TIME AND FREQUENCY REQUIREMENTS FOR RADIO INTERFEROMETRIC EARTH PHYSICS

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ABSTRACT

Two systems of VLBI (Very Long Baseline Interferometry) at J. P. L. are now applicable to earth physics: an intercontinental baseline system using antennas of the NASA Deep Space Network (DSN), now observing at one-month intervals to determine UTI for spacecraft navigation; and a shorter baseline system called ARIES (Astronomical Radio Interferometric Earth Surveying), to be used to measure crustal movement in California for earthquake hazards estimation. The DSN system is now regularly observing between Goldstone, California and Madrid, Spain, determining the Earth's integrated spin rate from fringe frequency measurements. This system will soon be improved by adding the capability to measure time delay and by extending the system to other stations of the DSN, making possible the determination of polar motion and of all three coordinates of the various intercontinental baselines. On the basis of experience with the existing DSN system, a careful study has been made to estimate the time and frequency requirements of both the improved intercontinental system and of ARIES. In this paper, such requirements for the two systems are compared and contrasted. The eventual requirements for the intercontinental system are a frequency stability of $\Delta f/f = 10^{-14}$ over an observing period of 24 hours, and a clock synchronization of 25 microseconds. The requirements on ARIES are less stringent in frequency, for reasons to be discussed in this paper. Over the shorter ARIES baselines, one must have a frequency stability of $\Delta f/f$ 3 $\cdot 10^{-14}$ over 3 hours, and a clock synchronization of 25 microseconds to attain accuracy in each baseline coordinate of 3 cm, using an optimal observing strategy and bandwidth synthesis technique.

INTRODUCTION

A new system of Very Long Baseline Interferometry (VLBI) is being developed at the Jet Propulsion Laboratory (JPL) for earthquake hazards estimation. This system, called ARIES (Astronomical Radio Interferometric Earth Surveying), will be used to monitor crustal motion and regional uplift in such earthquake-prone areas a sourthern California and perhaps northern Mexico. The time and frequency requirements for ARIES are determined by two sets of parameters. First, one must specify the accuracy with which the crustal movement must be measured to give useful information concerning the earthquake mechanism, and then one must determine how many observations must be taken with what precision and over how long a period of time in order to attain such accuracy. This paper outlines the logic by which time and frequency requirements are calculated for the system, and presents the results.

An earthquake occurs when the accumulated stress build up in the solid crust of the earth, believed to be caused by underlying convective currents in the mantle, exceeds the elastic limit of the crust and is relieved by a sudden fracture. If two crustal blocks were to slide smoothly past one another, then they would yield freely to the force which moved them, so that no stress would accumulate and no earthquakes need occur. But such large blocks as we speak of here are far from rigid; rock considered on a scale of hundreds of kilometers is elastic and compressible. It is possible for such blocks to be sliding freely at one point of their contact and sticking elsewhere, while stress builds up in a complicated three dimensional pattern characteristic of any deformable solid. Then to measure the accumulated stress at any point, and so to estimate the likelihood and magnitude of future earthquakes, it is not sufficient to measure the relative motion of a few points only, or even the relative motion along the entire line of the fault, but measurements must be taken over whole areas tens of kilometers to either side of the line of contact, and integrated in a model of crustal stress and strain. To secure such measurements, a system must be devised which enables portable devices to be moved from benchmark to benchmark over wide expanses of mountain and desert. Compare Figure 1, which illustrates how two blocks may be expected to deform according to the elastic rebound model of the earthquake mechanism.

The reality of southern California geology is even more complicated than the simple theory outlined above would indicate. Consider Figure 2, which shows the major faults, or lines of fracture, in this region. The most important single fracture is the San Andreas Fault, which runs southeast-northwest from the Gulf of California to Point Arena about 160 kilometers north of San Francisco. The motion along the San Andreas Fault, in its immediate vicinity, ranges from 0 to 3 centimeters per year. Broadly speaking, this fault is the boundary between two interacting plates, or rigid sheets of rock into which the uppermost layer of the earth, the lithosphere, seems to be divided, plates which are sliding past one another along this line of fracture. However, the plate motion has torn the crust, not along one fault only, but along a whole system of parallel faults which divide southern California into numerous small blocks which are presumably all moving with respect to one another, at least as seen on a geological time scale (see Fig. 3). Furthermore, another system of faults exists in this region which runs almost due east-west, examplified by the Garlock Fault north of Goldstone, California, which is very difficult to account for theoretically.

A very important symptom of an impending earthquake may be provided by the phenomenon of dilatancy. When the rock in a tectonically active region is

RELATIVE PLATE MOTION

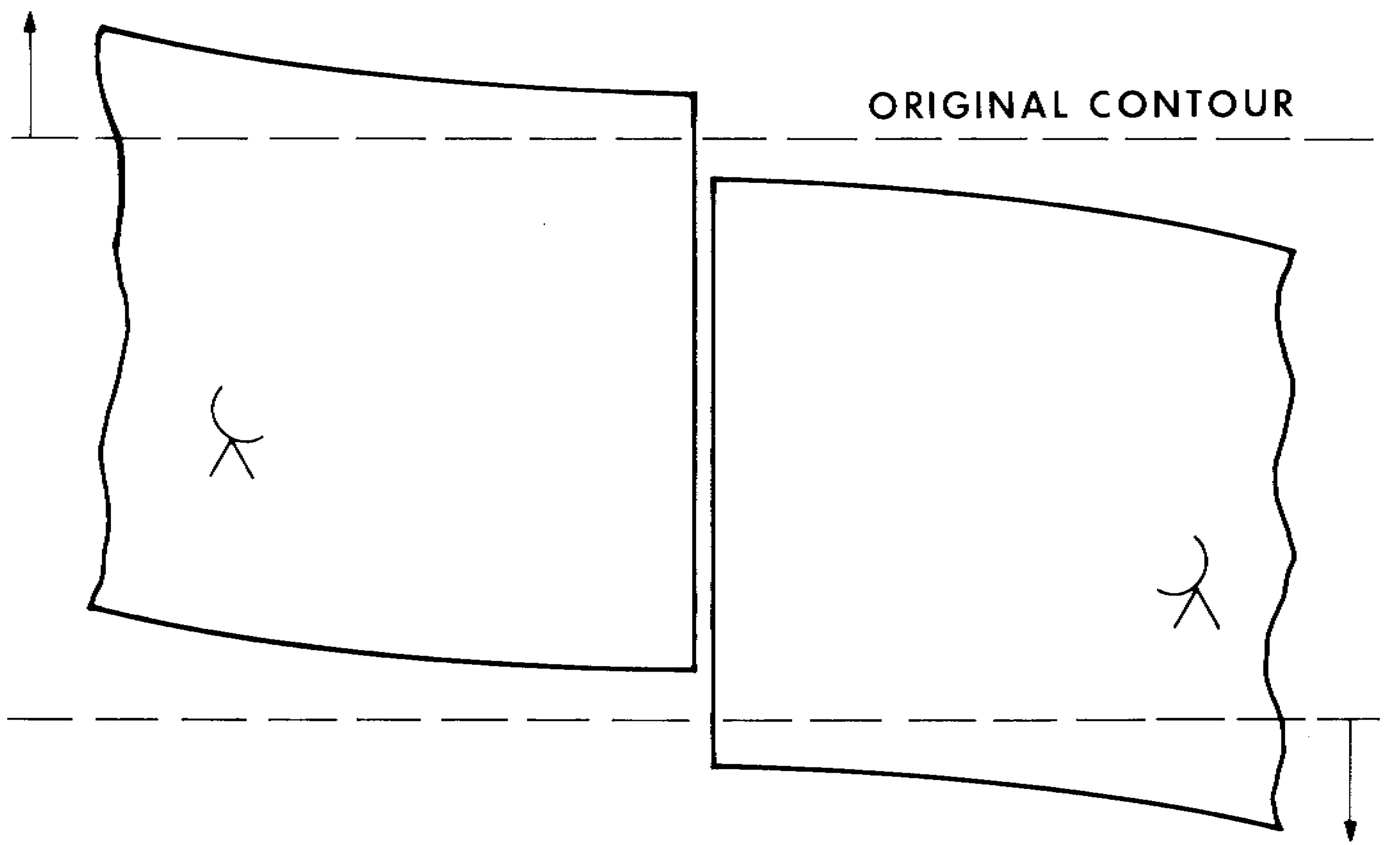

Figure 1. Schematic Diagram of Two Sideslipping Tectonic Plates Showing Deformation Due to Pressure and Fusion of Rock at the Boundary

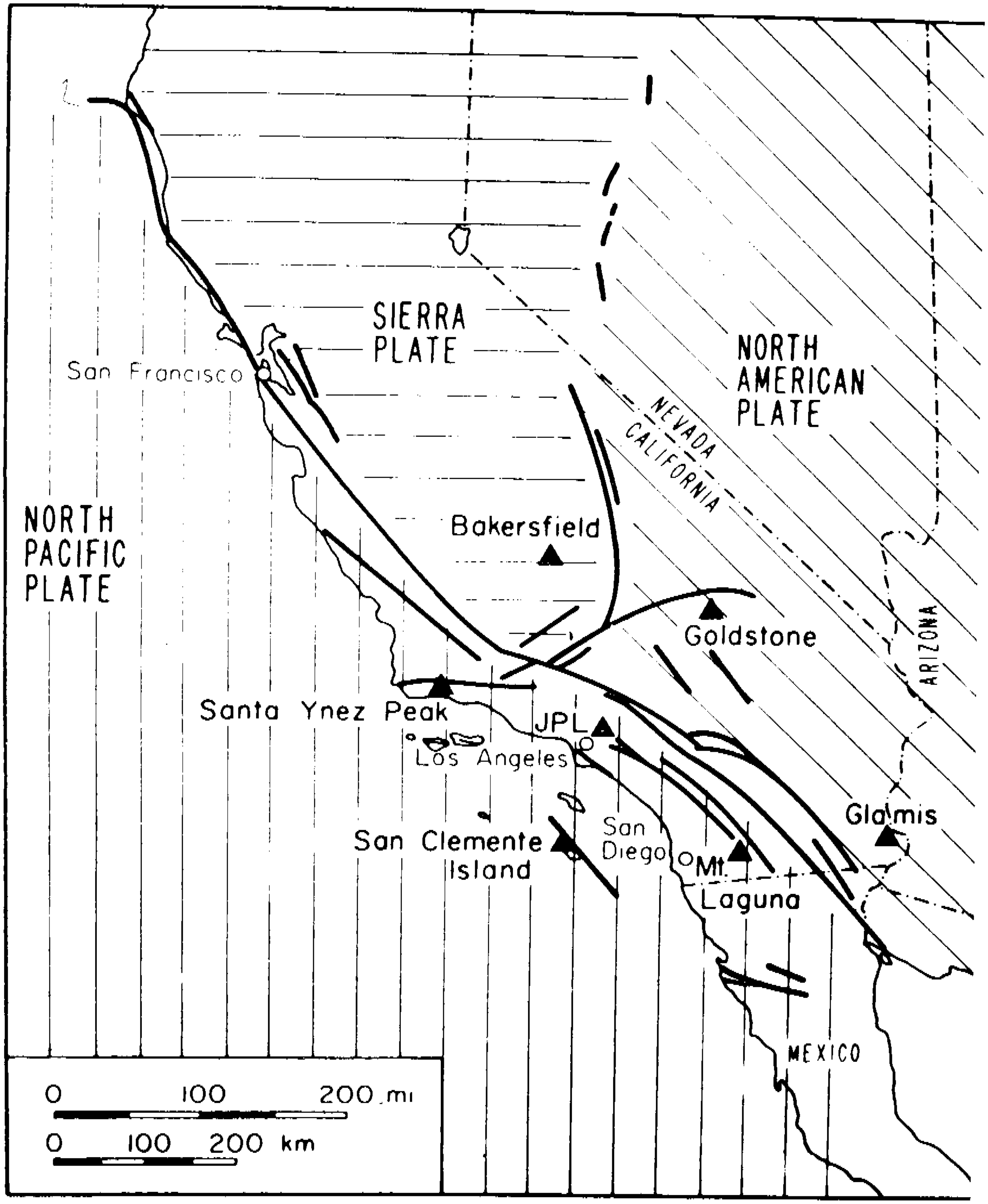

Figure 2. Simplified Fault Map of Southern California (Courtesy of Don L. Anderson, Seismological Laboratory, California Institute of Technology)

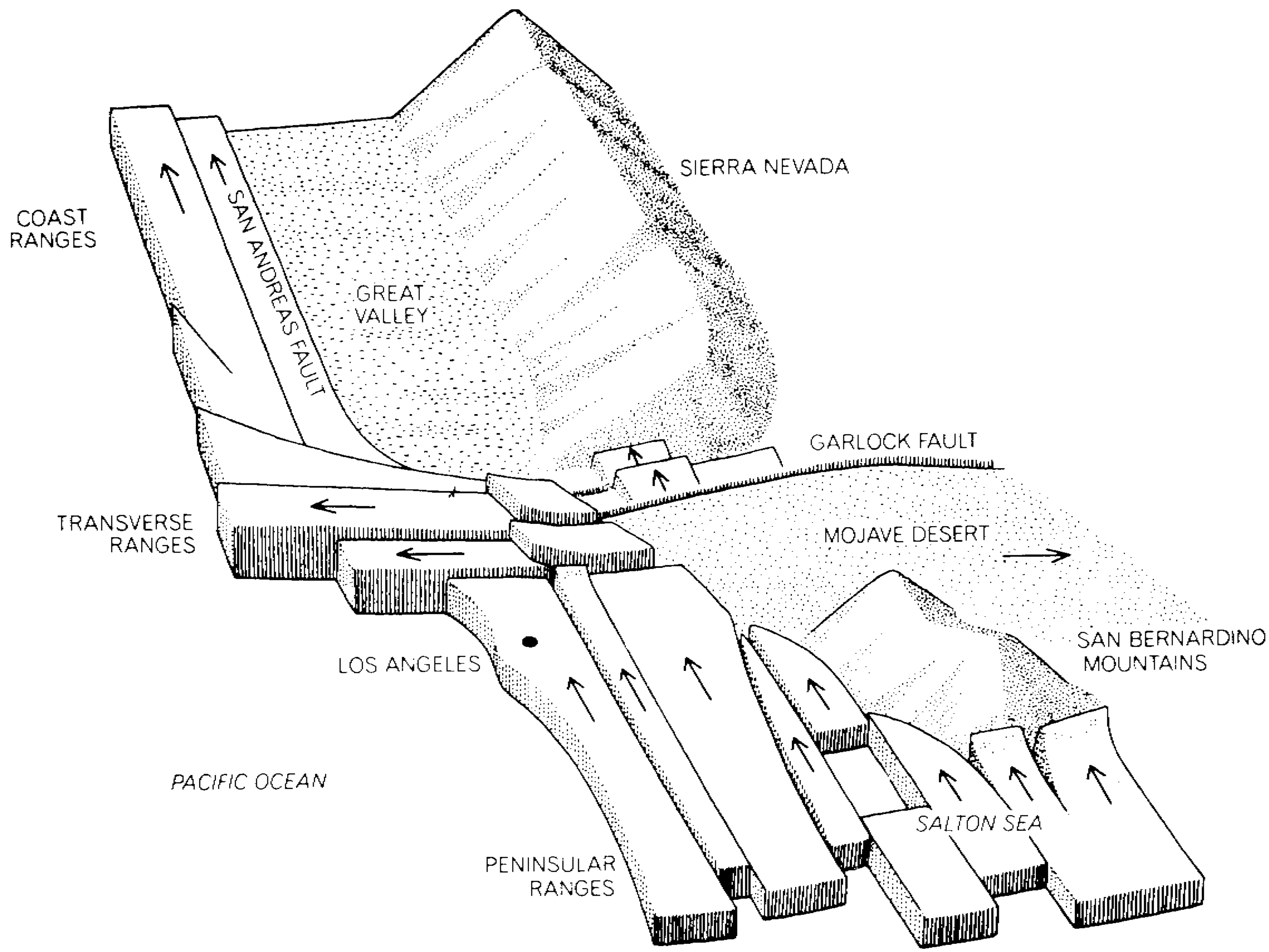

Figure 3. Schematic Diagram of Crustal Movements in Southern California
(Courtesy of Don Anderson)

subjected to a sufficient shearing stress, it develops a fine pattern of cracks, which ultimately fill with groundwater. This microscopic cracking of the rock has two effects: the velocity of pressure waves (P-waves) thru the rock decreases; and the rock increases in volume, which may be expected to induce a small regional uplift in the months or years prior to the earthquake which ruptures the rock completely and thus relieves the strain.

These geological phenomena determine the capability which a geodetic technique should have in order to be useful for earthquake hazards estimation. It should be capable of locating points of reference in 3 dimensions, so that regional uplift as well as horizontal movement will be detected, to an accuracy of 3 centimeters or better, in a coordinate system which permits results to be reproduced or changes measured over several decades. The VLBI technique is capable of measuring baseline vectors in 3 dimensions and with respect to an extragalactic frame of reference. It seems likely that the necessary accuracy can be attained. We confine ourselves here to the question: what are the time and frequency requirements to attain 3 centimeter accuracy?

The basic principles of VLBI geodesy are illustrated in Figure 4. The basic observable is the delay between the times of arrival of an electromagnetic wave at two antennas. If $\vec{s}_i$ is the unit vector from the ith celestial source to either antenna, and if $\vec{B}$ is the vector baseline from one antenna to the other, and c is the speed of light in vacuo, then the time delay τ_i for the ith observation is given by the equation

$$\tau_i = \frac{\vec{B} \cdot \vec{s}_i}{c}, \qquad (1)$$

which can be rewritten in the form

$$c\tau_i = x_i X + y_i Y + z_i Z, \qquad (2)$$

where (X, Y, Z) are the baseline coordinates, and (x_i, y_i, z_i) are the coordinates of the ith source. If the baseline coordinates (X, Y, Z) are not to be functions of time, then one must express Equation 2 and the source coordinates (x_i, y_i, z_i) in a frame of reference rotating with the earth, allowing for the effects of the variation in the rotation of the earth and of polar motion. JPL now operates an intercontinental interferometer between Goldstone, California, and Madrid, Spain, which determines changes in UT1 at intervals of approximately one month. At present, the Goldstone-Madrid interferometer is able to measure only the rate of change of time delay, which is proportional to a quantity called fringe frequency, but the system will soon be improved by adding the capability to measure time delay and by extending the system to other stations of the NASA Deep Space Network, making possible the determination of polar motion and of

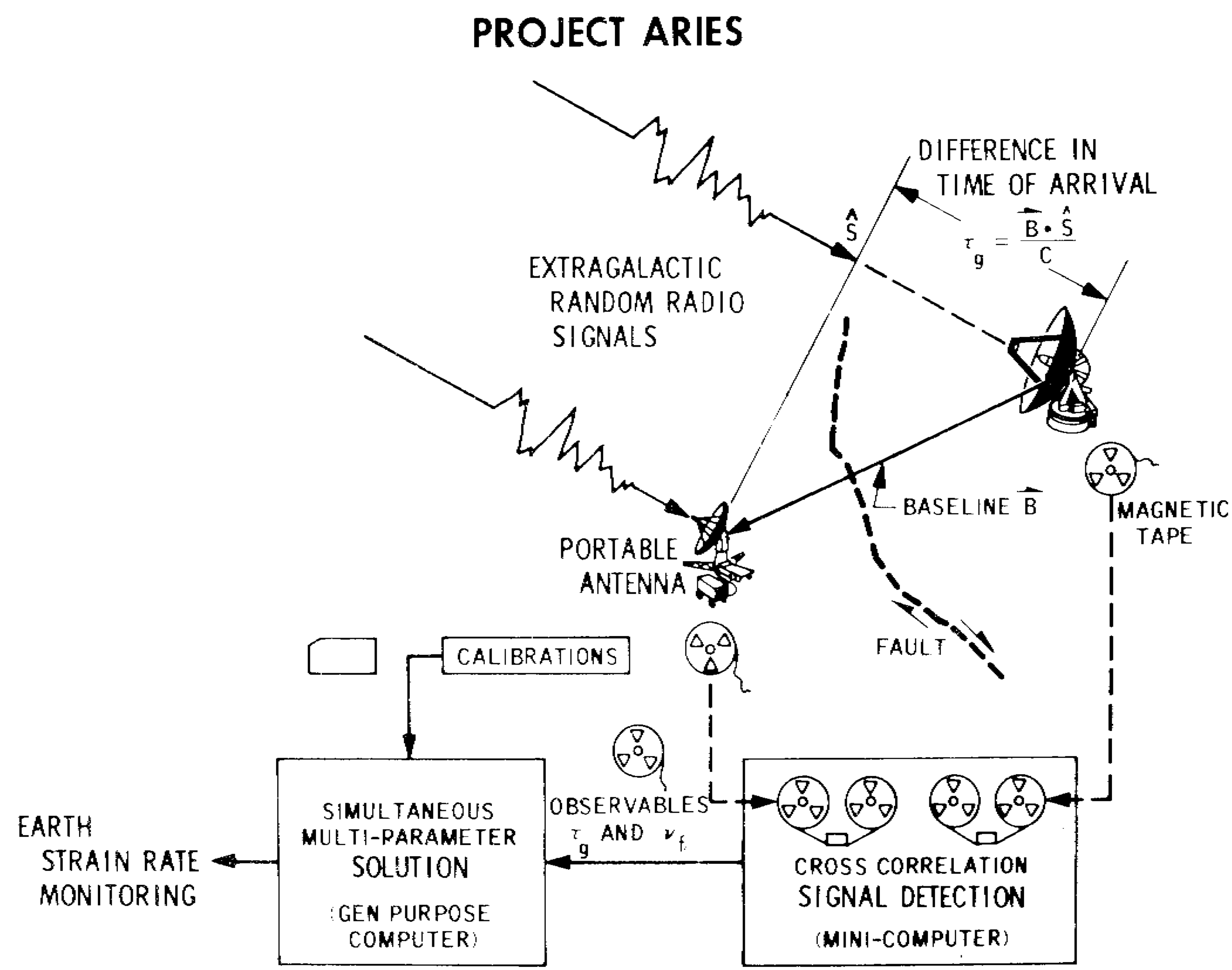

Figure 4. Sketch and Flow Diagram of a Working VLBI System for Earth Physics Using a Portable Antenna

all three coordinates of the various intercontinental baselines. It is planned to use the intercontinental interferometer at times not far removed from the dates of ARIES observations to determine UT1 and polar motion with high precision ($\sim$ 20 cm), and to determine a highly accurate catalog of celestial sources, so that the ARIES system, which will operate on relatively short baselines with a portable antenna, can input this information as known quantities. Thus, so far as the geometry is concerned, it will be possible to write Equation 2 above with only three unknowns, namely, ARIES baseline coordinates.

Apart from the geometry, it is possible and necessary to solve for two other parameters. In general, the two clocks at the two ARIES antennas will not be perfectly synchronized, and they will not have exactly the same rate. One may include the effects of clock offset and frequency offset by rewriting Equation 2 with two additional unknowns:

$$c\tau_i = x_i X + y_i Y + z_i Z + c \cdot \Delta T + cT \cdot (\Delta f/f), \qquad (3)$$

where ΔT and $(\Delta f/f)$ are the unknown clock offset and difference in clock rates, respectively, and T is time on a reference clock. Equation 3 is solvable with a minimum of five observations widely separated in azimuth. Notice that, in principle, VLBI can be used not only for geodesy but for clock synchronization in widely separated locations.

The basic mathematics, then, sets no requirement for clock synchronization, since one solves for clock offset. However, there is a practical requirement. As illustrated in Figure 4, the noise signal received from a celestial source is recorded at each antenna on magnetic tape, and the tapes are cross-correlated by matching the streams of binary digits, called bit streams, on the tapes. (The signal is received at whatever frequency the receivers are tuned, heterodyned to generate a sine-wave signal of frequency low enough to be recorded on video tape, and then re-heterodyned by a technique called "fringe stopping" to allow approximately for the difference in Doppler effect between the two antenna locations due to the rotation of the earth. The two resulting bit streams are then multiplied together.) It is necessary that the time-tags on the two tapes be reasonably accurate if excessive time is not to be wasted searching tapes in the cross-correlation process. Experience with the Goldstone-Madrid interferometer and the Mark II recorder of the National Radio Astronomy Observatory suggests an *a priori* requirement of 25 microseconds on clock synchronization. Once the cross-correlation is made, it is possible to deduce the clock offset with a precision of about 25 nanoseconds using an instantaneous recorded bandwidth of 2 megahertz on a single channel, or better using a technique called bandwidth synthesis.

Although one solves for frequency offset, Equation 3 presumes that the offset is a constant over the observing period. Since it is desired that Equation 3 be over-determined and well-conditioned, it requires about 3 hours = 10^4 seconds to observe a sufficient number of sources—say, 12 sources with about 10 minutes integration time on each, plus antenna moving time. A worst-case requirement for the frequency standards can be set by supposing that one neglects to include the term $cT \cdot (\Delta f/f)$ in Equation 3, and that the whole term is taken up in a single baseline parameter—say X. In that case, in order not to exceed 3 cm baseline errors, one would have

$$(\Delta f/f) = x/cT = 3\,\text{cm}/(3 \cdot 10^{10}\,\text{cm/sec}) \cdot 10^4\,\text{sec} = 10^{-14}$$

Numerical simulations in which the effect of a constant frequency offset is included in the equations and in which sources are observed well distributed around the sky suggest that frequency variations of from $3 \cdot 10^{-14}$ to $8 \cdot 10^{-14}$ can be tolerated without exceeding a standard deviation of 3 cm in any baseline coordinate or in baseline length. We have adopted a value of $(\Delta f/f) = 3 \cdot 10^{-14}$ as a reasonable requirement for the ARIES system.

The requirement for the intercontinental interferometer is approximately $(\Delta f/f) = 10^{-14}$, and is more stringent than for ARIES chiefly because a longer observing period is needed for a good solution, about 10 hours.

An attempt to compare the simple theory outlined above with experimental data illustrates both general similarities and striking differences. Two interferometry experiments were conducted on two separate days over a short (16 km) baseline, using a 24 kHz instantaneous recorded bandwidth, between the Mars (64 meter) and Echo (26 meter) antennas at the Goldstone DSN complex, as part of a series of tests of the ARIES technique. On the first day (18 October 1972), both stations were equipped with hydrogen masers; on the second (21 November 1972), one of the hydrogen masers was replaced by a rubidium oscillator. Each experiment consisted of 7 hours of observation, over which a single solution was made for ΔT and for $\Delta f/f$ via Equation 3. Residuals were formed of the observed fringe frequencies and time delays minus those calculated from the solution, and are displayed in Figures 5 and 6. The fringe frequency jitter for the rubidium-equipped station is about 0.5 millihertz, corresponding to $\Delta f/f \doteq 2 \cdot 10^{-13}$, since all observations were at S-band (~ 2.3 gigahertz). This figure equals or exceeds the expected performance of the rubidium oscillator. On the other hand, the jitter for the hydrogen maser equipped station is 0.12 millihertz, corresponding to $\Delta f/f = 5 \cdot 10^{-14}$, which fails to meet the expected short-term stability of the hydrogen maser by about a factor of 5. However, in this case, the observed fringe frequency reflects other sources of error, not only oscillator instability; and we suggest that the dominant error source is produced by the ionosphere. Also, in the case of the time delay residuals, the dominant error

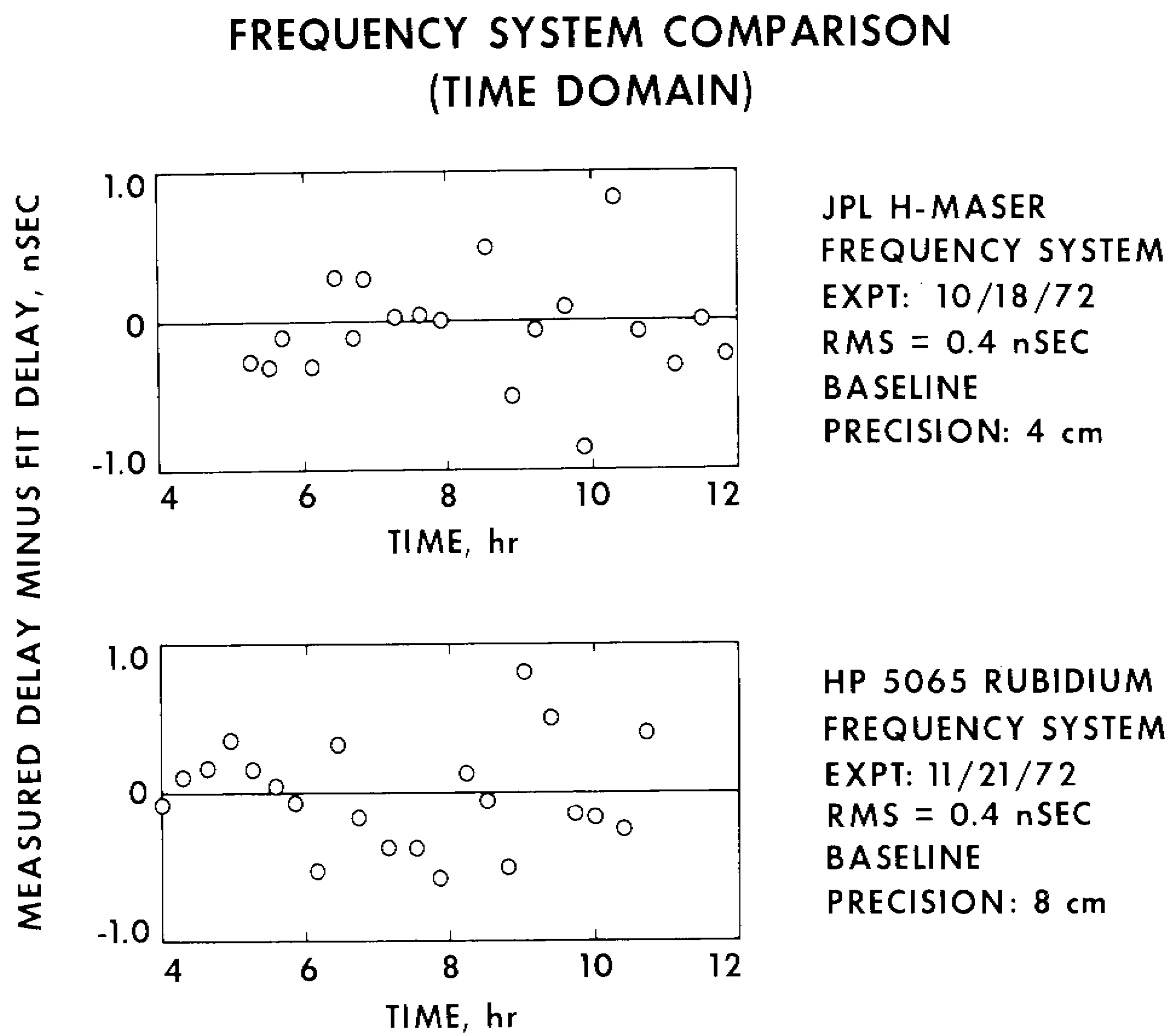

Figure 5. Residuals in Time for Two Experiments, One Using Hydrogen Masers, and the Other, a Rubidium Oscillator

FREQUENCY SYSTEM COMPARISON (FREQUENCY DOMAIN)

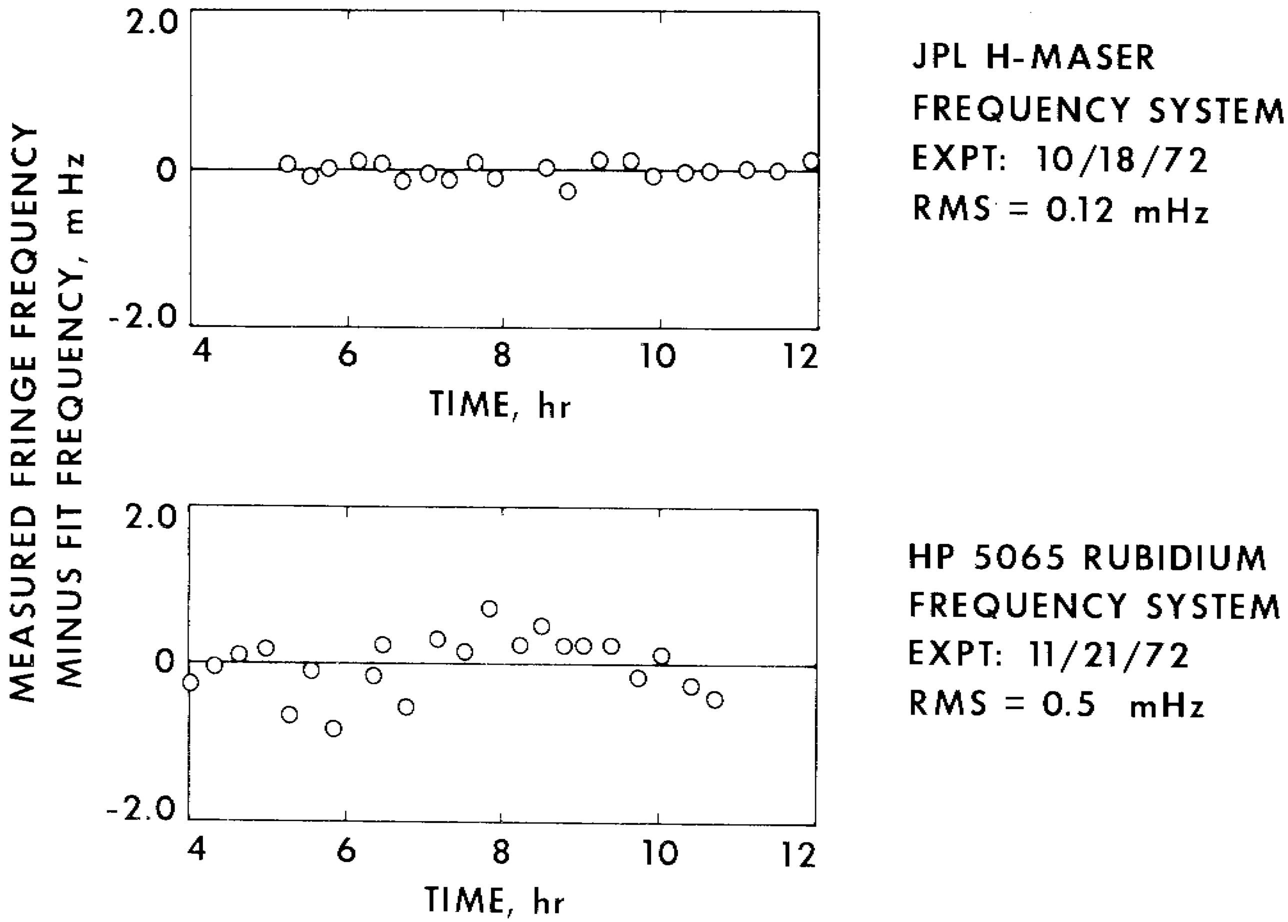

Figure 6. Residuals in Frequency (Same Experiments as Figure 5)

DR. KLEPCZYNSKI:

Is there a comment over here?

DR. HELWIG:

At the National Bureau of Standards, we have some results with a Hewlett-Packard Super Tube and I think it performs according to specs, and that is all I can say. I think a part in 10 to the 11th is at least an order of magnitude for a one second averaging time.

So this would be my comment, if you use the Super Tube just as a black box, and you assume that specs of H. P. are correct, I think you are in good shape. If you expect more than the specs say, you may not get it.

DR. KLEPCZYNSKI:

A comment from Dr. Winkler.

DR. WINKLER:

We will hear much more about it by Mr. Percival, who will report tomorrow on experiments which we have done in the observatory with 11 units.

There is one point in regard to Dr. Helwig's comment with which I completely agree. It is that I think one has to remember that the characteristics of the sigma, tau plot of the cesium standards are vastly different from that of a hydrogen maser, and of that of a rubidium standard.

These are three different kinds of animals. The cesium standard, in general, will follow a one over square root of tau performance, beginning from the time constant of the servo loop, which is in the order of a second to a minute or slightly shorter than a minute down to, and that is a point of some contention, to the flicker level.

It is here where the standards, according to our experience, seem to be better than the specifications. It is how far they will go down to long integration periods which determines their main quality for time keeping.

In the short time range, if you want to compare them with a rubidium standard for periods of 100 seconds or so, they will be slightly inferior, but again I completely agree with Dr. Helwig, there seems to be something wrong with that one standard referred to before which, incidentally, had completely stopped on its way from the observatory, and which has not performed according to what it should be.

But more about it tomorrow.

DR. KLEPCZYNSKI:

Dr. Helwig again.

DR. HELWIG:

As an example, we have reliable data on a comparison of one rubidium standard against our primary standard, we get 8 parts in 10 to the 12th one second, and it averages down as the square of tau, reaching a flicker level of about 2 parts in 10 to the 14th.

DR. KLEPCZYNSKI:

There was another comment over here? Yes.

When you ask a question, please identify yourself by name and place, so that people will get to be familiar with everybody in the audience.

DR. REINHARDT:

Dr. Reinhardt, from Harvard.

If you were to use an artificial signal source, let's say on a satellite, what performance could you get using hydrogen masers, and do you need that kind of performance? Would you benefit from that kind of performance?

DR. FLIEGEL:

I imagine that we could. We have no experience using artificial sources for the AIRES project, of course. We had to use natural sources, for obvious reasons, because we want our observations finally to be reduced to an inertial system, which the extra galactic framework provides us, or some kind of a fixed body reference.

I am certainly not ruling out the idea of using artificial sources, but we haven't used them up to now.

DR. REINHARDT:

What kind of performance did you get, signal to noise?