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A NEW MODEL OF 1/F NOISE IN BAW QUARTZ RESONATORS

F. L. Walls*, P. H. Handel#, R. Besson@, and J.-J. Gagnepain+

* Time and Frequency Division, National Institute of Standards and Technology, Boulder Colorado, USA; #Dept of Physics, University of Missouri, St Louis, Missouri, USA;@ Ecole Nationale Superieure de Mecanque et des Microtechniques, Besancon, France; +Laboratoire de Physique et Metrologie des Oscillateurs du CNRS, Besancon, France

Abstract

This paper presents a new model for predicting the 1/f (flicker) frequency noise in quartz resonators as a function of the unloaded resonator quality factor Q and volume under the electrodes for bulk acoustic wave (BAW) resonators. The functional form of this model originates from a quantum 1/f theory for scattering of phonons from the primary oscillator mode. Using this new model, we are able to match the 1/f frequency noise observed in the best quartz-controlled oscillators and resonators. Quite unexpectedly, this model indicates that the amplitude of 1/f frequency noise might be improved by making resonators with smaller electrodes. BVA resonators show approximately a factor of 3 improvement in 1/f frequency noise ($S_y(f)$ over electroded resonators with the same unloaded Q-factor and electrode volume.

Introduction

The amplitude of 1/f or flicker frequency noise in quartz resonators is a very important parameter of oscillators used in a wide range of applications. Although work has been done in this area for more than 20 years, it is difficult to find data where the operating conditions and all of the resonator parameters are well known. In addition, some workers report $S_\phi(1\text{ Hz})$, while others report $\sigma_y(1$ s or 100 s). For a given resonator Q, many different levels of frequency stability have been reported. Some of the variation may be due to random walk and drift which were not removed from the data and thereby bias the estimate the amplitude of 1/f frequency noise. In some cases the electronics may limit the noise. What is needed for accurate modeling is the amplitude of the 1/f frequency noise in the resonator, independent of electronic noise. The variation in $S_y(f)$ for the same Q may also indicate that some other variable significantly affects the 1/f frequency noise as the acoustic losses become small. Given these problems and uncertainties in the available data, it is difficult for any model to fit all the data. The 1/f contribution to frequency stability is best obtained by observing the stability over a range of measurement times in the time domain or a range of frequencies in the frequency domain. From the extended data we can fit a flicker-of-frequency model to the data that exclude the biases due to random walk FM and drift present in many resonators and oscillators (see Fig. 1).

Gagnepain [1] was one of the first to systematically study 1/f noise as a function of geometry, temperature, and Q. He found that the 1/f contribution to the spectral density of fractional frequency fluctuations, $S_y(f)$, varies as $1/Q^4$ for resonators between approximately 1 and 25 MHz. As the temperature of a resonator changes, Q changes. This makes it possible to exclude the effect of many other factors. Additional work by Parker, however, showed that the data from both BAW and Surface Acoustic Wave (SAW) devices could be roughly fit to the same model if one assumes a $1/Q^4$ dependence for $S_\phi(f)$ instead of $S_y(f)$ [2]. The fit is not particularly good, for the best resonators (see Fig. 2).

From a theoretical viewpoint, work on the general problem of 1/f fluctuations in systems has long pointed toward a $1/Q^4$ dependence for $S_y(f)$ [1,3,4,5]. Work on many systems other than quartz has yielded very good quantitative agreement between theory and experimental data for 1/f quantum noise [5]. The $1/Q^4$ dependence of the 1/f contribution to $S_y(f)$ is, however, in apparent conflict with the 1/f noise of the best quartz resonators over a wide frequency range where the dependence is between $1/Q^2$ and $1/Q^5$.

This paper refines the previous theoretical work on 1/f noise in quartz to suggest a better framework for predicting the amplitude of 1/f noise in quartz resonators over a wide range of frequencies and Q [1,3,4,5].

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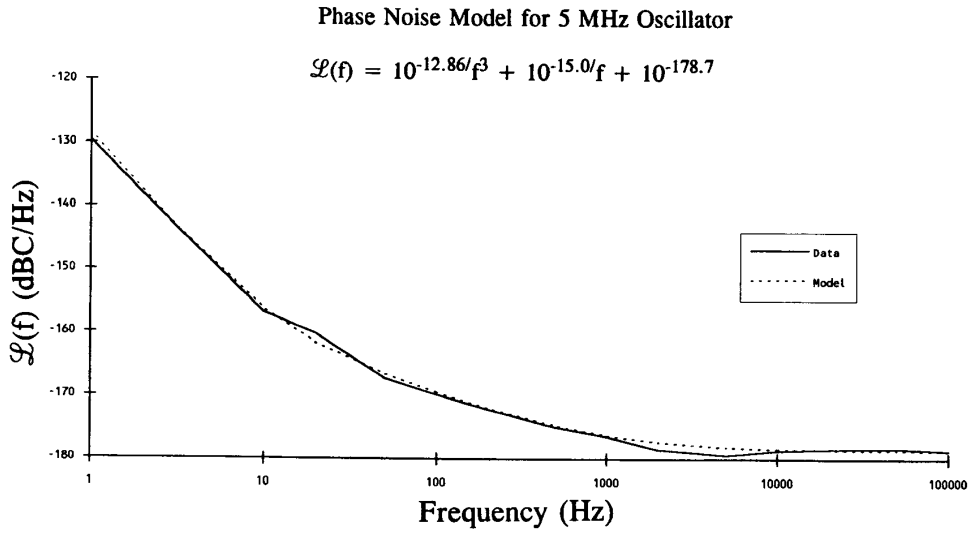

Figure 1. Phase noise of a 5 MHz quart z oscillator as a function of Fourier frequency. The coefficient of the f^{-3} component corresponds to 1/f (flicker) frequency noise.

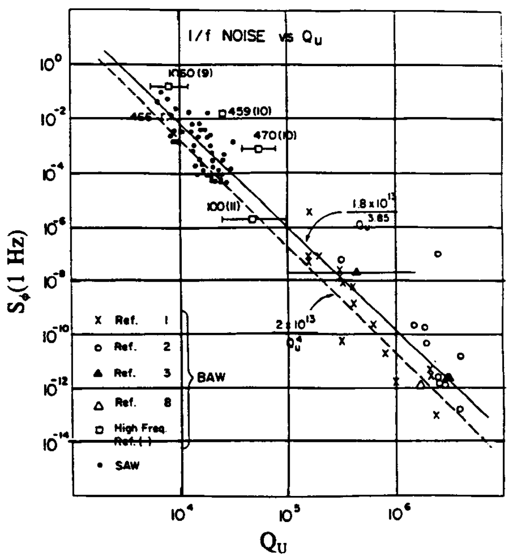

Figure 2. 1/f noise level at 1 Hz ($S_\phi(f)$ of quartz acoustic resonators as a function of unloaded Q [2]. Reference numbers are from [2].

Condensed Theory of 1/f Noise in Quartz Resonators

According to the general quantum 1/f formulation [5], $\Gamma^{-2}S_\Gamma(f) = 2\alpha A/f$ with $\alpha = e^2/\hbar c = 1/137$ and $A = 2(\Delta J/ec)^2/3\pi$ is the quantum 1/f effect in any physical process rate Γ. Setting $J = dP/dt = \dot{P}$, where P is the vector of the dipole moment of the quartz crystal, we obtain for the fluctuations in the rate Γ of phonon removal from the main resonator oscillation mode the spectral density

$$\Gamma^{-2}S_\Gamma(f) = 4\alpha(\Delta\dot{P})^2/(3\pi e^2c^2), \qquad (1)$$

where $(\Delta\dot{P})^2$ is the square of the dipole moment rate change associated with the process causing the removal of a phonon from the main oscillator mode. These fluctuations in the rate Γ are obtained by scattering on a phonon from any other mode of average frequency $<\omega>$, or through a two-phonon process at a crystal defect or impurity, involving a phonon of average frequency, $<\omega'>$. To calculate it, we write the energy W of the interacting resonator mode $<\omega>$ in the form

$$W = n\hbar<\omega> = 2(Nm/2)(dx/dt)^2 =$$

$$(Nm/e^2)(edx/dt)^2 = (m/Ne^2)\epsilon^2(\dot{P})^2. \qquad (2)$$

The factor 2 includes the potential energy contribution. Here m is the reduced mass of the elementary oscillating dipoles, θ their charge, ϵ a polarization constant, and N their number in the quartz crystal between the electrodes. Applying a variation $\Delta n = 1$, we get

$$\Delta n/n = 2|\Delta\dot{P}|/|\dot{P}|, \; or \; \Delta\dot{P} = \dot{P}/2n. \tag{3}$$

Solving Eq. (2) for $\dot{P}$ and substituting, we obtain

$$|\Delta\dot{P}| = (N\hbar<\omega>/n)^{1/2}(e/2\epsilon). \tag{4}$$

Substituting $\Delta\dot{P}$ into Eq. (1), we get

$$\Gamma^{-2}S_{\Gamma}(f) = N\alpha\hbar<\omega>/(3n\pi mc^2 f\epsilon^2) = \Lambda/f. \tag{5}$$

This result is applicable to the fluctuations in the loss rate Γ of the quartz. In the presence of a damping term Γ, the frequency of a harmonic oscillator of unperturbed angular frequency ω_0 is

$$\omega = \sqrt{\omega_o^2 - 2\Gamma^2} \; where \; Q = \frac{\omega}{2\Gamma}. \tag{6}$$

The fractional variation of ω_0 due to fractional changes in Γ is

$$\frac{\Delta\omega}{\omega_0} = \frac{2\Gamma^2}{\omega_0\sqrt{\omega_0^2-2\Gamma^2}}\frac{\Delta\Gamma}{\Gamma} \sim \frac{1}{2Q^2}\frac{\Delta\Gamma}{\Gamma}. \tag{7}$$

The spectral density of frequency fluctuations of the quartz resonator is [4]

$$\omega^{-2}S_{\omega}(f) = (1/4Q^4)(\Lambda/f) = N\alpha\hbar<\omega>/(12n\pi mc^2 f\epsilon^2 Q^4), \tag{8}$$

where Q is the unloaded quality factor of the single-mode quartz resonator considered, and $<\omega>$ is not the circular frequency ω_o of the main resonator mode, but rather the nearly constant frequency of the average interacting phonon, considering both three-phonon and two-phonon processes. The corresponding $\Delta\dot{P}$ in the main resonator mode also has to be included in principle, but is negligible because of the very large number of phonons present in the main resonator mode.

Eq. (7) can be written in the form

$$S_y(f) = \beta V/(fQ^4), \tag{9}$$

where, with an intermediary value $<\omega> \sim 2 \times 10^{11}$/s, $n = kT/\hbar<\omega>$, T = 300 K and $kT = 4 \times 10^{14}$, $\beta \approx (N/V)\alpha\hbar<\omega>/(12n\pi\epsilon^2 mc^2) = 10^{22}(1/137)(10^{-27}10^8)^2/(12kT\pi(2 \times 10^{-24}) 9\times10^{20}) \approx 1$.

The form of Eq. (8) shows that the amplitude of 1/f frequency noise depends not only on Q^4 as previously proposed but also on the volume between the electrodes. This model qualitatively fits the data of Gagnepain et al. [1,3] and the recent data of El Habti and Bastien [6], where Q was varied with temperature in the same resonator (but not frequency or volume).

The model also provides the basis for predicting how to improve the 1/f noise in resonators beyond just improving the Q, which has been known for many years. Since the amplitude of 1/f noise depends on active volume, we use the lowest overtone and smallest electrode diameter consistent with other circuit parameters.

Experimental Measurements and Analysis of 1/f Noise in Quartz Resonators

The 1/f frequency noise in quartz resonators has been measured using phase bridges and in complete oscillators [1,2,6-12]. Unfortunately much of the data in the literature are unusable for modeling because the unloaded Q is unknown. (Our case is even more restrictive because we also need to know the electrode size). The advantages of the phase bridge approach are that the unloaded Q can be easily measured and the noise in the measurement electronics can be evaluated independent of the resonator. If resonator pairs are used, driving source noise can generally be neglected and the pair can operate at virtually any frequency [9]. The oscillator approach makes it possible to compare many different resonators one at a time. The noise of individual oscillators can be derived by measuring the phase noise between 3 oscillators [14].

Figure 3 taken from [2] is one of several studies showing that the amplitude of 1/f frequency noise is virtually independent of the loaded Q. This is in complete agreement with the theoretical model. In practical oscillators dependence on loaded Q occurs only when the phase noise of the sustaining electronics contributes to the overall noise level.

We have analyzed 1/f frequency noise as a function of unloaded Q, volume under the electrodes, and frequency. For a given resonator geometry and manufacturer, we have taken the best values reported of $S_y(f)$ to remove the effects of poor crystals or electronics. In Fig. 4 we have taken all of the precise data available with unloaded Q, electrode volume, and frequency stability and plotted it

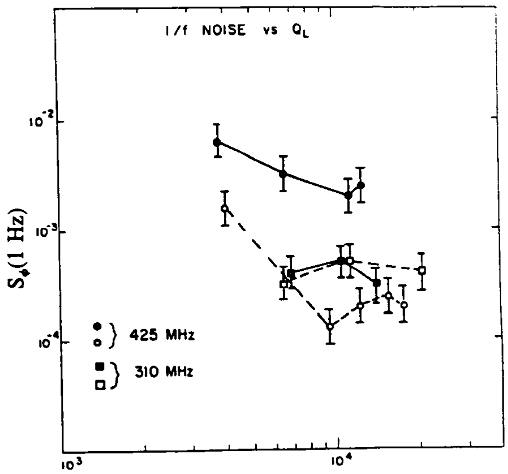

Figure 3. 1/f noise level at 1 Hz ($S_\phi(f)$ of 4 SA' resonators as a function of loaded Q [2].

according to the three models. Except for the 2.5 MHz resonator where $Q\nu_o$ = 0.95 x 10^{13}, the $Q\nu_o$ product for all resonators plotted is near 1.2 x 10^{13} (this is close to the material limit for AT and SC cut resonators). The curve labeled K_y shows the fit of the data to the model [1]

$$S_y(f) = K_y/f\ (3 \times 10^{-5}/Q^4). \quad (10)$$

K_y varies about a factor of 500 for Q between 10^5 and 3.8 x 10^6 (with resonator frequencies between 2.5 and 100 MHz). The curve labeled K_ϕ shows the fit of the same resonator data to the model [2]

$$S_\phi(f) = K_\phi/f^3\ (3 \times 10^{10}/Q^4). \quad (11)$$

K_ϕ varies about a factor of 10 for the same range in Q. Curves β_e and β_b show the fit of the same resonator data to the model

$$S_y(f) = \beta/f\ (Vol/Q^4), \quad (12)$$

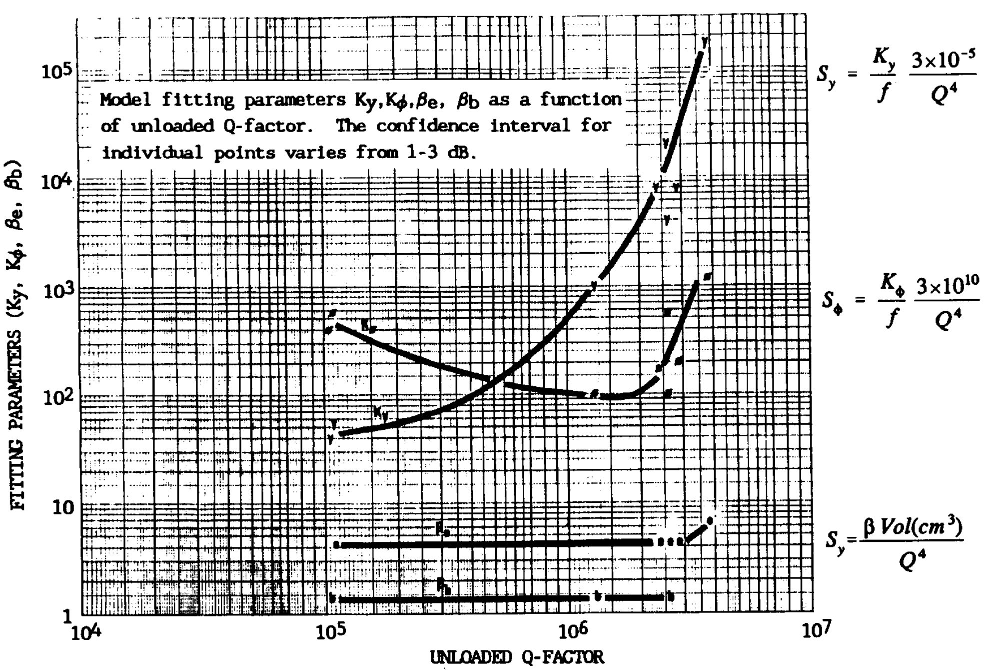

Figure 4. 1/f frequency noise for 9 resonators plotted according to the three different models using the fitting parameters Ky, Kϕ, βe, and βb as a function of the unloaded Q-factor. The confidence interval for the individual points varies from 1 to 3 dB. Only resonators with the lowest level of 1/f noise are reported for each type [8, 11, 13].

where $ß_e$ is for SC and AT resonators with electrodes plated on the resonator and $ß_b$ is for BVA-style AT and SC resonators [7]. Volume between the electrodes (in cm^3) is used to approximate the volume of quartz contributing to the scattering of power from the primary resonator mode. The ß factors are remarkably constant for Q from 10^5 to 3.8×10^6.

Figure 5 shows the dependence of β on Q for 3 different types of resonators as measured by Norton [8]. The wide variation in $ß_e$ and β_b for the same type of resonator and Q indicates that acoustic loss is not the only mechanism contributing to the noise level. The data for this graph were taken from measurements of $\sigma_y(100\ s)$ in similar oscillators. The difference between the various resonators of a given type can only be due to differences in the resonators. The reference oscillator for these measurements was a hydrogen maser. The data of Fig. 4 used only the smallest value of β for each resonator type.

Discussion

The 1/f frequency noise of the most stable resonators is in excellent agreement with the functional form of Eq. (8). The agreement between the theoretical and the experimentally measured values of β to the same order of magnitude is remarkable considering the rough approximation of both the average phonon frequency $<\omega>$ and the number N of oscillating dipoles (through volume) under the electrodes contributing to the scattering processes. The data suggest that the correct volume is that between the electrodes and not the volume of the oscillating mode. Further investigation of this question would help in interpreting the theory. One method would be to measure the 1/f frequency noise of a high performance resonator as a function of the diameter of the electrodes, since many other variables would be held constant. We are not surprised that $ß_e$ and $ß_b$ are different for the two types of resonators, since energy trapping and electrode stress are considerably different.

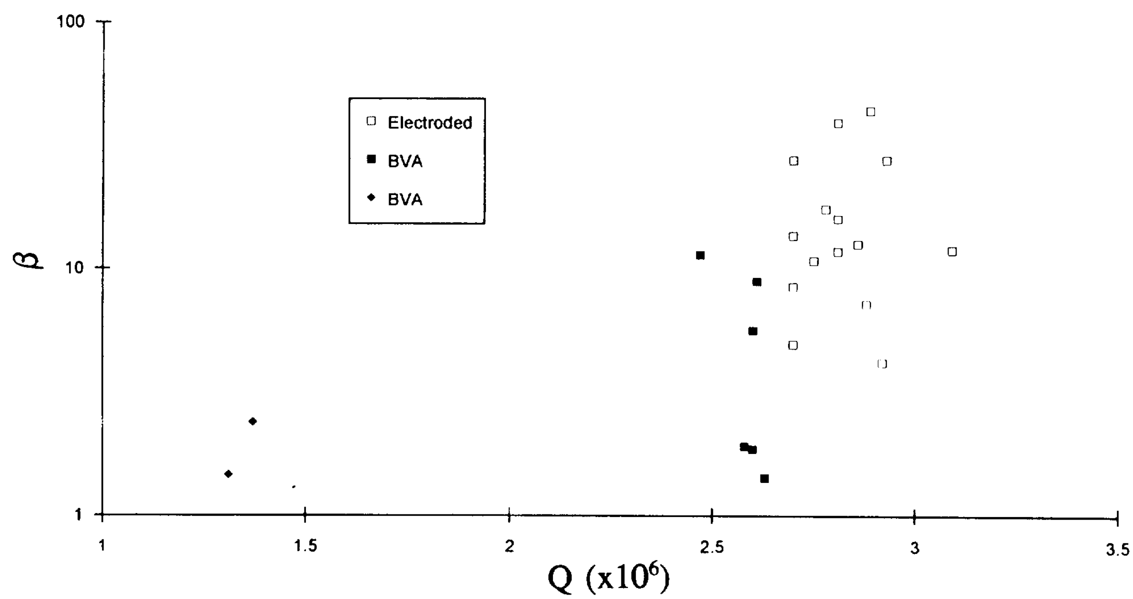

Figure 5. Fitting parameter β as a function of unloaded Q-factor for three types of resonators. The resonators in each group were matched in all known electrical parameters except Q-factor and 1/f noise [15].

Figure 5 shows that there is at least one other noise process besides acoustic loss that affects the 1/f frequency noise in some resonators. The magnitude of β in Fig. 5 is larger for the electroded resonators than for the BVA resonators. This suggests that the extra noise source is associated with the electrode-quartz interface. The fractional variation in β is roughly comparable for the two types of resonators. Much more data on resonators from the same material with the same surface preparation are needed to make any further conclusions.

Although we have analyzed only the data for a few resonators, the consistency of β_e and β_b over a factor of 40 in Q and resonator frequency and the general agreement for the magnitude of β between theory and experiment give us confidence that this new model can be used to predict the best performance of different resonator geometries and as a basis to analyze other 1/f noise processes in quartz resonators.

This new volume model predicts that a resonator having smaller electrodes would have lower 1/f frequency noise than another with the same frequency and Q but with larger-diameter electrodes. The decrease in electrode area would increase the impedance and degrade the wide-band noise somewhat. For most resonators the wideband noise is dominated by the electronics and not the resonator. The increase in series resistance, obtained by decreasing the electrode area by a factor of 4, would probably be tolerable from the standpoint of wideband noise, but might require a change in loop gain.

BT-cut resonators are potentially useful in that they offer a $Q\nu_o$ product approximately 3 times higher than that of AT- and SC-resonators. BT-cuts are nearly as sensitive to temperature transients as AT cuts. Therefore to achieve parts in 10^{-14} frequency stability with BT cuts would require temperature stabilities of order 10^{-9} K/s or 100 times better than is required for SC-cut resonators [14]. The phase noise requirements of the sustaining electronics would be less than for other resonators due to the increase in Q.

We conclude based on these early observations that the amplitude of 1/f frequency noise in quartz may yet be improved to 10^{-14} by applying one or more of the following techniques: reducing the electrode area, using lower-overtone resonators, using BVA type resonators, going to lower frequencies, and using BT-cut resonators. Acceleration-induced effects, however, become more dominant as the stability improves [16].

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