are respectively,

$$F(n) = \frac{1}{N}\sum_{k=1}^{N}[f_r(k) + if_i(k)]\exp[-2i\pi(n-1)(k-1)/N]\cdot t_{max} \tag{3.1a}$$

$$f^*(k) = \frac{1}{N}\sum_{n=1}^{N}[F_r(n) + iF_i(n)]^*\exp[-2i\pi(n-1)(k-1)/N]\cdot 2f_{max} \tag{3.1b}$$

Because the same subroutine is used to evaluate the transform F(n) (3.1a) and the inverse transform f(k) (3.1b), $f^*(k)$ (3.1b) is initially evaluated using the complex conjugate of the frequency domain samples $F^*(n)$ and f(k) is obtained by taking the complex conjugate of (3.1b).

The values of the N samples F(n) in the frequency domain are obtained from N + 1 values of $F(\alpha + i\omega)$ taken at N + 1 points equally spaced on the imaginary axis (ω) from $-\omega_{max}$ to $+\omega_{max}$. Samples n = 1,2...N/2 correspond to $F[(n-1)\Delta\omega]$ where $\Delta\omega = 2\omega_{max}/N$. The sample n = N/2 + 1 is given by $F(N/2 + 1) = [F(-i\omega_{max}) + F(i\omega_{max})]/2$. The samples n = N/2 + 2, N/2 + 3...N correspond to $F[(n-1-N)\Delta\omega]$. Similarly, in the time domain the numbering scheme is as follows:

$f(k) = f[(k-1)\Delta t]$, k = 1,...N, and $f(1) = f(N+1) = f(t_{max})$.

The increments in frequency $\Delta\omega$ and in time Δt between samples are determined by the Shannon criteria [8] which for this case is

$$N = 2^m = \frac{2\ f_{max}}{1/t_{max}} \tag{3.2a}$$

where

$$2\ f_{max} = N\Delta f \text{ and } t_{max} = N\Delta t \tag{3.2b}$$

4. EXCITATION-LORAN-C PULSE

The excitation (2.27) used in this work is the Loran-C pulse. This pulse can be represented as the real part of the sum of three damped sinusoids [9].

$$f(t) = \mathrm{Re}[f_s(t)] \tag{4.1a}$$

where

$$f_s(t) = \begin{cases} 0 & t < 0 \\ \sum_{j=1}^{3} A_j\ \exp(-\Gamma_j t) & t > 0 \end{cases} \tag{4.1b}$$

in which

$$A_1 = i/2 \quad \Gamma_1 = c + i\omega_1 \quad \omega_1 = \omega_o$$

$$A_2 = -i/4 \quad \Gamma_2 = c + i\omega_2 \quad \omega_2 = \omega_o + 2\omega_p$$

$$A_3 = -1/4 \quad \Gamma_3 = c + i\omega_3 \quad \omega_3 = \omega_o - 2\omega_p \tag{4.1c}$$

where ω_o is the radian frequency of the carrier and ω_p is the radian frequency of the envelope (the modulation frequency) and $c > 0$ is the damping coefficient.

For the Loran-C pulse excitation (4.1) $f(t)$ and its derivatives are continuous. Furthermore, for the values

$$c = 25000 \quad f_p = \omega_p/2\pi = 2500 \text{ Hz} \quad f_o = \omega_o/2\pi = 100 \text{ k Hz} \tag{4.1d}$$

99% of the input power is between 90 to 110 k Hz. This property of the Loran-C pulse considerably facilitates the numerical analysis since the Fourier transform $H(\omega)$ of the response $h(t)$ can be regarded as band-limited. The Fourier transform $H(\omega)$ is

$$H(\omega) = F(\omega)\, H_z(\omega) \tag{4.2}$$

where $F(\omega)$ is the Fourier transform of $f_s(t)$ (4.1b) and $H_z(\omega)$ is the transfer function, Section 2.

For the values of A_j and Γ_j given in (4.1c)

$$F(\omega) = \frac{2i\omega_p^2}{[i(\omega+\omega_o)+c][\{i(\omega+\omega_o)+c\}^2 + 4\omega_p^{\,2}]} \tag{4.3}$$

and

$$|F(\omega)|^2 = \frac{4\omega_p^4}{[(\omega+\omega_o)^2+c^2][\{c^2+4\omega_p^{\,2}-(\omega+\omega_o)^2\}^2 + 4c^2(\omega+\omega_o)^2]} \tag{4.4a}$$

Thus $|F(\omega)|^2$ peaks at $\omega = -\omega_o$, and

$$|F(\omega)|_{max} = 1/4[\omega_p D(1 + D^2)], \; D = c/2\omega_p \tag{4.4b}$$

If D is fixed, increasing ω_p (and c) decreases the magnitude of $|F(\omega)|_{max}$ and the bandwidth of $F(\omega)$ increases.

The sum of the three damped sinusoids (4.1b) can be expressed as follows:

$$f_s(t) = i \exp(-ct) \sin^2 \omega_p t \exp(-i\omega_o t). \tag{4.5a}$$

Thus

$$|f_s(t)| = \exp(-ct)\sin^2 \omega_p t \tag{4.5b}$$

and

$$f(t) = \text{Re } f_s(t) = \exp(-ct) \sin^2 \omega_p t \sin \omega_o t = |f_s(t)| \sin \omega_o t. \tag{4.5c}$$

Thus $|f_s(t)|$ is the envelope of $f(t)$. It peaks at $t = t_p$ and vanishes at $t = t_z$ where

$$t_p = \frac{1}{\omega_p} \tan^{-1}(1/D), \quad t_z = m\pi/_p, \quad m = 0, \pm 1, \pm 2 \ldots \tag{4.6a}$$

Hence,

$$|f_s(t_p)| = \exp(-ct_p)/(1 + D^2) \tag{4.6b}$$

The envelope of the input Loran C pulse $|f_s(t)|$ is shown in Fig. 2 for $f_o = 3 \times 10^2$ H_z, $f_p = f_o/30$, $f_o/40$, $f_o/50$, and $c = 5f_p$, $10f_p$, $15f_p$ to illustrate the effects of changing the modulation frequency f_p and the damping factor c.

5. SCATTERED RADIATION FIELD

The steady state scattered radiation field in db H_{db} as a function of the scatter angle θ^f is

$$H_{db} = 20 \log|H_z(x,y)/H_z^o| \tag{5.1a}$$

where $H_z(x,y)$ is given by (2.11a) and

$$H_z^o = H_z(\theta^f = \theta^i) \tag{5.1b}$$

for the case $\delta = 0$. (2.4). The plane wave at an angle of incidence θ^i is assumed to be excited by a magnetic line source $\overline{J}_m(\overline{r},t)$, (2.27).

The dips and peaks in the response H_{db} (as a function of θ^f) are primarily due to the minima and maxima of $|\text{sinc } v|$ (2.15)(except for near grazing incidences)

$$v = k_o L[S_o^f - S_o^i - \{(2p-1)C_1^f + (2q-1)C_1^i\} \sqrt{\varepsilon_r \delta}] = n\pi \tag{5.2}$$

where the principal maximum of θ_m^f corresponds to $n = 0$. The minima for $\theta^f < \theta_m^f$ and $\theta^f > \theta_m^f$ correspond to $n = -1,-2,-3\ldots$and $n = 1,2,3\ldots$ respectively. The maxima for $\theta^f < \theta_m^f$ and $\theta^f > \theta_m^f$ correspond to $n = -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2} \ldots$ and $n = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$ respectively.

For nondissipative overburdens S_o^f is real and

$$\theta^f = \sin^{-1}(S_o^f) \tag{5.3a}$$

For dissipative overburdens S_o^f is complex and

$$\theta^f = \text{Re}[\sin^{-1}(S_o^f)] \simeq \sin^{-1} \text{Re}(S_o^f), \quad \varepsilon_I << \varepsilon_R \tag{5.3b}$$

Using (2.11d), (2.11e), and (2.7), it follows that

$$H_m^i(2\pi/k_o\rho)^{\frac{1}{2}} \exp[i(\pi/4-k_o\rho)] = -iK \exp(-i\omega t_o)/(2\eta_o[\rho\rho_o]^{\frac{1}{2}}) \tag{5.4a}$$

in which

$$t_o = (\rho + \rho_o)/v_o \tag{5.4b}$$

and v_o is the velocity of light in free space. Thus, t_o is the time required for the signal to travel the distance $\rho + \rho_o$ (Fig. 1) in free space. For convenience set $K/(2\eta_o[\rho\rho_o]^{\frac{1}{2}}) = 1$.

The computer program evaluates the normalized response $h_{sN}(t)$ given by

$$h_{sN}(t) = h_s(t)/|f_s(t_p)|\cdot|H_z(\omega_o)| = h_s(t)/h_{os} \tag{5.5}$$

where $H_z(\omega_o)$ is the steady state response to $\exp(i\omega_o t)$ excitations (2.11a) for a specified reference case. Thus $|h_{sN}(t)|\max \simeq 1$ for the particular reference case considered.

For dissipative overburdens and when Z_s is a complex function of frequency the transfer function $H_z(s)$ is not an analytic function of s. In these cases it is convenient to express $h_s(t)$ as follows:

$$h_s(t) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \sum_{p,q} H^1_{pq}(s)\ H^2_{pq}(s)\ \sum_{j=1}^{3}[A_j/(s+\Gamma_j)]\exp(st)ds$$

$$= h_L(t) + h_H(t) \tag{5.6a}$$

where

$$H_z(s) = \sum_{pq} H_{pq}(s) = \sum_{pq} H^1_{pq}(s)\ H^2_{pq}(s) \tag{5.6b}$$

and $H_{pq}(s)$ is an individual term in the expression for $H_z(s)$. All the singularities of $H_{pq}(s)$ are contained in $H^1_{pq}(s)$ while $H^2_{pq}(s)$ is an analytic function of s. Furthermore

$$h_L(t) = \frac{1}{2\pi i}\sum_{j=1}^{3}\int_{-i\infty}^{i\infty} \sum_{pq} H^1_{pq}(-\Gamma_j)H^2_{pq}(s)[A_j/(s+\Gamma_j)]\exp(st)ds$$

$$= h_E(t)\exp(-i\omega_o t) \tag{5.6c}$$

$$h_H(t) = \frac{1}{2\pi i}\sum_{j=1}^{3}\int_{-i\infty}^{i\infty} \sum_{pq}[H^1_{pq}(s)-H^1_{pq}(-\Gamma_j)]H^2_{pq}(s)[A_j/(s+\Gamma_j)]\exp(st)ds \tag{5.6d}$$

In (5.6) $h_L(t)$ is due to the residues of the poles of F(s) and $h_H(t)$ is due to the contributions of the singularities of $H_z(s)$.

If $H^2_{pq}(s)$ in (5.6d) does not possess any singularities in the vicinity of the poles of F(s), (Γ_j), $|h_H(t)| \ll |h_L(t)|$.

For nondissipative overburdens $h_H(t) = 0$ and the envelope of h(t) is given by $|h_E(t)|$ (5.6c). For the general case $h_H(t) \neq 0$ (dissipative overburdens) and

$$h(t) = \mathrm{Re}\ h_L(t) + \mathrm{Re}\ h_H(t) \tag{5.7a}$$

Thus the envelope $h_T(t)$ of the total response is regarded as the envelope of $h_L(t)$ superimposed on the function Re $h_H(t)$. Hence

$$h_T(t) = \mathrm{Re}\ h_H(t) \pm |h_E(t)| \tag{5.7b}$$

The envelope (5.7b) is symmetric with respect to $\mathrm{Re}[h_H(t)]$.

In view of the singularities of $H_{pq}(s)$, the functions $h_s(t)$, $h_L(t)$ and $h_H(t)$ are evaluated by numerical methods. As a check on the numerical values, $h_L(t)$, the major contribution to the response $h_s(t)$ is also evaluated by analytical methods and $h_H(t)$ is evaluated directly using (5.6) and indirectly from

$$h_H(t) = h_s(t) - h_L(t) \tag{5.8}$$

An individual term $h_{Lpq}(t)$ of $h_L(t)$, (5.6c) is approximately given by

$$h_{Lpq}(t) = f_s(t_g) G_{pq}(-\Gamma_1)(1-G_p/T) \tag{5.9a}$$

in which

$$t_g = t-t_o - \frac{h_o}{v_o}\left[(2p-1)\sqrt{\varepsilon_R-(S_o^f)^2} + (2q-1)\sqrt{\varepsilon_R-(S_o^i)^2}\right] \tag{5.9b}$$

$$T = \tan \omega_p\ t_g, \tag{5.9c}$$

and

$$G_p = [1-G_{pq}(-i\omega_o)/G_{pq}(-\Gamma_1)]/D \tag{5.9d}$$

Thus the factor $(1-G_p/T)$ in (5.9a) represents the distortion of the envelope of $h_{Lpq}(t)$.

6. SURFACE WAVES EXCITED BY INCIDENT PLANE WAVES AND WKB SOLUTIONS FOR THE SURFACE WAVES

The steady state scattered surface wave in db, H_{db} at $x = L$, $y = 0$ as a function of the plane wave incident angle θ^i is

$$H_{db} = 20|\log H_z(L,0)/H^o_{zm}| \tag{6.1}$$

where $H_z(L,0)$ is given by (2.21). The normalization coefficient is $H^o_{zm} = H^o_z(\theta^i = \theta^i_m)$ in which $H^o_z = H_z(L,0)$ for $\delta = 0$ and θ^i_m is the angle at which H_z is maximum. The plane wave at an angle of incidence θ^i is assumed to be excited by a magnetic line source $\overline{J}_m(\overline{r},t)$ (2.27). Using (2.11d), (2.11e), and (2.7), it follows that

$$H^i_m(2\pi/\omega\varepsilon_o) = -K(\pi\ v_o/2\omega_o\rho_o)^{\frac{1}{2}} \exp(i\pi/4)\exp(-i\omega t_o) \tag{6.2}$$

here $t_o = \rho_o/v_o$ is the time required for the signal to travel the distance ρ_o (Fig. 1) and $K(\pi\, v_o/2\omega_o\rho_o)^{\frac{1}{2}} = 1$.

The steady state WKB solutions for the surface waves given by (2.27) are evaluated for an observation point at $x = L$, $y = 0$ and a line source at $x_o = L$, $y = 0$ and by setting the intensity of the line source $K = 2$.

The response to transient excitations is obtained by following the same procedures used to evaluate the scattered radiation fields (5.6).

7. ILLUSTRATIVE EXAMPLES

Unless otherwise specified the values of parameters used in illustrative examples presented in this section are as follows:

$$\begin{array}{llll} L = 5000 \text{ meters} & \Delta = -1 \text{ degrees} & h_o = 100 \text{ meters} & \\ \varepsilon_R = 8 & \varepsilon_I = 0 & Z_s = 0 \text{ ohms} & \\ f_o = 3\text{x}10^5 & f_p = f_o/40 & c = 10\, f_p \quad \theta^i = 85^\circ & \end{array} \tag{7.1}$$

and the number of terms of the series (2.11b) is taken to be 15, $(p-q \leq 6)$.

The response is normalized such that $H_z(\omega_o)$ in (5.16) is evaluated for $\Delta = 0^o$ and $\theta^f = \theta^i$ for the scattered radiation field and $\theta^i = \theta^i_m$ for the scattered surface wave.

The shape of the envelope depends strongly upon the incident and observation angles θ^i and θ^f. The envelope of the individual terms of the field expansions closely resemble the shape of the input Loran C pulse except for values of θ^f very near the minima of $H_{pq}(\theta^f)$. The minima of H_{pq} for different values of p and q do not occur for the same values of θ^f (unless $\Delta = 0^o$), and the envelope of the total field is not necessarily distorted only when the envelope of an individual term is distorted. The envelope of the response is usually distorted most strongly in the neighborhood of a minimum of $H_{db}(\theta^f)$. In Fig. (3) $|h_{LN}| \simeq |h_{sN}|$ is plotted for $\Delta = -1^o$, $\varepsilon_I = .5$ with $\theta^f = 44^o, 45^o, 46^o$, and 47^o. Major distortion of the envelope occurs when $\theta^f = 45^o$-46^o. In this case $H_{11}(\theta^f)$ is a minimum when $\theta^f = 44.6^o$ and $H_{db}(\theta^f)$ is minimum when $\theta^f = 46^o$.

In Fig. (4) $|h_{LN}| = |h_{sN}|$ is plotted for $\Delta = -1^o$ with $\theta^f = 46^o$. In an earlier comparison of the full wave steady state solutions with one derived using a geometrical optical approach it was pointed out that there are n terms for which $p + q - 1 = n$ $(n,p,q = 1,2,3...)$ in the full wave series expansion (2.11b), and associated with these n terms there exists only one term in the corresponding geometrical optical solution [3],[4]. The envelopes of the individual terms h_{pq} (5.21a) for $n = p + q - 1 \leq 3$ together with the sum of only these six terms are also given in Fig. (4). The time of arrival of the n terms associated with $p + q - 1 = n$ progressively increases as n increases. These n terms of the series correspond to waves that are reflected at the overburden-substrate interface n times before they emerge above the overburden. This particular property of the full wave solution assures that the reciprocity relationships in electromagnetic theory are satisfied.

The computed value of $|h_{LN}|$ for $p + q \leq 6$ (15 terms of the series 2.11b) is approximately equal to the sum obtained by taking $p + q \leq 4$. Notice that while the envelopes of the individual terms of the series are not double humped, the total response $|h_{LN}|$ is very distorted. In this case the minimum of $H_{db}(\theta^f)$ (which occurs at $\theta^f \approx 46^o$) is the result of the destructive interference between the individual terms.

In Fig. (5) $|h_{LN}| \approx |h_{sN}|$ and its significant terms are plotted for $\Delta = -.45^o$, $\theta^f = 57.9^o$. Due to the increased damping of the higher order terms when $\varepsilon_I = 10$, $|h_{LN}|$ is approximately equal to $|h_{LN11}|$.

The individual terms of the total response for θ^i near the Brewster angle are illustrated in Fig. (6) for $\Delta = -1^o$, $\theta^i = 70^o$, $\theta^f = 71.19^o$ (where $H_{11}(\theta^f)$ is minimum. Here $|h_{LNpq}| << |h_{LN}|$ for $p + q > 2$ since both R^i_{o1} and R^f_{o1} and R^f_{o1} are small.

In Fig. (7) the envelopes of the scattered surface waves h_{LN} and h_{LNpq} ($p + q \leq 4$) are plotted for $f_o = 1x10^5$ Hz, $\varepsilon_I = 5$. $\Delta = 1^o$, $\theta^i = 38.32^o$. In this case $H_{11}(\theta^i)$ is minimum for the value of θ^i chosen. The relative contributions of higher order terms increases as the overburden skin depth increases.

BIBLIOGRAPHY

[1] Bahar, E., Radio Sci., 5, pp. 1069-1075, 1970.

[2] Bahar, E., J. Geophys. Res., 70, pp. 1921-1928, 1971.

[3] Bahar, E., and G. Govindarajan, J. Geophys. Res., 78, pp. 393-406, 1973.

[4] Bahar, E., Radio Sci., 6, pp. 1109-1116, 1971.

[5] Beckmann, P., and A. M. Spizzichino, The Scattering of Electromagnetic Waves, Pergamon, London, and Macmillan, New York, 1963.

[6] Beckmann, P., The Depolarization of Electromagnetic Waves, Golem Press, Boulder, Colorado, 1968.

[7] Richmond, H. W., J. London Math. Soc., 19, pp. 31-38, 1944.

[8] Brigham, E. Oran, The Fast Fourier Transform, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1974.

[9] Johler, J. R., and S. Horowitz, Office of Telecommunications Report 73-20 (Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402), 1973.

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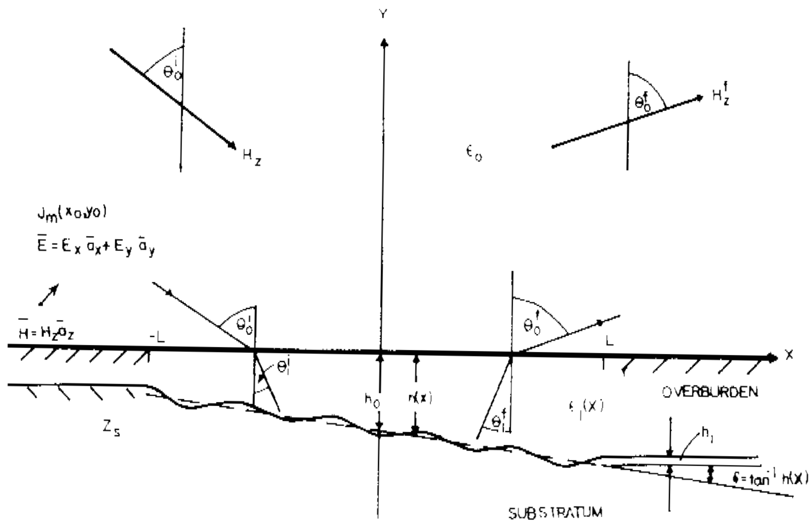

Fig. 1 Line source over a nonuniform overburden
$h(x) = h_o + \delta x + h_1 \cos \alpha x, \; -L < x < L$

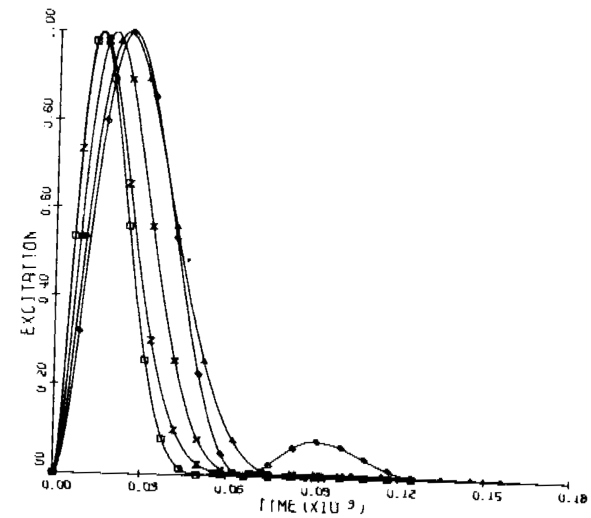

Fig. 2 Loran C pulse excitation
(□) $f_p = f_o/30$, $c = 10f_p$; (△) $f_p = f_o/50$, $c = 10f_p$;
(X) $f_p = f_o/40$, $c = 10\, f_p^{\frac{1}{2}}$ (◇) $f_p = f_o/40$, $c = 5f_p$;
(Z) $f_p = f_o/40$, $c = 15f_p$

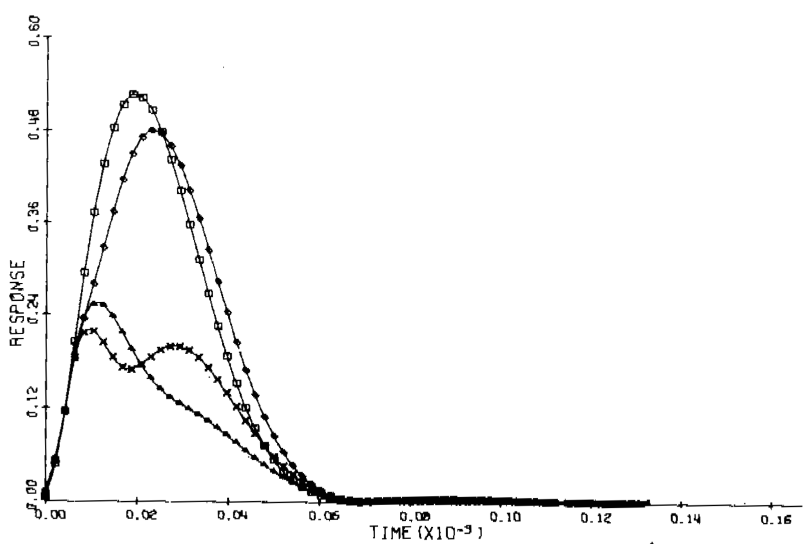

Fig. 3 The envelope of $h_{LN}(t'')$ for $\varepsilon_I = 0.5$,
(□) $\theta^f = 47^o$, (△) $\theta^f = 46^o$, (X) $\theta^f = 45^o$, (◇) $\theta^f = 44^o$

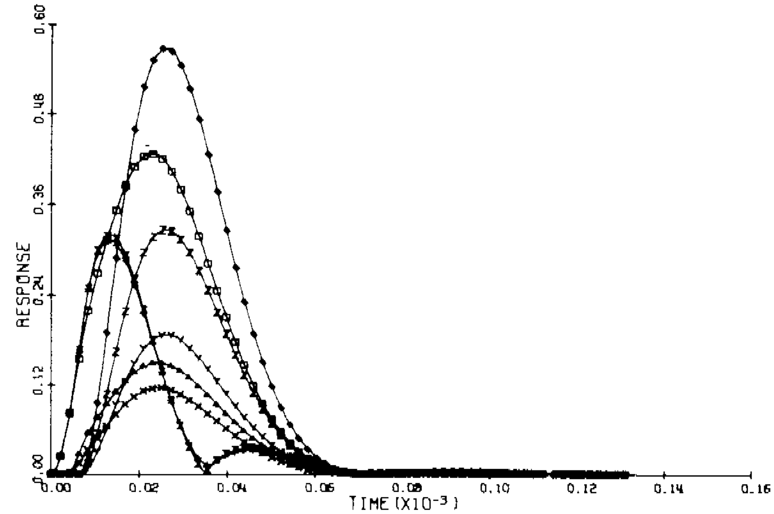

Fig. 4 The envelopes of (□) $h_{LN11}(t'')$; (△) $h_{LN12}(t'')$; (X) $h_{LN21}(t'')$; (◇) $h_{LN13}(t'')$; (Z) $h_{LN22}(t'')$; (Y) $h_{LN31}(t'')$; (X) $h_{LN}(t'')$, $p + q \leq 6$; (+) $h_{LN}(t'')$, $p + q \leq 4$ for $\theta^f = 46^\circ$

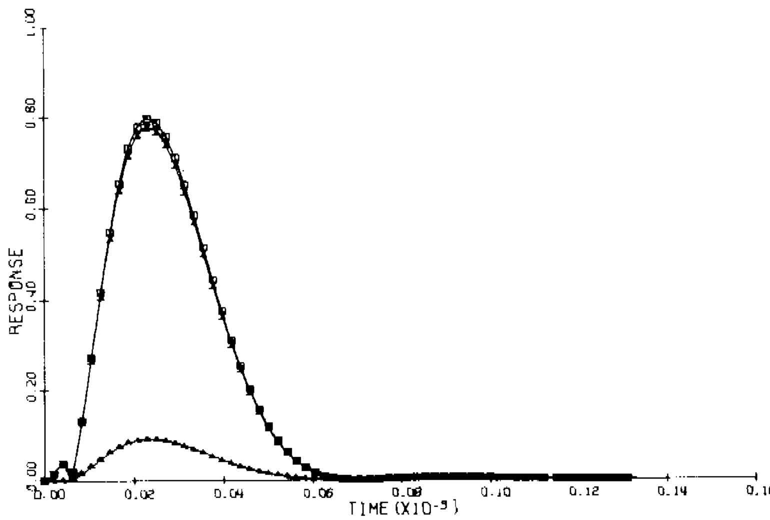

Fig. 5 The envelopes of (□) $h_{LN11}(t'')$; (△) $h_{LN12}(t'')$, (X) $h_{LN}(t'')$, $p + q \leq 6$ for $\Delta = -0.45^\circ$, $\theta^f = 57.9^\circ$, $\varepsilon_I = 10$, $Z_s = 10\ (1 + i)$ ohms

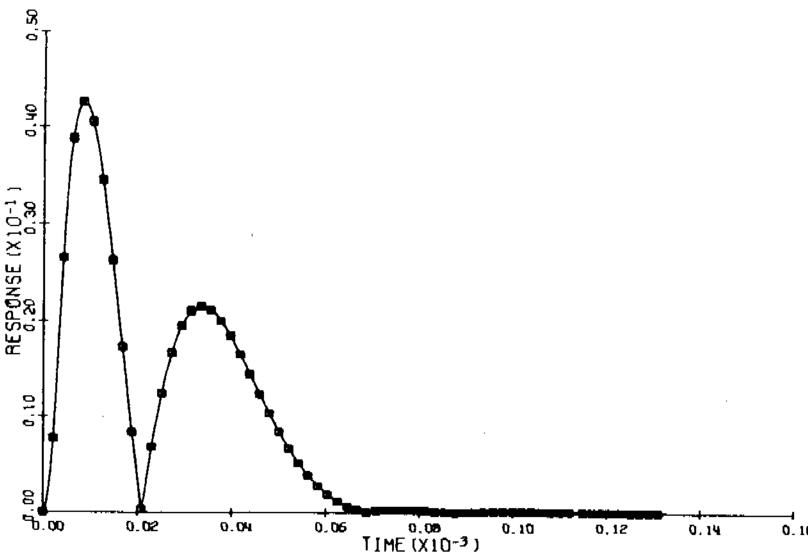

Fig. 6 The envelopes of
(□) $h_{LN11}(t'')$; (X) $h_{LN}(t'')$, $p + q \leq 6$ for $\theta^f = 71.19^o$, $\theta^i = 70^o$

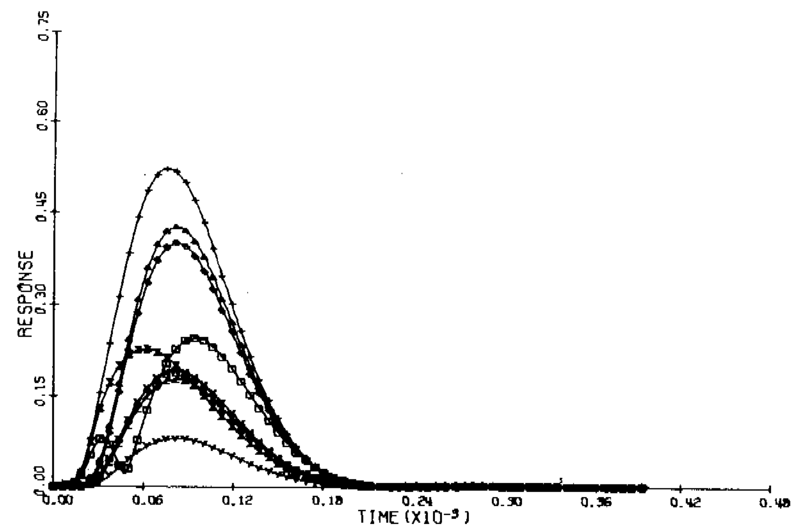

Fig. 7 The envelopes of (□) $h_{LN11}(t'')$; (△) $h_{LN12}(t'')$;
(X) $h_{LN21}(t'')$; (◇) $h_{LN13}(t'')$; (Z) $h_{LN22}(t'')$; Y $h_{LN31}(t'')$; (X) $h_{LN}(t'')$;
(+) $\sum_{pq}(t'')$, $p+q \leq 4$ for $f_o = 1x10^5$ Hz, $\varepsilon_I = 5$, $\Delta = 1^o$, $\theta^i = 38.32^o$

QUESTION AND ANSWER PERIOD

DR. REDER:

You have applied this to underground structure actually?

PROF. BAHAR:

This example here was for a non-uniform overburden. That means the earth's crust's thickness is changing, but we are now applying it also to the case where you simply have a hill or a valley without any layers to it.

DR. REDER:

How deep does the wave go in? What is your thickness?

PROF. BAHAR:

This is only 100 meters.

DR. REDER:

Is there any energy 100 meters down for Loran frequencies?

DR. BAHAR:

If you have seen the cases where I said the higher conductivity of the earth is, in those cases we had practically only one term, so the conductivity of the overburden is large, as in many cases, and indeed there is only one term, the dominant term, but, we have also examined the cases where the overburden conductivity is not large, in which case the 100 meters would be of the order of a skin depth, and then the high order terms are appreciable in their effect.

DR. REDER:

I do hope that you two gentlemen get together and maybe the theory of Professor Bahar can be applied to Mr. Roll's experimental data and vice versa.