STABILITY CHARACTERISTICS AND APPLICATION TECHNIQUES FOR PRECISION FREQUENCY SOURCES

by

Dr. L.S. Cutler

Dr. Cutler is Director, Physical Research Laboratory, Hewlett-Packard Company, Palo Alto, California.

We would like to go back to some fundamentals and present some of the aspects of noise and fluctuation on signals which lead to the characteristics of various types of precision signal sources.

First we will discuss representations of signals and go into some of the simple noise relationships. We will talk about how signals can become contaminated and then consider some of the various stability measures that have been proposed and are in present use and then talk about the effects of frequency multiplication on precision sources and means for achieving low noise frequency multiplication. In addition, we will consider some of the techniques for measuring stability and getting some of the numbers involved in stability measures. Last of all, I will present some of the characteristics of available sources -- by no means an exhaustive list, just a few.

Figure 1 is a representation of a pure signal. We have a signal which is a sinusoidal function of time and is depicted as a rotating vector. The real signal is the projection on the horizontal axis of this vector as it rotates at the angular velocity of ω_0. Figure 1 is the vector as viewed in a coordinate system that rotates at the angular velocity ω_0. In this coordinate system, the vector is fixed. This is the phasor representation.

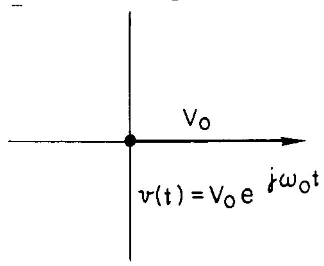


Figure 1. PURE SIGNAL

If we change the length of the vector (Figure 2) but do not rotate it about its position, then we get a representation of amplitude modulation. Again, remember that this total vector is rotating at the rate of ω_0, but its length is changing in time.

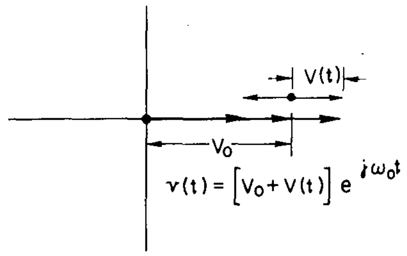


Figure 2. PURE AMPLITUDE MODULATED SIGNAL

If we take the vector, keep its length constant and swing it back and forth so that it advances and retards as it rotates at ω_0, we have pure phase modulation. This is directly related to frequency modulation and is shown in Figure 3. In this case the phase angle, $\phi(t)$ is a function of time and it appears in the argument of the exponential. If we do both things simultaneously, we obtain simultaneous amplitude and phase modulation as shown in Figure 4. There are cases where the amplitude --

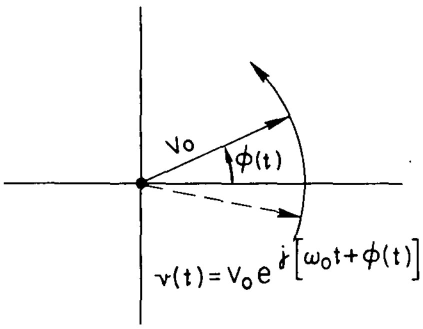


Figure 3. PURE PHASE MODULATED WAVE

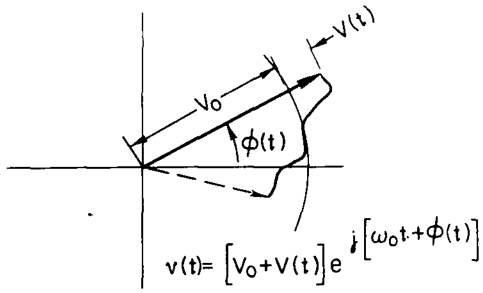


Figure 4. PHASE AND AMPLITUDE MODULATED SIGNAL

represented by V(t) -- might be correlated with the phase angle $\phi(t)$, so one would have a correlation between the amplitude and phase modulation that is present. In some cases this can be very important.

We consider amplitude modulation first. There is the carrier V_0 and two sidebands (Figure 5) which rotate at rates ω_m with respect to this carrier vector, in opposite directions and are so phased that their maxima -- when they add up on the same direction -- lies along V_0 . This gives us a representation in phasor language of pure sinusoidal amplitude modulation -- carrier plus two sidebands.

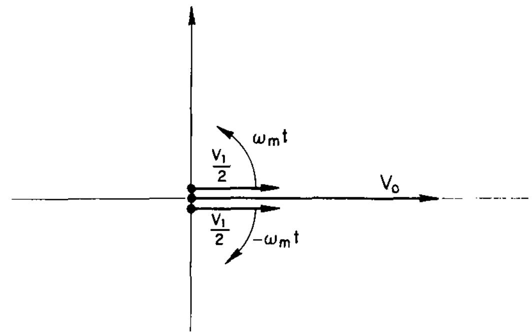


Figure 5. PURE SINUSOIDAL AMPLITUDE MODULATION-CARRIER PLUS TWO SIDEBANDS

You can picture these two sidebands as being added to the end of the vector and rotating in opposite directions so that the net resultant is just to change the lengh of this amplitude vector and not to change its phase angle.

If we go to a frequency domain representation (Figure 6) where we are talking about just the amplitudes of the carrier and sidebands, not the power, we represent the carrier at the center frequency ω_0 and the two sidebands equally spaced on either side of ω_0 by $\pm\,\omega_m$ where ω_m is the angular modulation frequency. The sidebands are shown as being in phase at the instant they lie along the carrier vector.

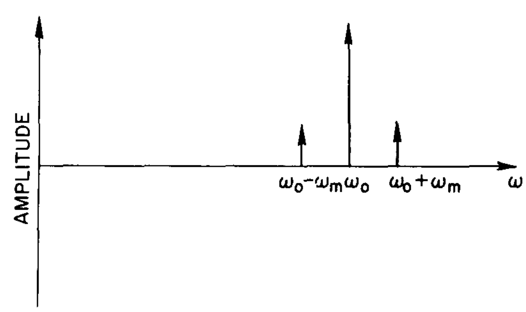


Figure 6. FREQUENCY DOMAIN REPRESENTATION FOR PURE SINUSOIDAL AMPLITUDE MODULATION

Now let us consider the case where we have two sidebands again, but phased differently from the case we had for amplitude modulation. This is shown in Figure 7. Here again the two sideband vectors rotate in opposite directions, but now when they point in the same direction the resultant would be to cause some phase shift and this is a representation of small index sinusoidal phase modulation. One can also see that if he added these vectors, the length of the total resultant vector would change slightly, and consequently a single pair of sidebands does produce some amplitude modulation in addition to phase modulation. Restraining the length of the vector to remain absolutely constant requires second-order and higher-order sidebands. One can go through and draw a diagram and see how all the Bessel function relations for the sideband amplitudes in phase modulation come about.

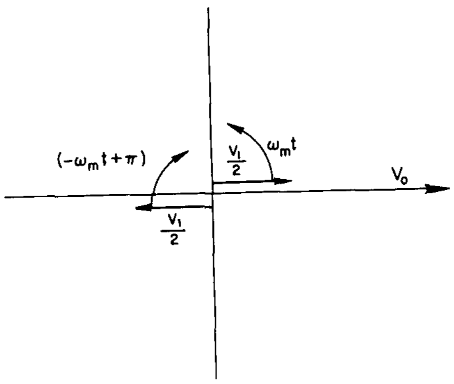


Figure 7. SMALL INDEX SINUSOIDAL PHASE MODULATION CARRIER PLUS TWO SIDEBANDS

Figure 8 is a frequency domain representation for pure sinusoidal phase modulation. This representation is to give the idea that the two sidebands are out of phase with respect to what they would be if it were amplitude modulation. Here again, if it were a power spectrum, one would have no indication of phase, but we have tried to preserve some phase here by having an amplitude spectrum rather than a power spectrum.

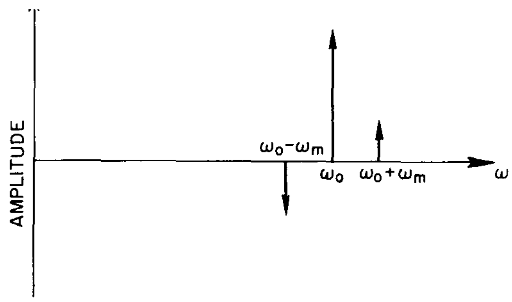


Figure 8. FREQUENCY DOMAIN REPRESENTATION FOR PURE SINUSOIDAL PHASE MODULATION (Small Index)

In Figure 9 we consider a pure signal and one sideband. This has been broken down, as the second line in Figure 9 shows, into a phase and amplitude modulated wave and the phase and amplitude modulations are correlated, so a carrier plus a single sideband does give correlated phase and amplitude modulation. The phase modulation is relatively pure if the sideband amplitude as compared to the carrier amplitude is fairly small. One can

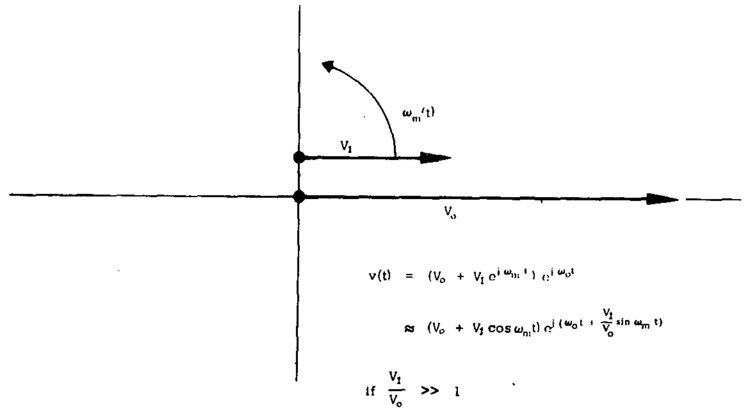


Figure 9. AMPLITUDE AND SMALL INDEX PHASE MODULATION -- CARRIER PLUS ONE SIDEBAND

picture this if he adds the sideband to the tip of the carrier vector and lets it rotate; it performs a circle out there, and the resultant vector then grows in length and shrinks in length at the same time it changes angle. It is apparent that the angle and length are correlated. This all leads to the point that any time there is an asymmetric power spectrum, there will be correlation between the amplitude and the phase modulation, even in the case of noise.

So far we have considered mainly pure sinusoidal modulations. Now let us look at random processes, including noise. Here one has to invoke such things as autocorrelation functions, probability densities, power spectral densities, and phase and frequency power spectral densities. Let us investigate some of these things.

Figure 10 shows some very basic definitions. If one can assume some things about the signal, he may say that the autocorrelation function is the average value of the product of a time function with itself at a time τ later as shown in the first part of the equation in Figure 10. v(t) is a real function and the average brackets mean either time or statistical average in the case where these two are equivalent. The spectral density $S(\omega)$ of this same function is the one-sided Fourier transform of the autocorrelation function R as shown in the second line. Conversely, there is the inverse Fourier transform relation between R and S shown in the third line. These relations are very useful; they come up time and time again in this sort of work. A lot of what we are covering is treated in fair detail in the National Bureau of Standards' Technical Memo 394, which was also published in the <u>IEEE Transactions on Instrumentation and Measurement</u> entitled, "Characterization of Frequency Stability."

$$R_v(\tau) = \langle v(t)\, v(t+\tau) \rangle$$

(v real)

$$S_v(\omega) = 4 \int_0^\infty R_v(\tau) \cos\omega\tau \, d\tau$$

$$R_v(\tau) = \frac{1}{2\pi} \int_0^\infty S_v(\omega) \cos\omega\tau \, d\omega$$

$\langle \; \rangle$ means time or statistical average

Figure 10. RELATIONS BETWEEN AUTOCORRELATION FUNCTION, $R_V(\tau)$, AND SPECTRAL DENSITY, $S_V(\tau)$

Now we would like to define a couple of useful quantities related to phase and frequency as shown in Figure 11. It is useful to normalize both phase and frequency by dividing by the angular frequency of the oscillator itself. We define a variable x which is equal to the phase divided by ω_0 and a variable y which is equal to the angular frequency (the time derivative of ϕ) divided by ω_0 so y is equal to $\dot{x}$.

$$x(t) \equiv \frac{\varphi(t)}{\omega_0}$$

$$y(t) \equiv \frac{\dot{\varphi}(t)}{\omega_0} = \dot{x}(t)$$

Figure 11. DEFINITIONS OF x(t) AND y(t)

We have to work with spectral densities. Figure 12 shows some relationships between the spectral densities of x and y and ϕ. Inasmuch as frequency is essentially the time derivative of phase, the spectral density of frequency will be ω^2 times the spectral density of phase. $S_y(\omega)$, for example, is equal to $\omega^2 S_x(\omega)$. That is a very useful relationship.

$$S_x(\omega) = \frac{S_\varphi(\omega)}{\omega_0^2} = \frac{S_{\dot{\varphi}}(\omega)}{\omega^2 \omega_0^2} = \frac{S_y(\omega)}{\omega^2}$$

$$S_y(\omega) = \frac{S_{\dot{\varphi}}(\omega)}{\omega_0^2} = \frac{\omega^2 S_\varphi(\omega)}{\omega_0^2} = \omega^2 S_x(\omega)$$

Figure 12. RELATIONSHIPS BETWEEN $S_\varphi(\omega)$, $S_{\dot{\varphi}}(\omega)$, $S_x(\omega)$, AND $S_y(\omega)$

Let us talk now about contamination of signals. Signals can be contaminated through noise by two processes: (1) multiplication or modulation, and (2) additive noise. By multiplication, we mean multiplication by some time function rather than frequency multiplication, although contamination does occur in that process. Additive noise is added to a pure signal and contaminates it when it is localized in frequency around the signal. This effectively adds sidebands to the signal and can be construed as a mixture of amplitude and and phase noise sidebands.

We should look at various types of modulators. Figure 13 represents a simple amplitude modulator. The function A(t) is real, and consequently, one gets a signal which is not contaminated in phase. Its amplitude is now a function of time, therefore it is a pure amplitude modulator.

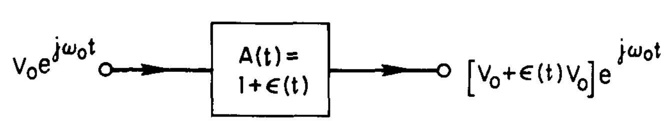


Figure 13. AMPLITUDE MODULATOR

Figure 14 shows a pure phase modulator. $V_0 e^{j\omega_0 t}$, a pure signal, is multiplied by a complex function with a constant magnitude, producing a signal with a constant amplitude, V_0, but a phase which is now a function of time.

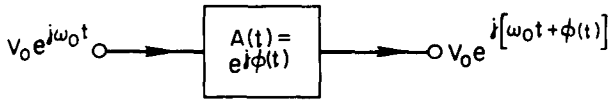


Figure 14. PHASE MODULATOR

Obviously, we can combine the two things and obtain simultaneous amplitude and phase modulation (Figure 15), and if one wants to make it correlated, he certainly can.

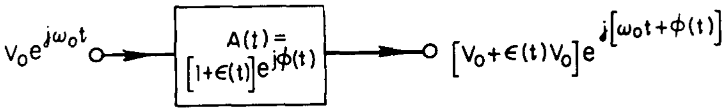


Figure 15. AMPLITUDE AND PHASE MODULATOR

Now let us look at additive noise -- the other contaminator of signals. Figure 16 is a representation of a narrow band random noise centered about the frequency ω_0. It has a randomly varying amplitude and a randomly varying phase. The vector rotates around in some random fashion and changes its length in a random fashion. If the noise were not centered at the frequency ω_0, one would find there would be some net

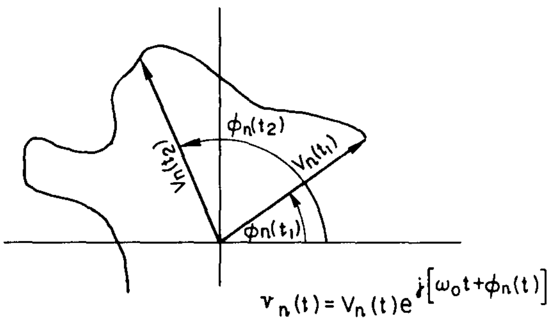
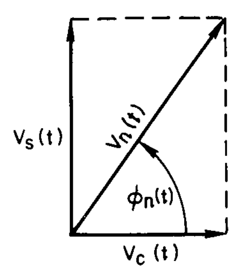


Figure 16. NARROW-BAND RANDOM NOISE CENTERED ABOUT ω_0

average rotation rates corresponding to the difference in frequency between the center of the spectrum and the frequency of reference, ω_0.

We can resolve this narrow-band noise as shown in Figure 17 into an in-phase part $V_c(t)$ and a quadrature part $V_s(t)$ both of which are random time functions. For most types of noise sources derived from a lot of statistically independent sources, one can prove that $V_c(t)$ and $V_s(t)$ will be gaussian random variables and will, in general, be independent processes provided we have chosen our center frequency properly. We can represent the instantaneous voltage, shown on the bottom line of Figure 17, as an in-phase and a quadrature part times our pure sinusoidal signal that is rotating at the frequency of ω_0. Now, if we add a pure signal to this, we get the pure signal modulated



$$V_c(t) = V_n(t) \cos \phi_n(t)$$

$$V_s(t) = V_n(t) \sin \phi_n(t)$$

$$v_n(t) = V_n(t)\, e^{j[\omega_o t + \phi_n(t)]}$$

$$= [V_c(t) + j V_s(t)]\, e^{j\omega_o t}$$

Figure 17. RESOLUTION OF NARROW-BAND RANDOM NOISE INTO AMPLITUDE (IN-PHASE) AND PHASE (QUADRATURE) COMPONENTS

as shown in Figure 18. The resultant signal can be resolved into an equivalent phase modulation and equivalent amplitude modulation.

Figure 19 represents the signal plus random additive noise in terms of $V_c(t)$ and $V_s(t)$. The relationships given hold true if the noise voltages are small compared with the signal voltage.

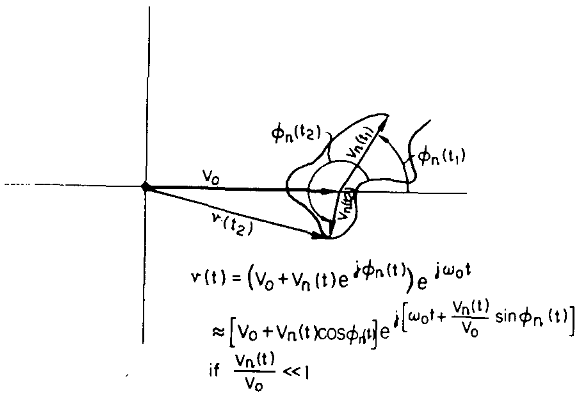


Figure 18. PURE SIGNAL AT ω_0 PLUS NARROW-BAND RANDOM NOISE

$$v(t) = \left[V_o + V_c(t) + jV_n(t)\right] e^{j\omega_o t}$$

$$\approx \left[V_o + V_c(t)\right] e^{j\left[\omega_o t + \frac{V_s(t)}{V_o}\right]}$$

$$\text{if} \quad \frac{V_c(t)}{V_o} \ll 1 \quad \text{and} \quad \frac{V_s(t)}{V_o} \ll 1$$

Figure 19. SIGNAL PLUS RANDOM ADDITIVE NOISE

Figure 20 is a representation of the RF power spectral density of a pure signal with added noise. The pure signal is an infinitely thin spectral line that is infinitely tall. Presumably, it has area P_0 under it and the additive noise has area P_n which is equal to the integral of the spectral density of the noise power over all frequencies. Here we are using one-sided spectral densities in terms of ω (rather than frequency f or ν).

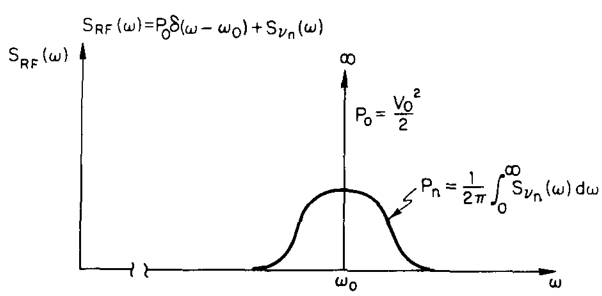


Figure 20. RF POWER SPECTRAL DENSITY OF PURE SIGNAL PLUS ADDED NARROW-BAND NOISE

If one has additive noise, how does he relate the effective phase modulation or the effective amplitude modulation of that noise back to things one can measure, such as spectral density of a noise itself, the signal power, etc.? Figure 21 shows some of the mathematics of how this is done. The spectral density, $S_\varphi(\omega)$, is given by the spectral density of the sine or quadrature part of the noise voltage divided by V_0^2 and that can be given in terms of the spectral density of the noise itself, as shown in the second line. We can say that the spectral density of phase is given by the bottom line, the spectral density of the noise voltage divided by the power in the signal. This is a very useful and important relation.

$$S_\varphi(\omega) \approx \frac{S_{V_s}(\omega)}{V_o^2} \quad \text{for} \quad \frac{P_n}{P_o} \ll 1$$

$$\text{But} \quad S_{V_s}(\omega) = S_{v_n}(\omega_o + \omega) + S_{v_n}(\omega_o - \omega)$$

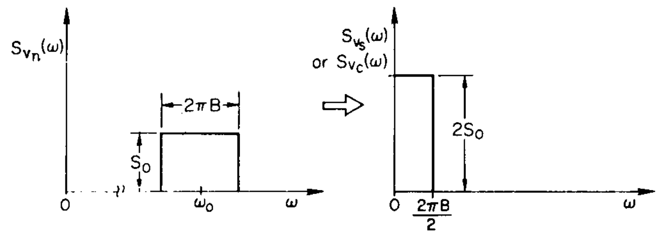
$$= 2\, S_{v_n}(\omega_o + \omega) \qquad \text{if } S_{v_n}(\omega)$$

is symmetric about ω_o

$$\text{Then} \quad S_\varphi(\omega) \approx \frac{S_{v_n}(\omega_o + \omega)}{P_o}$$

Figure 21

Figure 22 shows the effect of a spectrum of added noise that has even symmetry about ω_0. Suppose we have a rectangular band of added noise that is $2\pi B$ wide and S_0 tall. Therefore, the total noise power is $S_0 B$. This transforms to the effective modulation spectral density, either phase, $S_{V_S}(\omega)$, or amplitude, $S_{V_C}(\omega)$, as shown in the right half of Figure 22.



$$\sigma_v^2 = \sigma_{V_s}^2 = \sigma_{V_c}^2 = S_o B$$

Spectrum symmetric about $\omega_o \Rightarrow V_s$ & V_c uncorrelated

Figure 22

There one sees the spectral density is twice as high and it extends out to half the bandwidth, but the total area under it is the same, so that the total area under either the sine part or the cosine part is equal to the total area under the additive noise. This is a very important relationship. This is a case where we have a symmetric spectrum, and that implies immediately that the sine part and cosine part, or, equivalently, the phase and amplitude modulations produced by this additive noise, are uncorrelated.

Figure 23 is an example of an asymmetric power spectrum of the additive noise. This leads to the power spectral density for the sine or cosine part corresponding to the phase or amplitude modulation parts as shown on the right half of the figure. The spectral density is obtained essentially by taking the even part of the RF power spectral density of the added noise and translating it down to zero frequency. For the case of asymmetric spectral densities, one can show that the effective amplitude and phase modulations are correlated.

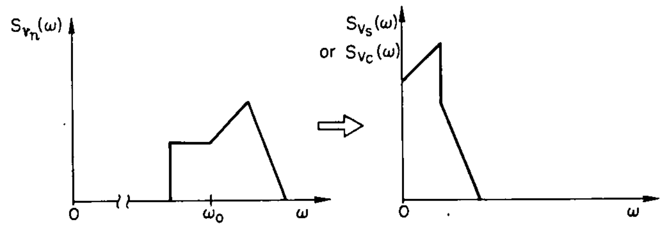


Figure 23. ADDITIVE RANDOM NOISE; SPECTRUM ASYMMETRIC ABOUT $\omega_0 \Rightarrow V_s$ AND V_c CORRELATED

If one has any kind of a signal with a zero mean, as in Figure 24, the variance of that signal is equal to the auto-correlation function at zero lag which is equal to the integral over the power spectral density. In other words, it is the total power in the signal. We are interested in the mean square value of the phase as given by the bottom line in Figure 24. If one has a signal that is quite pure but is contaminated with additive noise, then the total variance in phase produced by that additive noise is just equal to the noise-to-signal power ratio as shown in the bottom line.

For v with 0 mean,

$$\sigma_v^2 = R_v(0) = \frac{1}{2\pi}\int_0^\infty S_v(\omega)\,d\omega$$

so that

$$\sigma_\varphi^2 = \frac{\sigma_{v_n}^2}{P_o} = \frac{P_n}{P_o}$$

Figure 24

Obviously, one way to get a better signal or equivalently a smaller variance in phase, is to shrink the power spectrum of the additive noise by narrow-band filtering and, therefore, reduce the total noise power. This, of course, will reduce the total phase angle that the signal is swinging.

Let us look at Figure 25 which shows the spectral density of x for a typical signal that might be derived from a precision oscillator or a cesium standard, hydrogen maser, or something else. It has proven to be useful and accurate to represent this as a series in inverse powers of ω, and generally most sources will exhibit a number of these components. Remember $S_x(\omega)$ is essentially a measure of the spectral density phase -- not frequency.

Typical sources will have phase spectral densities that can be represented as

$$S_x(\omega) = \underbrace{h_{-4}\,\omega^{-4}}_{\text{random walk frequency}} + \underbrace{h_{-3}\,\omega^{-3}}_{\text{flicker frequency}} + \underbrace{h_{-2}\,\omega^{-2}}_{\text{random walk phase}} + \underbrace{h_{-1}\,\omega^{-1}}_{\text{flicker phase}} + \underbrace{h_0}_{\text{white phase}}$$

Figure 25

Most signals can be broken down as shown in Figure 25, and one can assign values to the coefficients by making measurements. In the process of designing equipments, one can get some idea as to how to reduce some coefficients, perhaps at the expense of others.

In addition to spectral densities, there is another very useful measure, the Allan Variance shown mathematically in Figure 26. It is a measure of of frequency fluctuations for some averaging time τ. Figure 26 shows a special case of the more general Allan Variance which involves N samples with the time between samples of T and averaging time τ. There are also some cases involving high frequency divergences, so sometimes one must consider the high frequency cutoff either in the apparatus he is measuring or the apparatus with which he is making the measurement.

Another useful measure is the Allan Variance, $\sigma_y^2(\tau) \equiv \langle \sigma_y^2(2, \tau, \tau) \rangle$ which is the rms fractional frequency fluctuation averaged over time τ for pairs of adjacent samples.

$$\sigma_y^2(\tau) = \frac{1}{\pi} \int_0^\infty d\omega \; S_y(\omega) \; \frac{\sin^4\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)^2}$$

This is a special case of $\langle \sigma_y^2(N, T, \tau) \rangle$

Figure 26

Figure 27 shows some very simple relationships which have been derived for the power law spectra. These relationships are mentioned in detail in Technical Note 394 of the National Bureau of Standards. For a phase spectral density proportional to $\omega^{(\alpha-2)}$, the Allan Variance is proportional to τ^{μ} where τ is the averaging time and the graph on the right of Figure 27 gives a relationship between μ and α. For example, for flicker-frequency noise, α would be -1 in which case μ is 0. That is, the Allan Variance is then proportional to τ^0; and so is a constant. Therefore, if one takes Allan Variances as a function of averaging time, and finds that they do not change, he knows that he is looking at a flicker frequency noise spectrum.

For $S_x(\omega) = h_{(\alpha-2)}\,\omega^{(\alpha-2)}$

$\sigma_y^2(\tau) = C_\alpha \tau^{\mu}$

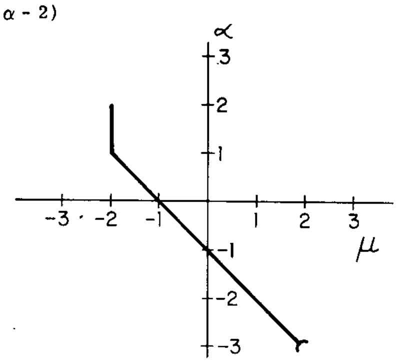


Figure 27. RELATIONSHIPS FOR POWER LAW SPECTRA

Another case that commonly occurs is white frequency noise. This is a commonly observed type of frequency fluctuation for moderate averaging time in cesium beam standards or rubidium standards. For this case, $\alpha = 0$ and $\mu = -1$.

Now let us talk about frequency multiplication. If a frequency multiplier multiplies by a number N, N does not necessarily have to be an integer. For example, in a well-designed frequency synthesizer, N can be some rational fraction. In any case, upon frequency multiplication by the factor N, the instantaneous phase angle is multiplied by N. Usually amplitude limiting occurs so that mose of the noise on the output is phase noise.

Let us now consider some of the techniques for frequency multiplication. Figure 28 shows one scheme for achieving a spectrally-pure signal after frequency multiplication. Of course, if one had a pure source and it was contaminated with some noise so that it represented a real physical source -- namely our precision frequency source -- then we would like to remove as much of the noise modulation as possible by using a narrow-band filter, which could be either a passive filter or active filter such as a high-level phase-locked oscillator. This filters the signal and improves the signal to phase noise ratio. The filtered signal can then be fed into a low-noise frequency multiplier. In order to have a decent RF power spectrum, the

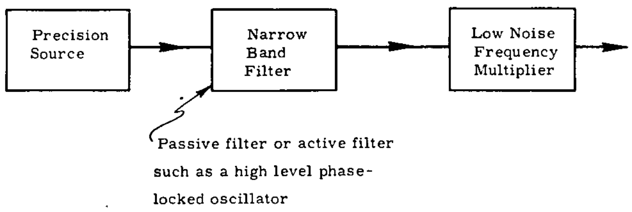


Figure 28. TECHNIQUE FOR ACHIEVING SPECTRALLY PURE SIGNALS

signal-to-noise power ratio after multiplication must be much greater than 1, so one must start out with a signal-to-noise ratio before multiplication which will satisfy this. For example, assume a 60-db signal-to-noise ratio prior to multiplication. Multiplying the frequency by 1,000 times, corresponds to increasing the phase modulation by 60 db. The result would be a signal-to-noise ratio of 0 db after multiplication, and there would, consequently, be great spreading out of the noise, higher-order mixing products, etc. In addition, the power in the signal which we would have liked to preserve would have gone down and been smeared out into the sidebands. It is always necessary to look at what you wish to achieve; what frequency multiplication you want to do; and then satisfy the criterion that the signal-to-noise power ratio, after multiplication, be much less than 1. This places very stringent requirements on the signal-to-noise ratio prior to multiplication.

Figure 29 demonstrates one technique for achieving low noise frequency multiplication. An input signal is fed to a filter or a phase-locked oscillator to clean up the signal as much as possible before it goes into the first frequency multiplier. Somewhere along the chain is another phase-locked oscillator, which acts as an active filter, again cleans the noise off the sides of the signal with an optimum bandwidth such that the total spectral density of noise is minimized. This is again followed by frequency multiplication and filtering. The whole process is continued until the desired output frequency is achieved. This is probably one of the best techniques for achieving low noise frequency multiplication.

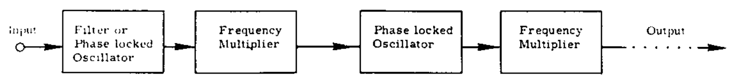


Figure 29. TECHNIQUE FOR LOW NOISE FREQUENCY MULTIPLICATION

A technique for achieving relatively low noise frequency multiplication if a fairly noise free signal is available is shown in Figure 30. It is essentially a phase-locked oscillator in which the phase comparison is done by sampling the output oscillator, the VCO, with a sampler run from the input signal. Loop bandwidth is determined by the sampling rate and the type of filter used. The loop bandwidth cannot be higher than one-half of the

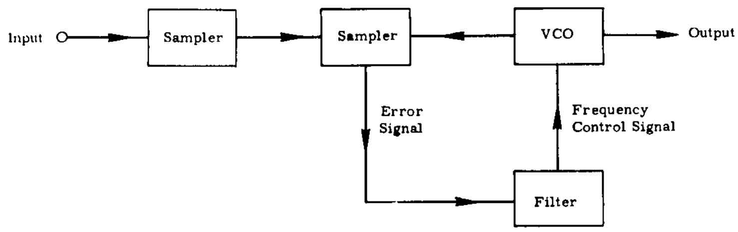


Figure 30. SAMPLED SIGNAL PHASE LOCK LOOP FOR FREQUENCY MULTIPLICATION

sampling rate. There are many samplers available now which will operate up into X-band regions and higher. This technique is useful for phase locking some of the very high frequency oscillators.

Now let us consider some methods of measuring the noise. Figure 31 shows a phase detector technique which is useful for measuring higher frequency noise. Two sources, presumed to be identical, and multiplied in frequency by optional frequency multipliers are fed into a phase detector. The output of the phase detector then can be fed back through a low-pass filter and thus make a very loose, narrow band phase-lock loop to keep the signals essentially in average phase quadrature for long periods of time.

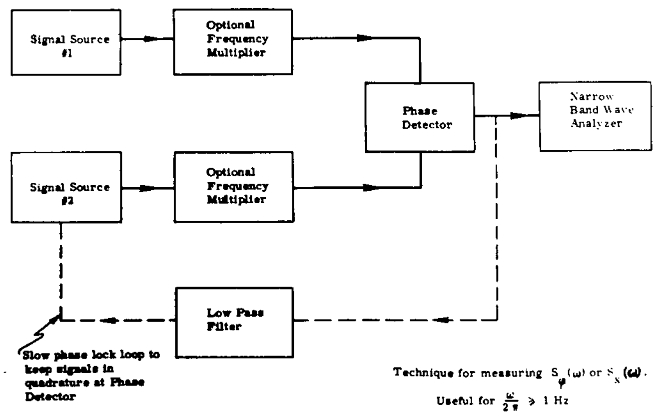
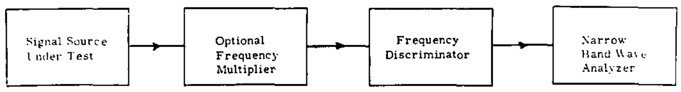


Figure 31. PHASE DETECTOR TECHNIQUE FOR MEASURING HIGH FREQUENCY NOISE

Operation is then on the linear slope portion of the phase detector. Its output can be spectrally analyzed by a narrow-band wave analyzer, and the output can be related back to $S_\varphi(\omega)$ or $S_x(\omega)$. This technique is well documented in the literature and very often used.

In Figure 32 is shown a technique for measuring $S_{\dot{\varphi}}(\omega)$ or $S_y(\omega)$. The signal source under test is passed through an optional frequency multiplier, then into a frequency discriminator. (One may use a cavity type discriminator or something similar.) The output will be a voltage that is proportional to the frequency swings on an instantaneous basis and may be analyzed with a narrow-band wave analyzer. It gives spectral information for the frequency components above about 1 hertz.

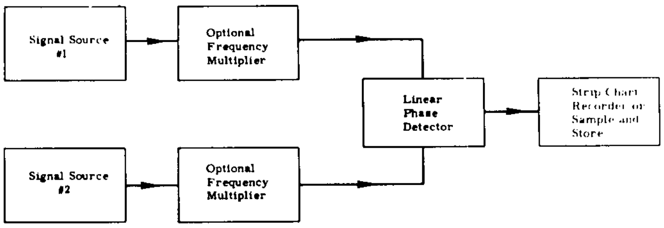


Technique for measuring $S_{\dot{\varphi}}(\omega)$ or $S_y(\omega)$

Useful for $\frac{\omega}{2\pi} \geq 1$ Hz

Figure 32. TECHNIQUE FOR MEASURING $S_{\dot{\varphi}}(\omega)$ or $S_y(\omega)$

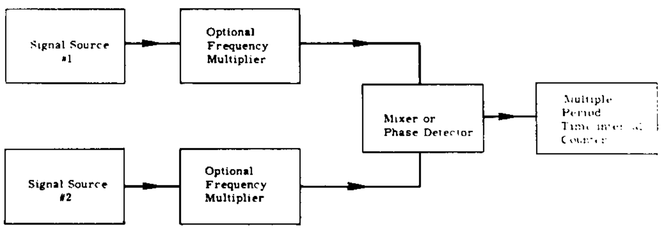
In the case of very low frequency noises where direct spectral analysis techniques cannot be used, one must actually record the phase (Figure 33). A plot should be made of the phase versus time. The autocorrelation function of the phase may then be obtained. $\sigma_y^2(\tau)$ can be obtained from the phase data by taking successive phase differences. Here again, we have two signal sources with optional frequency multipliers and linear phase detector. In this case it is desirable to have a linear phase detector because the phase will make very large excursions.



Direct Phase Recording - Suitable for getting $R_x(t)$ or $\sigma_y^2(t)$

Figure 33. DIRECT PHASE RECORDING

Another technique using two sources assumed to be identical is shown in Figure 34. The two sources are slightly offset in frequency to get a beat note for the counter measurement. This is called the "Beat Period Technique." The signals should be fed from the signal sources through the optional frequency multipliers into a mixer or phase detector followed by a counter. The period of beat note or its frequency is measured with an averaging time determined by the time interval counter. Applying the appropriate statistics to a number of such measurements gives $\sigma_y^2(\tau)$ directly.



Technique for measuring $\sigma_y(\tau)$. The two signal sources are offset in frequency to get a suitable beat note for the counter measurement.

Figure 34. TECHNIQUE FOR MEASURING $\sigma_y(\tau)$

Let us now look at a few of the actual characteristics of some of the sources. As mentioned previously, this is by no means an exhaustive list and there are probably great errors of omission.

Figure 35 shows σ versus τ for a number of sources. The 100-megahertz crystal oscillators have fairly low phase modulation in the high frequency range. Their σ is flat versus time for times much longer than about 10 milliseconds. This indicates that they suffer from flicker frequency noise. The higher the fundamental oscillation frequency, the worse the low frequency noise characteristic (or long-time performance) of an oscillator will be. Conversely, the higher the frequency of the oscillator, the better its

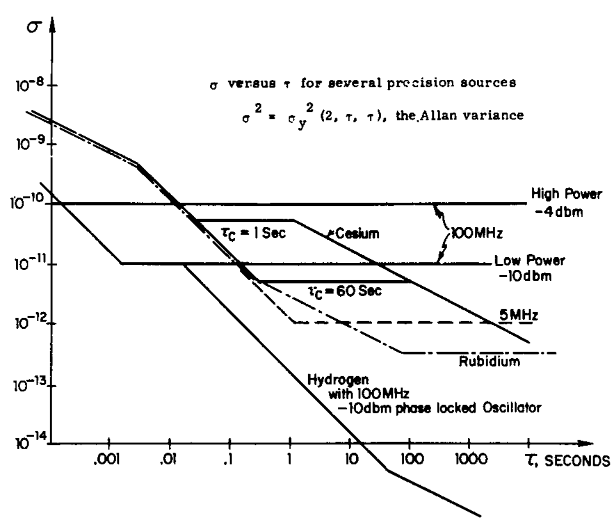


Figure 35. σ VERSUS τ FOR SEVERAL PRECISION SOURCES

high frequency noise characteristics (or its short-time performance) will be. Note the curves for cesium and rubidium. These are typical of the Hewlett-Packard rubidium and cesium standards. The short-time performance is dominated by the 5-megahertz crystal oscillator that is locked to the atomic resonance. The behavior for long times and intermediate times is determined by the atomic resonator. We have two time constants noted -- 1-second and

60-second time constants. One can get some control of the behavior by changing the loop time constant with which the oscillator is locked to the atomic resonator. We have shown rubidium flattening out into a flicker noise region, something below a part in 10^{12}.

The cesium curves on the right side of Figure 35 indicate the shot noise region. That is essentially the noise determined by the number of cesium atoms per unit time that one detects. Cesium performance has been greatly improved with the development of a new 16" tube that is about ten times better than that of the present 16" tube. The hydrogen maser is shown locked to a low-power, 100-megahertz oscillator. One can see where the loop crossover takes place.

The spectral density plots in Figure 36 are for some low-frequency oscillators. The solid curves are from the data sheet on the Hewlett-Packard 105 oscillator. The performance characteristics in the frequency range we measured were equivalent or better than our guarantee. The broken line represents the performance of a new low noise Ebauches oscillator which was recently reported by Brandenburger and Kartaschoff. The circled points are an experimental 10-megahertz Hewlett-Packard oscillator. The scale on the left is $S_x(\omega)$ in db and if one adds 216 db to the scale, it gives the performance

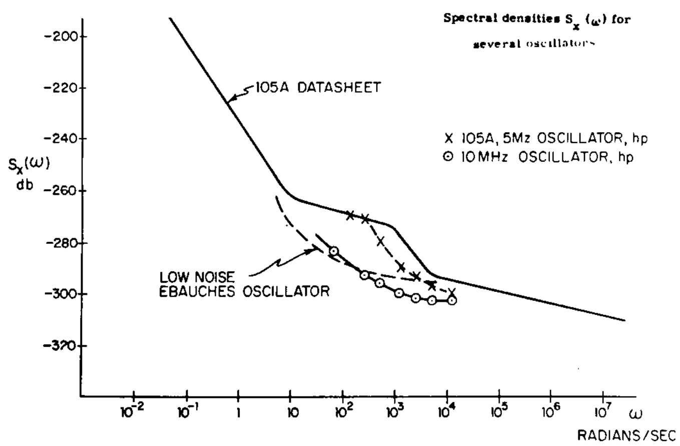


Figure 36. SPECTRAL DENSITIES $S_x(\omega)$ FOR SEVERAL OSCILLATORS

at X-band. Adding 206 db gives the performance at the S-band. Evidently one can scale the chart for any frequency that is of interest by adding the appropriate number of db to the ordinate.

Figure 37 gives spectral plots for other types of oscillators, including some microwave oscillators. The right-hand side of the figure represents high frequency behavior. Note that the two-cavity klystron, the UHF-cavity oscillator and X-band Gunn oscillator, and the UHF voltage controlled oscillator have very good performance for these short times. That bears out the earlier statement that if one wants very high spectral purity at high frequency for very short times, a high frequency oscillator should be used. Notice also

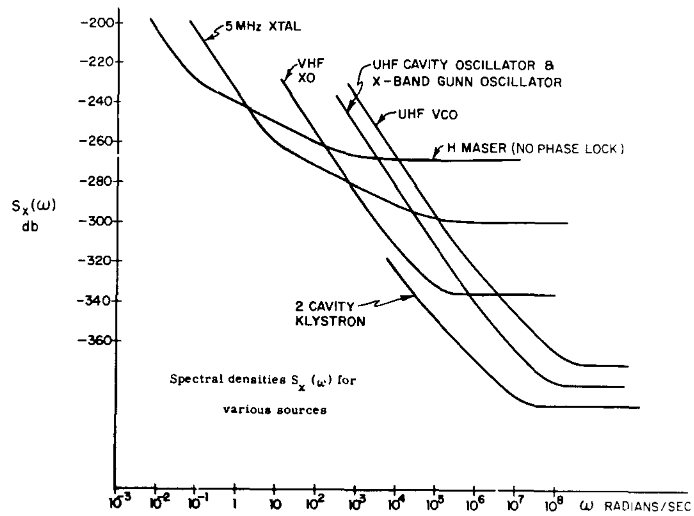


Figure 37. SPECTRAL DENSITIES FOR VARIOUS SOURCES

that these sources have spectral densities that go up quite steeply in the flicker noise region and the performance for low modulation frequencies with corresponding long averaging times is poor. We have plotted, for comparison, a 5-megahertz crystal which has rather intermediate performance high frequency wise, and considerably better performance low frequency wise. We have also plotted a hydrogen maser -- the purest oscillator. It has the best low frequency performance, but rather poor high frequency performance. This is because its power level is so low.

Figure 38 represents $S_x(\omega)$ for several precision sources. These sources are generally composite sources, such as two oscillators locked together to achieve the best results of both, a rubidium standard, a cesium standard, or a hydrogen maser with a phase lock. Again, if the output oscillator is a high frequency oscillator like the 100-megahertz oscillator, it has better high frequency performance than something that has a 5-megahertz oscillator tied to it. Over on the low frequency end, we see that hydrogen has the best performance.

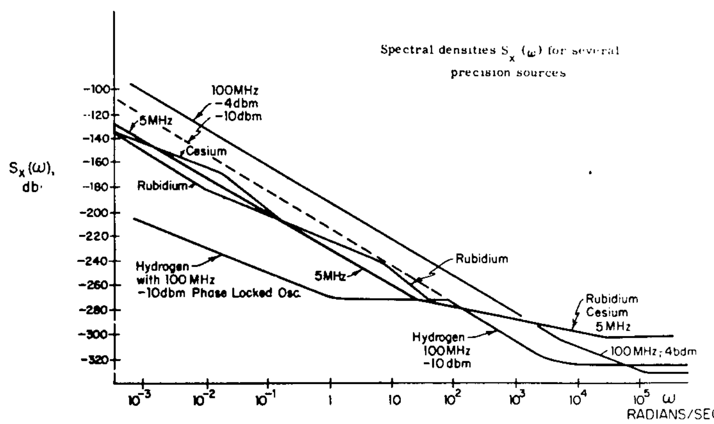


Figure 38. SPECTRAL DENSITIES FOR SEVERAL PRECISION SOURCES

Figure 39 shows spectral densities for two high-performance frequency synthesizers. One can see, here again, if we were to multiply these up to X-band, the 300 db point at the modulation frequency of 10^3 radians/second would be degraded to 84 db.

One word of caution on looking at frequency synthesizers: since they usually cover a very wide frequency range and the output frequency is very often achieved by mixing two high frequency signals, the spectral density does not improve as the frequency goes down. Therefore, $S_x(\omega)$ would get much, much worse at lower frequencies. The curves in Figure 39 are plotted for frequencies close to the top of the frequency range of each synthesizer, so they must be used with caution.

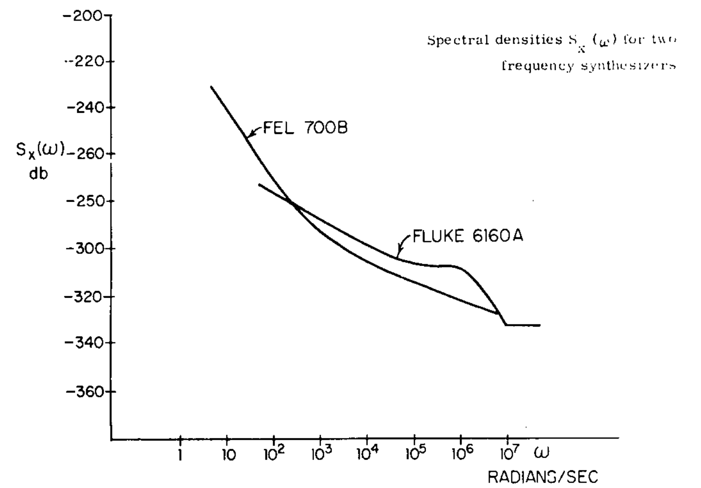


Figure 39. SPECTRAL DENSITIES FOR TWO FREQUENCY SYNTHESIZERS

We have tried to lead you very quickly through some of the basics that determine the characteristics of signal sources. We have also discussed some of the measuring techniques and some characteristics of various available precision sources.

DISCUSSION

DR. VESSOT: I think we can give you some fairly firm data on hydrogen masers beyond 100 seconds. The people at Jet Propulsion Labs have seen 7 parts in 10^{15} onward to 10^6 seconds; and recently -- which is extraordinary since it shows flicker behavior that is flat over about four decades starting at 100 seconds going to 1,000,000 seconds -- we have seen 3 to 5 parts in 10^{15} at 1,000 seconds, in recent measurements with small masers and slightly different cavities.

DR. CUTLER: That is a very respectable performance.