INTERMEDIATE TERM FREQUENCY MEASUREMENTS WITH THE HP COMPUTING COUNTER IN THE USNO CLOCK TIME SYSTEM

Gernot M. R. Winkler
U.S. Naval Observatory

I. INTRODUCTION

High precision frequency measurements with various integration times τ can be made with conventional counters by using variable gate times. The Hewlett-Packard (HP) computing counter allows such measurements over a very wide range of frequencies and measurement times. This instrument can also convert period measurements into frequency by means of its arithmetic capabilities: The program library contains programs to facilitate frequency stability measurements $[\sigma_y(\tau)]$ in the most direct and convenient way.

An important limitation exists, however, in that measurement intervals are restricted to $\tau < 100$ seconds.

For long measurement intervals ($\tau \geqslant 1$ day), conventional phase difference recording provides a simple and economic frequency measurement capability at described frequencies (Reference 1).

At the USNO this method was used as the basis for all time scale computations (Reference 2), until requirements for highest resolution justified the development of an automatic data acquisition system as described by K. Putkovich (Reference 3). The phase measurements are now being made with the HP computing counter (with time interval plug-in unit) under program control from the HP "System Programmer," which in turn is interfaced with an IBM "1800" system. The "1800" controls the coaxial switch system through which the start and stop signals can be directed to the time interval meter.

Each phase measurement consists of an average of 256 individual measurements. This allows such a precision of measurement and a flexibility of operation that evaluation of frequency standards and clocks can be performed on line for $\tau > 100$ seconds, thereby closing the gap mentioned above.

This paper will discuss details of this phase measurement technique and its application in the evaluation of precision oscillators.

II. PHASE MEASUREMENTS WITH THE TIME INTERVAL UNIT (HP COMPUTING COUNTER, MODEL 5360)

1. The "Fly-Back" Subroutine

Successive phase measurements that are to be averaged must be checked against the previous measurement (φ-) to avoid averaging of widely divergent values near the "zero" or "full-period" point. The flow chart of our "fly-back" subroutine is shown in Figure 1. It occupies locations 160-200 in the system programmer and is called after every time interval measurement which contributes to an average. P is the period used (one second for tick-to-tick, 2×10^{-7} seconds for five-MHz signals) and is entered through the "external data" (switch) input.

NO CORRECTION REQUIRED IF $\varphi - \varphi_- < P/2$

AND $\varphi_- - \varphi < P/2$

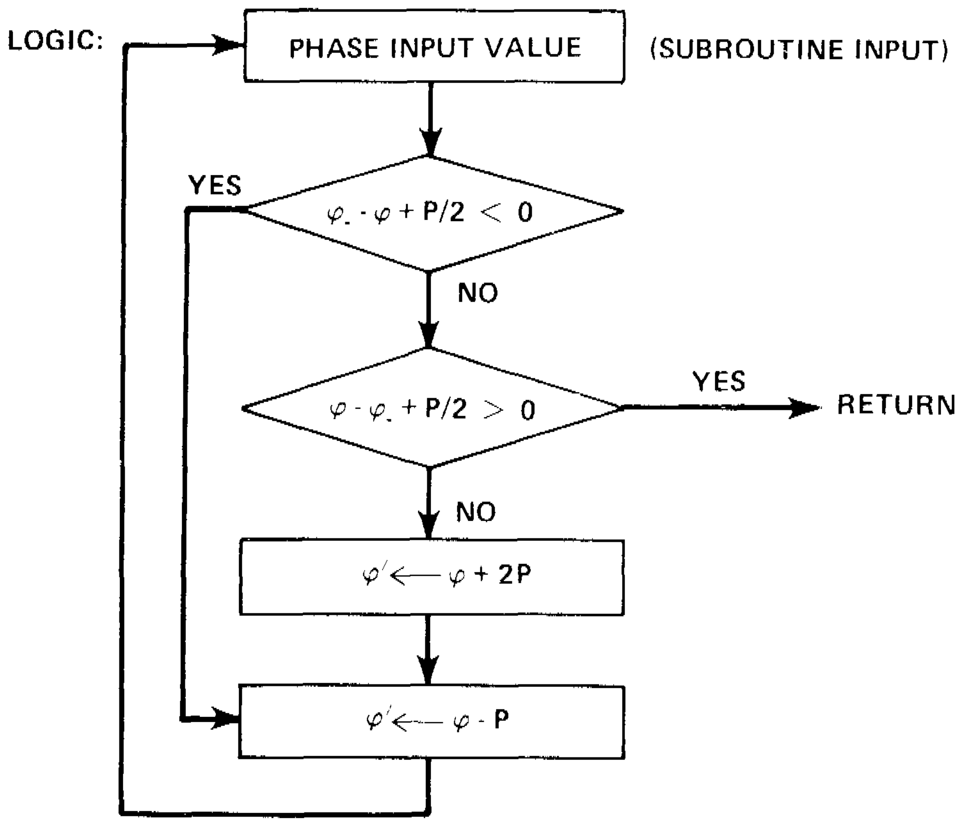


Figure 1. Fly-back subroutine.

2. Trigger Level Instability

The "start" and "stop" trigger levels are set at zero for phase measurements. Even very small additive noise will, however, contribute to measurement scatter. In order to minimize this noise contribution, five-MHz signals for phase measurements are used. Figure 2 compares time interval measurement scatter as a function of rise time (frequency of the sine wave input).

During preliminary performance tests, significant differences between our HP 5360 counters became evident. Table 1 summarizes our computing counters as far as it is pertinent to this report.

	Sine Waves 1 V_{rms}		
	100 khz	1 MHz	5 MHz
Peak to peak variation of Δt (10^4 samples)	2.8	1.0	0.7
σ of these p-p values	0.5	0.1	0.2
σ ($\overline{\Delta t}$)	0.4	0.14	0.1

Figure 2. Measurement scatter of a short time interval Δt as a function of rise time (taken with SER #863) (values in nanoseconds, Δt = 10).

Table 1
HP 5360 Counters in Use at the USNO.

Serial No.	Application
312	Used in preliminary tests until April 1972. Low calibration noise. Warranty repair of display module.
603	Used by Hefele and Keating in their global clock experiments. Average noise. (Resolution 10^{-15} in 1/2 day.) Presently on line.
863	Used as "system unit" May to October 1972. High calibration noise (10^{-14} in 1/2 day).
1048	Warranty repair (high noise level).

3. Quantization and Interpolator Noise (Q&I)

The resolution of a simple time interval measurement with the HP 5360 is 0.1 nanosecond. Since two interpolators provide input to the time interval routine, a naive expectation would be that Q&I noise could contribute as much as four nanoseconds peak to peak, if the interpolators are fully stabilized. Table 2 was derived with counter #312. A comparison of the computed σ (average) with the actually measured σ (by measuring a number of groups of n measurements each) indicates the presence of long-term instability which cannot be improved upon with our 256 measurement averaging routine.

The operational program which we use allows the collection of both calibrator readings (nominal value 1000 ± 1) during each measurement cycle as a routine check of the counter. Figures 3 and 4 give samples of calibrator readings of counter #863. It is evident that particularly "N2" is affected by additional noise. Figures 5 and 6 give the corresponding probability density functions and the power spectrum. The noise is white and nearly Gaussian (note the small side peak of N1). The power spectrum is given in relative values over a time period in days.

It should be emphasized that this additional noise encountered is the worst case we saw.

Counter #312 (Table 2) performed about ten times better. Figure 7 gives an updated version of Figure 6 of Reference 2 and is based on the measured σ of an average of 256 measurements as listed in Table 2. The presently used Counter #603 is only slightly inferior in the actual performance (10^{-15} in 1/2 day, Figure 8).

4. Cycle Resolution

The choice for five-MHz signals for the phase measurements was dictated by the desire to minimize trigger level noise contribution. A price to be paid for this benefit is the rather short period of only 200 nanoseconds. The Observatory has adopted the positive cycle crossover as the time reference mark; one-pulse-per-second ticks are used to identify a particular cycle. Caution must be exercised, however, in the use and adjustment of all

Table 2
Measured Time Delays of a Four-Foot RG58 Cable
With a One-MHz Signal From an HP Rubidium Oscillator

Number of Measurements (picosec)	$\overline{\Delta t}$	σ	$(\sigma AV)^{Comp}$	$(\sigma AV)^{Meas}$
40	6819	65	19	47
400	6809	75	3.5	7
4K	6805	76	1.00	3.5
40K	6811	77	0.33	4.2
400K	6813	77	0.1	2.3

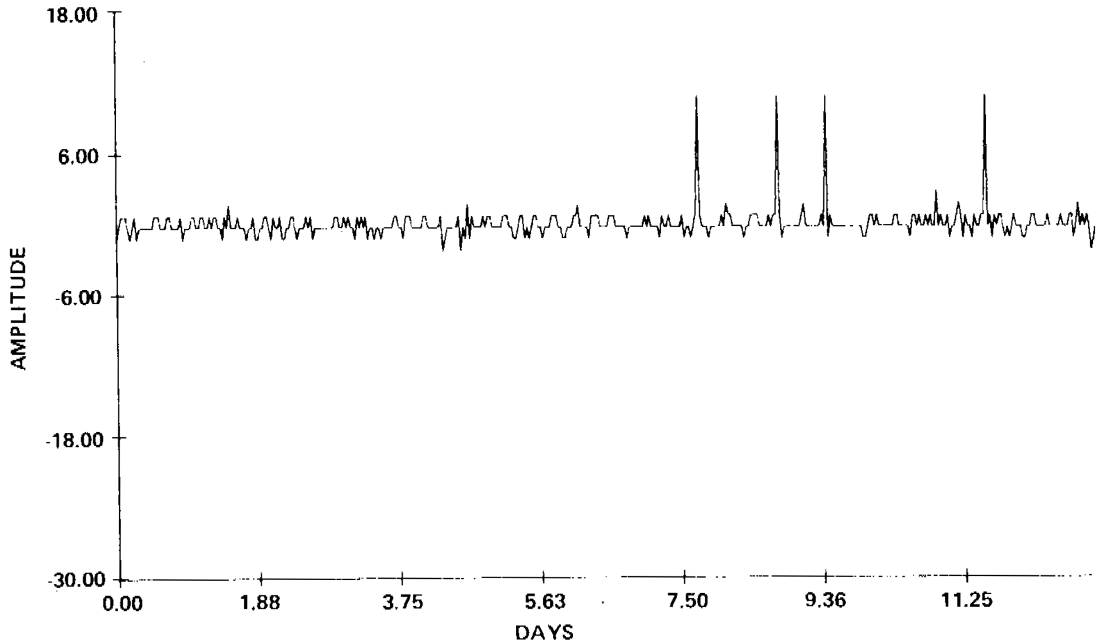


Figure 3. N1 calibration.

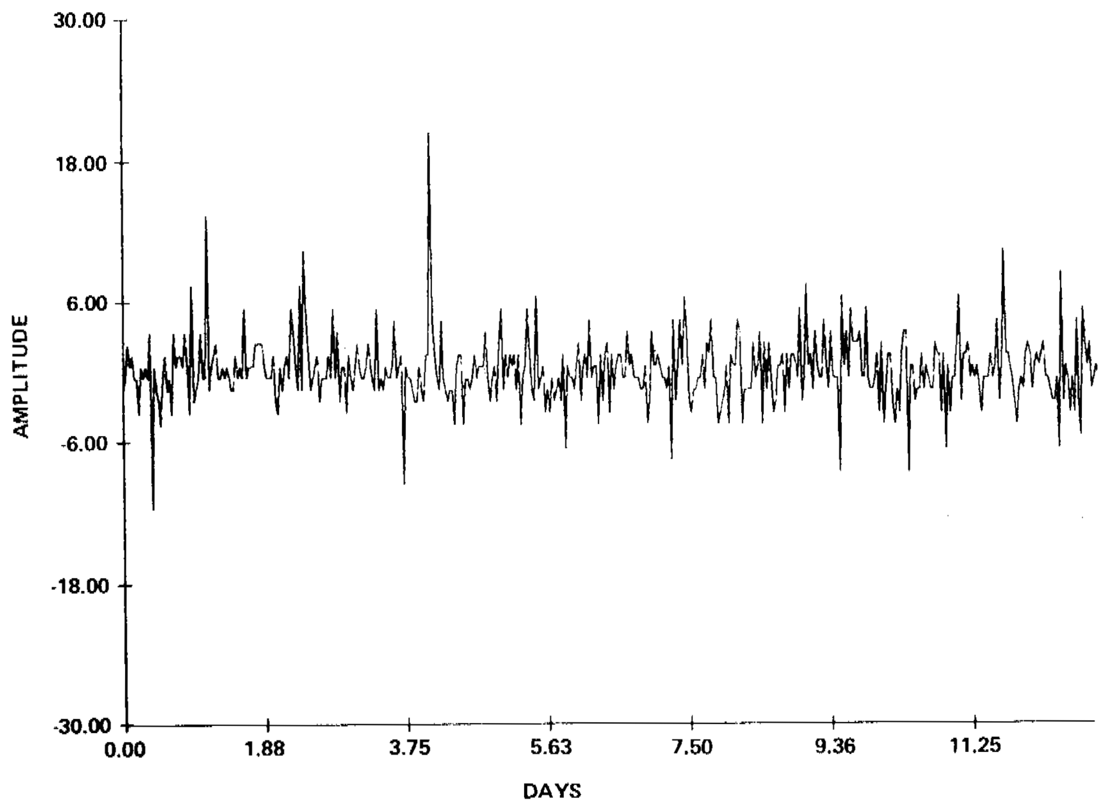


Figure 4. N2 calibration.

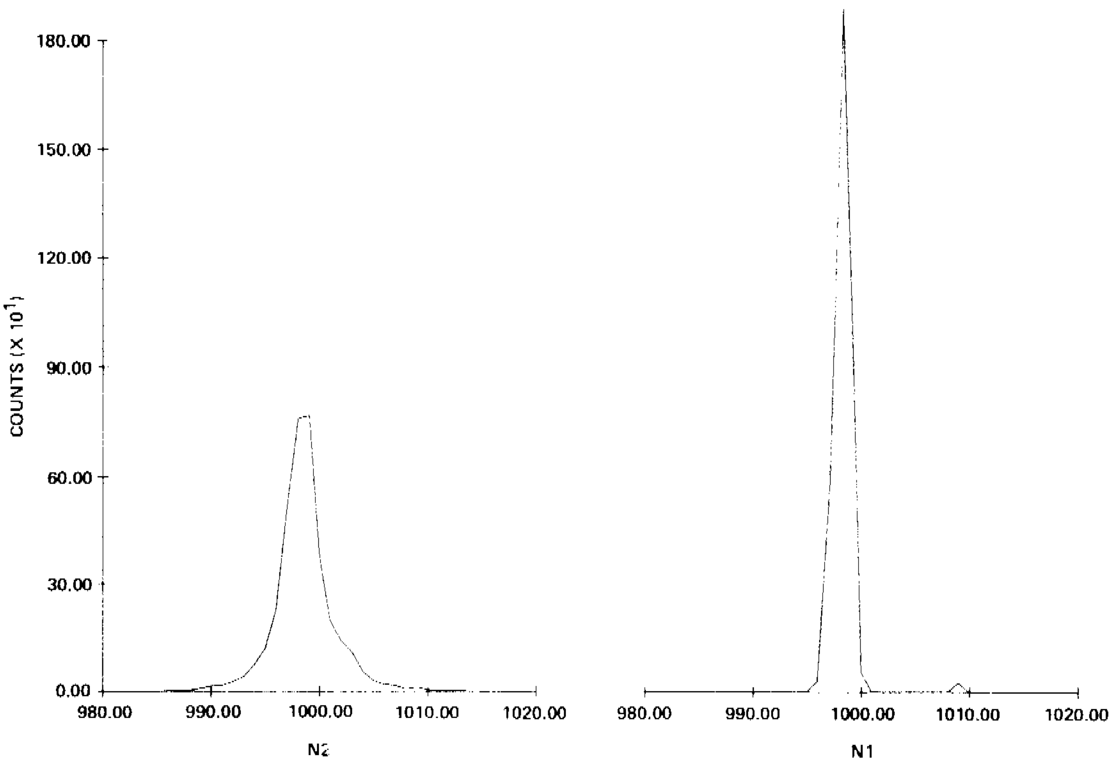


Figure 5. N2, N1 frequency distribution.

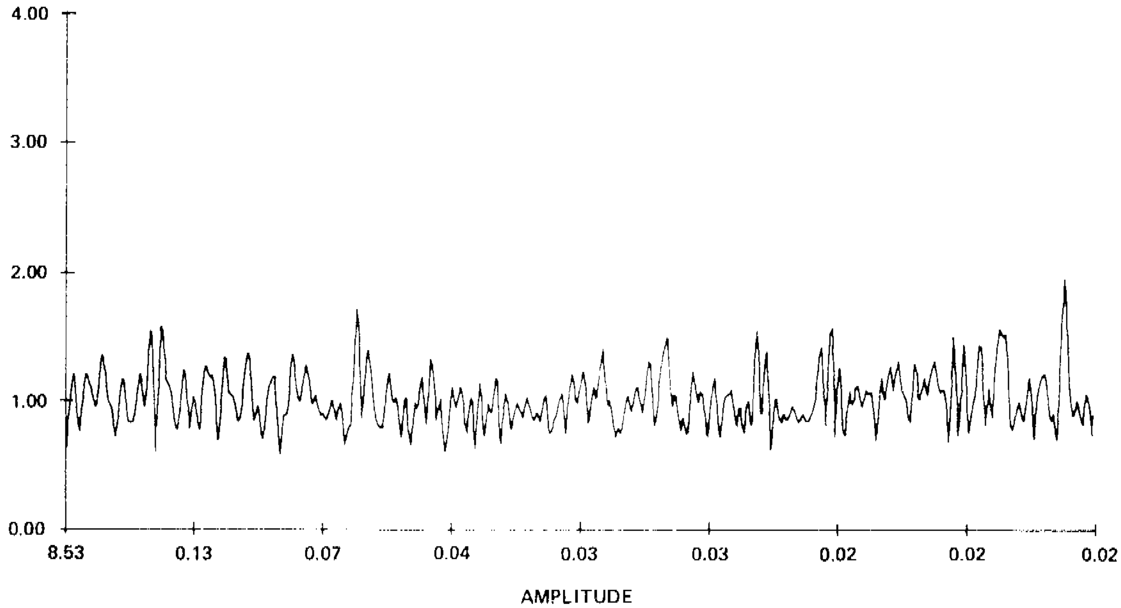


Figure 6. N2 calibration.

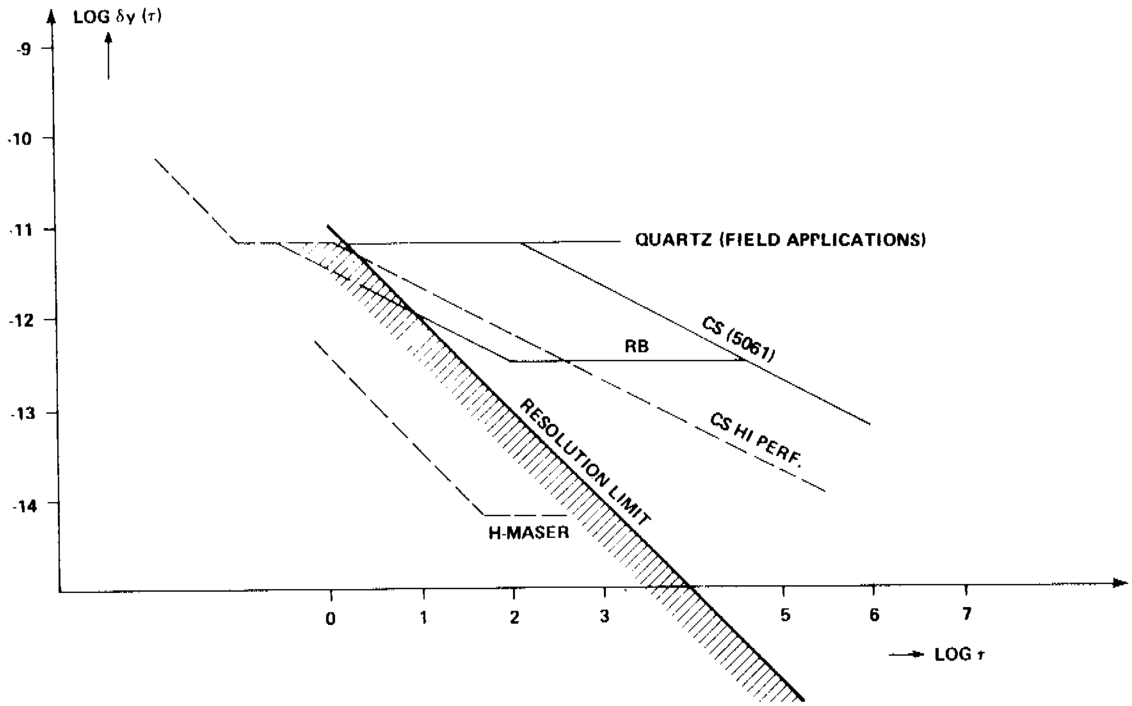
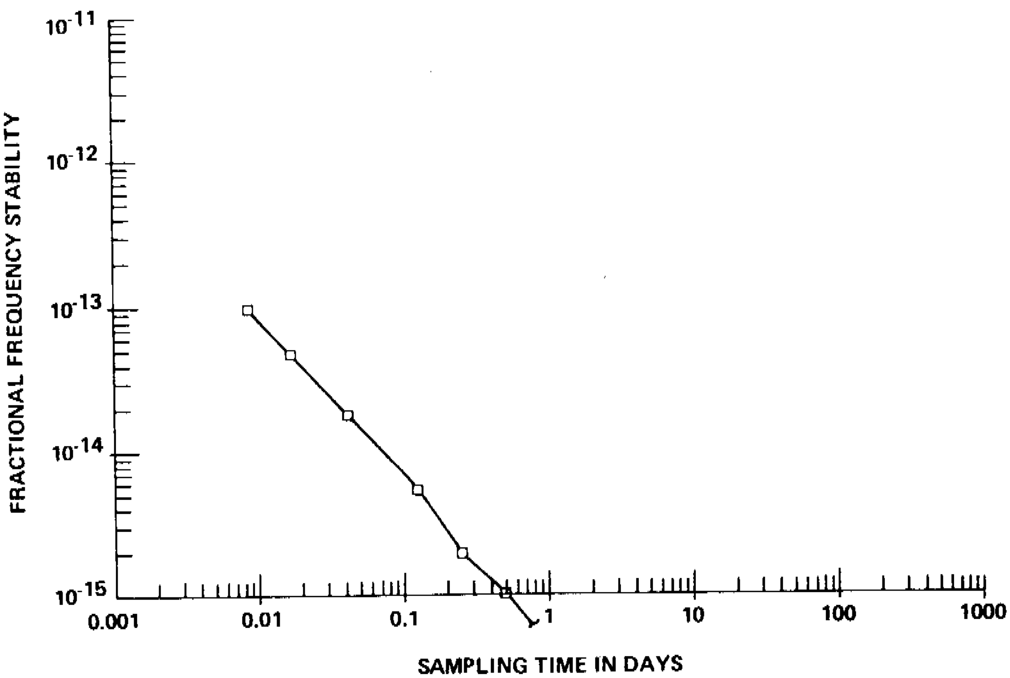


Figure 7. Schematic of expected limit of resolution in comparison with clock performances.



DATA SAMPLE FROM 15. O. U. T. ON MJO 41617 TO 8. 8. U.T. ON MJO 41626

Figure 8. Overall system noise with counter #603.

distribution amplifiers in the system. Figure 9 gives an example of variations in the tick-to-phase relationship of our former reference system #2 (it has since been replaced with an improved version similar to system #1). These tick-to-phase measurements are simple (not averaged) data points. The interpolator noise, if present, enters in full magnitude in this case.

III. EXAMPLES OF SIGMA-TAU MEASUREMENTS

The present data collection at the USNO is programmed for a full measurement cycle of all clock differences and environmental parameters every hour on the hour. This measurement takes about four minutes. Each average of 256 phase-to-phase measurements takes about 1/2 second. The more time-consuming measurements are the tick measurements, where only 16 are being made for each average (17 seconds each). The tick-to-phase measurements are presently single.

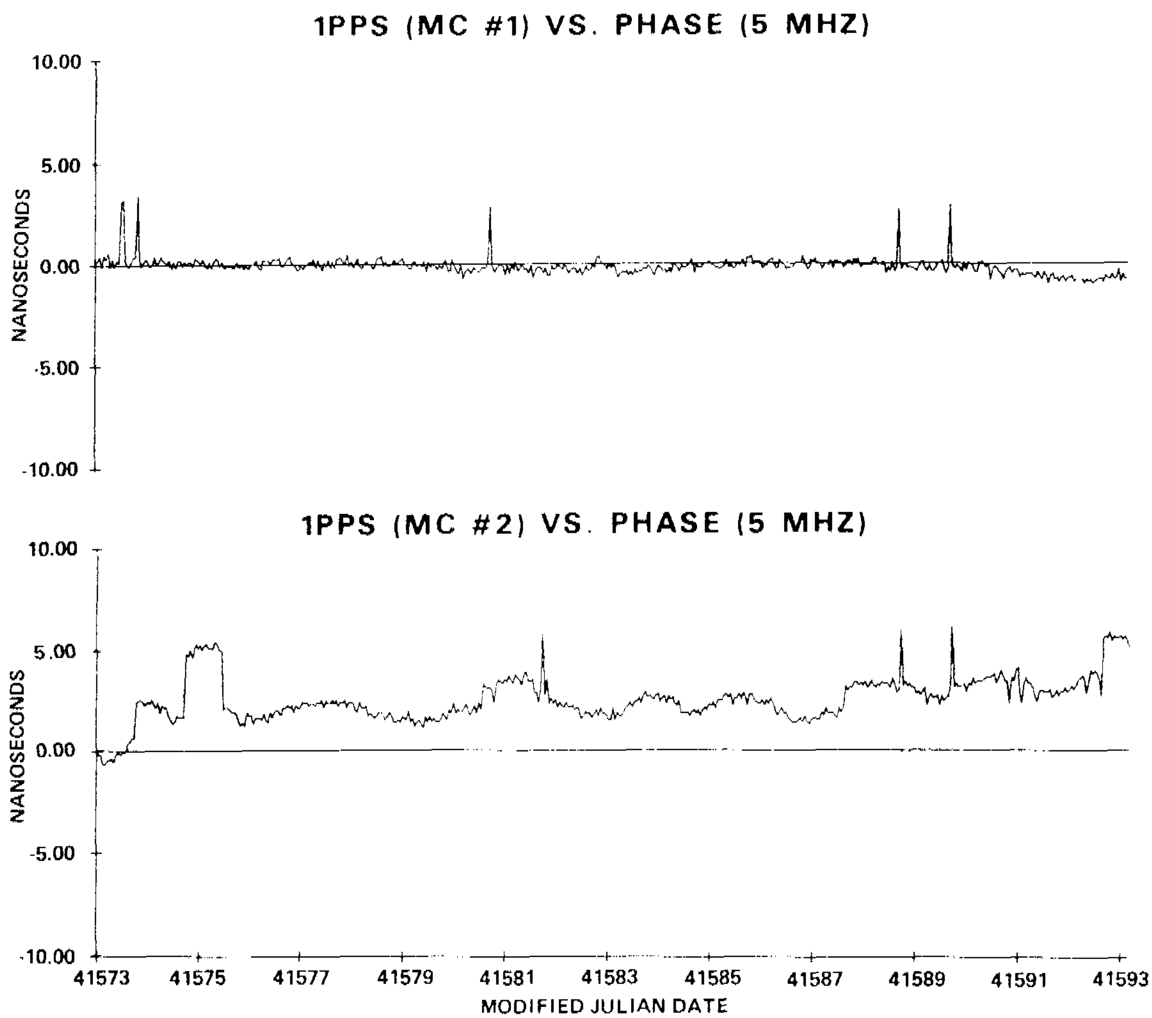


Figure 9. Examples of variations in the tick-to-phase relationship.

1. Experimental Scan

In addition to the above, a fast scan of 1×10 measurements is being done every 720 seconds (five times per hour). The NP4 hydrogen maser, on loan from the Goddard Space Flight Center (H. Peters design), was used as the "start" input against itself through a cable loop (for noise-level checks of the overall system) and against about ten other "stop" signals. This is a fast measurement (six seconds), and it allows stability measurements of clocks entirely independent of the USNO time scale. The reference signal has recently been replaced with the H-10 USNO maser.

As explained in Reference 2, the USNO clock time scale "MEAN (USNO)" is computed in five-day intervals. Its purpose is the provision of a clock time scale of superior reliability and long-term stability. Within the five-day intervals, however, the low frequency filtering inherent in the iterative procedure (Reference 2, Appendix) loses its effectiveness as we go to shorter averaging times τ.

For stability measurements $\sigma(\tau)$ where a hydrogen maser can be considered superior, direct comparisons of the clock with a hydrogen maser will produce reliable estimates of this clock's frequency stability. In cases where we deal with clocks of comparable performance, we can evaluate them in groups of three. By measuring rate variations of pairs of clocks (1 and 2, 2 and 3, 3 and 1) and by allowing for the contribution of the system noise (expressed as variance σ_N^2), one can solve the equations as listed in Figure 10.

In practice, several problems arise. First, one must use data which are homogeneous for the whole set; i.e., they must be collected nearly simultaneously. Second, one should expect frequent failures (imaginary results) due to the statistical variance in the measured estimates of $\sigma(\tau)$. One should not include clocks of widely different performance if the noise is high. Thirdly, one must realize that the environment is never perfect nor are the clocks truly stationary in their behavior. Any comparison of performance must take into account these factors. Nevertheless, this method can give consistent estimates of $\sigma(\tau)$.

MEASURED:

$$\sigma^2_{1,2}(\tau) = \sigma^2_{11} + \sigma^2_{22} + \sigma^2_{NOISE}$$

$$\sigma^2_{2,3}(\tau) = \sigma^2_{22} + \sigma^2_{33} + \sigma^2_{NOISE}$$

$$\sigma^2_{3,1}(\tau) = \sigma^2_{33} + \sigma^2_{11} + \sigma^2_{NOISE}$$

COMPUTED:

$$\sigma_{11} = \sqrt{\frac{\sigma^2_{12} + \sigma^2_{13} - \sigma^2_{23} - \sigma^2_{N}}{2}}$$

ETC.

Figure 10. Frequency stability measurements with clock triplets.

2. Examples

Examples of $\sigma(\tau)$ measurements (defined according to Reference 1) are given in Figures 11-13 taken with computing counter #863, and in Figure 14, from computing counter #603. Estimates of individual stabilities are listed in Table 3.

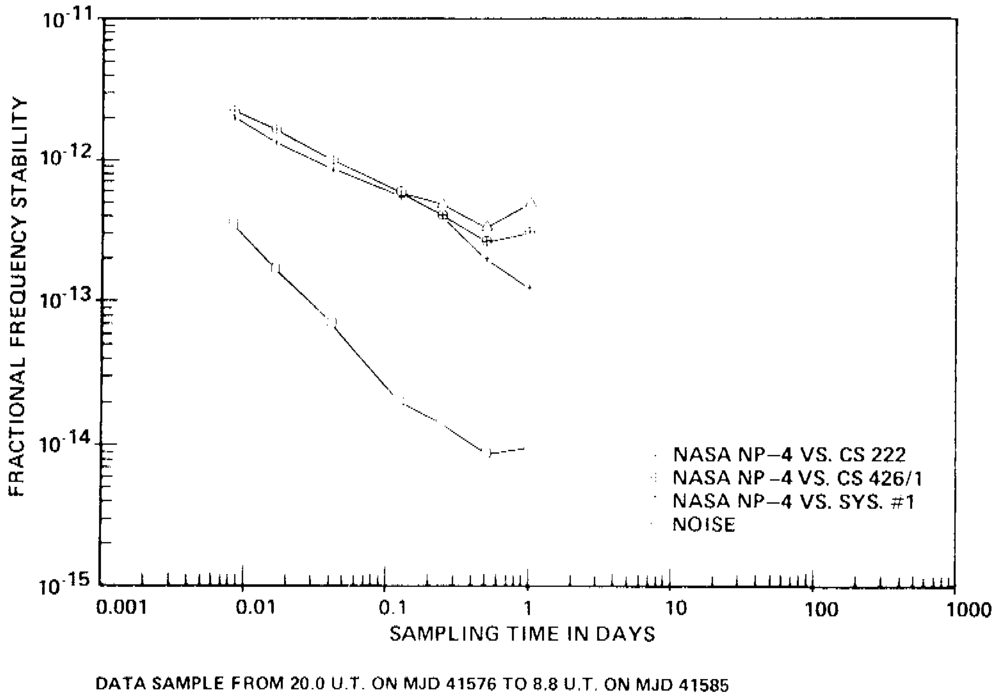
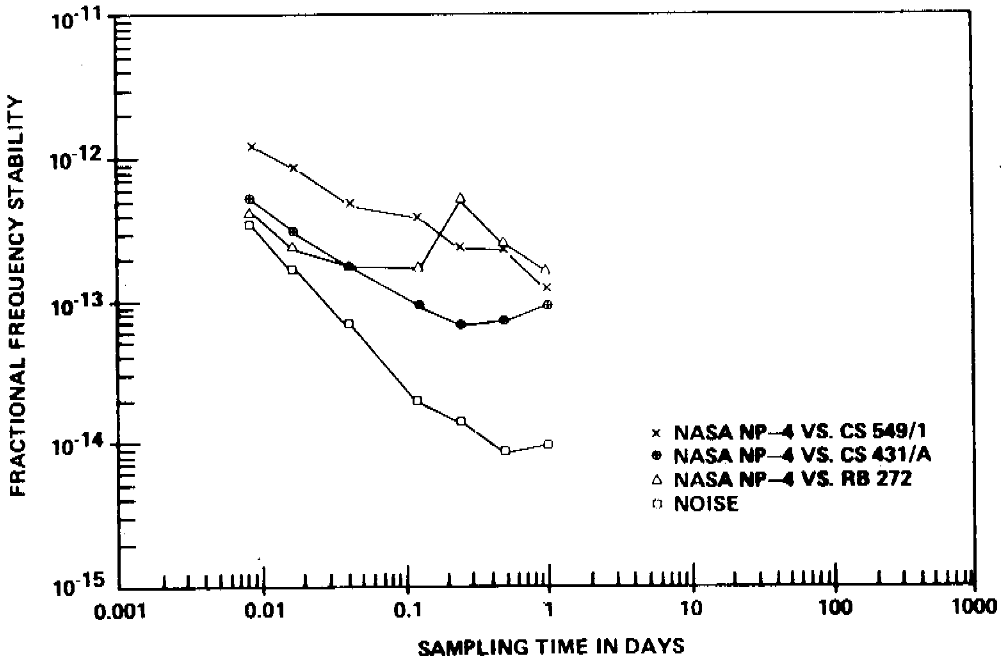


Figure 11. Frequency stabilities measurements – set 1.

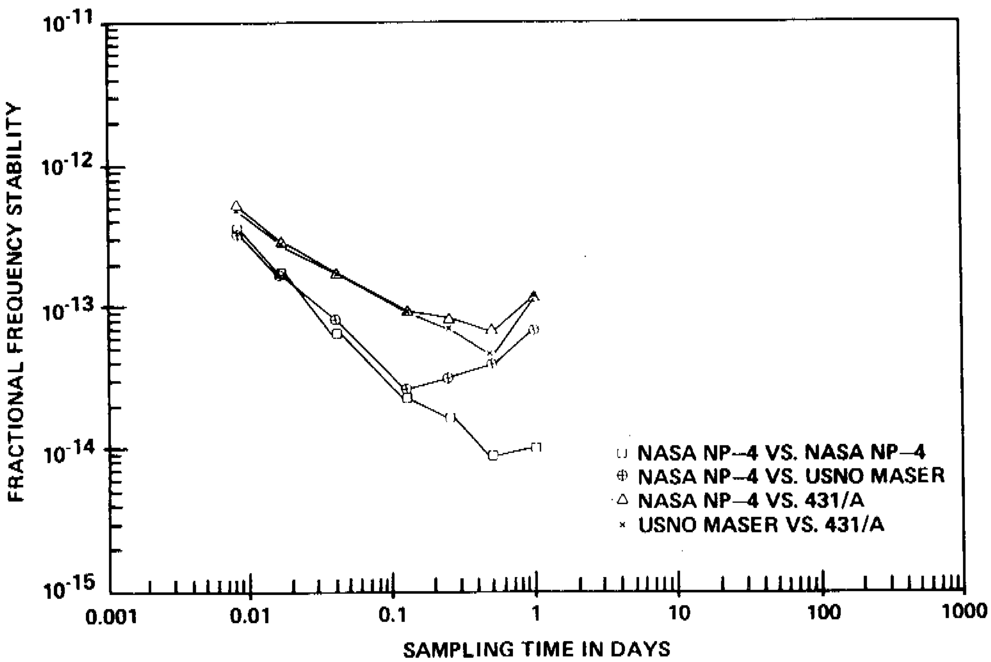
Table 3
$\sigma(\tau)$ of Six Oscillators

Sec	Days	NP4 (NASA)	Cs(HP) 431	Rb(HP) 272	Cs(HP) 549	Cs(HP) 426	H_{USNO}	Noise
720	0.008	(0.9)	3.66	1.9	11.7	21.3	- ≤0.06	3.35
1,440	0.017	0.7	2.3	1.4	8.2	15.7	- ≤0.06	1.62
3,600	0.042	0.5	1.6	1.7	9.5	9.5	- ≤0.06	0.62
10,800	0.125	0.3	0.9	1.8	3.8	5.6	≤0.06	0.19
21,600	0.250	0.3	0.7	2.0	2.2	3.8	0.1	0.14
43,200	0.500	0.3	0.5	2.0	2.4	0.2	0.08	
86,400	1.000	(0.4)	0.6	2.0	1.0	(0.6)		



DATA SAMPLE FROM 20.0 U.T. ON MJD 41576 TO 8.8 U. T. ON MJD 41585

Figure 12. Frequency stabilities measurements – set 2.



DATA SAMPLE FROM 20.0 U.T. ON MJD 41576 TO 20.6 U.T. ON MJD 41581

Figure 13. Frequency stabilities measurements – set 3.

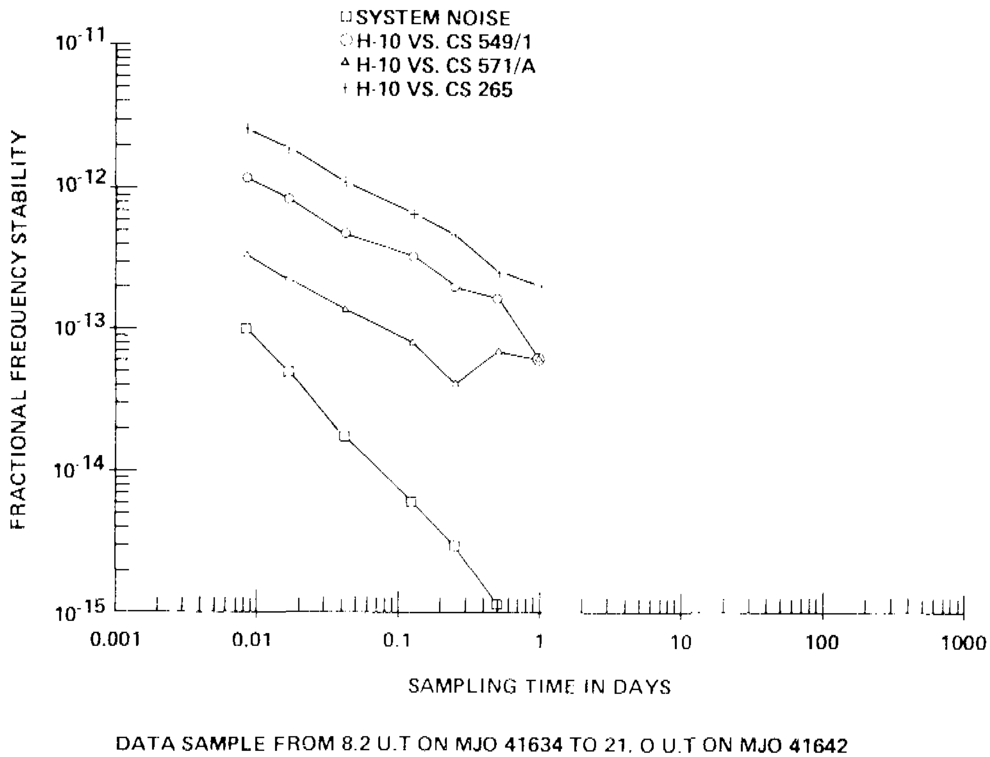


Figure 14. Frequency stabilities measurements – computing counter #603.

The routines used in preparing the $\sigma(\tau)$ plots mentioned above also include a tabulation of variances for the parameter $N \neq 2$ in order to allow for another indication of the noise type actually affecting the clock performance as originally discussed by David Allan (Reference 4). Almost all the practical utility of such stability parameters seems to reside, however, in the column labeled $N = 2$ (Tables 4 and 5).

IV. DISCUSSION AND CONCLUSIONS

The practical results of the stability measurements described can be summarized (at least for the commercially available cesium and rubidium clocks) in two parameters, if we restrict our interest to "clock" applications (we ignore "spectral purity") for $\tau > 100$ seconds.

The first parameter κ_s describes an oscillator's performance in the (Gaussian & White) FM region according to the model

$$\sigma(\tau) = \frac{\kappa_s}{\tau_s} \qquad \text{(Reference 2, p. 128)}$$

Table 4
Results of Sigma Computations
for USNO Maser versus 431/A (Old High-Performance Cesium).

τ (days)	N = 2 RMS Sigma		N = 4 RMS Sigma		N = 8 RMS Sigma	
	Complete Groups	($\times 10^{-13}$)	Complete Groups	($\times 10^{-13}$)	Complete Groups	($\times 10^{-13}$)
0.017	150	2.74	75	2.66	37	2.67
0.042	60	1.74	30	1.68	15	1.63
0.125	20	0.90	10	0.94	5	0.93
0.250	10	0.72	5	0.69	2	1.05
0.500	5	0.44	2	1.01		

Data sample from 20.0 UT on MJD 41576
to 20.6 UT on MJD 41581

Reduced on November 8, 1972.

Computing Counter No. 863.

Table 5
Results of Sigma Computations
for H-10 versus CS 571/A (New High-Performance Cesium).

τ (days)	N = 2 RMS Sigma		N = 4 RMS Sigma		N = 8 RMS Sigma	
	Complete Groups	($\times 10^{-13}$)	Complete Groups	($\times 10^{-13}$)	Complete Groups	($\times 10^{-13}$)
0.017	255	2.24	127	2.23	63	2.20
0.042	102	1.36	51	1.33	25	1.31
0.125	34	0.79	17	0.72	8	0.85
0.250	17	0.40	8	0.65	4	0.75
0.500	8	0.68	4	0.74	2	0.75

Data sample from 8.2 UT on MJD 41634
to 21.0 UT on MJD 41642

Reduced on November 25, 1972

Computing Counter No. 603

The second parameter is value τ_F, where the σ (τ) plot levels off into a flicker noise performance. (We know that for a very large τ, all clocks will show noise behaviors that correspond to models of even larger dispersion, random walk FM, "drift" noise, etc.)

In general then, we found evidence that aging of atomic standards will increase κ_s. We also found that inclement environmental conditions (except vibration, which also increases κ_s) will only reduce τ_F. Table 6 attempts to give representative values for these parameters.

Table 6
The Parameters κ_s and τ_F
($\tau_F{}'^{1}$ refers to less than ideal environments).

Clock	κ_s	τ_F	τ_F
Rb HP #272	5.3×10^{-12}	1/2 day	1/2 hour**
Cs HP #265 (5060, 1967 vintage with original beam tube)	7×10^{-11}	20 days	Unknown
Cs HP #549 (5061, vintage 1972)	3.2×10^{-11}	10 days	5 days***
Cs HP #431 (5061, with Hi-Perf tube, 1-1/2 years old)	1×10^{-11} *	10 days	1/2 day**
HP #571 (5061 with option 04) new	8×10^{-12}	10 days***	1 day***

*Cs 431 achieved 1.- $\times 10^{-14}$ for $\tau = 10$ days in Spring 1972 ($\kappa_s = 9.3 \times 10^{-12}$).

**The main cause of this poor showing is temperature sensitivity.

***Insufficient data.

V. ACKNOWLEDGEMENT

I owe all data preparation to D. B. Percival, and all of the indispensable hardware design and operation to K. Putkovitch, both of the USNO.

REFERENCES:

1. J.A. Barnes, et al., "Characterization of Frequency Stability," *IEEE Transactions on Instrumentation and Measurement*, vol. IM-20 (May 1971), pp. 105-20.

2. G.M.R. Winkler, R.G. Hall, and D.B. Percival, "The USNO Clock Time Reference and the Performance of a Sample of Atomic Clocks," *Metrologia*, 6 (no. 4, October 1970), pp. 126-34.

3. K. Putkovich, "Automated Timekeeping," *IEEE Transactions on Instrumentation and Measurement*, vol. IM-21, (November 1972). pp. 40-405.

4. David W. Allan, "Statistics of Atomic Frequency Standards," *Proc. IEEE*, vol. 54 (no. 2, February 19, 1966). pp. 221-30.