Laser Stabilization to a Single Ion.

J. C. Bergquist, W. M. Itano and D. J. Wineland

Time and Frequency Division, National Institute of Standards and Technology
Boulder, CO 80303

I. – Spectrally narrow Optical Oscillators.

I.1. – Frequency references.

An unperturbed optical resonance in an atom or molecule provides a good absolute frequency reference, but practically there are limitations. The signal-to-noise ratio is limited by the number of atoms in the interrogation region and by saturation of the resonance. If the probe laser is spectrally broad, then the measured atomic line is broadened, which degrades the stability. Furthermore, atoms recognize interactions and collisions with neighboring atoms, usually with shifts to the internal energy level structure. The motion of the atoms also produces Doppler shifts and broadening. A single laser-cooled atom removes many of these problems but only with a severe penalty in the signal-to-noise ratio [1]. Even so, by detecting each transition in the single atom, it should be possible to stabilize the frequency of a laser oscillating in the visible to better than $10^{-15}\,\tau^{-1/2}$ with an accuracy approaching 10^{-18} [2], if the laser were sufficiently stable (or spectrally narrow) for times that are comparable to the interrogation time of the transition in the atom.

A two-step approach might then be appropriate; spectrally narrow the laser by some scheme that offers good short- to medium-term stability (*e.g.*, 1 ms to 10 s), then stabilize the frequency of the laser for longer times to a narrow resonance in a single atom. The reference for the short- to medium-term stabilization of the laser not only needs good stability in this time frame, but also low phase and frequency noise. If a Fabry-Perot cavity is used as the frequency discriminator, then the response of the frequency discriminator can be nearly linear as a function of power and the signal-to-noise ratio can be high [3]. Frequency shifts and fluctuations arise due to thermal distortion in the mirror coatings caused by absorption of light in a small volume in the dielectric stack and due to photochemical processes at the surfaces of the coatings. Also, practical limits are reached at powers that saturate the detector or at powers (circulating in the cavity) that cause radiation pressure noise, but the signal-to-noise ratio from a cavity can be many orders of magnitude larger than that obtained with atoms. Although there are other types of optical frequency references, the most widely used is the Fabry-Perot interferometer, principally because the frequency excursions to error-voltage can be extremely large in a high-finesse cavity. We will spend time in the next few sections discussing some of the details and limits of a suitably stable reference cavity, and then turn our attention to the single atom.

I.2. – Reference cavity limitations.

We can begin with a brief look at the demands that a spectral purity of 1 Hz places on the physical stability of the reference cavity. If the cavity has a length of 30 cm and the optical wavelength is 500 nm, then the optical-path length between the mirrors must not change by more than 10^{-15} m, the approximate size of the nucleus of any of the constituent atoms in the dielectric coatings. Researchers who study parity-nonconserving interactions in atomic systems sometimes use the analogy that a human hair added to the radius of the Earth is equivalent to the distortion in an atom caused by the parity-nonconserving part of the Hamiltonian. By the same taken, if the spacer for an optical cavity were the Earth, a human hair added to the diameter would cause a frequency shift of about 300 Hz! In the first part of this section, we investigate some of the fundamental limits to the attainment of an average spacing between two mirrors that is stable to better than $1 \cdot 10^{-15}$ and to the achievement of a laser that is spectrally narrowed to better than 1 Hz. We will also address some of the limitations imposed by various environmental factors. We will see that, although fundamental limits come from quantum mechanics and thermodynamics, important practical limitations come from mechanics and gravitational coupling to the noisy terrestrial environment. In particular, at low Fourier frequencies (< 100 Hz), seismic noise and pendulum motion dominate the noise budget.

At the quantum level, the measurement of the length of the cavity to which the laser is locked brings about the inevitable competition between the measurement precision and the perturbation of the measurement to the system. The measurement precision can be improved by a factor of $1/\sqrt{N}$ by increasing the number N of (signal) photons in the measurement interval, whereas the shot noise of the radiation pressure on the mirrors increases as $\sqrt{N}$. The optimum flux of signal photons, or input power (assuming 100% of the light is coupled into the cavity on resonance and that the detection efficiency is unity), is given when both effects are equal in magnitude [4]. For the laser interferometers used in gravity wave detection, the mirrors are suspended as pendulums and the optimum power is calculated for frequencies $\omega/2\pi$ well above the resonance frequency $\omega_0/2\pi$ of the pendulum support. As a function of frequency, this limit length uncertainty is given by [5]

(1) $$\Delta x \approx (4\hbar \Delta f/m)^{1/2}/\omega ,$$

where m is the mass of suspended mirrors and Δf is the measurement bandwidth. Interestingly, this is the same measurement precision allowed by the standard quantum limit [4-7]. In our case the resonator is composed of a single bar, or spacer, to which the mirrors are rigidly attached. The resonator is then suspended, often by small-diameter wires, inside a temperature-regulated, evacuated housing. The resonator bar can also be treated as a harmonic system, but now the frequency of the lowest mechanical resonance frequency $\omega_0/2\pi$ is typically greater than a few kHz. The noise spectrum of length fluctuations that is of particular importance to us is at Fourier frequencies that are below the lowest resonance frequency of the bar. At frequencies lower than $\omega_0/2\pi$ and for optimum power, the measurement precision limit is independent of Fourier frequency

(2) $$\Delta x \approx (4\hbar \Delta f/m)^{1/2}/\omega_0 .$$

For $\omega_0/2\pi = 10\,\text{kHz}$, a cavity finesse of 100000 at $\lambda = 500\,\text{nm}$ and a near optimum input power of about 3 W, this limit corresponds to a fractional length uncertainty of less than $(5 \cdot 10^{-22}\ m/\sqrt{\text{Hz}})(\Delta f)^{1/2}$ for a typical bar mass of 4 kg. Thus, with as little as 100 μW incident on the cavity, the quantum fluctuations in the radiation pressure acting on the mirrors are negligible, yet there is sufficient signal-to-noise ratio that the frequency of a laser can be made to track the resonance of the cavity to well below 1 Hz [8] (if only limited by the shot noise of the detected signal).

The technical fluctuations in the laser light that is coupled into the cavity must also be considered. If the finesse is 100 000 and the corresponding power enhancement as high as 30 000, then 100 μW of input power translates into 3 W of *circulating* power when the laser is resonant with the cavity. This in turn gives a force W on each mirror of about $2 \cdot 10^{-8}$ N. If the mirrors are treated as clamped disks of thickness t, the deflection of each mirror, δ, is given by [9]

(3) $$\delta = 3Wr^2(1-\rho^2)/4\pi E t^3 ,$$

where the light force is assumed to act uniformly over a concentric area much less than the area of the mirror. The radius r of the mirror is measured from the center to the clamped edge, ρ is Poisson's ratio and E is Young's modulus of elasticity. For a mirror substrate with a material composition similar to fused silica, ρ is about 0.17 and E is about $7 \cdot 10^{10}\ \text{N/m}^2$. If t is 5 mm and r is 10 mm, then the deflection at the center of the mirror is about $1.7 \cdot 10^{-16}$ m for a radiation mode size (w_0) of 200 μm. For a cavity that is 30 cm long, the corresponding fractional length change ($2\delta/L$) is about $1 \cdot 10^{-15}$, or about 0.5 Hz. The dimensions and physical properties of the resonator used in this example are typical. Cavities have been constructed that have been shorter, that have used thinner mirrors, and that have coupled in more light; all of these conspire to degrade the frequency stability through fluctuations of the intracavity light intensity. In our example, if 1 mW of power is used to stabilize the laser to the cavity, then the amplitude of the circulating light must be constant to about 10% to achieve a laser stability better than 1 Hz. The radiation pressure also works to stretch the entire bar. The strain, or fractional length change, induced by a force acting normal to the end of a cylindrical bar of cross-sectional area A is given by [9]

$$\Delta L/L = F/AE\,. \tag{4}$$

The fractional length change for 3 W of circulating power is about $2.5 \cdot 10^{-17}$. This is smaller than the elastic distortion of our typical mirror, and even power fluctuations as large as 10% would cause only millihertz frequency fluctuations through this term. The clear indication is that the fluctuations in the circulating power must be controlled if the laser frequency is to be stabilized to much better than 1 Hz. Cavity power fluctuations are caused by intensity fluctuations of the laser light external to the cavity and by variations in the amount of light that is coupled into the cavity. The latter fluctuations are caused, for example, by any motion of the resonator with respect to the input beam.

Another problem is the local heating of the dielectric mirror coating from the light circulating in the cavity. With high-finesse low-loss mirror coatings, one might assume that this would not be an important concern. However, as we have seen, even with as little as 100 μW of power coupled into the cavity that has a finesse of 100 000, the circulating power inside the cavity can exceed 3 W. If the absorption losses are as little as a few p.p.m., $(5 \div 10)$ μW are absorbed in the coating. Most of this power is absorbed in the first few layers of the dielectric stack where the light intensity is the highest. When the light amplitude fluctuates, there is a transient response followed by relaxation to a steady-state condition. For a radiation mode size of 200 μm, we have measured a 2 Hz/μW shift of the cavity resonance to higher frequencies with increased power. The magnitude has been measured to be as much as 20 Hz per μW change in the power of the input coupled light [10]. Both the thermal distortion of the mirror and the light pressure problem could be reduced by adjusting the mirror radii and the cavity length so that the mode size is larger at the mirrors (for example, by using a near-spherical resonator).

The thermal noise in the bar or spacer must also be considered. We can think of this as the weak coupling of the fundamental mode of the spacer to its environment, *e.g.*, the residual background gas, the wire suspension, radiation from the walls of the vacuum vessel, etc. If the bar is thermalized to this background or thermal bath at some physical temperature, T, then the weak coupling to the thermal bath causes the mode's amplitude to execute a random walk in the domain $|\Delta x| \leq \chi_{\text{r.m.s.}}$. $\chi_{\text{r.m.s.}}$ is the average deviation of one end of the spacer from the equilibrium position assuming the other end is fixed. The magnitude of this deviation can be found by equating the energy in the harmonic motion of the fundamental mode to $k_B T$. After rearranging, this gives

$$\chi_{\text{r.m.s.}} = \{2k_B T/M_{\text{eff}}\,\omega_0^2\}^{1/2}\,, \tag{5}$$

where k_B is Boltzmann's constant and M_{eff} is the effective mass of the spacer. The fluctuation-dissipation theorem states that the time scale on which this random walk produces changes of order $\chi_{\text{r.m.s.}}$ is the same as the time scale given by the decay time of the fundamental mode [11]. The decay time τ_0 is related to the quality factor of the fundamental mode by $\tau_0 = 2Q/\omega_0$. Q is inversely proportional to the fractional energy loss per cycle. For times τ shorter than τ_0, the mean-square change in the mode's amplitude is $\chi_{\text{r.m.s.}}$ reduced by the ratio τ/τ_0, $\Delta x = \chi_{\text{r.m.s.}}\ \tau/\tau_0$ [12]. Physically, this expresses the fact that a harmonic oscillator is a tuned system that responds to noise and other perturbations in a narrow range of frequencies. So, while the noise is proportional to temperature, the fluctuating forces cannot appreciably change the mode amplitude in times short compared to the decay time.

To reach an appreciation of the size of the length fluctuations due to thermal noise, we can calculate the frequency of the lowest mode of the spacer and solve for $\chi_{\text{r.m.s.}}$. Alternatively, since M_{eff} is only estimated, we can equate the work necessary to stretch or compress a cylindrical bar by $\chi_{\text{r.m.s.}}$ to the energy $k_B T$. The force necessary to elastically stretch a bar by a small amount x is given by eq. (4),

$$F = EAx/L\,. \tag{6}$$

$F\,\mathrm{d}x$ is the incremental work done by this force in going from x to $x + \mathrm{d}x$. Integrating from $x = 0$ to $x = \chi_{\text{r.m.s.}}$, the work done is

$$W = EA(\chi_{\text{r.m.s.}})^2/2L\,. \tag{7}$$

When this is equated to $k_B T$, $\chi_{\text{r.m.s.}}$ is simply related to the temperature of the spacer and to its physical properties,

$$\chi_{\text{r.m.s.}} = \{2Lk_B T/EA\}^{1/2}\,. \tag{8}$$

Recall that, for a spacer made from a material comparable in its properties to fused silica, E is about 10^{11} N/m^2. If the spacer is 30 cm long and 10 cm in diameter, then $\chi_{\text{r.m.s.}} \approx 1.8 \cdot 10^{-15}$ m at a thermal-bath temperature of 300 K. The frequency of a laser in the mid-optical locked to this cavity would move by about 3 Hz for a length change of this magnitude. However, this excursion occurs dominantly at the vibrational frequency of the lowest fundamental (mechanical) mode of the spacer. For a fused-silica spacer of this size, the resonance frequency of its lowest mode is about 10 kHz. Hence, there is little power in the thermally induced sidebands at ± 10 kHz since the modulation index is so small [13] ($\approx 3 \cdot 10^{-4}$). It is worth noting that the resonant Q in a fused-silica bar at room temperature is only about 10^5 [14]. Therefore, the decay time for the lowest mode is about 10 s, and, unlike our colleagues looking for gravity waves with high-Q resonant-bar detectors, we are sensitive to the full change in the mode amplitude on time scales of critical interest to us. However, a few hertz at 10 kHz driven by the Brownian motion of the cavity is not a limitation to the frequency stability nor spectral purity of a 1 Hz laser.

The temperature sensitivity of low-expansion materials suitable for spacers can be better than 10^{-8}/K, which implies μK control at the cavity in order to achieve stabilities of a few hertz. However, if the cavity is suspended in a thermally massive, evacuated chamber, then the thermal coupling to the environment is primarily radiative. The time constant can exceed a day. Therefore, if the temperature fluctuations of the walls of the vacuum vessel never exceed 10 mK, then the frequency fluctuation rate of the laser will be less than 1 Hz/s, independent of the time rate of charge of the wall temperature.

Pressure fluctuations in the gas between the mirrors produce density variations which in turn cause refractive-index changes. This causes an effective optical-length change to the cavity, nL, where n is the index of refraction for air. Near room temperature, the index of refraction of dry air is linearly related to its pressure, P, by

$$n - 1 \approx 3 \cdot 10^{-9} P, \tag{9}$$

where P is in Pa (133 Pa = 1 Torr). Thus, even if the cavity is evacuated to 10^{-5} Pa, the absolute shift in the cavity resonance for optical frequencies near $\lambda \approx 500$ nm is about 15 Hz (from that of 0 Pa). 10% fluctuations in this pressure cause frequency excursions of the laser of approximately 1.5 Hz. The air pressure at 10^{-5} Pa also causes a strain in the bar but the length compression is negligible. The pressure at 10^{-5} Pa is less than 10^{-5} N/m^2, which produces an axial strain in the spacer of about $1 \cdot 10^{-16}$. The fractional length change is not fully this magnitude since the pressure-induced stress in the axial direction is somewhat compensated by the radial stress. Consequently, in a reasonably stiff spacer, fluctuations in the cavity length due to fluctuations in the pressure are dominated by index-of-refraction changes.

Additional limitations to the stability of a laser locked to a Fabry-Perot resonator come from technical problems such as optical feedback, intensity fluctuations, beam pointing stability, etc. [15], but seismic noise or ambient vibrations that cause changes in the cavity length are the most important practical problems limiting the frequency stability for times longer than a few ms. Generally, there are two distinct effects: high-frequency vibrations, which may excite fundamental mechanical resonances of the bar, and low-frequency vibrations, which tend to produce nonresonant distortions of the bar. The first effect typically occurs at frequencies in the range of $(100 \div 1000)$ Hz and often can be effectively eliminated, for example, by mounting the system on alternating layers of shock-absorbing material and passive masses, such as thin rubber and cinder block. Low-frequency vibrations ($(0.1 \div 100)$ Hz), which are typically driven by ground noise and building vibrations, are much harder to eliminate. Some sophisticated vibration isolation systems, both active and passive, have been developed to reduce noise in this frequency interval [16]. While an active vibration isolation system may ultimately be necessary for sufficient attenuation of seismic noise to reach laser spectral purity below 1 Hz, we, for the moment, isolate only with passive systems. The simplest method of passive vibration isolation consists of mounting the device to be isolated on a resilient support, such as a pendulum or spring. A pendulum isolates in the horizontal plane, using gravity as its spring constant (note that this spring can be nearly lossless). A spring can isolate horizontally and vertically simultaneously. It is relatively simple to show that the attenuation in the amplitude of motion between the support and the bar increases as ω^2 for motion at frequencies higher than the resonance frequency ω_0 ($\omega_0^2 = k/m$, spring; $\omega_0^2 = g/l$, pendulum). Damping must be included to limit the amplitude of motion, a_1, at resonance. The response of a damped system is

$$a_1 = a_2(1 + i\Gamma\omega)/\{1 - (\omega/\omega_0)^2 + i\Gamma\omega\}, \tag{10}$$

where a_1 is the amplitude of the motion at the bar, a_2 is the amplitude of motion of the support at frequency ω, and Γ is the coefficient of damping. The amplitude a_1 is complex (phase shifted) and everywhere bounded. In practice the choice of damping is a compromise between low resonant amplitude and sufficient high-frequency isolation. There are more complex passive systems that offer better high-frequency isolation while at the same time reduce the resonance peaking to a factor of 2 or less [16]. Calculations of the vibration isolation produced by various mechanical suspensions have been driven by the work done on gravity wave detectors; a good treatment is that of Veitch [17].

The most important function of the isolation system is to reduce fractional length changes of the reference cavity to below (ideally, well below) $1 \cdot 10^{-15}$. If the cavity is suspended with its axis horizontal by wires that act as vertically stiff pendulums, then it has been demonstrated that the isolation from horizontal vibrations in the direction of the cavity axis can be adequate to attain a stability approaching 1 Hz [18]. However, a suspended bar is subjected to a distributed load resulting from the pull of gravity, which produces considerable stress to the support structure and to the bar. In addition, the bar is not perfectly rigid and must distort at some level under its weight and this causes additional stress. All these stresses can produce sudden acoustical emission [19] at the rate of up to several per second and at frequencies that may or may not coincide with the eigenfrequencies of the bar. Also, since the wires are essentially stiff to vertical vibrations, these perturbations can be coupled into the bar. If, though, the wires were connected to the nodal points of the bar, then the external force would be unable to excite that mode (or modes) through that nodal position. Unfortunately, there are many bending modes and vibrational modes with disparate nodal positions, so, although it is possible to reduce some of the cavity sensitivity to vertical perturbations, single suspension points are not sufficient to eliminate excitation of all modes. Further improvement can be achieved if the cavity is isolated vertically as well. We now turn our attention to some of the experimental studies pursued over the past few years in the ion storage group at NIST.

I.3. – Experimental results: cavity comparisons.

In some of our studies, a direct heterodyne comparison of two independently frequency-stabilized light beams was made. The linewidths and frequency stabilities were analyzed in various ways. The beat note was recorded in an r.f. or microwave spectrum analyzer, thereby directly displaying the combined linewidth of the two sources. The signal-to-noise ratio was improved and the long-term relative frequency drift was studied by averaging many successive scans of the beat note. It was also possible to use two spectrum analyzers in tandem to identify the specific frequency noise components that contribute to the laser linewidth. This was particularly useful toward unraveling the noise sources that cause fluctuations to the length of the cavities. For instance, if the seismic noise was independently studied with a seismometer, a correspondence between the seismic-noise terms and the dominant contributors to the laser linewidth could be made. If the length fluctuations of the two cavities were similar in frequency and amplitude, but otherwise independent, the linewidth of either frequency-stabilized system is smaller than the recorded beat note by $\sqrt{2}$. If the effective length stability of one cavity had been worse than that of the other, the measured linewidth would have been be dominated by the frequency fluctuations of the laser locked to the noisier cavity.

Either the transmitted light beam or the reflected beam that interferes with the light re-emerging from the cavity can be used to stabilize the frequency of the laser. In our experiments, we used the beam reflected from the cavity. The error signal can be derived either near zero frequency or at some higher frequency. Since technical-noise terms are present at low frequencies, it is better to modulate and detect at a frequency at which the signal-to-noise ratio is limited by the shot noise in the detected light beam. This is the Pound-Drever-Hall reflected-sideband technique that has been treated in detail elsewhere [8, 20]. Attention to the optical layout and to the electronic-noise terms is critical to achieving a spectral linewidth that is smaller than a few hertz. This also has been discussed in papers by HOUGH *et al.* [21] and by SALOMON *et al.* [15]. By several separate measurements, the electrical problems in our work were determined to be unimportant to the attainment of a laser linewidth below 1 Hz; the dominant contribution to the laser linewidth were mechanical (and perhaps optical) perturbations that caused length changes in the reference cavity.

The frequency-stabilized light beams were derived from a home-built ring dye laser oscillating at 563 nm. The wavelength of the laser was chosen because its frequency would eventually be doubled into the ultraviolet to probe a narrow transition in $^{199}Hg^+$. Historically, the dye laser was locked in a two-step process. The laser was pre-stabilized to the order of a few hundred Hz by locking the laser to a lower-finesse (about 800) cavity. The cavity resonance was probed by the reflected-sideband technique using a modulation frequency of about 10 MHz. Rapid frequency fluctuations of the laser were removed by a fast (bandwidth > 2 MHz) servo driving an intracavity E/O modulator, and the lower-frequency fluctuations were corrected by a second-order servo-loop driving an intracavity PZT-mounted mirror. The frequency-stabilized light was then transmitted through optical fibers to the high-finesse cavities. Since the laser linewidth was less than 1 kHz, a smaller-bandwidth (100 kHz), lower-noise servo could be used to strip the remaining noise from the laser beams. Again, the reflected-sideband technique was used to probe the resonance of the high-finesse cavities. In the second stage, the frequency-correcting element was an acousto-optic modulator through which the laser beam was singly or doubly passed. The frequency corrections were simply written onto the acousto-optically shifted beam by the servo. Long-term corrections were fed back to the low-finesse cavity to maintain frequency alignment of the laser with one of the high-finesse cavities. The laser power was also stabilized by adjusting the r.f. power applied to the acousto-optic modulator. The overall intensity regulation was better than 0.1%, but this was applied to the beam before it entered the cavity. From the arguments made in sect. I.2, intensity variations in the light circulating in the cavity cause length fluctuations by light pressure changes and by heat variations in the mirror coatings. Therefore, although we have not done so yet, it may be better to stabilize the intensity of the light circulating in the resonator by using the light transmitted from the cavity. This should give a first-order insensitivity of the frequency of the laser to power fluctuations caused by relative motion between the cavity and the injected light beam.

The cavities were constructed from a cylindrical, Zerodur [22] rod that was cut and rough ground to a diameter of about 10 cm and a length of about 27 cm. A 1 cm round hole was bored along the axis of the spacers, and a smaller one was bored through the center of the rod midway from the ends and normal to its axis. The smaller bore permitted evacuation of the space between the mirrors. The length of the cavities and the mirror radii of curvature were chosen so that the cavities were highly nondegenerate for the spatially transverse modes. In particular, the frequency of the TEM_{01} and TEM_{10} modes are separated from the lowest-order TEM_{00} mode by approximately 30% of the free spectral range.

Even at the 12th transverse order, the frequencies are still separated from the lowest-order fundamental mode (modulo $c/2L$) by several percent of the free spectral range. This gives good immunity to any line pulling if any light power is coupled into the higher-order modes. Even so, great care is taken to mode match into the cavities. Note that this high immunity to line pulling would be somewhat compromised by going toward a near spherical resonator as suggested in sect. I.2.

Each cavity is suspended by two thin (250 μm) molybdenum wires inside a thick-walled (19.1 mm) aluminum vacuum vessel that has an inner diameter of 26.7 cm. The wires are placed as slings under either end of the spacer, about 1/5 the cavity length from each end, in an attempt to support at the nodal positions for the lowest-order bending mode. The ends of each wire are attached to the inner wall of the vacuum vessel by small clamps. Each wire either travelled vertically upward from either side of the spacer to the wall («U» shaped), or opened slightly away from the bar («V» shaped). This allowed free movement of the cavity along the direction of its axis and restricted, but not rigidly, the motion perpendicular to the axis. Damping of the pendulum motion was weak and dominantly into the table and its padding through the aluminum housing. The aluminum vessels are thermally insulated and temperature regulated to the order of a few mK. The temperature coefficient of the spacers is approximately $6 \cdot 10^{-9}\,K^{-1}$ at $T = 300$ K. The thermal time constant from the walls of the evacuated aluminum housing to the spacer is on the order of one day, giving adequate isolation to small temperature fluctuations at the vessel walls. Since the variations in the wall temperature were controlled to less than 10 mK for any time period, the maximum rate of change in the temperature of the bar did not exceed 100 pK/s. This corresponds to a frequency fluctuation rate of about 0.3 Hz/s and a maximum frequency change of about 50 kHz. A pressure of $1 \cdot 10^{-6}$ Pa ($8 \cdot 10^{-9}$ Torr) is maintained in each vacuum vessel by an ion pump that is rigidly attached to the vessel. This is adequate to give frequency fluctuations of less than 1 Hz for pressure fluctuations of 10%.

The length of the longer rod corresponds to a free spectral range of 622 MHz and the shorter rod has a free spectral range of 562 MHz. The mirrors are highly polished Zerodur substrates that are coated to give high finesse and good efficiency and then optically contacted to the polished ends of the spacer. The finesses of the cavities are about 60 000 and 90 000, respectively, and, for both cavities, the transmission on resonance exceeded 30%. The high finesse F, or the long optical storage time, translates into a narrow fringe whose HWHM is given by $c/2LF$. Consequently, the ratio of error voltages to frequency excursions can be very high, even for short cavities. Shorter cavities have been constructed from ULE [22] that have measured finesses that exceed 130 000 (and, recently, finesses exceeding 10^6 have been reported [23]). The frequencies of the mechanical resonances of these shorter, stiffer bars will be about a factor of 3 higher than those of the Zerodur bars. As long as the frequencies of the mechanical vibrations are high enough, a narrow optical resonance can be probed with high resolution by the unperturbed carrier of the laser spectrum.

The aluminum vacuum vessels rested on Viton [22] rubber strips attached to v-blocks made of aluminum. The v-blocks were secured to a rigid acrylic plastic plate. Each reference cavity system was mounted on a separate optical table that (initially) consisted of surface plates that were deadened by damping their internal vibrations into sand. The sandbox sat on soft rubber pads and cinder blocks in one case and on low-pressure inner tubes and cinder block legs in the second. Noise vibrations from the floor and on the table tops were monitored with moving-coil seismometers that had a sensitivity of 629 V·s/m from approximately (1 ÷ 100) Hz. Measurements of the floor motion in our laboratory revealed bright resonances at 14.6 Hz, 18.9 Hz and 29.2 Hz on top of a broad pedestal from about 1 to 40 Hz. The average amplitude of the resonant motion was greater than 10^{-6} m/s and the pedestal peak was about 10^{-7} m/s. Isolation from mechanical vibrations began at frequencies above about 5 Hz for both table tops. By 100 Hz the isolation from floor noise for either table had improved by a factor of 40 or better.

For heterodyning, a small fraction of the frequency-stabilized light from each cavity was combined on a beam splitter and detected with a fast diode. The heterodyned signal was amplified and analyzed with two spectrum analyzers used in series. This allowed us to look directly at the beat note and also Fourier-analyze the noise terms that contributed to its linewidth. The first analyzer could be used to observe the beat signal, or as a frequency discriminator. As a frequency discriminator, the scan was stopped and the analyzer was used as a tunable r.f. filter/receiver with a variable bandwidth. The center frequency of the analyzer was then shifted so that the heterodyned signal lay at the half-power point of the response curve. The bandwidth was adjusted so that the frequency excursions of the beat signal were much less than the bandwidth. This produced a one-to-one map of frequency excursions-to-voltage excursions whose Fourier power spectrum could be analyzed in the second low-frequency ((0 ÷ 100) kHz) spectrum analyzer. The noise power spectrum in the second analyzer helped to reveal the nature and origin of the vibrational noise that contributes to the linewidth of the stabilized lasers.

The width of the heterodyne signal between the two stabilized lasers was less than 50 Hz. The noise power spectrum disclosed that low-frequency fluctuations in the range from near zero to 30 Hz dominated this linewidth. The vibrational-noise spectrum measured by the seismometers on the table tops matched the largest noise components of the beat note. The frequencies of the pendulum motion of the suspended cavities were about 1.4 Hz and 1.48 Hz which gave FM at these frequencies. There were also bright features in the laser power spectra that came from the floor motion at 14.6 Hz, 18.9 Hz and 29.2 Hz. The modulation indices of the latter three noise components were about one, so they all had enough power to contribute to the beat note linewidth. When the pendulum motions of the bars were quiet, their FM contributed little to the laser linewidths. However, the integral of the nearly featureless noise power spectrum from about 0 Hz to 10 Hz contributed about 15 Hz to the combined spectral purity of the lasers. Some, if not all, of this noise was mechanical, but it was not clear how it coupled to the suspended cavity. To help elucidate the connection, we drove one of the table tops in either the horizontal or vertical direction with a small loudspeaker connected to an audio signal generator. The motion of the speaker diaphragm was coupled to the table by a rod glued to the diaphragm and gently loaded against the table. The table could be driven at frequencies from a few hertz to about 100 Hz with enough power to be 40 dB above background noise. When the loudspeaker drove the table in the horizontal plane in a direction parallel to the axis of the cavity, the isolation of the suspended cavity was sufficiently good that the beat signal showed little evidence of the perturbation even at the high drive levels. However, when the drive was applied vertically at a level barely perceptible above the vertical background noise, the heterodyne signal showed added noise power at the drive frequency. The stiff support in the vertical direction strongly coupled vertical motion into effective cavity length changes. The sensitivity of the Fabry-Perot cavity to vertical motion was orders of magnitude higher than for horizontal motion parallel to the

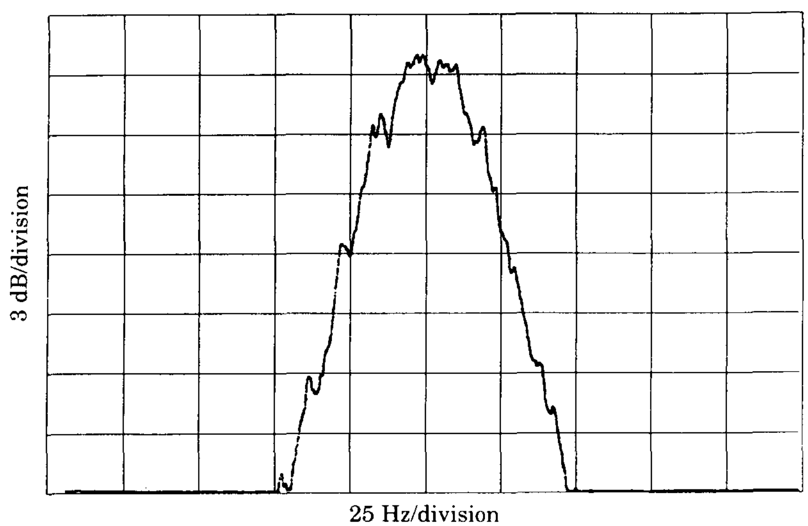

Fig. 1. – Spectrum of the beat frequency between the two independent, cavity-stabilized lasers discussed in the text. The resolution bandwidth is 30 Hz. Total integration time for these data is about 70 s (the relative linear drift between the two cavities is removed by mixing the beat note with the frequency from a synthesizer that is swept in time). The partially resolved sidebands at 14.6 Hz are due to a resonant floor vibration. The apparent linewidth is about 30 Hz; the linewidth of the better-stabilized laser is at least $\sqrt{2}$ narrower.

cavity axis. (For practical reasons, we did not drive the table in the horizontal plane in a direction perpendicular to the cavity axis.)

In order to improve the vertical isolation, one table was suspended just above the floor with latex-rubber tubing attached to the ceiling. The resonance frequencies for both the vertical motion and the horizontal pendulum motion of the suspended table were near 0.33 Hz. These were damped to the floor with two small dabs of grease, but the damping did not significantly change the isolation afforded by the latex tubing at higher frequencies. The isolation from vibrational noise above 1 Hz was more than an order of magnitude better than that of the quietest sandbox table. This was partially reflected in the linewidth of the heterodyne signal between the laser radiation stabilized to the cavity supported on this table and the laser radiation stabilized to the cavity supported on the best sandbox table; it dropped from about 50 Hz to less than 30 Hz. In fig. 1 the spectrum of the beat frequency is shown. Zerodur is known to temporally contract with a time constant of years [8, 24]. The creep rate for both cavities corresponds to a linear frequency drift of $(3 \div 5)$ Hz/s, but the rates are not identical. The linear, relative cavity drift is removed by mixing the beat frequency with the frequency from a synthesizer that was swept in time. The spectrum of fig. 1 represents an integration time of 70 s. The noise vibrational sidebands at 14.6 Hz are partially resolved. Note that the linewidth of the best stabilized laser is at least $\sqrt{2}$ narrower. The Fourier noise power spectrum from 0 to 10 Hz is shown in fig. 2. The beat note linewidth obtained by integrating the power spectrum is about 15 Hz. We suspect that the laser stabilized to the cavity on the sandbox table is the dominant contributor to the width of the heterodyne signal since the vibrational noise measured on this table is greater. Perhaps the linewidth of the laser stabilized to the cavity on

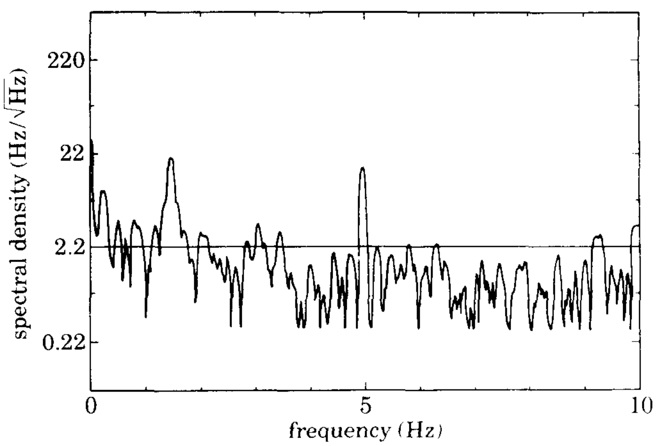

Fig. 2. – Fourier noise power spectrum of the laser heterodyne signal (shown in fig. 1) from 0 to 10 Hz.

the table suspended by the latex tubing is below 1 Hz. We are working to verify this and to build better cavities.

II. – Single-atom Spectroscopy.

II.1. – Single-ion results.

The ion trapping and laser cooling relevant to our experiments have been described elsewhere [25, 26]. A ^{199}Hg atom is ionized and trapped in the harmonic pseudopotential well created by an r.f. potential applied between the electrodes of a miniature Paul trap. The separation between the endcap electrodes ($2z_0$) is about 650 μm. The frequency of the r.f. potential is about 21 MHz. Its amplitude can be varied up to 1.2 kV; at the maximum r.f. amplitude, the quadratic pseudopotential is characterized by a secular frequency of nearly 4 MHz. The ion is laser-cooled to a few millikelvin by a few microwatt of radiation from two 194 nm sources. One source drives transitions from the $5d^{10}6s\ ^2S_{1/2}(F=1)$ to the $5d^{10}6p\ ^2P_{1/2}(F=0)$ level (see fig. 3). This is essen-

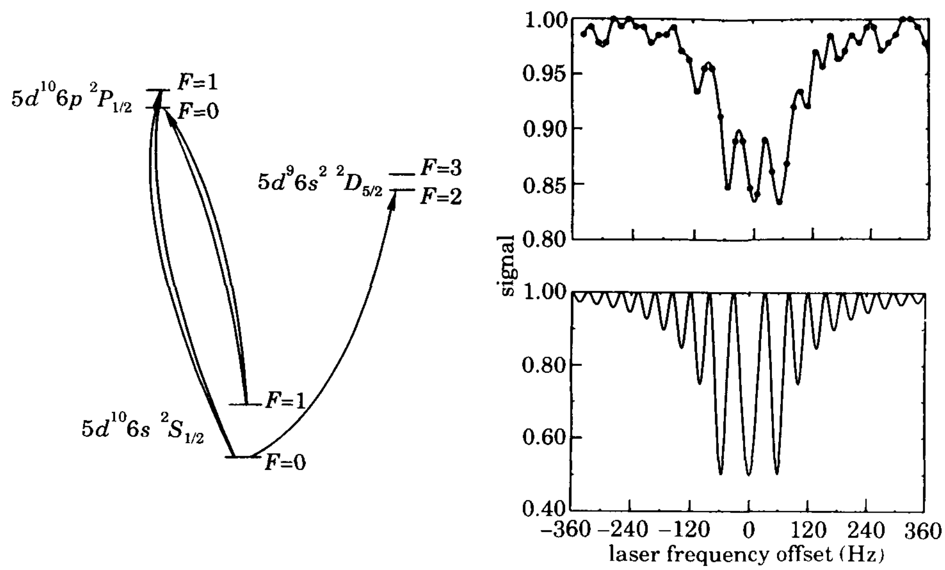

Fig. 3. – On the left is a simplified energy level diagram for $^{199}Hg^+$ at zero field. Shown in the upper figure on the right is the power-broadened lineshape obtained by scanning through the Doppler-free resonance of the $^2S_{1/2}(F=0,\ m_F=0)$-$^2D_{5/2}(F=2,\ m_F=0)$ transition in a single laser-cooled $^{199}Hg^+$ ion. A 563 nm laser that is stabilized to a high-finesse reference cavity, which in turn is long-term stabilized to the ion, is frequency-doubled and stepped through the resonance for 138 consecutive sweeps. The step size is 15 Hz at 563 nm (30 Hz at 282 nm). The lower right figure shows the lineshape calculated for conditions similar to the experimental conditions for the upper figure, except that the ion is assumed to have zero temperature and the laser is assumed to have zero linewidth.

tially a two-level system suitable for laser cooling, except for weak off-resonance pumping into the $^2S_{1/2}(F=0)$ state. The second 194 nm source, tuned to the $^2S_{1/2}(F=0)$ to $^2P_{1/2}(F=1)$ transition, returns the ion to the ground-state $F=1$ hyperfine level. The frequency separation between the two radiation sources is equal to the sum of the ground- and excited-state hyperfine splittings (about 47 GHz). The two 194 nm beams propagate collinearly and irradiate the ion at an angle of 55° with respect to the symmetry (z) axis of the trap. In this way, all motional degrees of freedom are cooled to near the Doppler-cooling limit of 1.7 mK. 194 nm fluorescence from the ion, collected in a solid angle of about $5 \cdot 10^{-3}\, 4\pi$ sr, is detected with an efficiency of 10% to give a peak count rate on resonance of about 25 000/s. The complication of laser cooling with two lasers is brought about by the hyperfine structure of $^{199}Hg^+$. Only an isotope with nonzero nuclear spin can have first-order, field-independent transitions, which give great immunity to magnetic-field fluctuations. In $^{199}Hg^+$, the nuclear spin is 1/2. Near $B=0$, the narrow $5d^{10}6s\ ^2S_{1/2}$-$5d^9 6s^2\ ^2D_{5/2}$ transition at 282 nm is first-order field-independent. The decay rate of the metastable $^2D_{5/2}$ state corresponds to an optical linewidth of less than 2 Hz—certainly, a suitably challenging test for the stabilized dye laser.

The 282 nm radiation is obtained by frequency doubling the radiation from the dye laser that is stabilized to the Fabry-Perot cavity on the sandbox table. (The cavity comparisons were done subsequent to the single-ion studies and we had not yet suspended a table with latex tubing.) Prior to being frequency-doubled, the 563 nm radiation (beam 1) is passed through an acousto-optic modulator (A/O-1) so that its frequency can be tuned through the *S-D* resonance. We also used A/O-1 to suppress the linear drift of the cavity and the frequency fluctuations caused by relative motion between the cavity and the ion trap (which are supported on different tables separated by 3 m). These Doppler effects can be removed in a fashion similar to that used by VESSOT to remove Doppler frequency shifts between a ground-based microwave source and a rocket-borne microwave oscillator [27]. Another acousto-optic modulator (A/O-2) is placed in an auxiliary laser beam (beam 2) near the ion trap. The frequency of beam 2 need not be stabilized. A/O-2 generates a frequency-shifted beam (beam 3) that is sent to the cavity table and returned on a path very close (< 2 cm) to that followed by beam 1. The light paths need not be overlapping in order to reach a frequency stability of 1 Hz. Beam 3 is recombined with its carrier to produce a beat note at the r.f. frequency of A/O-2. However, because the shifted beam traveled over to the cavity and back to the trap, the frequency fluctuations caused by relative motion between the tables and atmospheric turbulence are impressed on the beat note. Dividing the beat frequency by 2 gives the one-way path noise information carried at half the radiofrequency of A/O-2. If this frequency is summed with the right quadrature to the frequency that sweeps the stabilized laser through the *S-D* resonance, then path noise is eliminated. This is equivalent to bringing the cavity and trapped ion together. Stepping the frequency of the stabilized laser through the S-D resonance and removing the linear cavity drift are accomplished with an r.f. drive frequency to A/O-1 obtained by summing the output of two synthesizers. The frequency of one synthesizer sweeps opposite to the cavity drift and the frequency of the second synthesizer is stepped back and forth, sweeping the frequency of the stabilized laser through the narrow atomic resonance.

The 282 nm radiation and the two-frequency 194 nm source are turned on and off sequentially using shutters and the acousto-optic modulator. This prevents any broadening of the narrow S-D transition due to the 194 nm radiation. Electron shelving [25, 28] is used to detect each transition made to the metastable D state as a function of the frequency of the 282 nm laser. At the beginning of each cycle, both 194 nm lasers irradiate the ion. The fluorescence counts in a 10 ms period must exceed a minimum threshold (typically 20 counts) before the interrogation sequence can continue. The 194 nm beams irradiate the ion for sequential 10 ms periods until the threshold is met. The 194 nm radiation tuned to the ${}^2S_{1/2}(F=0)$-${}^2P_{1/2}(F=1)$ transition is chopped off for 5 ms. During this time, the 194 nm radiation tuned to the ${}^2S_{1/2}(F=1)$-${}^2P_{1/2}(F=0)$ transition optically pumps the ion into the ${}^2S_{1/2}(F=0)$ ground state. Then this 194 source is turned off. One millisecond later, the 282 nm radiation, tuned to a frequency resonant or nearly resonant with the ${}^2S_{1/2}(F=0,\ m_F=0)$-${}^2D_{5/2}(F=2,\ m_F=0)$ transition, irradiates the ion for an interrogation period that is varied up to 25 ms. At the end of this period, the 282 nm radiation was turned off and both 194 nm sources were turned on. Another 10 ms detection period was used to determine whether a transition to the D state had been made (fluorescence counts $>$ threshold, no; fluorescence counts $<$ threshold, yes). The result was recorded as a 1 or 0 (no or yes) and averaged with the previous results at this frequency. Then the frequency of the 282 nm radiation was stepped and the measurement cycle repeated.

Since the frequency drift of the 282 nm laser depended not only on the reference cavity contraction rate, but also on small pressure and temperature changes, on laser power variations, and so on (as discussed in sect. I.2), we locked the frequency of the laser to the narrow S-D transition to remove long-term frequency drifts. To do this, we modified the measurement cycle to include a locking cycle. We began each measurement cycle by stepping the frequency of the 282 nm radiation to near the half maximum on each side of the resonance N times (N varied from 8 to 32). At each step, we probed for 5 ms and then looked for any transition with the electron-shelving technique. We averaged the N results from each side of the resonance line, took the difference and corrected the frequency of the synthesizer used to compensate the cavity drift. The gain of this lock needed to be properly adjusted to avoid adding frequency noise to the laser. In this way, variations in the frequency of the 282 nm laser for time periods exceeding a few seconds were reduced.

In fig. 3, we show the spectrum obtained by scanning in this drift-free way through the Doppler-free resonance of the ${}^2S_{1/2}(F = 0, m_F = 0)$-${}^2D_{5/2}(F = 2, m_F = 0)$ transition. The lineshape shown is the result of 138 consecutive scans, each of which included a locking cycle. The probe period was 15 ms, and the step size was 15 Hz at 563 nm (30 Hz at 282 nm). The resonance shows a clearly resolved triplet with the linewidth of each component less than 40 Hz (< 80 Hz at 282 nm). We first thought that the triplet structure might be due to 60 Hz modulation of the frequency of the 563 nm laser either due to grounding problems, line pickup or inadequate servo gain. However, when the radiation from two independently stabilized laser beams was heterodyned together, the 60 Hz modulation index was far too small to account for the sideband structure observed on the *S-D* resonance. In addition, the frequency separation of the peaks is nearer to 50 Hz, not to 60 Hz. We now think that, most likely, the triplet structure is caused by Rabi power broadening. The 282 nm radiation is focussed to a spot size of about 25 μm; therefore, on resonance, fewer than 10^6 photons/s (< 1 pW) will saturate the transition. Below, the data is a theoretical lineshape calculated for an ion a rest, for no broadening due to collisions or laser bandwidth, for a pulse length of 15 ms and for sufficient power at resonance to give a 3.5π pulse (which roughly corresponds to the power used).

Qualitatively, the figures compare well. The fluctuations from measurement cycle to measurement cycle in the quantum occupation number of the ion in the harmonic well of the trap cause variations in the transition probability of the ion. This, and the finite laser linewidth, likely cause the general broadening and weakening of the signal. Current efforts are devoted to measuring the narrow *S-D* transition using the laser stabilized to the cavity on the suspended table. A cryogenic, linear r.f. Paul trap has been constructed and will soon be tested. With this trap, it should be possible to laser-cool many ions and to store them without attrition for days. The increased numbers of trapped ions will give a better signal-to-noise ratio (thereby better stability), but it will still be possible to have a small second-order Doppler shift. We also plan to investigate the lineshape and the effects of power broadening in more detail in future experiments.

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