The Spectrum of Earth Strain From 10^{-8} to 10^{2} Hz

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We have measured the power spectrum of the earth strain fluctuations over 10 decades in frequency from 10^{-8} to 10^{2} Hz using data from three strain observatories. Although the strain meters were widely separated and of different design, they produced records whose power spectra are in close agreement with each other. We find that the composite power spectrum shows an approximate inverse square dependence on frequency over the entire band investigated.

In general, the motion of the earth's surface behaves like a stochastic process with the notable exception of earthquakes and earth tides [*Haubrich,* 1965]. In the discussion of such wide band stochastic phenomena the concept of power spectrum is an extremely useful one. Indeed the fruitful comparison of data obtained by different investigators demands some objective measure of the signal levels involved. We have chosen to present our results as power spectra normalized to unit bandwidth, that is, in units of $(\text{strain})^2$ per hertz. In this way, data recorded by using widely differing sampling rates and record lengths can be compared easily.

The ability of a strain meter to detect geophysically significant changes in the local strain tensor is limited by the seemingly random fluctuations in strain that are recorded by all instruments. Although in principle this noise could come from either the instrument or the earth, the data we present here show that instruments of widely differing design and operating in widely separated locations record strain fluctuations whose power spectra are remarkably alike. We are therefore led to hypothesize that all of these instruments are in fact recording the random fluctuations of the local strain tensor and that the data presented here represent a close approximation (or at least an upper bound) to the true strain fluctuations in the earth. This hypothesis is not unreasonable in that the power levels of strain noise should be approximately the same anywhere on the earth's surface.

The results presented here represent averages of the power spectra obtained over a period of several years and represent our best estimates of 'typical data.' Of course, it is recognized that there is a certain amount of variability of the earth strain spectrum in space and in time, particularly at the extreme ends of the spectral band investigated.

SOURCES OF THE DATA

The strain data used in this study come primarily from the laser strain meters operated by the University of California at San Diego and by the National Bureau of Standards in Boulder, Colorado. The instruments [*Berger and Lovberg,* 1970; *Levine and Hall,* 1972] are wide band interferometers that measure earth strain in terms of the wavelength of light. The data are recorded digitally at sample rates varying from 1000 samples/s to 10 samples/h, depending upon the frequency band of interest.

The University of California at San Diego strain meters are surface installations that have been operating for some 3 years at two sites, one 15 km from the Pacific coast and the other some 100 km inland. The National Bureau of Standards strain meter is located 60 m underground in an unused gold mine some 1600 km from the coast.

DATA ANALYSIS

The spectrum has been divided (somewhat arbitrarily) into four frequency bands: (1) high-frequency, $f > 0.5$ Hz; (2) seismic frequency, 10^{-4} Hz $< f < 0.5$ Hz, which covers the band in which most seismic energy is found; (3) tidal frequency 10^{-5} Hz $< f < 10^{-4}$ Hz, a fairly narrow band that contains the main semidiurnal and diurnal tidal lines; and (4) low-frequency, $f < 10^{-5}$ Hz (Table 1).

Basically, the analysis consists of band-passing the data to include only the frequencies in the band of interest and then performing a Fourier analysis of the result.

The band-pass filtering is done in two steps. Low-pass filtering is first used to set the upper bound to the spectral region of interest. Decimation in time can then be used to reduce the normally oversampled data to a sampling rate fast enough to satisfy the Nyquist criterion (at least two samples per cycle of the highest frequency present). High-pass filtering of the data is then usually required so that frequencies below the region of interest do not contaminate the spectrum through side band effects. This is especially important in the current work, since the strain power per unit bandwidth generally increases as we go to lower and lower frequencies.

The power spectrum is then computed by using conventional techniques. The spectrum is smoothed to increase the stability of the estimates at the expense of frequency resolution, and the resulting smoothed spectra are shown in Figure 1. Owing to the presence of the tidal lines in band 3, these data must be treated differently, as is discussed below.

Analysis in band 1. The analysis of the spectral region above 0.5 Hz is conveniently divided into the regions 0.5–5 Hz and 5–60 Hz.

In the first region we used a sampling rate of 10 samples/s. The length of the record was typically 65536 samples long, although records as long as 500,000 samples have been used.

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TABLE 1. Sample Intervals and Recording Media Used to Obtain the Data

Band	Frequencies	Sample Interval	Recording Medium
1	>0.5	1 to 100 ms	digital magnetic tape
2	10^{-4} to 0.5	1 s	digital magnetic tape
3	10^{-5} to 10^{-4}	6 min	digital paper tape
4	$<10^{-5}$	6 min	digital paper tape

Low-pass filtering of these data is not very important, since the power spectrum falls off quite rapidly above 5 Hz. However, high-pass filtering is essential, since there is appreciable power in the microseismic band (~0.16 Hz) and below.

The high frequencies in the upper part of the band made it impractical to sample rapidly enough to satisfy the Nyquist criterion. In addition, it is relatively simple to construct analog filters at these frequencies. These factors led us to construct a single side band superheterodyne receiver. In this system the signal is band-passed in a tunable narrow band filter ($Q \sim 100$) and is then heterodyned against a stable local oscillator. The difference frequency (~0.1 Hz) is digitized and recorded at a sampling rate of 1 sample/s. In this way it was possible to obtain long-term estimates of the spectrum.

In addition, extreme precautions were taken to minimize power line pickup, especially in recording near 60 Hz or any of its subharmonics. The residual power line pickup resulted in an apparent strain fluctuation at 60 Hz of $\Delta L/L = 8 \times 10^{-15}$. This produced a contribution to the power spectrum that was of the same order of magnitude ($\sim 10^{-27}(\Delta L/L)^2$/Hz) as the broad band noise in this frequency region.

At the highest frequencies in this band (>60 Hz) the system noise in the Poorman mine strain meter begins to dominate the earth noise, since frequencies in this region are not very far below the unity gain frequency of the servo. Thus although the measured power spectrum is essentially flat, there is no reason to suppose that the earth noise does not continue to decrease at a rate approximately given by $1/f^2$.

Analysis in band 2. This band is divided into two subbands, one extending from 10^{-4} to 2.5×10^{-2} Hz and the other from 2.5×10^{-2} to 0.5 Hz by band-passing the data through sharp digital filters. The upper band was analyzed during a period of abnormally high microseismic activity and during a period of normal activity. Most of the microseismic energy is concentrated in a narrow band centering on 0.16 Hz (6-s period), and the levels elsewhere in the band are essentially constant. Indeed the noise levels in the entire seismic band are virtually identical at the three laser strain meter sites. We have included in this band the spectral levels observed during a magnitude 7.5 earthquake near Valparaiso, Chile, July 9, 1971 [*Berger,* 1972]. These levels are typical for events of this magnitude, and thus we record the normal modes with a signal to noise ratio varying from about 20 dB at 2.8×10^{-4} Hz (1 cph) to 45 dB at 8.3×10^{-3} Hz (30 cph). For comparison, *Benioff et al.* [1961] reported a signal to noise ratio varying from 5 dB at 2.8×10^{-4} Hz to 20 dB at 8.3×10^{-3} Hz, using data obtained following the magnitude 8.5 earthquake in Chile in May 1960.

Analysis of band 3. For the spectral analysis of the tidal band the tides were calculated by using the convolution techniques of *Munk and Cartwright* [1966]. The earth tides are computed as the convolution of the geopotential $V(\theta, \phi; t)$ with a set of weights $W(s)$. Thus

$$T = \sum_s W(s) V(t - \tau_s)$$

where T is tide, and one chooses the $W(s)$ so as to minimize in the least squares sense the differences between the observed series and T. In the computation, V is expanded as a sum of spherical harmonics

$$V(\theta, \lambda, t) = g \sum_{n=0}^{N} \sum_{m=0}^{n} [a_n{}^m(t) V_n{}^m(\theta, \lambda) + b_n{}^m(t) V_n{}^m(\theta, \lambda)]$$

and then the tide is given by

$$T = \sum_{m,n} \sum_s u_n{}^m(s) a_n{}^m(t - \tau_s) + v_n{}^m(s) b_n{}^m(t - \tau_s)$$

The best fit T is then subtracted point by point from the observed data, and the residuals are spectrally analyzed. The advantage of this method over notch filtering of the original data to remove energy at the tidal frequencies is simply that it allows one to examine the data in the neighborhood of the tidal spectral peaks without being plagued by the side band energy from those peaks. Further, it allows one to make a signal to noise estimate at the tidal frequencies themselves.

Analysis of band 4. Figure 2 shows the data gathered to date from the three-component strain meter at Piñon Flat and the single-component meter at the Poorman mine. These data reveal the lowest secular strain rates yet reported to our knowledge. The secular strain rate may be expected to vary from site to site according to the local tectonic activity, but from the limited data taken to date, there does not seem to be an order of magnitude difference between the Poorman site and the Piñon Flat site, which is located in an area of high tectonic activity. An examination of Figure 2 also shows that the secular strain rate depends quite strongly on the orientation of a strain meter as well as on its location, since the three instruments at Piñon Flat show quite different strain rates.

SUMMARY

It should be noted that the body of data collected at these sites over a period of some 3 years strongly disputes results reported elsewhere [*Wideman and Major,* 1967; *Romig,* 1970; *Major et al.,* 1972; *Butler,* 1972; *Butler and Brown,* 1972; *Stewart et al.,* 1973; *Bufe and Nason,* 1973]. We have consistently observed secular strain rates less than 2×10^{-7}/yr, and these rates are based on data taken over the period indicated, not extrapolations of a few weeks or months run. We have never observed the large 'strain episodes' widely reported elsewhere [*Major et al.,* 1972; *Butler,* 1972; *Butler and Brown,* 1972; *Bufe and Nason,* 1973], nor have we observed anomalous strain steps such as those reported by *Wideman and Major* [1967] and by *Press* [1965]. Our experience, which includes long-term observations and observations of many large seismic events, both near and far, natural and nuclear, does not give us any reason to doubt the predictions of classical elastic theory and seismology. The earth, as we have observed it, does not exhibit strain rates in excess of those produced by tidal potentials (2×10^{-8} amplitude with a 12-hour period), and in fact for longer periods the rates have always been less than 10^{-8}/day. The exception, of course, is during seismic disturbances, but in these observations, which include events as large as magnitude 8.0, no anomalous strain steps have been observed.

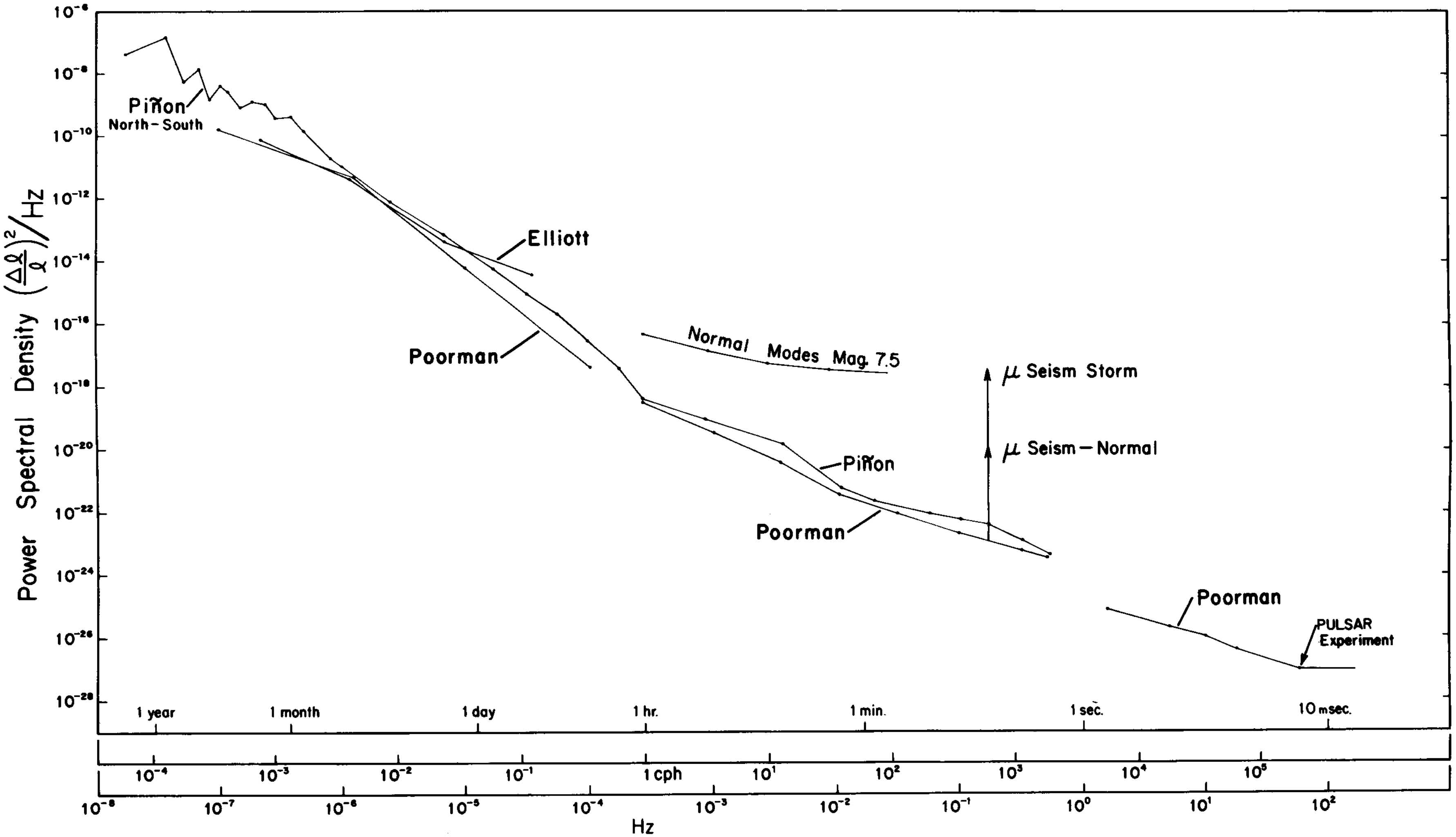

Fig. 1. Strain noise spectrum obtained by using laser strain meters. The power in the normal mode band following a magnitude 7.5 earthquake and typical microseismic powers are shown for comparison.

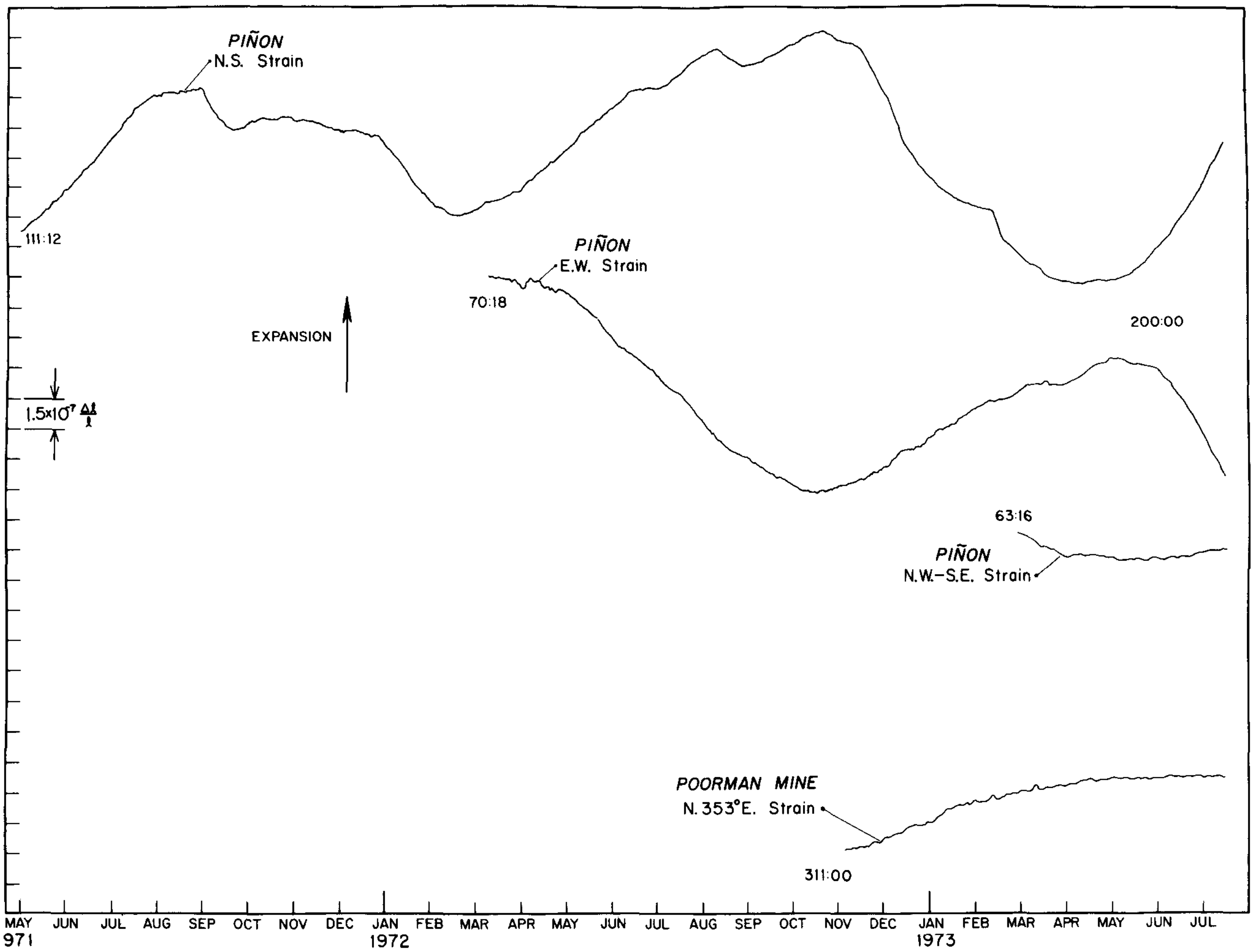

Fig. 2. The secular strain recorded at the three sites. Each vertical division corresponds to a strain change of 1.5×10^{-7}

It should be pointed out, moreover, that the Piñon Flat observatory is situated in an active tectonic area, being some 15 km from the San Jacinto fault (historically the most active fault in southern California) and 25 km from the main trace of the San Andreas fault. The Poorman mine is situated in the same general area, where both large secular strain rates [*Romig*, 1970] and anomalous strain steps [*Wideman and Major*, 1967] have been reported.

Data taken on a wire strain meter at the Queensbury tunnel in Yorkshire, England [*Bilham et al.*, 1972], exhibit a noise level some 20 dB higher than the levels reported herein in the frequency band from approximately 10^{-7} to 10^{-4} Hz. We suspect that such a high noise level is due to either instrumental noise or some anomalous local conditions.

In general, the strain noise that has been observed at Piñon Flat and Poorman mine constitutes the quietest observations reported to date to our knowledge. The similarity of the power levels at these three sites leads us to the conclusion that these levels represent an upper limit to the strain noise and that this limit does not vary by more than 10 dB over a frequency band from 10^{-8} to 1 Hz between these three sites.

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